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Extending Newton's Apsidal Theorem: Effective Angular Momentum

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Abstract

Apsidal Shift, or the angular displacement of the orbital apses, is an important quantity in the study of the orbits of planets, starts, and other celestial bodies. The apparent lack of shift of our nearby planets is that on which Sir Isaac Newton based his assumption of the inverse-square relationship in Universal Gravity. The extremely small shift in the orbit of Mercury is one of the many measurements that was used to support Albert Einstein's General Relativity.
Potentials of the form $\mu r^{-(2-n)}$ are of great importance to current and historic scientific research, the Kepler Potential being an example where n=1 (μr^{-1}). The shape of an orbit, while normally thought of as a conic section, only is a conic section for orbits in the Kepler Potential.

To approximate the Apsidal Shift in these power-law central forces, Newton derived his Theorem of Revolving Orbits, also known as Newton's Apsidal Theorem, which states: $\Delta\theta = 2\pi n^{-0.5}$, in the limit that the eccentricity goes to zero (circular orbits). We generalize the method used to correct Mercury's Orbit from General Relativity by approximating the potential as Keplerian with an effective angular momentum. The effective angular momentum defines a rotating reference frame in which we approximate the shape of the orbit as an ellipse. We are able to find excellent approximations for the Apsidal Shift and use our results to generalize Newton's Apsidal Theorem.