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James A. Lock
Cleveland State University, j.lock@csuohio.edu

Leiming Yang

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Interference between diffraction and transmission in the Mie extinction efficiency

James A. Lock and Leiming Yang*

Department of Physics, Cleveland State University, Cleveland, Ohio 44115

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We give simple analytic and numerical demonstrations showing that the interference structure in the Mie extinction efficiency of a sphere is caused by the interference of the light waves that are diffracted and transmitted in the near-forward direction.

When the Mie extinction efficiency for a spherical particle is calculated and graphed as a function of its radius a for a fixed wavelength λ of the incident light, the resulting graph possesses both an interference structure and a ripple structure. The ripple structure is caused by scattering resonances of individual partial waves in the multipole expansion of the scattered fields. The interference structure has been interpreted as being caused by the interference either of (i) the forward-scattered light and the incident beam[1–3] or (ii) the diffracted and the transmitted light waves in the near-forward direction.[4–6]

Recently there has been renewed interest in the interference structure.[7] In this paper our purposes are to comment on the connection between the two interpretations of the interference structure and to give a simple demonstration of it by employing the diffraction-plus-transmission interpretation.

When plane-polarized electromagnetic waves of wavelength λ are scattered by a sphere of radius a and refractive index n located at the origin of coordinates, the total electric field outside the sphere is

$$\mathbf{E}_{\text{outside}}(\mathbf{r}, t) = \mathbf{E}_{\text{incident}}(\mathbf{r}, t) + \mathbf{E}_{\text{scattered}}(\mathbf{r}, t), \tag{1}$$

where the field of the incident plane wave is

$$\mathbf{E}_{\text{incident}}(\mathbf{r}, t) = \mathbf{E}_0 \hat{u}_x \exp(ikz - i\omega t) \tag{2}$$

and the far-zone asymptotic form of the spherically outgoing scattered field is

$$\lim_{r\to\infty} \mathbf{E}_{\text{scattered}}(\mathbf{r}, t) = \mathbf{E}_s(\theta, \phi)\exp(ikr - i\omega t)/r, \tag{3}$$

where

$$\mathbf{E}_s(\theta, \phi) = \frac{-i}{k}\left\{\sum_{l=1}^{\infty} \frac{2l+1}{l(l+1)}\left[a_l \tau_l(\theta) + b_l \pi_l(\theta)\right]\right\}\cos\phi\,\hat{u}_\theta + \frac{i}{k}\left\{\sum_{l=1}^{\infty} \frac{2l+1}{l(l+1)}\left[a_l \pi_l(\theta) + b_l \tau_l(\theta)\right]\right\}\sin\phi\,\hat{u}_\phi \tag{4}$$

and where the partial wave coefficients a_l and b_l and the partial wave angular functions $\pi_l(\theta)$ and $\tau_l(\theta)$ have been derived by many authors.[8] The extinction efficiency of the sphere is expressed in terms of a_l and b_l by

$$\epsilon_{\text{ext}} = \frac{2}{x^2}\sum_{l=1}^{\infty}(2l+1)(\text{Re}\,a_l + \text{Re}\,b_l), \tag{5}$$

where the size parameter x is given by

$$x = 2\pi a/\lambda. \tag{6}$$

If the refractive index of the sphere is real, no absorption occurs, and the extinction efficiency and the scattering efficiency

$$\epsilon_{\text{scat}} = \frac{2}{x^2}\sum_{l=1}^{\infty}(2l+1)(|a_l|^2 + |b_l|^2) \tag{7}$$

are equal.

The partial wave coefficients a_l and b_l may be written as an infinite series of interactions of the various spherical multipole waves with the surface of the sphere[9,10]:

$$\left.\begin{matrix} a_l \\ b_l \end{matrix}\right\} = \frac{1}{2}\left[1 - R_l^{22} - \sum_{m=0}^{\infty} T_l^{21}(R_l^{11})^m T_l^{12}\right], \tag{8}$$

where the interior of the sphere is region 1 and the exterior is region 2. The term R_l^{ii} is the amplitude for the radially propagating multipole wave l to be reflected at the sphere surface from region i back into region i, and T_l^{ij} is the amplitude for the multipole wave l to be transmitted through the sphere surface from region i into region j. The first term in Eq. (8) is independent of the refractive index of the sphere and represents diffraction.[11] This series is known as the Debye series. It is analogous to a plane wave incident upon a thin film, for which the transmitted and reflected fields may again be written as an infinite series of interactions of the incident wave with the two surfaces of the film.[12] The partial wave reflection and transmission amplitudes for both the a_l and b_l polarizations in Eq. (8) are given in Refs. 9 and 10.

In interpretation of the interference structure (i), the incident field and the forward-scattered field are given by Eq. (2) and by the near-zone expression for $\mathbf{E}_{\text{scattered}}(\mathbf{r}, t)$ for $\theta = \phi = 0$, respectively. In interpretation (ii), the diffracted and transmitted fields in the far zone are given by the first term and by the $m = 0$ term of Eq. (8), respectively. The relation between the two interpretations is as

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follows: The incident-plus-forward-scattered fields in a near-zone plane immediately beyond the sphere act as a source field for Fraunhofer diffraction over the circular aperture consisting of the projected area of the sphere. The Fraunhofer diffraction of this source field generates the diffracted-plus-transmitted fields in the far zone.[13]

A demonstration of the interference structure employing the diffracted-plus-transmitted fields in the far zone has been outlined by van de Hulst[14] and has been performed with the use of the complex angular momentum analysis of the Debye series by Nussenzveig and Wiscombe,[5,6] in which the sums over partial waves are converted into integrals over an impact parameter by the modified Watson transform.

A shorter derivation employing the Debye series alone proceeds as follows: We retain only the diffraction and transmission terms of Eq. (8):

$$\left.\begin{matrix} a_l \\ b_l \end{matrix}\right\} \approx \frac{1}{2}\left(1 - T_l^{21}T_l^{12}\right). \tag{9}$$

Since the amplitudes T_l^{21} and T_l^{12} are proportional to each other, expression (9) may be rewritten as

$$\left.\begin{matrix} a_l \\ b_l \end{matrix}\right\} \approx \frac{1}{2}\left[1 - \frac{(r_l^{21})^2}{n}\exp(2i\phi_l^{21})\right], \tag{10}$$

where r_l^{21} and ϕ_l^{21} are the magnitude and the phase of T_l^{21},

$$\tan \phi_l^{21} = \frac{t_l^1 + t_l^1}{t_l^3 - t_l^4}, \tag{11}$$

and where the t_l^i are sums of products of spherical Bessel and Neumann functions given by Eqs. (2.11) and (2.13) of Ref. 10. However, truncating the scattering amplitudes by Eq. (9) spoils their unitarity. As a result, the extinction efficiency obtained from expressions (5) and (9) is in error by a factor of 2. Instead, we exploit the fact that ϵ_{scat} and ϵ_{ext} are equal when n is real, and we calculate the scattering efficiency associated with the truncated scattering amplitudes of expression (9). From expression (10) we obtain

$$\left.\begin{matrix} |a_l|^2 \\ |b_l|^2 \end{matrix}\right\} \approx \frac{1}{4}\left[1 + \frac{(r_l^{21})^4}{n^2} - \frac{2}{n}(r_l^{21})^2 \cos 2\phi_l^{21}\right]. \tag{12}$$

The last term in expression (12) is the diffraction–transmission interference in the partial wave l.

In the geometrical optics limit of Mie scattering, one associates a narrow range of partial waves of the incident, interior, and scattered fields with the impact parameter of a geometrical light ray.[15,16] Transmission of such rays with minimal refraction occurs only for small impact parameters, i.e., for $l \ll x$. Using the asymptotic forms of the spherical Bessel and Neumann functions in Eq. (11) in this limit,[17] we obtain

$$\phi_l^{21} = (n - 1)x, \tag{13}$$

independent of l for both the a_l and the b_l polarizations. For both polarizations, the interference maxima of expression (12) occur when

$$(n - 1)x = (N + 1/2)\pi \tag{14}$$

for integer N, and the interference minima occur when

$$(n - 1)x = N\pi. \tag{15}$$

Since these results are independent of l, they produce interference maxima and minima in ϵ_{scat} and ϵ_{ext} when all the partial wave contributions to Eqs. (5) and (7) for $l \ll x$ are added together. These results agree with the calculations outlined by van de Hulst[14] and those of Nussenzveig and Wiscombe.[6] At the same level of approximation, the relative maxima and minima of the exact Mie scattering amplitudes occur at the same values[18] of $(n - 1)x$.

As a check of this calculation, the scattering efficiency of a dielectric sphere of real refractive index $n = 1.33$ was calculated with the use of the Mie theory and the diffraction-plus-transmission approximation of expressions (7) and (9). The results are shown in Fig. 1. The periodicities of the interference structure of the exact scattering efficiency and of the diffraction–transmission scattering efficiency are identical.

When the sphere radius a is comparable with λ, well-defined geometrical rays are not formed within the sphere, and each term of the Debye series represents a component of the scattering that is large over a substantial range of scattering angles. Since the scattering efficiency is the normalized scattered intensity integrated over all angles, the truncation of Eq. (8) by expression (9) omits a large percentage of the scattered field. As a result, the scattering efficiency obtained from expressions (7) and (9) for $a \approx \lambda$ deviates greatly from the exact result. For a large sphere with $a \gg \lambda$, well-defined geometrical rays are formed within the sphere, and the scattered intensity is strongly peaked in the forward direction. Thus the scattered intensity integrated over all angles is dominated by the forward peak. In the near-forward direction, diffraction and transmission are dominant, while direct reflection and multiple internal reflections are weaker.[19] Thus the interference character of diffraction and transmission

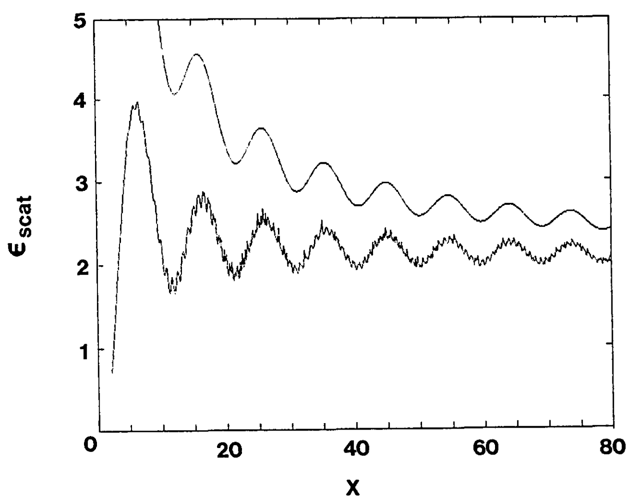

Fig. 1. Mie scattering efficiency of a sphere (lower curve) as a function of the size parameter x and the Debye series diffraction-plus-transmission scattering efficiency of expressions (7) and (9) (upper curve) for $n = 1.33$. The ripple structure in the exact scattering efficiency is superimposed upon the interference structure.

in the near-forward direction is imprinted on the scattering and extinction efficiencies, and, as is seen in Fig. 1, expressions (7) and (9) are a good approximation to the exact scattering efficiency.

The diffraction-plus-transmission interpretation of the extinction efficiency gives a simple explanation of three computationally obtained results[6,7]: (1) the damping of the interference structure when the refractive index of the sphere has a large imaginary part, (2) the absence of the interference structure for scattering by a metal sphere, and (3) the complexity of the interference structure when the refractive index of the sphere has a large real part. For case (1), as the sphere becomes absorptive the diffracted waves are unaffected. But the transmitted waves are attenuated as they pass through the sphere. This effect gives weaker interference in the near-forward direction and a decrease in the amplitude of the interference structure in the extinction efficiency. For case (2), the scattering consists only of diffraction in the near-forward direction and direct reflection from the surface that occurs primarily away from the near-forward direction. As a result, there is no diffraction–transmission interference and no interference structure. The large real refractive index of case (3) produces substantial refraction of the transmitted waves away from the near-forward direction, which decreases their interference with the diffracted waves. Substantial internal reflection also causes multiply internally reflected waves to exit in the near-forward direction and interfere with the diffracted waves, complicating the periodicity of the interference structure.

*Permanent address, Department of Physics, New Mexico State University, Las Cruces, New Mexico 88003.

REFERENCES AND NOTES

1. H. C. van de Hulst, *Light Scattering by Small Particles* (Dover, New York, 1957), p. 175.
2. M. Kerker, *The Scattering of Light and Other Electromagnetic Radiation* (Academic, New York, 1969), p. 108.
3. C. F. Bohren and D. R. Huffman, *Absorption and Scattering of Light by Small Particles* (Wiley, New York, 1983), pp. 75, 106.
4. Ref. 1, pp. 174, 177–178, 258.
5. H. M. Nussenzveig, "High-frequency scattering by a transparent sphere. II. Theory of the rainbow and the glory," J. Math. Phys. **10**, 125–176 (1969), Sec. 6E.
6. H. M. Nussenzveig and W. J. Wiscombe, "Efficiency factors in Mie scattering," Phys. Rev. Lett. **45**, 1490–1494 (1980).
7. P. Chýlek and J. Zhan, "Interference structure of the Mie extinction cross section," J. Opt. Soc. Am. A **6**, 1846–1851 (1989).
8. Ref. 1, pp. 123–124; Ref. 2, pp. 45, 47; Ref. 3, pp. 94, 101.
9. H. M. Nussenzveig, "High-frequency scattering by a transparent sphere. I. Direct reflection and transmission," J. Math. Phys. **10**, 82–124 (1969), Sec. 3A.
10. J. A. Lock, "Cooperative effects among partial waves in Mie scattering," J. Opt. Soc. Am. A **5**, 2032–2044 (1988). The right-hand sides of Eqs. (2.17) and (2.18) in this paper should both be multiplied by nx^2.
11. Ref. 1, pp. 209–210.
12. J. R. Reitz, F. J. Milford, and R. W. Christy, *Foundations of Electromagnetic Theory,* 3rd ed. (Addison-Wesley, Reading, Mass., 1979), Sec. 18-5.
13. Ref. 1, pp. 175–176.
14. Ref. 1, pp. 184–185, second method.
15. Ref. 1, pp. 208–209.
16. J. R. Probert-Jones, "Surface waves in backscattering and the localization principle," J. Opt. Soc. Am. **73**, 503 (1983).
17. A. Messiah, *Quantum Mechanics* (Wiley, New York, 1966), Vol. I, p. 489.
18. Ref. 3, pp. 105–106.
19. A. Ungut, G. Grehan, and G. Gouesbet, "Comparisons between geometrical optics and Lorentz–Mie theory," Appl. Opt. **20**, 2911–2918 (1981).