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# $M$-ary Amplitude Shift Keying OFDM System

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***Abstract*—Coherent $M$-ary amplitude-shift keying (MASK) is proposed for use in orthogonal frequency-division multiplexing (OFDM) systems. The frequency separation between subcarriers is only $1/2T$ instead of $1/T$. With a slightly wider bandwidth, an $\sqrt{M}$-ary ASK OFDM can achieve the same bit-error rate (BER) of $M$-ary quadrature amplitude modulation (QAM) OFDM and a better BER than that of $M$-ary phase-shift keying (MPSK) OFDM. The $\sqrt{M}$-ary ASK OFDM has the same peak-to-average-power ratio as that of the $M$-ary QAM OFDM. The MASK OFDM can be implemented digitally and efficiently by fast cosine transform and demodulated by inverse fast cosine transform. Comparisons show that implementation complexity is reduced for additive white Gaussian noise channels with the use of the new scheme.**

***Index Terms*—Amplitude shift keying (ASK), $M$-ary amplitude-shift keying orthogonal frequency-division multiplexing (MASK OFDM), orthogonal frequency-division multiplexing (OFDM).**

## I. Introduction

It is well known that the minimum subcarrier frequency separation is $1/\mathrm{T}$, where T is the symbol duration, for quadrature amplitude modulation (QAM) or $M$-ary phase-shift keying (MPSK) orthogonal frequency-division multiplexing (OFDM). However, it is less known that if the subcarriers differ only in frequencies and amplitudes, and their phases are the same (0, $\pi/2$ or $\pi$), then the minimum frequency spacing is only $1/2\mathrm{T}$ for orthogonality. Thus, this smaller spacing can be used with coherent $M$-ary amplitude-shift keying (MASK) or $M$-ary frequency-shift keying (MFSK). For bandwidth-efficient applications, MFSK is excluded since its bandwidth occupation is bigger. Only coherent MASK-OFDM is worth considering. This letter proposes MASK for OFDM. It will be shown that in an additive white Gaussian noise (AWGN) channel, the MASK OFDM achieves comparable power and bandwidth efficiency with less computational and system complexity in comparison with ordinary OFDM that uses QAM or MPSK.

## II. MASK OFDM

We define a MASK OFDM signal as

$$s(t) = \sum_{k=0}^{N-1} A_k \cos 2\pi \frac{k}{2T} t \qquad (1)$$

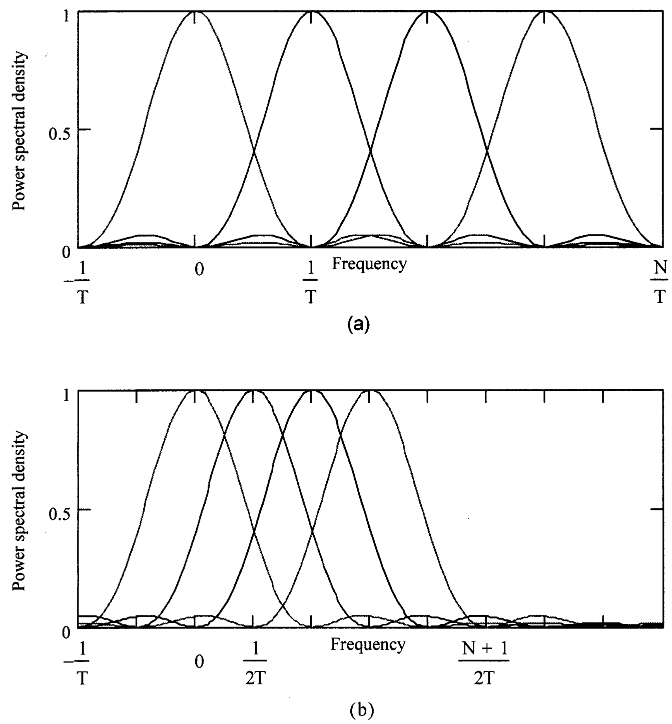


Fig. 1. Spectra of OFDM subcarriers. (a) QAM or PSK OFDM, the subcarrier separation is $1/\mathrm{T}$. (b) MASK OFDM, the subcarrier separation is $1/2\mathrm{T}$.

where $A_k$ is one of the $M$-ary amplitudes and $N$ is the number of carriers. Each subcarrier frequency $f_k = k/2T$; $k$ are contiguous integers. The frequency separation is $1/2\mathrm{T}$. The orthogonality of the signals can be easily verified by performing the integration

$$\int_0^T A_i A_j \cos 2\pi \frac{i}{2T} t \cos 2\pi \frac{j}{2T} t dt = 0, \quad i \neq j.$$

Fig. 1 shows the spectra of four-channel OFDM systems with $1/\mathrm{T}$ spacing (QAM or MPSK) and $1/2\mathrm{T}$ spacing (MASK), respectively. From the figure, we can see that the total null-to-null bandwidth (BW) is

$$\mathrm{BW}_{QP} = \frac{N+1}{T} \qquad \text{for QAM or MPSK OFDM} \qquad (2)$$

and

$$\mathrm{BW}_{A} = \frac{N+3}{2T} \qquad \text{for MASK OFDM} \qquad (3)$$

respectively. The bandwidth savings is the ratio of the former over the latter

$$\text{Bandwidth Savings} = \frac{2(N+1)}{N+3}$$

which approaches two when $N$ goes to infinity.

However, for the same modulation order $M$, coherent MASK has loss in power efficiency compared with coherent MPSK or QAM. But coherent MASK still may be considered for OFDM since the bandwidth savings can be traded for power efficiency. For an approximately fixed bandwidth occupancy, when coherent MASK is used for OFDM, the number of bits per symbol ($k = \log_2 M$) can be reduced to half due to its half subcarrier frequency spacing, in comparison with MPSK or QAM. That is, $M$ can be reduced to $\sqrt{M}$. By doing so, the loss in MASK can be completely recovered against QAM and overly recovered against MPSK, as we will see shortly.

Assume that the QAM is the most popular square QAM with amplitudes of $\pm 1, \pm 3, \ldots, \pm(\sqrt{M} - 1)$ on both I and Q channels, and the amplitudes of the MASK are $\pm 1, \pm 3, \ldots, \pm(M - 1)$. For high signal-to-noise ratios (SNRs) and Gray mapping, the BER expressions of MASK and QAM for the coherent (optimum) receiver in an AWGN channel are [1, p. 416, 439]

$$P_b \approx \frac{2(M-1)}{kM} Q\left(\sqrt{\frac{6k}{(M^2-1)} \frac{E_b}{N_o}}\right), \qquad \text{(MASK)} \quad (4)$$

$$P_b \approx \frac{4(\sqrt{M}-1)}{k\sqrt{M}} Q\left(\sqrt{\frac{3k}{(M-1)} \frac{E_b}{N_o}}\right), \qquad \text{(QAM)}. \quad (5)$$

In (4), substituting $M$ with $\sqrt{M}$ and $k$ with $k/2$, we obtain (5) exactly. That is, reducing MASK's order to $\sqrt{M}$ leads to the exact same power efficiency as that of QAM.

Reducing MASK's order to $\sqrt{M}$ leads to a better power efficiency than MPSK. For high SNRs and Gray mapping, the MPSK's BER for the coherent (optimum) receiver in an AWGN channel is given as

$$P_b \approx \frac{2}{k} Q\left(\sqrt{\frac{2kE_b}{N_o}} \sin\frac{\pi}{M}\right), \qquad \text{(MPSK)}.$$

Comparing MASK and MPSK based on the BER expressions is not straightforward. They are compared in Fig. 2. From the figure, it is seen that reducing MASK's order to $\sqrt{M}$ leads to 0, 4, 10, and 16 dB improvement in power efficiency compared with 4, 16, 64, and 256 PSK, respectively.

MASK's symbol rate is double that of QAM (since $\log_2 M = 2\log_2 \sqrt{M}$). Thus, in terms of QAM's symbol rate, (3) becomes $(N+3)/T$, and the bandwidth ratio (BWR) of MASK over QAM (or PSK) is

$$\text{BWR} = \frac{N+3}{N+1} = 1 + \frac{2}{N+1}. \qquad (6)$$

For $N = 8$, the bandwidth increase is about 22%. When $N$ becomes very large, the bandwidth increase is negligible. For example, when $N = 256$, $\text{BWR} = 1.008$; the bandwidth of both schemes are essentially the same.

For digital implementation, choice of sampling frequency must be carefully considered. Refer to Fig. 1. For QAM/MPSK OFDM (where "/" means "or"), the highest null point in its power spectral density (PSD) is $f_h = N/T$, the lowest null point frequency is $f_l = -1/T$. To avoid severe aliasing in the spectrum of the sampled modulated signal, the sampling frequency must be

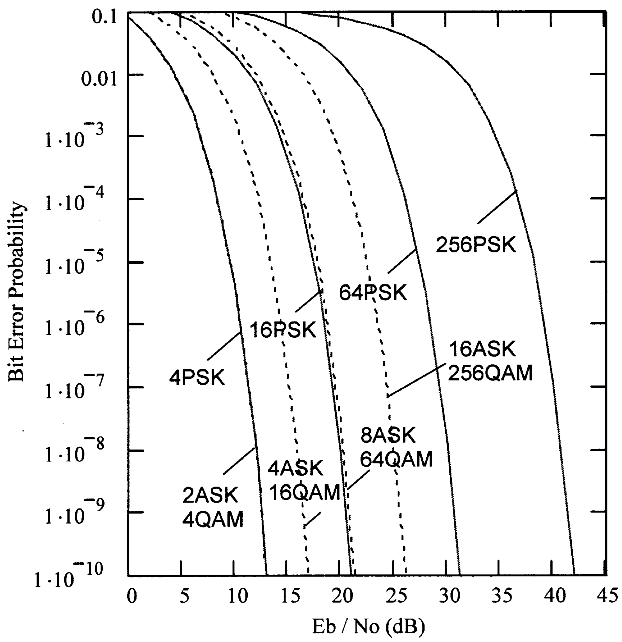


Fig. 2. BER comparison between MASK, MQAM, and MPSK.

$$f_s \geq (f_h - f_l) = \frac{N+1}{T} = \frac{(N+1)R_b}{\log_2 M}$$

where $R_b$ is the bit rate of each channel. To further reduce aliasing, usually $f_s$ is chosen much higher than this. For example, $f_s$ is often chosen as $2N/\text{T}$. If $N$ is a power of two, $2N$ samples in a symbol period can be conveniently and efficiently generated by a $2N$-point fast Fourier transform (FFT) with radix-2 algorithm. In terms of bit rate $R_b$

$$f_s = \frac{2N}{T} = \frac{2NR_b}{\log_2 M}, \qquad \text{(QAM/MPSK OFDM)}. \quad (7)$$

For ASK OFDM, the highest null point in its PSD is $f_h = (N+1)/(2T)$. The lowest null point frequency is $f_l = -1/T$. To avoid aliasing in the spectrum of the sampled modulated signal, the sampling frequency must be $f_s \geq (N+3)/(2T)$. For $N \geq 3$ (this is usually satisfied in a practical OFDM system)

$$\frac{N+3}{2T} \leq \frac{N}{T}.$$

Thus, the sampling frequency for a $\sqrt{M}$-ary ASK OFDM can be chosen as

$$f_s = \frac{N}{T} = \frac{NR_b}{\log_2 \sqrt{M}} = \frac{2NR_b}{\log_2 M}, \qquad (\sqrt{M}\text{- ary ASK OFDM}). \quad (8)$$

This is the same as that of QAM/MPSK OFDM in (7). Note that for $\sqrt{M}$-ary ASK OFDM, by using $f_s = N/T$ instead of $f_s = (N+3)/(2T)$, for big $N$, the sampling frequency almost doubles what is required, just like what is specified for QAM/MPSK OFDM in (7). However, the samples per symbol is $N$ for $\sqrt{M}$-ary ASK OFDM instead of $2N$ for

TABLE I

PAPR COMPARISON OF MASK OFDM VERSUS QAM AND MPSK OFDM

| *Order* | $PAPR_{(PO)}$ | $PAPR_{(AO)}$ | $PAPR_{(QO)}$ | *Increase: AO/QO* | | *Increase: AO/PO* | |
|---|---|---|---|---|---|---|---|
| M | N | $3N\frac{(M-1)}{(M+1)}$ | $3N\frac{(\sqrt{M}-1)}{(\sqrt{M}+1)}$ | $\frac{(\sqrt{M}+1)^2}{(M+1)}$ | dB | $\frac{3(M-1)}{(M+1)}$ | dB |
| 4 | $N$ | $1.8N$ | $N$ | 1.8 | 2.533 | 1.8 | 2.553 |
| 16 | $N$ | $2.647N$ | $1.8N$ | 1.471 | 1.675 | 2.647 | 4.228 |
| 64 | $N$ | $2.908N$ | $2.333N$ | 1.246 | 0.956 | 2.908 | 4.635 |
| 256 | $N$ | $2.977N$ | $2.647N$ | 1.125 | 0.510 | 2.977 | 4.737 |

QAM/MPSK OFDM. This will reduce the complexity of the digital implementation of ASK OFDM as compared with that of QAM/MPSK OFDM, as will be seen in Section IV.

## III. PEAK-TO-AVERAGE POWER RATIO (PAPR) COMPARISON

It can be easily shown that due to the orthogonality between different subcarriers, the total power in an OFDM is the sum of the powers of all subcarriers $P_i$, where

$$P_i = \frac{1}{T}\int_0^T [A_i \cos(\omega_i t + \phi_i)]^2 dt = \frac{1}{2}A_i^2. \quad (9)$$

From this, we can infer that the total average power is also equal to the sum of average powers of all subcarriers, that is

$$P_{\text{avg(OFDM)}} = E\{P_{\text{total}}\} = \sum_{i=0}^{N-1} E\{P_i\} = \sum_{i=0}^{N-1} P_{\text{avg}i}$$

where $E\{x\}$ denotes the expectation of $x$. This conclusion will be used in the following.

The average power of an equal-amplitude spaced, bipolar MASK signal on a subcarrier is $P_{\text{avg}} = (1/3T)(M^2-1)A_o^2$ [1, p. 416], where $A_o$ is the smallest amplitude on a normalized cosine (or sine) signal (i.e., $\sqrt{2/T}\cos\omega t$). For the amplitude assignment we have assumed above, $A_o = \sqrt{T/2}$ and the average power of the OFDM signal on $N$ subcarriers is

$$P_{\text{avg(AO)}} = \frac{1}{6}N(M^2-1) \quad (10)$$

where the subscript AO stands for ASK OFDM, and similarly, we will use QO for QAM OFDM and PO for PSK OFDM in the following. The peak power occurs when all the subcarriers have the same maximum amplitudes $A_{\max(\text{MASK})} = (M-1)$ and the same phase (0, $\pi/2$ or $\pi$). Thus, from (1), it is seen that the maximum envelope of the MASK OFDM signal occurs at $t = 0$, and is equal to $A_{\text{peak(AO)}} = N(M-1)$. The peak power is defined as the power of a sine (or cosine) wave with an amplitude equal to the maximum envelope value. That is, from (9) $P_{\text{peak(AO)}} = (1/2)N^2(M-1)^2$. Thus, the PAPR is

$$\text{PAPR}_{(\text{AO})} = \frac{P_{\text{peak(AO)}}}{P_{\text{avg(AO)}}} = 3N\frac{M-1}{M+1}. \quad (11)$$

For the QAM, the maximum amplitude is $A_{\max(\text{QAM})} = \sqrt{2}(M-1)$ (the outermost point in the constellation) and the maximum OFDM envelope is $A_{\text{peak(QO)}} = N\sqrt{2}(M-1)$, and the peak power is $P_{\text{peak(QO)}} = N^2(\sqrt{M}-1)^2$. The average power of the square QAM signal on a single subcarrier is $P_{\text{avg}} = (1/3)(M-1)P_o$ [1, p. 432], where $P_o$ is the power of the smallest signal. For the amplitude assignment we have assumed above, $P_o = (1/2)(\sqrt{2})^2 = 1$, thus the average power of the QAM OFDM signal on $N$ subcarriers is $P_{\text{avg(QO)}} = (1/3)N(M-1)$, and the PAPR is

$$\text{PAPR}_{(\text{QO})} = \frac{P_{\text{peak(QO)}}}{P_{\text{avg(QO)}}} = \frac{3N(\sqrt{M}-1)}{\sqrt{M}+1}. \quad (12)$$

For the MPSK, all the amplitudes are the same, assuming it is $A_{\text{MPSK}}$. The maximum OFDM envelope is $A_{\text{peak(PO)}} = NA_{\text{MPSK}}$, and the peak power is $P_{\text{peak(PO)}} = 1/2N^2A_{\text{MPSK}}^2$. The average power of the MPSK signal is equal to the power of each individual signal, and the average power of the MPSK OFDM signal on $N$ subcarriers is $P_{\text{avg(PO)}} = 1/2NA_{\text{MPSK}}^2$. Thus, the PAPR is

$$\text{PAPR}_{(\text{PO})} = \frac{P_{\text{peak(PO)}}}{P_{\text{avg(PO)}}} = N. \quad (13)$$

Comparing (11) with (12) and (13), it is seen that the PAPR of the MASK OFDM is increased by a factor of $(\sqrt{M}+1)^2/(M+1)$ against QAM and $3(M-1)/(M+1)$ against MPSK. Table I shows the PAPRs and PAPR increases.

From Table I, we can see that a $\sqrt{M}$-ary ASK OFDM has the exact PAPR of an $M$-ary QAM OFDM. In fact, substituting $\sqrt{M}$ into (11) results in (12). Thus, by using $\sqrt{M}$-ary ASK OFDM, not only the power efficiency loss can be recovered, but also the PAPR will remain the same in comparison with QAM OFDM. When compared with MPSK OFDM, using $\sqrt{M}$-ary ASK OFDM will increase PAPR (so does QAM OFDM), but will improve power efficiency significantly, as we showed earlier (Fig. 2).

## IV. DIGITAL IMPLEMENTATION OF MASK OFDM BY DISCRETE COSINE TRANSFORM

Ordinary QAM/MPSK OFDM can be implemented using the inverse discrete Fourier transform (IDFT), whereas MASK OFDM can not be implemented by IDFT due to the frequency separation, which is $1/2T$ instead of $1/T$. Fortunately, there is the discrete cosine transform (DCT) that comes to the rescue. DCT and inverse discrete cosine transform (IDCT) is a pair of orthogonal transforms that is popular in image compression coding [2], [3]. The DCT algorithm is the basis for the widespread coding standards such as JPEG, MPEG, etc. The similarity between the discrete MASK OFDM signal and the DCT expression leads this author to investigate the feasibility of using DCT/IDCT for modulating and demodulating MASK OFDM signals. It turns out that it is not only feasible, but also it is even more efficient than the IFFT/FFT pair for QAM/MPSK OFDM, due to the existence of fast cosine transform (FCT) algorithms.

The DCT/IDCT pair is [4]

$$X(n) = \frac{2}{N}\varepsilon(n)\sum_{k=0}^{N-1} x(k)\cos\frac{\pi n(2k+1)}{2N}, \quad n = 0,1,\ldots,N-1 \quad \text{(DCT)} \tag{14}$$

$$x(k) = \sum_{n=0}^{N-1} \varepsilon(n)X(n)\cos\frac{\pi n(2k+1)}{2N}, \quad k = 0,1,\ldots,N-1 \quad \text{(IDCT)} \tag{15}$$

where

$$\varepsilon(n) = \begin{cases} \frac{1}{\sqrt{2}}, & n = 0 \\ 1, & \text{otherwise.} \end{cases} \tag{16}$$

Let $t = n\cdot\Delta t$, $T = N\cdot\Delta t$ in (1), where $\Delta t$ is the sampling interval, the discrete-time MASK OFDM signal is

$$s(n) = \sum_{k=0}^{N-1} A_k \cos\frac{\pi n(2k)}{2N}. \tag{17}$$

Comparing (14) and (17) reveals that (14) is basically a frequency-shifted version of (17). The shift is $1/4N$ in discrete-time signal or $1/4T$ in continuous-time signal. Thus, by allowing the first subcarrier frequency to be $1/4T$ instead of 0, the MASK OFDM signal can be written in the form of the DCT (14).

In order to use the DCT, we redefine the MASK OFDM signal as

$$s(t) = \varepsilon(t)\sum_{k=0}^{N-1} A_k \cos\frac{\pi(2k+1)t}{2T} \tag{18}$$

where

$$\varepsilon(t) = \begin{cases} \frac{1}{\sqrt{2}}, & 0 \le t \le \Delta t \\ 1, & \Delta t \le t \le T \end{cases}. \tag{19}$$

The subcarrier frequencies now are $1/4T, 3/4T, 5/4T, \ldots, (2N-1)/4T$. The separation is still $1/2T$. The total signal bandwidth is shifted up by $1/4T$, which is a small amount.

Note that $\varepsilon(t)$ is a symbol pulse-shaping function that is almost rectangular, except that the amplitude is smaller for the first sampling interval. It can be easily shown that the spectrum of the pulse is given by

$$\varepsilon(f) = T\frac{\sin\pi fT}{\pi fT}e^{-j\pi ft} - \left(1 - \frac{1}{\sqrt{2}}\right)\frac{T}{N}\frac{\sin(\frac{\pi fT}{N})}{\frac{\pi fT}{N}}e^{-j\pi fT/N} \tag{20}$$

where the first term accounts for the original rectangular pulse, and the second term is for the small notch in the first sampling interval. The second term is about $3.4N$ times smaller than the first one. Therefore, for practical systems where $N$ is usually big, the effect of $\varepsilon(t)$ on the signal spectrum is negligible. In addition, due to the fact that $\varepsilon(t)$ is almost rectangular, the effect of $\varepsilon(t)$ on the signal PAPR is also negligible for large $N$. Thus, for practical systems where $N$ is usually big, the conclusions on the spectrum and PAPR obtained above are still valid.

The discrete form of (18) is (14) with $x(k) = A_k$. That is

$$s(n) = \frac{2}{N}\varepsilon(n)\sum_{k=0}^{N-1} A_k \cos\frac{\pi n(2k+1)}{2N}, \quad n = 0,1,\ldots,N-1 \quad \text{(ASK OFDM modulation)} \tag{21}$$

where $2/N$ is just a constant mandated by the DCT expression, which has no effect on the signal shape and spectrum shape. The sampling frequency is still $N/T$. DCT and IDCT are a pair of orthogonal transforms. That is, MASK OFDM in the form of (18) can be generated by an $N$-point DCT using (21) and demodulated by the following $N$-point IDCT (22):

$$A_k = \sum_{n=0}^{N-1} \varepsilon(n)s(n)\cos\frac{\pi n(2k+1)}{2N}, \quad k = 0,1,\ldots,N-1 \quad \text{(ASK OFDM demodulation).} \tag{22}$$

Note that due to (16), at the demodulator the first signal sample $s(0)$ must be premultiplied by a factor of $1/\sqrt{2}$ before performing the IDCT.

Many fast algorithms exist for computing DCT/IDCT efficiently [4], [5]. Lee's FCT algorithm [4] follows the concept of FFT by decomposing the $N$-point DCT or IDCT into two smaller $N/2$-point DCT or IDCT, and decomposing can be repeated as needed. Lee's algorithm requires $(N/2)\log_2 N$ *real* multiplications and $(3N/2)\log_2 N - N + 1$ *real* additions. Other FCT/IFCT algorithms either increase the speed further or have some new features.

An $N$-point FFT or IFFT needs $(N/2)\log_2 N$ *complex* multiplications and $N\log_2 N$ *complex* additions [6]. Recall that an $N$-subcarrier QAM/MPSK OFDM requires a $2N$-point IFFT/FFT, which requires $N(\log_2 N + 1)$ *complex* multiplications and $2N(\log_2 N + 1)$ *complex* additions. If Lee's FCT algorithm is used for MASK OFDM, not only the number of multiplications and addition is reduced to about half of that of QAM/MPSK OFDM, but also the multiplications and additions are real instead of complex. This makes the modulation or demodulation of MASK OFDM require about 1/4 the computation of that of QAM/MPSK OFDM. Since the symbol rate of MASK OFDM is double that of QAM/PSK OFDM, the computation speed-reduction factor can only be about two. Note that the above comparison is made for the case where the channel is AWGN. If the new scheme is to be used in a time-dispersive channel, a guard time is to be inserted between symbols. In this case, due to the shorter symbol duration of MASK OFDM, the guard time will occupy a higher percentage of the symbol duration, which makes the speed reduction factor lower than two. The degree of reduction depends on the specific system configuration, especially the percentage of the guard time length in the total symbol length. Nonetheless there is speed reduction, which could be substantial if the guard time is small in comparison with the total symbol length (guard time can be zero for some systems, for example, satellite communication systems). Speed reduction translates into either less hardware complexity and power consumption for the same data rate or higher data rates for the same hardware complexity and power consumption.

## V. Conclusion

In this letter, coherent MASK OFDM is investigated. The subcarrier frequency separation is only $1/2T$ for orthogonality. However, to achieve the same BER performance, the bandwidth savings must be traded for power efficiency. By reducing the modulation order from $M$ to $\sqrt{M}$, with a slightly wider bandwidth, the MASK OFDM can achieve a BER performance that is the same as or better than that of an $M$th-order QAM or MPSK OFDM, respectively. The PAPR of the $\sqrt{M}$-ary ASK OFDM is the same as that of the $M$-ary QAM OFDM. MASK OFDM can be digitally and efficiently implemented by FCT/IFCT pair, which requires less computational or circuit complexity in comparison with the IFFT/FFT pair for QAM/MPSK OFDM.

It should be emphasized that the analysis and results in this paper are for AWGN channels. The performance of the new scheme is expected to degrade in time-dispersive channels and fading channels, as well as in cases of imperfect carrier and symbol synchronization. In these impaired channels or adverse conditions, it remains to be studied to what extent the implementation advantages stated above can be preserved. Remedial measures, such as channel estimation and correction for time-dispersive channels and fading channels, which can be used to reduce the degradation, are to be investigated.

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