10-19-2021

How Random Incidents Affect Travel-Time Distributions

Melike Baykal-Gürsoy

Andrew Reed Benton

Pedro Cesar Lopes Gerum
Cleveland State University, p.lopesgerum@csuohio.edu

Marcelo Figueroa CAndia

How Random Incidents Affect Travel-Time Distributions

Melike Baykal-Gürsoy, Andrew Reed Benton, Pedro Cesar Lopes Gerum, and Marcelo Figueroa Candia

***Abstract*—We present a novel analytical model to approximate the travel-time distribution of vehicles traversing a freeway corridor that experiences random quality of service degradations due to non-recurrent incidents. The proposed model derives the generating function of travel times in terms of clearance time, incident frequency, and severity, as well as other traffic characteristics in closed-form. We validate the model using data from a freeway corridor where weather events and traffic accidents serve as the principal causes of service degradation. The resulting model is equivalent in performance to widely used methodologies while uniquely providing a clear connection on how incidents affect travel time distribution. Through this connection, the model readily yields travel time reliability measures for alternative roadway behaviors, providing crucial information for long-term planning.**

***Index Terms*—travel time reliability, stochastic models, random incidents, Markov modulated service.**

I. Introduction

THE travel time experienced by travelers is one of the most critical performance measures in a transportation system. However, it can be highly variable in the face of non-recurrent events that lead to congestion. Travelers often expect recurrent events such as morning or evening rush hours. They adapt by leaving early or choosing a different route. Conversely, non-recurrent events occur randomly. Not being able to predict such events, travelers do not prepare or plan for them in advance. As a result, non-recurrent congestion leads to delay [1] and overall traveler dissatisfaction.

Travel time reliability is a measure of travel time variability due to non-recurrent events [2], [3] and is directly associated with travelers' quality of service. To estimate several travel time reliability metrics, we require a travel time distribution, typically estimated through statistical analysis of observed or simulated data, or analytical models such as the ones derived from queuing theory. Theoretically, the true distribution could be directly determined using empirical travel time observations. However, such an approach can only capture measurable behaviors of a system particular to its present configuration. For example, travel time distributions immediately generated from data provide no information on the effect of incidents on travel time. Moreover, if one is interested in what-if scenario analysis under uncertainty, travel-time data must be obtained through simulations since such data may not exist.

This paper is the first that focuses on deriving analytical approximations for the travel-time distribution under congestion resulting from non-recurrent events. The resulting model explicitly considers the stochastic nature of traffic deteriorating circumstances and requires simple calibration. The analytical solution can be readily employed in most scenarios. A noteworthy innovation of the model is its flexibility to account for either frequent or infrequent deteriorating events with any level of severity. Hence, the travel time distribution can be forecast under vastly different scenarios without extensive simulations. By analyzing incidents with different severities, durations, and frequencies, decision-makers can develop better incident management practices in the field [4].

The paper is structured as follows: section II surveys the related literature; section III introduces the notation and describes the analytical model, as well as presents our main result; section IV shows the validation and calibration methodology; section V demonstrates the calculation of the reliability indices and how to perform scenario analysis; finally, section VII discusses the findings and future work.

II. Literature Review

The last decade has seen increased attention to the study of travel time reliability [5]. The calculation of statistical indices of travel time reliability has experienced prominence due to their direct relation to the level of service (LOS) provided by traffic systems [1], [6]–[9]. Examples of travel-time reliability measures include the

The work of M. F. Candia was partly funded by the Tayfur Altiok Scholarship of the Industrial & Systems Engineering Department, Rutgers University, and by Fulbright. *(All authors contributed equally to this work.) (Corresponding author: Melike Baykal-Gürsoy.)*

M. Baykal-Gürsoy is with the Department of Industrial and Systems Engineering, Rutgers University, Piscataway, NJ 08854 USA (email: gursoy@soe.rutgers.edu).

A. R. Benton is with Rutgers University, Industrial and Systems Engineering Dept, (email:andrew.benton@rutgers.edu).

P. C. L. Gerum is with Cleveland State University, Operations and Supply Chain Management Dept, (email:p.lopesgerum@csuohio.edu).

M. Figueroa Candia is with the Boston Consulting Group, Chile (email:marcelo@figueroacoronado.cl).

90th and 95th travel time percentiles, the Buffer Index (BI), the Median based BI (M-BI), the Planning Time Index (PTI), the Percentage of Trips on Time (PTT) [2], α-reliable mean excess travel time [10], Fosgerau's reliability ratio [11] and others [12]. The computation of travel time reliability indices often requires travel time distributional information.

The literature has not yet agreed on a distribution to fit travel time. Traditional choices include normal, lognormal, Weibull, and gamma. Susilawati *et al.* [13], and Taylor [14] show how Burr (type XII) often outperforms the traditional choices. Zang *et al.* [12] consider an approach for approximating the travel time percentile function. In [15], the authors investigate two and three-component normal mixtures. The two-component model sets the two mixture components to represent the non-congested state and represent the congested state. This model is further validated for lognormal distributions in [16]. Yang and Wu [17] consider a gamma mixture model and verify the difference in performance when more components are added to the mixture.

All these models assume that no change in traffic condition happens mid-route. They work well in situations where roadway conditions remain constant for long periods, but not when changes may occur. Moreover, despite their good performance, mixture models provide little insight into the impact particular types of incidents have on travel time. They cannot be tuned for different incident rates and severities.

This paper determines travel time distributions through an analytical model that incorporates various levels of service and causes of travel time variability. Our approach has some similarity to those that also use a Markovian approach [18]–[20]. However, there are significant differences. In [18], the authors model the vehicle speed as a finite state continuous-time Markov Chain (MC). In our model, the speed (or the service time) is itself a random variable and experiences external random effects. Also in the others, the authors model the traffic speed on arterial networks [19] or travel-time state within each link over time [20], and from one link to another, as a discrete time MC to obtain route travel times.

III. Notation and Analytical Model

The proposed model derives the probability distribution of the travel time of a single-vehicle traveling on a corridor subject to random quality of service degradation. Because the speed depends on the driver's preferences, state regulations, and roadway conditions, vehicles take a random amount of time to traverse the corridor.

The terminology *service time* refers to the time taken to traverse the corridor when the system remains under the same traffic conditions, either normal or deteriorated. Because circumstances may change mid-course, we consider the *travel time* to refer to the vehicle's overall traversing time. Travel time thus incorporates possible traffic condition shifts that could occur while traveling.

Under normal traffic conditions, the service time required to complete the trip, S^u, is distributed randomly with cumulative distribution function (CDF) $F_{S^u}(t)$. We denote the mean service time under normal conditions by $\mathbb{E}[S^u] = 1/\mu$. Here, $\mu = \frac{1}{\mathbb{E}[S^u]}$ represents the rate of service completion, i.e., service rate under normal conditions.

When an incident happens, the vehicle's service rate decreases to $\alpha\mu$ with $0 < \alpha < 1$. Hence, the mean service time under normal conditions, $1/\mu$, increases to $1/\alpha\mu \equiv \mathbb{E}[S^d] \geq 1/\mu$. We define $\mu' = \alpha\mu$ as the service rate under incidents.

Note that these service times depend on roadway conditions. As such, they are affected by link location, traffic volume level, curvature on the roadway, roadway quality, distance from ramps, and other exogenous factors. The combined effect of these factors on service times are represented by the service time parameters, which are assumed to be constant.

These degradations in the quality of service take place randomly. We assume that whenever the system is under normal traffic conditions, it remains in this state for an exponentially distributed amount of time with mean $1/f$. Here, f represents the *incident* rate. *Up periods* define such periods, which are independent and identically distributed (iid). Similarly, *down periods* indicate the periods in which the roadway is experiencing a degradation of service. Down periods are also assumed to follow an iid exponential distribution with mean duration $1/r$, where r is the *repair* rate. This type of service process is said to be *Markov modulated* [21]. Section IV verifies that the exponentiality assumptions for the up and down periods are valid with data.

Suppose one assumes that system changes do not occur during service, meaning that a car travels either under normal or deteriorated conditions. In that case, the travel time distribution can be written as a mixture of two random variables corresponding to normal and deteriorated service times. Our analytical model does not make this assumption. We assume changes occur randomly and may happen during service. Moreover, whenever a service regime change occurs, the new service time is resampled. The new sampled service time value is then added to the travel time already expended. This behavior is also present in other applications, such as the CPU processing times under processor sharing service discipline [22], [23]. We show that this assumption does not impact the validity of the results for our data.

Experiencing changes mid-service may be unrealistic in short corridors. Travel times for short traffic-homogenous road sections are usually much quicker than the times between incidents or incident durations. In contrast, travelers may experience multiple system changes in long corridors. The analytical model proposed in this paper is sufficiently general to cover both situations.

A. Full Travel Time Distribution

This section derives a closed-form expression for the moment generating function of the travel time random variable, which we denote by T.

The moment generating function (MGF) of a random variable uniquely determines its distribution, and immediately provides all its moments. The MGF of a nonnegative continuous random variable X is derived from the Laplace transform of its density function f_X. Because it transforms convolutions into multiplications, Laplace transforms considerably facilitate the tractability of distributions that involve sums of random variables [24].

Proposition 1 presents the resulting closed-form Laplace transform, $L_T(\cdot)$, of the travel time T. Although a procedure to derive the Laplace transform of a job completion time r.v. in general service systems can be found in the works of Kulkarni et al. [25], [26], we use a much simpler counting argument for a particular class of server systems yielding more detailed results.

Proposition 1. Consider a two-service-regime server as described above. Let $L_{S^u}(\cdot)$ and $L_{S^d}(\cdot)$ represent the Laplace transforms of the service time distributions during up and down periods, respectively. Then, the travel time random variable T has a distribution with Laplace transform for complex variable $s \geq 0$ given as

$$\begin{aligned} L_T(s) = \mathbb{E}[e^{-sT}] &= \frac{1}{1-V(s)} \\ &\left\{ \frac{r}{f+r} \cdot L_{S^u}(s+f) \left(1 + \frac{f}{s+r} \left[1 - L_{S^d}(s+r) \right] \right) \right. \\ &\left. + \frac{f}{f+r} \cdot L_{S^d}(s+r) \left(1 + \frac{r}{s+f} \left[1 - L_{S^u}(s+f) \right] \right) \right\}, \end{aligned} \tag{1}$$

where,

$$V(s) = \frac{rf \left[1 - L_{S^u}(s+f) \right] \left[1 - L_{S^d}(s+r) \right]}{(s+f)(s+r)},$$

and with mean

$$\begin{aligned} \mathbb{E}[T] = &\frac{\frac{1}{r}(1 - L_{S^d}(r) \left[1 - \frac{r}{r+f} L_{S^u}(f) \right]}{L_{S^d}(r) + L_{S^u}(f) - L_{S^u}(f) L_{S^d}(r)} \\ &+ \frac{\frac{1}{f}(1 - L_{S^u}(f)) \left[1 - \frac{f}{r+f} L_{S^d}(r) \right]}{L_{S^d}(r) + L_{S^u}(f) - L_{S^u}(f) L_{S^d}(r)}. \end{aligned} \tag{2}$$

Proof. A proof of is provided in Appendix A. □

Under particular circumstances, the travel time Laplace transform given in proposition 1 can be inverted analytically. In general, numerical Laplace inversion methods are necessary [27]. Section IV-D further discusses numerical inversion methods.

The two terms in Eq. (1) are weighted by the probabilities that a traveler arrives to the system during an up period, $r/(f + r)$, or arrives during a down period, $f/(f+r)$. The multipliers of these probabilities describe the Laplace transforms of the total travel time for a traveler arriving during an up or down period, respectively. The above structure of $L_T(s)$ informs us that these two travel times are statistically independent, suggesting that the travel time distribution is a mixture of two random variables. Because deterioration may randomly happen during service, the mixing distributions differ from the original service time distributions.

As a matter of comparison, if we were to rely solely on renewal arguments, we might have written the Laplace transform and mean of the travel time as follows. Considering only the up and down process in steady-state, the probability of the system being in an up period is $r/(r + f)$, and in a down period is $f/(r + f)$. Then, ignoring the switch overs between up and down periods, we could write the Laplace transform of the mixture service time as

$$L_M(s) := \frac{r}{r+f} L_{S^u}(s) + \frac{f}{r+f} L_{S^d}(s).$$

This model corresponds to the mixture models used in literature. But, clearly, this is quite different than the formula given in Eq. 1.

It is worth mentioning that the arguments employed to obtain the analytical results permit a more general result than the one shown in proposition 1. For example, Baykal-Gürsoy and Duan [28] analyze a generally distributed service system with general down periods and partial breakdowns.

Vehicular traffic averages and incident frequency and duration provide the estimates of the four parameters for the model (r, f, $\vec{\theta}$, and $\vec{\theta'}$ – where $\vec{\theta}$, and $\vec{\theta'}$ denote the parameters vectors of the service time distributions during up and down periods, respectively). Then, the only missing input is a profile for the service time distribution under normal traffic conditions $f_{S^u}(t)$. For the sake of simplicity, we assume the profile of the distribution for the service time under deteriorated conditions will be the same, only with different parameters.

The analytical model immediately gives the travel time distribution in the Laplace domain $L_T(s)$. This result is then inverted to the time domain $f_T(t)$, often via numerical methods.

IV. Model Implementation

Implementing the proposed model for an actual traffic corridor requires statistical information about (1) the vehicle's service time under normal traffic conditions and (2) the events that deteriorate/deviate the traffic from these conditions. This section validates our model's technical assumptions using vehicular traffic data and shows how it can be reasonably calibrated using readily available data averages.

A. *Traffic, accident, and weather datasets*

We use traffic data from an 8.5-mile freeway segment depicted in Fig. 1.

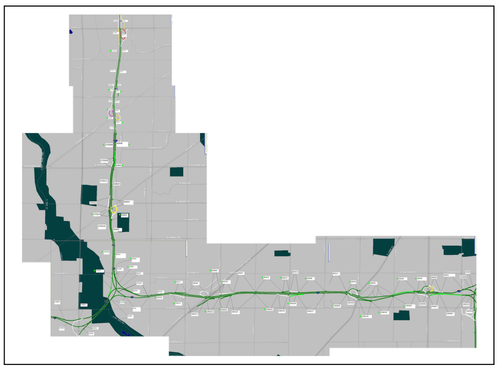

Fig. 1: Interstate 894, Milwaukee, WI

The traffic data is reported by detector stations located near or at interchanges with eighteen detectors in the South-East direction and seventeen in the West-North direction. The average spacing between detector stations is half a mile. Every sensor gives minute-time stamped speed, volume, and occupancy data for 14 months from January 1, 2008, to February 28, 2009. The speed data is truncated (censored) at the local speed limit of either 55 or 60 miles per hour. The proportion of truncated speed data ranges from 1% to 70%, with an average of 51%.

The original accident data comes from reports of both the local police authorities and the State Traffic Operations Center (STOC) of the Wisconsin DOT. The data includes the start and end times of each accident, together with the identification of the nearest detector station. We note that the impact of accidents may cause delays in adjacent sections, despite only being recorded by one detector.

We obtain weather event data from the Climate Data Online system of the National Oceanic and Atmospheric Administration [29]. This data is given as the hourly amount of precipitation in hundredths of inches recorded at the Milwaukee Mitchell International Airport weather station during the same period as the traffic data. The data also indicate days with snow, fog, and thunderstorms.

These datasets are then combined by labeling any time interval from our selected time windows with no accident and adverse weather reports as an up period. The service time distribution during the up periods provides information on how the system behaves under non-congested conditions. Similarly, we define as a down period any time interval with reported accidents or serious weather events (snow or over half an inch of hourly rain). This combined dataset will allow us to estimate the parameters of normal service time, deteriorated service time, uptime, and downtime distributions in the later sections. For example, the deterioration frequency f and the restoration frequency r can be obtained from up and down intervals' average duration. Let $\bar{U}$ denote the mean uptime duration of the system and $\bar{D}$ the mean downtime duration. Then $f = 1/\bar{U}$ and $r = 1/\bar{D}$.

B. *Validation of Model Assumptions*

Our principal assumption is that the uptime and downtime are exponentially distributed. Although it is clear that neither the uptime or downtime processes will be perfectly memoryless, we illustrate below that they provide a close fit to our dataset.

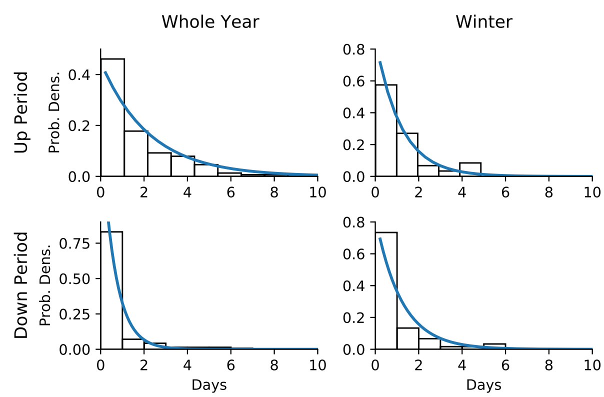

Fig. 2: Scaled event duration histograms with fitted exponential distributions.

For demonstration purposes, we select a southbound half a mile stretch between Cleveland and Oklahoma avenues. This stretch is one of the most incident-prone sections of the road, with 137 recorded incidents in the 14 months of the study. Fig. 2 shows normalized histograms of the durations of the up and down periods for this road section, as well as the corresponding exponential distributions fitted to the data. It considers both the whole year together and the winter months separately from the rest of the year. In the summer, incidents seldom occur, providing little information on the uptime and downtime duration behavior.

Note that, although the down durations include accidents and weather events, which have different behaviors in winter, the plots do not indicate multimodality. Nevertheless, we will also consider only weather related incidents in winter. For both up and down time durations, exponential fit seems to be appropriate.

In our example, we obtain the mean uptime duration as $\bar{U} \approx 41$ hours and the mean downtime duration as $\bar{D} \approx 21.5$ hours for the complete period. In the summer, we see longer periods without incidents, $\bar{U}_{\text{sum.}} \approx 96$ hours, and faster clearance times, $\bar{D}_{\text{sum.}} \approx 0.6$ hours. This occurs because most incidents reported are accidents and rainfalls, whose impact often lasts a short time. Finally, winter data shows that the uptime duration and downtime duration are similar, $\bar{U}_{\text{win.}} \approx \bar{D}_{\text{win.}} \approx 28$ hours. Because accidents may impact neighboring sections despite being recorded in a single section, and weather affects the whole roadway, we use these values for validation across all sensors in our validation tests depending on the incident type involved. We also consider including only weather-related incidents in the winter data for downtime durations and use $\bar{U}_{\text{win.}} \approx \bar{D}_{\text{win.}} \approx 70$ hours.

The failure frequency f and the repair frequency r correspond to the adjusted exponential distributions' rate parameters. Since we consider a half-mile section of the road, service times are on the scales of seconds, thus much shorter than the duration of up or down periods. However, for longer segments, the service times may be relatively similar to incident durations.

C. Calibration

When modeling traffic conditions, $1/\mu$ and $1/\mu'$ are parameters that are straightforward to obtain. They are the mean service times for the normal and deteriorated traffic conditions, respectively. The required service times are approximated from the speed data by "walking the speed field" of the specific road section [30], which yields sufficiently good service time estimations for our statistical analysis. In order to obtain $1/\mu$, we use the mean service time during up periods of the system, and, similarly, $1/\mu'$ is the mean service time for down periods of the system as described before. Finally, because we focus on non-recurrent congestion, we only consider non-rush hour times.

Figures 3 show probability-scaled service time histograms for up and down periods respectively in the half a mile test road section. The mean service time is $1/\mu = 29.60$ seconds during the uptimes and it is $1/\mu' = 32.67$ seconds during the downtimes, giving $\alpha = \frac{29.60}{32.67} = 0.906$. As expected, $1/\mu < 1/\mu'$, i.e., the mean service time increases under deteriorated traffic conditions.

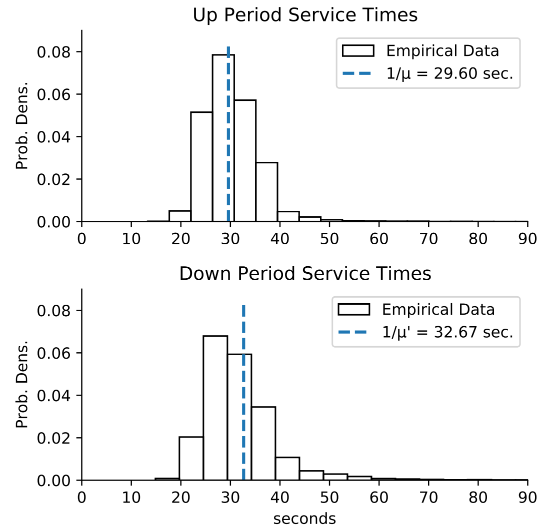

Fig. 3: Scaled service time histograms and their means.

The final step to complete the implementation of our analytical model is to select a distribution to the service times, S^u, under the normal traffic conditions which correspond to our up-periods (see Fig, 3). If non-parametric distributions are to be used, they can be transformed numerically to the frequency domain in order to be used in proposition 1.

For convenience, we use triangular and gamma distributions as examples in this paper. These two distributions are flexible and relatively easy to compute. Their overall shapes also match what one would expect from service time under normal conditions during non-peak hours. However, we emphasize that different roadways may have different distributions of service time. Each case must be analyzed separately.

Once properly defined, the travel time distribution described in proposition 1 can be inverted numerically to the time domain. We now discuss this process and the computation of reliability metrics using our distribution.

D. Computing Travel Time Distribution and Reliability Measures

We now show how to compute various travel time reliability measures under our model. The simplest case is moment-based reliability measures. These can be found using the property

$$\mathbb{E}[T^n] = (-1)^n \frac{d^n}{ds^n} L_T(s)\bigg|_{s=0}. \quad (3)$$

This will provide us with the mean, variance, skewness, and kurtosis of travel time that could directly be used in approximating the percentile function as in [12].

Most other travel time reliability measures require the distribution function. For general distributions, this will require numerical inverse Laplace transform methods - a particularly robust method comes from Abate and Whitt [27], who write the Bromwich integral as a Fourier series. Other popular algorithms can be found in [31], [32].

Provided a numerical evaluation of $f(t)$, we find the cumulative distribution function $F(t)$ by numerically integrating f over the interval $[0, t]$. Once f and F have been found, the remaining reliability measures, including α-quantile, Buffer Index, Planning Time Index [2], α-reliable mean excess travel time [10], and Fosgerau's reliability ratio [11], are immediate from their definitions.

V. Results

This section compares our model to several probability distributions proposed in literature. We find that our model is competitive.

A. An approach for fitting censored data

The data set displays truncated speed observations at 55 miles per hour for some sensors and 60 miles per hour for some others. That is, for speeds less than 55 mph (60 mph), we observe the speed exactly. For vehicle speeds greater than 55 mph (60 mph), we only know that the speed exceeded 55 mph (60 mph).

We refer to this concept as right-censoring, as it resembles the common problem faced in survival analysis. Because these speeds are then inverted to compute travel times, the resulting travel time observations are left-censored. As is the case in survival analysis, censored observations can occur frequently and belong to the overall population, so they should not be filtered as outliers.

We provide the maximum likelihood estimator, which only uses the left-censored travel-time observations. Suppose that out of k travel-time observations, n are left-censored at T– that is, we do not know their exact value, only that their value was less than or equal to T. We can include this information by using the CDF $F(T) = P\{\text{service time} \leq T\}$. To do so, we consider the likelihood function

$$\mathcal{L}(\theta) = (F(T))^n \prod_{i=1}^{k-n} f(t_i),$$

where the pdf and cdf are parameterized by θ. From here, we proceed as usual by maximizing this function either analytically or numerically. This is the best parametric estimator we can construct without reconstructing speeds with supplemental data and carries all the properties of a maximum likelihood estimator.

Unfortunately, even with a modified estimator, there is irreparable data loss due to this left-censoring. As discussed in [33] and [34], overly censored data sets can result in degenerate estimators, possibly bringing parameters to their extreme values (e.g., variance $\approx$ 0). This is not realistic in most cases. Throughout our experiments, we will avoid using overly censored data sets to mitigate this problem.

B. Model Validation Across Sensors

To evaluate our distribution's relative performance, we fit some alternative distributions to the pooled travel time empirical distribution and compute the Akaike information criterion. The results are displayed in Tables I and II.

This information measure is calculated as shown in equation (4), where k is the number of parameters in the model, and $\mathcal{L}$ is the likelihood of the model given the calibration data.

$$\text{AIC} = 2k - 2\ln(\mathcal{L}). \tag{4}$$

This criterion ranks models by relative comparisons, and it favors parsimonious models. Smaller values of the AIC indicate that the quality of the model may be higher. In the referenced tables, our models have a similar AIC value compared to the other distributions considered.

Model	1	2	3	4	5	6	7	8	9	10	11	12
$f_T(t)$, gamma	82	84	89	47	63	37	44	47	74	53	81	74
Gamma	84	86	110	55	65	41	45	50	81	56	110	96
Log-normal	83	85	110	55	64	39	44	48	78	54	100	92
Log-logistic	81	83	95	52	63	38	44	45	70	50	90	80
Weibull	87	89	130	58	67	41	45	51	88	58	120	110
Burr XII	79	82	84	50	63	38	44	44	68	48	80	70
$f_T(t)$, gamma mixture	77	80	82	47	62	30	43	44	63	45	99	66
Gamma mixture	77	81	82	47	63	37	43	44	65	47	81	72
Log-normal mixture	77	81	80	45	63	36	43	42	62	43	75	66
Log-logistic mixture	77	81	95	45	63	35	43	41	62	42	90	80
Weibull mixture	77	81	84	46	63	36	43	43	63	44	79	71
Burr XII mixture	77	81	80	43	55	34	43	39	57	34	72	62

TABLE I: Wintertime AIC across sensors for various distributions, all values in Thousands. All observations are Tuesday - Thursday, 10:00 am - 4:00 pm, December - March. $f = r \approx 10^{-5} s^{-1}$

We use the gamma distribution for both service time distributions in our model because the gamma distribution is a sufficiently flexible distribution for our purposes. For comparison, we consider several distribution families that have been used for travel time distributions [7]. To show how our model performs relative to mixtures, we also evaluate 2-component mixtures for each individual distribution. To our knowledge, the Burr XII mixture has not been considered as a travel-time distribution before. Similarly, we also use 2-component gamma mixtures as service time distributions to provide an even comparison between our model and the alternatives.

Model	1	2	3	4	5	6	7	8	9	10	11	12
$f_T(t)$, gamma	82	84	89	47	63	37	44	47	74	53	81	74
Gamma	84	86	110	55	65	41	45	50	81	56	110	96
Log-normal	83	85	110	55	64	39	44	48	78	54	100	92
Log-logistic	81	83	95	52	63	38	44	45	70	50	90	80
Weibull	87	89	130	58	67	41	45	51	88	58	120	110
Burr XII	79	82	84	50	63	38	44	44	68	48	80	70
$f_T(t)$, gamma mixture	77	80	82	47	62	31	43	45	63	45	100	66
Gamma mixture	77	81	82	47	63	37	43	44	65	47	81	72
Log-normal mixture	77	81	80	45	63	36	43	42	62	43	75	66
Log-logistic mixture	77	81	95	45	63	35	43	41	62	42	90	80
Weibull mixture	77	81	84	46	63	36	43	43	63	44	79	71
Burr XII mixture	77	81	80	43	55	34	43	39	57	34	72	62

TABLE II: Wintertime AIC across sensors for various distributions, all values in Thousands. All observations are Tuesday - Thursday, 10:00 am - 4:00 pm, December - March. Incidents are restricted to snow. $f = r \approx 4 \times 10^{-6} s^{-1}$

To ensure that overly censored sensors do not corrupt our results, we include all sensors with less than 80% censorship rate in both seasons. We consider only Tuesday through Thursday, 10:00 am - 4:00 pm, as rush-hour, late-hours, and Monday / Friday travel times are significantly different in their distribution. We restrict our dataset to winter season so that the failure, f, and clearance, r, rates are relatively constant across observations, both of which are computed by taking the mean up and downtime from the supplemental incident dataset discussed in Section IV-B.

Each service time distribution is fit using the maximum likelihood estimator discussed in section V-A. We compute separate parameters and AIC for each sensor, which produces the results shown in Tables I and II. For illustration purposes, the gamma variants of each model, using their MLE's, are depicted in Figure 4. We see that our model captures both the peak and the tail portions of travel time distribution more accurately and seems to fit the data more closely than the alternatives.

Table I displays AIC in wintertime including all incidents, e.g., accidents and inclement weather with $r = f = 10^{-5} s^{-1}$, while Table II only includes snow events not accidents. Because each sensor has a different number of observations, the computed AICs are not comparable across sensors but only across models within each sensor. Our model has a similar AIC to the distributions considered, suggesting our model has an equivalent fit across all sensors.

With results being comparable to the ones in the literature, we emphasize that the proposed model innovatively allows for a direct understanding of how different incidents affect travel time. This unique characteristic helps decision-makers study hypothetical scenarios on how various changes in incident behavior alter the travel time distribution without the need for time-consuming simulations, as discussed in section VI.

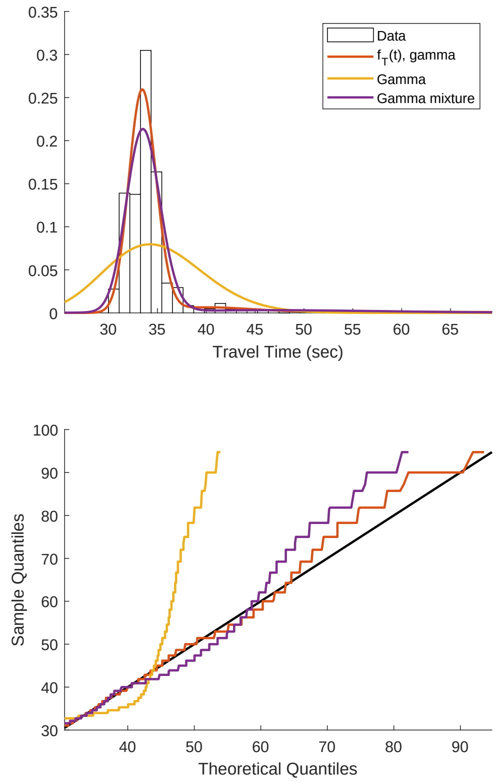

Fig. 4: Plot of each model based on the gamma distribution. The proposed model more accurately captures both the peak and tail, when compared to a simple gamma or gamma mixture.

VI. Scenario Analysis

In this section, we illustrate the flexibility of our model as the interpretable f, r and α parameters may be adjusted. This allows for what-if scenarios, and long-term traffic deterioration sensitivity studies.

To showcase how incident behavior variations affect travel time, we consider two scenarios:

1) Roadways with infrequent but severe incidents. For this, we vary α;
2) Roadways with frequent incidents that cause any level of disruption. For this, we vary f and r.

In each scenario, we compute the mean ($\mathbb{E}[T]$), standard deviation (σ_T) and 95th percentile directly from the obtained travel time distribution. We also include three reliability metrics proposed by the Federal Highway Administration [2]. The first is The Buffer Index (BI), the fraction of time that travelers should add to their mean travel time to ensure on-time arrival at the 95% level. One shortcoming of BI is that it may potentially overestimate the reliability when travel time distributions are heavily right-skewed [35]. To provide a more accurate picture, the second reliability metric we obtain is the median based Buffer Index (M-BI), the fraction of time that travelers should add to their median travel time to ensure on-time arrival at the 95% level. The third is the

Planning Time Index (PTI), the factor of free-flow time that travelers should consider to ensure on-time arrival. The top of Fig. 5 depicts our model's travel time distribution using triangular service time distributions for one particular sensor. This distribution is straightforward to parameterize, and we choose it as the baseline (with parameters $a = 22.13,\ b = 40.91,\ c = 25.77$) for each of the scenarios discussed in this section.

A. Varying the severity of the incident

We start by showing the impact of the severity of the traffic deterioration on the travel time. We vary the α parameter, defined as

$$\alpha = \frac{1/\mu}{1/\mu'} = \frac{\mu'}{\mu} = \frac{110.17}{121.60} = 0.906.$$

This is the ratio of expected service time under normal and deteriorated traffic conditions.

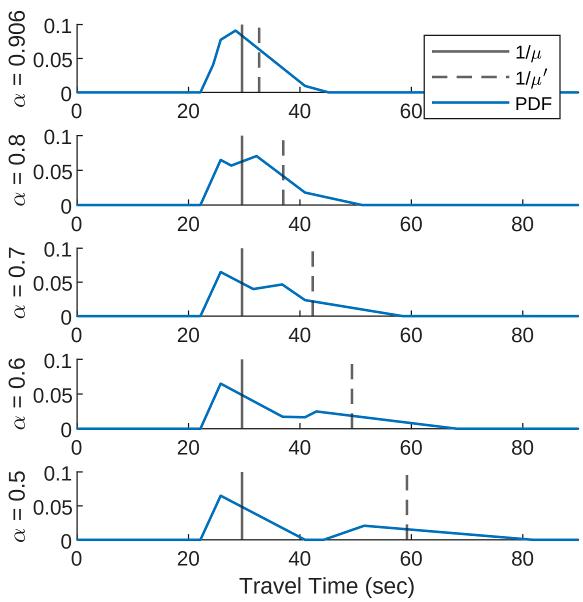

Fig. 5: Travel time PDFs for varying α.

Fig. 5 displays how the travel time PDF changes according to decreases in α. This is equivalent to keeping μ fixed and having the service rate under deteriorated conditions decrease. We plot the corresponding values of $1/\mu$ and $1/\mu'$. As α drops, the distribution becomes bimodal, with some proportion of the travelers experiencing more significant travel times. The distribution becomes disconnected for small values of α.

Table III shows the reliability indices calculated for each experiment. Notice from Table III that, as expected, all reliability indices increase as α decreases. For the case with $\alpha = 0.5$ BI (M-BI) indicates that a 69% (106%) buffer needs to be added to the median travel time to ensure on-time arrival at the 95% level of confidence. Also, PTI demonstrates that the driver should expect to travel more than double the distance at free-flow speed to ensure on-time arrival at the same confidence level.

	$\mathbb{E}[T]$ [sec]	σ_T [sec]	95th [sec]	BI	M-BI	PTI
$\alpha = 0.906$	30.81	4.50	39.01	0.27	0.29	1.32
$\alpha = 0.800$	32.50	5.77	43.60	0.34	0.37	1.47
$\alpha = 0.700$	34.56	7.85	49.82	0.44	0.49	1.68
$\alpha = 0.600$	37.31	10.99	58.11	0.56	0.72	1.96
$\alpha = 0.500$	41.15	15.64	69.73	0.69	1.06	2.36

TABLE III: Reliability indices for travel times distribution.

B. Varying the frequency of incidents

This section shows how the tail of travel time distribution thickens as the frequency of incidents increases. Traditional mixture models fail to capture this behavior without data observed under an increased incident rate. Consider a roadway corridor of 30 miles, 60 times longer than our previous example. The time parameters ($1/\mu$, $1/\mu'$) are scaled by a factor of sixty and are now read in minutes. For example, $1/\mu$ is now 29.60 min and $1/\mu' = 32.67$ min (α remains the same). Suppose that the mean downtime duration is 30 min, $\bar{D} = 1/r = 30$ min. This says that, on average, incidents are cleared out after 30 minutes. Let us now analyze the impacts of having an increasing incident frequency. This could result from changes in speed limits, sudden increases in demand, or environmental changes such as the variation in the deer population. We calculate the travel time distribution for three different values of the uptime duration (the inverse of incident frequency): $1/f = 30$, $1/f = 120$ and $1/f = 240$ min. In the most extreme case, failures and repairs are expected to happen every 30 minutes.

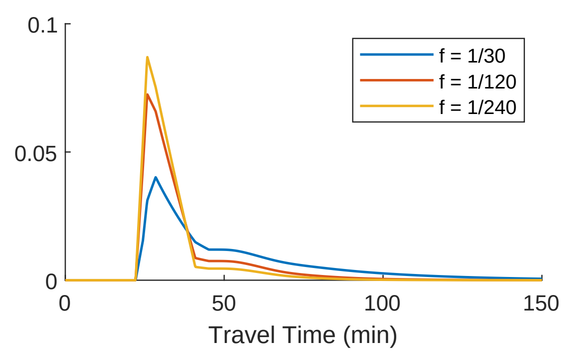

Fig. 6: Effect of changes in the failure frequency.

Fig. 6 depicts the travel time density functions for these three cases. We observe that the more frequent the incidents, the thicker the tail becomes, indicating that such systems become more unstable with higher probabilities of abnormally long travel times.

Table IV shows that the reliability indices increase, consistent with the decrease in service quality. Standard deviation grows rapidly, and the 95th percentile moves drastically to the right. For the most severe case, BI gives a value larger than one. This indicates that the mean travel time is smaller than half the 95th percentile. Also, the PTI value is almost 4, which means that a driver who expects to get on time at the 95% confidence level should picture a trip four times longer than the original one (that is, a 120-mile trip) to be traveled at free-flow speed.

	$\mathbb{E}[T]$ [min]	σ_T [min]	95th [min]	BI	M-BI	PTI
$1/f = 30$	53.72	31.72	117.52	1.19	2.48	3.97
$1/f = 120$	37.16	15.73	69.87	0.88	1.07	2.36
$1/f = 240$	33.58	11.52	58.86	0.75	0.74	1.99

TABLE IV: Reliability indices for travel times distribution.

This section shows that our model can compute the travel time distribution with varying incident frequencies. As seen in Fig. 6, our model's outcome can be largely different from the traditional mixtures currently proposed in the literature [15]–[17].

VII. Discussion and Conclusions

We present an analytical model and calibration strategy to approximate a road segment's travel time distribution subject to random degradations of service. By focusing on off-peak hours, the model incorporates the stochasticity of traffic degradation due to non-recurrent events. This analytical approach provides the travel time distribution, from which a variety of reliability indices can be readily computed.

Accident and weather reports serve as non-recurrent events for the validation of the assumptions of our analytical model and the justification of its application to a travel time reliability study. The resulting travel time distributions adequately approximate empirical data. Standard reliability indices can be calculated directly from the full analytical travel time distribution.

We present scenarios to illustrate the flexibility of the model for other non-recurrent events. We verify that our model accurately describes the travel time distribution when the frequency of failure is high. Otherwise, our model is in line with traditional single or mixture models. This model is the first to analyze travel time distribution in roadways subject to random incidents (currently restricted to a single type of degradation). For future work, we plan to extend the model to multiple levels of degradation. We will also consider numerical implementations that allow the use of service time distributions ($f_{S^u}(t)$ and $f_{S^d}(t)$) that do not have a closed-form Laplace transform (e.g., lognormal distribution). For this, we need to reliably and efficiently approximate the Laplace transform for truncated versions of the distributions.

An additional limitation is that this model only captures the travel time along individual links. For more realism in modeling route travel time distributions, the dependence of travel times between adjacent links could be incorporated into the model. However, such an inclusion brings about numerous questions unique to Markovian environments, particularly regarding how link up-down periods should be modeled, which we plan to investigate in our future research.

Finally, the ability to compute travel time distributions (or even the tail portion of the distribution) aids the estimation of the value of travel time reliability. The proposed model provides the distribution of the travel time regardless of the vulnerability of the corridors. With this information, we can study the effect of incidents on the value of travel time reliability according to the expected utility. We supply example scenarios that decision-makers can use to explore without the need for extensive simulations of the effect perceived when increasing the frequency of incidents or how the level of deterioration α affects the perceived utility.

Appendix A
Proof of Proposition 1

Proof. A first key observation is that the travel time of a traveler is calculated differently depending on the traveler's arrival epoch. Let us call G^1 as the event that an arrival happens during an up period, and G^2 as the event that an arrival happens during a down period. In particular, by renewal arguments, it holds that:

$$P\{G^1\} = \frac{r}{f+r}, \qquad P\{G^2\} = \frac{f}{f+r}. \tag{5}$$

The Laplace transform of the conditional travel time can be calculated separately for each one of these events. From these, the Laplace transform of the unconditioned travel time distribution can be calculated.

We start by studying the travel time of a travelers' arrival under G^1. Consider a traveler arriving during an up period, and denote the subsequent length of up and down periods as U_i and D_i, respectively, for $i = 1, 2, 3, \ldots$. Let S_j^u denote the jth up period service time requirement, and similarly, let S_j^d denote the jth down period service time requirement. Fig. 7 shows a sample path of how the system could evolve. We assume the traveler arrives at time zero, and we denote with the crosses on the time-axis some possible departure times (realizations of the travel time).

Define the following events:

- A_n, for $n = 0, 1, 2, \ldots$ denotes an event during which exactly n complete up and down periods pass before the travel time is completed during the $(n+1)$-st up period. The second cross in Fig. 7 is an example of event A_2. Notice that the occurrence of event A_n implies that necessarily:
  1) $S_i^u > U_i$, for all $i = 1, 2, \ldots, n$,
  2) $S_i^d > D_i$, for all $i = 1, 2, \ldots, n$,

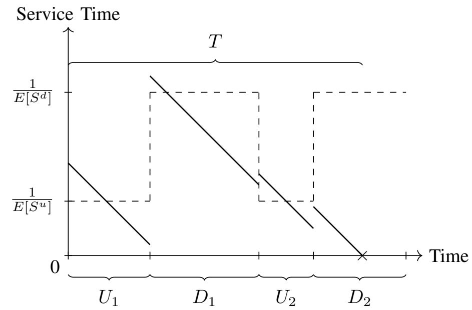

Fig. 7: Sample Path of Service System for a Traveler Arrival Under G^1.

3) $S^u_{n+1} < U_{n+1}$.

- E_n, for $n = 0, 1, 2, \ldots$ denotes an event during which exactly $n+1$ complete up and n complete down periods pass before the travel time is completed during the $(n+1)$-st down period. The first cross in Fig. 7 is an example of event E_1. Here, necessarily:
  1) $S^u_i > U_i$, for all $i = 1, 2, \ldots, n+1$,
  2) $S^d_i > D_i$, for all $i = 1, 2, \ldots, n$,
  3) $S^d_{n+1} < D_{n+1}$.

Since clearly S^u_i's, S^d_i's, U_i's, and D_i's are all independent and respectively identically distributed,

$$P\left\{A_n|G^1\right\} = P^n\{S^u > U\}P^n\{S^d > D\}P\{S^u < U\}, \tag{6}$$

$$P\left\{E_n|G^1\right\} = P^{n+1}\{S^u > U\}P^n\{S^d > D\}P\{S^d < D\}. \tag{7}$$

Notice that the probabilities defined in (6)-(7) were obtained by simple enumeration of the number of system state transitions. Similarly, the conditional travel time $\{T|G^1\}$ under each event stated above is:

$$\{T|G^1\} = \begin{cases} \sum_{i=1}^{n}(U_i + D_i) + S^u_{n+1}, & \text{event } A_n, \\ \sum_{i=1}^{n+1} U_i + \sum_{i=1}^{n} D_i + S^d_{n+1}, & \text{event } E_n, \end{cases} \forall n.$$

Let an indicator random variable under some event H is defined as

$$\mathbf{1}\{H\} = \begin{cases} 1, & \text{event } H, \\ 0, & \text{otherwise.} \end{cases}$$

Then, the conditional travel time Laplace transform is derived as

$$\begin{aligned} &\mathbb{E}[e^{-sT}\mathbf{1}\{A_n\}|G^1] = \mathbb{E}[e^{-sT}|A_n, G^1] \cdot P\{A_n|G^1\} \\ &= \Big\{ \Big(\mathbb{E}\left[e^{-sU}|S^u > U\right] P\left\{S^u > U\right\}\Big) \qquad (8) \\ &\quad \cdot \Big(\mathbb{E}[e^{-sD}|S^d > D]P\left\{S^d > D\right\}\Big) \Big\}^n \\ &\quad \cdot \mathbb{E}[e^{-sS^u}|S^u < U]P\{S^u < U\}, \qquad (9) \end{aligned}$$

where s is a complex number with positive real part, U is an exponentially distributed up period random variable with parameter f, i.e., $U \sim \text{Exp}(f)$, D is also an exponentially distributed random variable with parameter r, i.e., $D \sim \text{Exp}(r)$, S^u denotes the generally distributed service time random variable under normal conditions with cumulative distribution function denoted as F_{S^u}, and S^d denotes the generally distributed service time random variable under deteriorated conditions with cumulative distribution function denoted as F_{S^d}.

Laplace transform of the up period for uptimes lasting less than the service requirement is derived below as

$$\begin{aligned} &\mathbb{E}[e^{-sU}|S^u > U]P\{S^u > U\} = \mathbb{E}[e^{-sU}\mathbf{1}\{S^u > U\} \\ &= \int_{x=0}^{\infty}\int_{y=0}^{x} e^{-sy} f e^{-fy} dy\, dF_{S^u}(x) \\ &= \frac{f}{s+f}[1 - L_{S^u}(s+f)]. \end{aligned}$$

The last equality follows directly from the properties of Laplace transforms or by application of integration by parts to the integral term. Similarly, the following holds

$$\begin{aligned} &\mathbb{E}[e^{-sD}|S^d > D]P\{S^d > D\} = \frac{r}{s+r}[1 - L_{S^d}(s+r)], \\ &\mathbb{E}[e^{-sS^u}|S^u < U]P\{S^u < U\} = L_{S^u}(s+f), \end{aligned}$$

from where (9) becomes:

$$\begin{aligned} \mathbb{E}[e^{-sT}\mathbf{1}\{A_n\}|G^1] &= \left\{\frac{f}{s+f}[1 - L_{S^u}(s+f)]\right\}^n \\ &\cdot \left\{\frac{r}{s+r}[1 - L_{S^d}(s+r)]\right\}^n \cdot L_{S^u}(s+f). \end{aligned} \tag{10}$$

For the case E_n, similar to the case A_n, we have:

$$\begin{aligned} &\mathbb{E}[e^{-sT}\mathbf{1}\{E_n\}|G^1] = \mathbb{E}[e^{-sT}|E_n, G^1] \cdot P\{E_n|G^1\} \\ &= \Big(\mathbb{E}[e^{-sU}|S^u > U]P\{S^u > U\}\Big)^{n+1} \\ &\quad \cdot \Big(\mathbb{E}[e^{-sD}|S^d > D]P\{S^d > D\}\Big)^n \\ &\quad \cdot \Big(\mathbb{E}[e^{-sS^d}|S^d < D]P\{S^d < D\}\Big), \end{aligned}$$

where the last term is,

$$\begin{aligned} &\mathbb{E}[e^{-sS^d}|S^d < D]P\{S^d < D\} = \mathbb{E}[e^{-sS^d}\mathbf{1}\{S^d < D\}] \\ &= \int_{t=0}^{\infty}\int_{x=0}^{t} e^{-sx} dF_{S^d}(x) r e^{rt} dt \\ &= \int_{x=0}^{\infty}\int_{t=x}^{\infty} r e^{-rt} dt e^{-sx} dF_{S^d}(x) = L_{S^d}(s+r), \end{aligned}$$

giving,

$$\begin{aligned} \mathbb{E}[e^{-sT}\mathbf{1}\{E_n\}|G^1] &= \left\{\frac{f}{s+f}[1 - L_{S^u}(s+f)]\right\}^{n+1} \\ &\cdot \left\{\frac{r}{s+r}[1 - L_{S^d}(s+r)]\right\}^n \cdot L_{S^d}(s+r). \end{aligned} \tag{11}$$

Then, from (10), we obtain the Laplace transform of the conditional travel time, given that the traveler arrives during an up period as shown in (12).

$$\begin{aligned} &\mathbb{E}\left[e^{-sT}|G^1\right] = \sum_{n=0}^{\infty}\mathbb{E}[e^{-sT}\mathbf{1}\{A_n\}|G^1] \\ &\quad + \sum_{n=0}^{\infty}\mathbb{E}[e^{-sT}\mathbf{1}\{E_n\}|G^1] = \frac{1}{1 - V(s)} \\ &\left\{L_{S^u}(s+f) + \frac{f}{s+f}\cdot[1 - L_{S^u}(s+f)]\cdot L_{S^d}(s+r)\right\}, \end{aligned} \tag{12}$$

where

$$V(s) = \left[\frac{f}{s+f}[1 - L_{S^u}(s+f)]\right]\left[\frac{r}{s+r}[1 - L_{S^d}(s+r)]\right].$$

Consider now a traveler under G^2, that is who arrives during a down period. Fig. 8 shows a sample path of a run for the system for a traveler arriving during a down period.

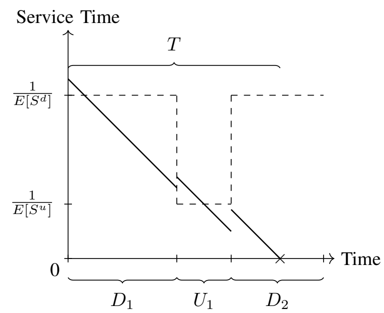

Fig. 8: Sample Path of Service System for a Traveler Arrival Under G^2.

Let again A_n and E_n denote the similar events under G^1 but interchanging up and down periods. Hence, for example, under A_n, there are exactly n down and up periods before the travel time is completed during $(n+1)$-st downtime. Then, the conditional travel time given that a traveler arrives during a down period is:

$$\{T|G^2\} = \begin{cases} \sum_{i=1}^{n} D_i + \sum_{i=1}^{n} U_i + S_{n+1}^d, & \text{event } A_n, \forall n, \\ \sum_{i=1}^{n+1} D_i + \sum_{i=1}^{n} U_i + S_{n+1}^u, & \text{event } E_n, \forall n. \end{cases} \tag{13}$$

We write the Laplace transform of the conditional travel time under each event as,

$$\mathbb{E}[e^{-sT}\mathbf{1}\{A_n\}|G^2] = \left\{\frac{r}{s+r}[1 - L_{S^d}(s+r)]\right\}^n \cdot \left\{\frac{f}{s+f}[1 - L_{S^u}(s+f)]\right\}^n \cdot L_{S^d}(s+r), \tag{14}$$

$$\mathbb{E}[e^{-sT}\mathbf{1}\{E_n\}|G^2] = \left\{\frac{r}{s+r}[1 - L_{S^d}(s+r)]\right\}^{n+1} \cdot \left\{\frac{f}{s+f}\left(1 - L_{S^u}(s+f)\right)\right\}^n \cdot L_{S^u}(s+f). \tag{15}$$

From there, we obtain the Laplace transform of the conditional travel times as,

$$\mathbb{E}[e^{-sT}|G^2] = \frac{1}{1 - V(s)} \cdot \left\{L_{S^d}(s+r) + \frac{r}{s+r} \cdot [1 - L_{S^d}(s+r)] \cdot L_{S^u}(s+f)\right\}. \tag{16}$$

Finally, by combining the conditional travel times (12) and (16) using the corresponding probabilities $P\{G^1\}$ and $P\{G^2\}$, the unconditional travel time Laplace transform is obtained as shown in proposition 1. Taking the first derivative of $\mathbb{E}[e^{-sT}]$ in Eq. 1 with respect to s and then evaluating at $s = 0$ provides the negative of the mean travel time as given in equation 2. $\square$

Acknowledgement

The authors are grateful to Peter J. Jin for sharing his data and Eric Gonzales for many fruitful discussions.

References

[1] J. Kwon, T. Barkley, R. Hranac, K. Petty, and N. Compin, "Decomposition of travel time reliability into various sources: incidents, weather, work zones, special events, and base capacity," *Transportation Research Record: Journal of the Transportation Research Board*, vol. 2229, no. 2229, pp. 28–33, 2011.

[2] Federal Highway Administration, *Travel Time Reliability: Making it there on time, all the time*. USA: Federal Highway Administration, 2006.

[3] C. Carrion and D. Levinson, "Value of travel time reliability: A review of current evidence," *Transportation research part A: policy and practice*, vol. 46, no. 4, pp. 720–741, 2012.

[4] W. Yue, C. Li, S. Wang, Z. Xu, and G. Mao, "Towards enhanced recovery and system stability: Analytical solutions for dynamic incident effects in road networks," *IEEE Transactions on Intelligent Transportation Systems*, 2020.

[5] Institute for Transportation Research & Education, "Establishing Monitoring Programs for Travel Time Reliability," United States. National Transportation Library [distributor], Tech. Rep., 2013.

[6] C. Chen, A. Skabardonis, and P. Varaiya, "Travel-time reliability as a measure of service," *Transportation Research Record: Journal of the Transportation Research Board*, vol. 1855, no. 1855, pp. 74–79, 2003.

[7] H. Rakha, I. El-Shawarby, and M. Arafeh, "Trip travel-time reliability: issues and proposed solutions," *Journal of Intelligent Transportation Systems*, vol. 14, no. 4, pp. 232–250, 2010.

[8] Y. Guessous, M. Aron, N. Bhouri, and S. Cohen, "Estimating travel time distribution under different traffic conditions," *Transportation Research Procedia*, vol. 3, pp. 339–348, 2014.

[9] A. Pankaj, "A comprehensive study on the estimation of freeway travel time index and the effect of traffic data quality," *Theses and Dissertations*, vol. 2234, 2019. [Online]. Available: https://dc.uwm.edu/etd/2234

[10] A. Chen and Z. Zhou, "The α-reliable mean-excess traffic equilibrium model with stochastic travel times," *Transportation Research Part B: Methodological*, vol. 44, no. 4, pp. 493–513, 2010.

[11] M. Fosgerau, "The valuation of travel time variability," *Quantifying the Socio-economic Benefits of Transport*, p. 39, 2017.

[12] Z. Zang, X. Xu, C. Yang, and A. Chen, "A closed-form estimation of the travel time percentile function for characterizing travel time reliability," *Transportation Research Part B: Methodological*, vol. 118, pp. 228–247, 2018.

[13] S. Susilawati, M. A. Taylor, and S. V. Somenahalli, "Distributions of travel time variability on urban roads," *Journal of Advanced Transportation*, vol. 47, no. 8, pp. 720–736, 2013.

[14] M. A. Taylor, "Fosgerau's travel time reliability ratio and the burr distribution," *Transportation Research Part B: Methodological*, vol. 97, pp. 50–63, 2017.

[15] F. Guo, H. Rakha, and S. Park, "Multistate model for travel time reliability," *Transportation Research Record*, vol. 2188, no. 1, pp. 46–54, 2010.

[16] E. Kazagli and H. N. Koutsopoulos, "Estimation of arterial travel time from automatic number plate recognition data," *Transportation research record*, vol. 2391, no. 1, pp. 22–31, 2013.

[17] S. Yang and Y.-J. Wu, "Mixture models for fitting freeway travel time distributions and measuring travel time reliability," *Transportation Research Record*, vol. 2594, no. 1, pp. 95–106, 2016.

[18] J. P. Kharoufeh and N. Gautam, "Deriving link travel-time distributions via stochastic speed processes," *Transportation Science*, vol. 38, no. 1, pp. 97–106, 2004.

[19] M. Ramezani and N. Geroliminis, "On the estimation of arterial route travel time distribution with Markov chains," *Transportation Research Part B: Methodological*, vol. 46, no. 10, pp. 1576–1590, 2012.

[20] J. Tang, J. Hu, W. Hao, X. Chen, and Y. Qi, "Markov chains based route travel time estimation considering link spatio-temporal correlation," *Physica A: Statistical Mechanics and its Applications*, vol. 545, p. 123759, 2020.

[21] V. F. Nicola, V. G. Kulkarni, and K. S. Trivedi, “Queueing analysis of fault-tolerant computer systems,” *Software Engineering, IEEE Transactions on*, vol. 3, no. 3, pp. 363–375, 1987.
[22] O. Boxma and I. Kurkova, “The M/M/1 queue in a heavy-tailed random environment,” *Statistica Neerlandica*, vol. 54(2), 2000.
[23] P. R. Jelenković and J. Tan, “Can retransmissions of superexponential documents cause subexponential delays?” in *INFOCOM 2007. 26th IEEE International Conference on Computer Communications. IEEE*, 2007, pp. 892–900.
[24] S. M. Ross, J. J. Kelly, R. J. Sullivan, W. J. Perry, D. Mercer, R. M. Davis, T. D. Washburn, E. V. Sager, J. B. Boyce, and V. L. Bristow, *Stochastic processes*. Wiley New York, 1996, vol. 2.
[25] V. G. Kulkarni, V. F. Nicola, and K. S. Trivedi, “On modelling the performance and reliability of multimode computer systems,” *Journal of Systems and Software*, vol. 6, no. 1-2, pp. 175–182, 1986.
[26] ——, “The completion time of a job on multimode systems,” *Advances in Applied Probability*, vol. 19, pp. 932–954, 1987.
[27] J. Abate and W. Whitt, “Numerical inversion of Laplace transforms of probability distributions,” *ORSA Journal on computing*, vol. 7, no. 1, pp. 36–43, 1995.
[28] M. Baykal-Gürsoy and Z. Duan, “Completion time analysis for M/G/∞ queue with two service speeds.” *presented at the 3rd Madrid Conference on Queueing Theory, Toledo, Spain*, 2010.
[29] National Climatic Data Center, NOAA. (2015, Apr.) Climate data online. [Online]. Available: http://www.ncdc.noaa.gov/cdo-web/
[30] T. Oda, “An algorithm for prediction of travel time using vehicle sensor data,” in *Third International Conference on Road Traffic Control*. IET, 2002, pp. 40–44.
[31] F. R. D. Hoog, J. H. Knight, and A. N. Stokes, “An Improved Method for Numerical Inversion of Laplace Transforms,” *SIAM Journal on Scientific and Statistical Computing*, vol. 3, no. 3, pp. 357–366, 1982. [Online]. Available: http://epubs.siam.org/doi/abs/10.1137/0903022
[32] K. J. Hollenbeck, “INVLAP.M: A Matlab function for numerical inversion of Laplace transforms by the de Hoog algorithm,” 1998. [Online]. Available: http://www.isva.dtu.dk/staff/karl/invlap.htm
[33] M. Woodroofe, “Estimating a distribution function with truncated data,” *The Annals of Statistics*, vol. 13, no. 1, pp. 163–177, 1985.
[34] D. Murdoch and A. Benjamin, “The probability of degenerate maximum-likelihood estimates with randomly truncated survival data,” *Canadian Journal of Statistics*, vol. 21, no. 2, pp. 223–231, 1993.
[35] W. Pu, “Analytic relationships between travel time reliability measures,” *Transportation Research Record*, vol. 2254, no. 1, pp. 122–130, 2011.

Melike Baykal-Gürsoy is a Professor and the Director of Laboratory for Stochastic Systems, and of GRIST-Game Research for Infrastructure SecuriTy Lab, in the I&SE department at Rutgers University. She received her BS in Electrical Engineering and her MS in Electrical Engineering with a major in Control from Bogazici University, Istanbul, Turkey. Her doctorate is in Systems Engineering from the University of Pennsylvania, Philadelphia. Her specific fields of interest include stochastic modeling, queueing, Markov decision processes, stochastic games, and their applications to security and transportation problems. Dr. Baykal-Gürsoy is the co-author of a book entitled ”An Introduction to Probability and Statistics.”

Andrew Reed Benton is a Ph.D. student in industrial and systems engineering from Rutgers University. His fields of interest include stochastic modeling, Markov decision processes, as well as their applications to security and transportation problems.

Pedro Cesar Lopes Gerum received a Ph.D. in Industrial and Systems Engineering from Rutgers University. He is currently an Assistant Professor in the Operation and Supply Chain department at Cleveland State University. His research focuses on transportation optimization, stochastic modeling, data science, and machine learning. He has experience developing machine-learning applications for finance problems at American Express, and has helped create deep-learning models for the classification of exoplanets at NASA Ames in California.

Marcelo Figueroa-Candia , Fulbright Fellow, holds a Ph.D. in Industrial and Systems Engineering from Rutgers University and a Masters in Computer Science from Universidad de Concepción, Chile. His research focuses on stochastic programming applied to transportation optimization. He currently works as a Senior Data Scientist for Boston Consulting Group, where he specializes in deploying Advanced Analytics solutions in Industrial Goods processes.