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A Nonlinear Model for Prediction of Dynamic Coefficients in a Hydrodynamic Journal Bearing

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This paper investigates the variation of nonlinear stiffness and damping coefficients in a journal orbit with respect to equilibrium position. The journal orbit is obtained by the combined solution of equations of motion and Reynolds equation. In the linearized dynamic analysis, dynamic pressure is written as a perturbation of static pressure and pressure gradients at equilibrium position. However, in order to obtain nonlinear dynamic coefficients about equilibrium position, the dynamic pressure gradients in the orbit are also written as the first order perturbation of static pressure gradients and higher order pressure gradients for displacement and velocity perturbations. The dynamic coefficients are functions of bearing displacement and velocity perturbations. The higher order pressure gradients at equilibrium position are evaluated at various eccentricity ratios and L/D ratios of 0.5 and 1.0. The variation of nonlinear dynamic coefficients is analyzed for three Sommerfeld numbers of a two-axial groove journal bearing under the action of an external synchronous load along and perpendicular to the radial journal load. Results indicate that the oil film nonlinearities affect the journal motion at lower eccentricity ratios (higher Sommerfeld numbers) with wide variation in stiffness and damping coefficients.

Keywords Hydrodynamic journal bearing, Dynamic coefficients, Nonlinear model, Journal orbits

Linearized stiffness and damping coefficients are widely used for the stability and journal response of rotor bearing systems. In the linearized analysis, bearing stiffness and damping coefficients can be evaluated at the journal equilibrium position. However, the linearized dynamic coefficients do not provide insight into the variation of dynamic coefficients for large amplitude journal motion about equilibrium position. Fluid film forces in a journal bearing are nonlinear functions of journal center positions and velocities. The large amplitude journal (transient motion is generally obtained by nonlinear analysis which is based on the combined solution of governing Reynolds equation and journal equations of motion at each time step. However, it is important to estimate the variation of dynamic coefficients along the journal orbit in order to determine the degree of nonlinearity of the orbital response.

Lund's (1978, 1987) infinitesimal perturbation method or finite perturbation approach (Qiu and Tieu, 1996) are used for evaluation of linearized stiffness and damping coefficients about journal equilibrium position. Lund (1987) reported that although the dynamic coefficients are evaluated by infinitesimal approach, they are valid up to a 0.4 of the bearing clearance. Qiu and Tieu (1996) calculated the dynamic coefficients at different perturbation amplitudes. They concluded that perturbation displacements and velocities should be within 5% and 4%, respectively, to keep the difference between the coefficients obtained from finite and infinitesimal perturbations under 2.5%. Hattori (1993) analyzed the variation of stiffness and damping coefficients with large dynamic loads over one rotor revolution of a journal bearing of rotary compressor. The dynamic coefficients are calculated along the journal center locus under unsteady state. It is observed that the coefficients vary by one order of magnitude over the complete cycle and the nonlinearity caused by the fluid film forces is evident in the system. Choy et al. (1991) calculated nonlinear bearing stiffness coefficients of the order of odd power of perturbation displacements at various locations from the equilibrium position. The nonlinearity of the bearing was evaluated from the deviation of exact stiffness from linear coefficients. Linear stiffness was evaluated at the equilibrium position, while exact stiffness was obtained by the finite perturbation

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approach. They showed that for displacements far away from the equilibrium position, nonlinearity in the oil film forces is significant, and can be modeled closely by higher order stiffness and damping coefficients. Choy et al. (1992) examined the nonlinear effects on the dynamic performance of the journal bearing under various operating conditions such as low and high eccentricity ratios, high speeds, and journal bearing axial misalignment. Results obtained at both high and low eccentricity ratios indicated that degree of nonlinearity is high when the journal is operating at lower eccentricity ratios. Chu et al. (1998) included the higher order terms in the bearing reaction expansion to obtain the oil film nonlinearity in journal bearing undergoing large amplitude dynamic motion. Error evaluation scheme is adopted to setup confidence bounds on the higher order solutions. The nonlinear dynamic model is applied to both conventional and smart slider bearing. Results using the above nonlinear model indicate that the linearized bearing coefficients are valid for 0.06 of displacement perturbations. Muller-Karger and Granados (1997) presented a methodology wherein the dynamic coefficients are adjusted using minimum square method for one orbit. Their studies indicated that nonlinearity depends on the size and shape of the orbital motion.

In this paper, the stiffness and damping coefficients are evaluated along the journal orbit in terms of first order and higher order dynamic coefficients about equilibrium position. First order perturbation of dynamic pressure in unsteady Reynolds equation gives one unsteady pressure term and four unsteady pressure gradients. Further perturbation of the one unsteady pressure and four unsteady pressure gradients about static equilibrium position, yields static pressure, first order and higher order pressure gradients about equilibrium position. A finite difference method with successive over relaxation scheme is used to solve the unsteady two dimensional Reynolds equation. Journal center trajectory is obtained by the combined solution of Reynolds equation and equations of motion. The first and higher order stiffness and damping coefficients are obtained at journal equilibrium position using the Lund's (1978, 1987) infinitesimal perturbation approach. For a given journal trajectory, by making use of perturbation displacements and velocities, variation of dynamic coefficients along the journal orbit can be obtained.

THEORY

Stiffness and damping coefficients are obtained from the Taylor series expansion of bearing fluid film forces in terms of perturbation displacements and velocities. Nonlinearities in the bearing forces are obtained by including the higher order displacement and velocity perturbations in the bearing force expansion and thereby nonlinear stiffness and damping coefficients are evaluated. In this case, the nonlinear dynamic coefficients are predicted with respect to perturbation displacements and velocities. In general, the comparisons are obtained for (i) coefficients obtained from finite perturbation amplitudes, (ii) coefficients including first and higher order perturbation amplitudes, and (iii) coefficients from infinitesimal perturbation approach. However, for bearings acted upon by external dynamic loads journal undergoes an orbital motion about static equilibrium position. Under these conditions it is important to determine the deviation of the coefficients along the journal orbit to predict the range of validity of the linearized coefficients along the journal orbit. Hence, there is a need to evaluate the coefficients at the unsteady state position along the journal locus with respect to the static equilibrium position rather than the evaluation of coefficients with respect to the corresponding perturbation amplitudes, as in the case of traditional approach.

GOVERNING EQUATIONS

The two dimensional isoviscous, laminar and incompressible Reynolds lubrication equation under dynamic conditions for the two-axial groove bearing shown in Figure 1 takes a form:

$$\frac{\partial}{\partial\theta}\left(\frac{H^3}{12}\frac{\partial P}{\partial\theta}\right)+\left(\frac{R}{L}\right)^2\frac{\partial}{\partial Z}\left(\frac{H^3}{12}\frac{\partial P}{\partial Z}\right)=\frac{1}{2}\frac{\partial H}{\partial\theta}+\Omega\frac{\partial H}{\partial T}\quad[1]$$

Under dynamic conditions, the transient motion of the journal is defined by the journal center position and velocity, such that the film thickness is expressed as:

$$H = 1 + X\cos\theta + Y\sin\theta \quad [2]$$

The fluid film reaction forces along the locus of the journal trajectory are functions of journal center displacements and velocities. The first order perturbation of pressure and film thickness about the unsteady state journal position under dynamic conditions is:

$$P = P + P_x\Delta X + P_y\Delta Y + P_{\dot{x}}\Delta\dot{X} + P_{\dot{y}}\Delta\dot{Y} \quad [3]$$

$$H = H + \Delta X\cos\theta + \Delta Y\sin\theta \quad [4]$$

Substituting the first order perturbation of pressure (Equation 3) and film thickness (Equation 4) into the Reynolds

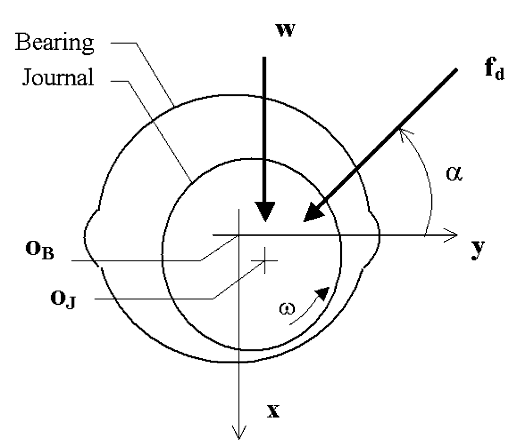


FIGURE 1
Geometry and configuration of two-axial groove bearing.

equation (Equation 1) yields one unsteady pressure (Equation 5) and four unsteady pressure gradients (Equations 6–9). These are:

$$\frac{\partial}{\partial\theta}\left(\frac{H^3}{12}\frac{\partial P}{\partial\theta}\right)+\left(\frac{R}{L}\right)^2\frac{\partial}{\partial Z}\left(\frac{H^3}{12}\frac{\partial P}{\partial Z}\right)=\frac{1}{2}\frac{\partial H}{\partial\theta}+\Omega\frac{\partial H}{\partial T} \quad [5]$$

$$\frac{\partial}{\partial\theta}\left(\frac{H^3}{12}\frac{\partial P_x}{\partial\theta}\right)+\left(\frac{R}{L}\right)^2\frac{\partial}{\partial Z}\left(\frac{H^3}{12}\frac{\partial P_x}{\partial Z}\right)$$
$$=-\frac{1}{2}\sin\theta-\frac{\partial}{\partial\theta}\left(\frac{H^2}{4}\cos\theta\frac{\partial P}{\partial\theta}\right)$$
$$-\left(\frac{R}{L}\right)^2\frac{\partial}{\partial Z}\left(\frac{H^2}{4}\cos\theta\frac{\partial P}{\partial Z}\right) \quad [6]$$

$$\frac{\partial}{\partial\theta}\left(\frac{H^3}{12}\frac{\partial P_y}{\partial\theta}\right)+\left(\frac{R}{L}\right)^2\frac{\partial}{\partial Z}\left(\frac{H^3}{12}\frac{\partial P_y}{\partial Z}\right)$$
$$=\frac{1}{2}\cos\theta-\frac{\partial}{\partial\theta}\left(\frac{H^2}{4}\sin\theta\frac{\partial P}{\partial\theta}\right)$$
$$-\left(\frac{R}{L}\right)^2\frac{\partial}{\partial Z}\left(\frac{H^2}{4}\sin\theta\frac{\partial P}{\partial Z}\right) \quad [7]$$

$$\frac{\partial}{\partial\theta}\left(\frac{H^3}{12}\frac{\partial P_{\dot{x}}}{\partial\theta}\right)+\left(\frac{R}{L}\right)^2\frac{\partial}{\partial Z}\left(\frac{H^3}{12}\frac{\partial P_{\dot{x}}}{\partial Z}\right)=\Omega\cos\theta \quad [8]$$

$$\frac{\partial}{\partial\theta}\left(\frac{H^3}{12}\frac{\partial P_{\dot{y}}}{\partial\theta}\right)+\left(\frac{R}{L}\right)^2\frac{\partial}{\partial Z}\left(\frac{H^3}{12}\frac{\partial P_{\dot{y}}}{\partial Z}\right)=\Omega\sin\theta \quad [9]$$

Similar to Equations (3) and (4), the first order perturbation of unsteady pressure, pressure gradients, and film thickness with respect to journal equilibrium position, result in:

$$P=P_o+P_{xo}\Delta X+P_{yo}\Delta Y+P_{\dot{x}0}\Delta\dot{X}+P_{\dot{y}o}\Delta\dot{Y} \quad [10]$$
$$P_j=P_{jo}+P_{jx}\Delta X+P_{jy}\Delta Y+P_{j\dot{x}}\Delta\dot{X}+P_{j\dot{y}}\Delta\dot{Y} \quad [11]$$

for $j=x,y,\dot{x},\dot{y}$, where $P_{ij}=P_{ji}$

$$H=H_o+\Delta X\cos\theta+\Delta Y\sin\theta \quad [12]$$

Calculation of stiffness and damping coefficients in journal bearing dynamic analysis is performed with respect to the steady state position. Hence, considering the perturbation of unsteady pressure (Equation 5) and unsteady pressure gradients (Equations 6–9) about steady state equilibrium position, and collecting one zeroth order and four first order terms for each of Equations (5–9), yields a set of pressure and/or pressure gradients with respect to the equilibrium position.

The equations of motion in non-dimensional form for two-degree of freedom rotor bearing system considering the hydrodynamic and external forces are:

$$M\Omega^2\frac{d^2X}{dT^2}=1-\frac{F_x}{W}+F_d\cos(\Omega T)\sin\alpha \quad [13]$$
$$M\Omega^2\frac{d^2Y}{dT^2}=-\frac{F_y}{W}-F_d\cos(\Omega T)\cos\alpha \quad [14]$$

The hydrodynamic forces are obtained by integrating the pressure profile. When a small amplitude motion of the journal about equilibrium position is considered, these forces are proportional to the journal displacements and velocities, wherein the constants of proportionality are referred as stiffness and damping coefficients, respectively. The boundary conditions used for the evaluation of first order and higher order stiffness and damping coefficients are same as those enunciated by Lund and Thomsen (1978) and Lund (1987).

The nonlinear stiffness coefficients K_{xx} and K_{yx} are evaluated by integration of nonlinear pressure gradient term P_x (Equation (11)) obtained using the first order perturbation of the unsteady Reynolds equation about the unsteady state position as well as about the equilibrium position:

$$-\begin{Bmatrix}K_{xx}\\K_{yx}\end{Bmatrix}=\int\int P_x\begin{Bmatrix}\cos\theta\\\sin\theta\end{Bmatrix}d\theta \quad [15]$$

where $P_x=P_{xo}+P_{xx}\Delta X+P_{xy}\Delta Y+P_{x\dot{x}}\Delta\dot{X}+P_{x\dot{y}}\Delta\dot{Y}$, and the integration is carried out for each of the pressure gradient terms P_{xo}, P_{xx}, P_{xy}, $P_{x\dot{x}}$, and $P_{x\dot{y}}$, respectively.

The nonlinear stiffness coefficients are written as:

$$-\begin{Bmatrix}K_{xx}\\K_{yx}\end{Bmatrix}=\begin{Bmatrix}K_{xxo}\\K_{yxo}\end{Bmatrix}+\begin{Bmatrix}K_{xxx}\\K_{yxx}\end{Bmatrix}\Delta X+\begin{Bmatrix}K_{xxy}\\K_{yxy}\end{Bmatrix}\Delta Y$$
$$+\begin{Bmatrix}C_{xxx}\\C_{yxx}\end{Bmatrix}\Delta\dot{X}+\begin{Bmatrix}C_{xxy}\\C_{yxy}\end{Bmatrix}\Delta\dot{Y} \quad [16]$$

SOLUTION PROCEDURE

For the half bearing considered in the analysis, the computational grid has 73 nodes in circumferential direction and 7 nodes in axial direction.

The procedure for predicting the first order and higher order dynamic coefficients about steady state equilibrium position of the journal can be summarized as follows:

1. The eccentricity ratio and slendemess ratio are the given variables.
2. The two-dimensional Reynolds equation is solved using the finite difference method with successive over relaxation to obtain the steady state pressure, attitude angle, and Sommerfeld number.
3. Perturbation of the unsteady pressure and four unsteady pressure gradients with respect to equilibrium position is conducted.
4. Using the infinitesimal perturbation method and the boundary conditions given by Lund (1978), the first order and higher order stiffness and damping coefficients are determined.

The procedure for predicting the nonlinear dynamic coefficients for a journal orbit and along the locus of the orbit can be outlined as follows:

(a) Given the Sommerfeld number, the eccentricity ratio and attitude angle are predicted.

(b) The steps (2) and (3) in the above procedure for predicting the dynamic coefficients about steady state equilibrium position are evaluated.
(c) Incorporating the reaction forces in equations of journal motion, the new journal center position and velocities are determined. The improved Euler method (Abdul-Wahel et al., 1982) is used for numerical integration of equations of motion, and the trajectory is obtained.
(d) The perturbations of displacements and velocities are calculated with reference to equilibrium position for each journal position along the locus of journal center trajectory.
(e) The nonlinear stiffness and damping coefficients are calculated along the locus of the journal trajectory using the perturbation amplitudes, first order and higher order dynamic coefficients.

RESULTS AND DISCUSSION

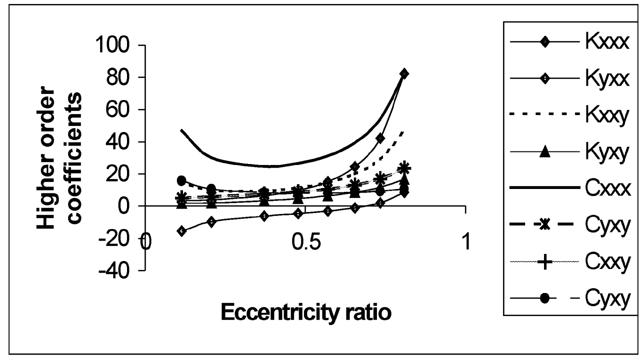
The higher order coefficients in the nonlinear expansion of stiffness and damping coefficients are evaluated at journal equilibrium positions. The variation of first order dynamic coefficients for four different multi-lobe bearing configurations is discussed extensively for various L/D ratios by Lund and Thomsen (1978). The results obtained for higher order coefficients with several eccentricity ratios and for L/D ratios from 0.5 to 1.5 are presented in this paper. The geometry and configuration of the two-axial groove journal bearing, considered for the determination of first order and higher order coefficients about equilibrium position using infinitesimal perturbation method, are same as that of studied by Lund and Thomsen (1978). The bearing parameters and operating conditions are given in Table 1.

Figure 2 shows the higher order nondimensional coefficients for displacement perturbation in the x-direction (along the direction of radial load) for L/D ratios of 0.5, 1.0 and 1.5. These higher order coefficients are included in the first order Taylor series expansion of nonlinear stiffness coefficients K_{xx} and K_{yx}.

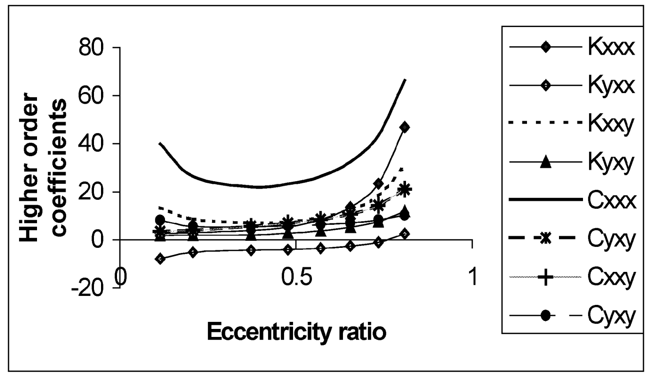
The variation of the higher order coefficients K_{xxx}, K_{yxx}, K_{xxy}, K_{yxy}, C_{xxx}, C_{yxx}, C_{xxy}, C_{yxy} with the increase of eccentricity ratio is depicted in the Figure 2. Also it is observed that all the higher order coefficients show a decrease in the range of their values with the increase of L/D ratios from 0.5 to 1.5.

TABLE 1
Bearing Characteristics and Operating Conditions (Lund and Thomsen, 1978)

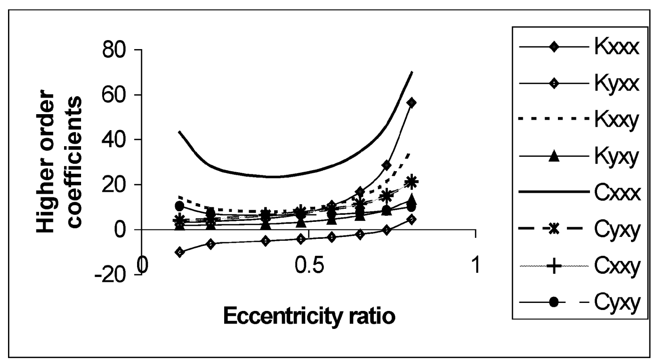
Bearing type	Two-axial groove
Pad arc	160°
Groove angle	20°
Preload	0.0
Offset factor	0.5
L/D	0.5, 1.0 and 1.5
Eccentricity Ratios	0.114 to 0.809



(a) L/D =0.5



(b) L/D =1.0



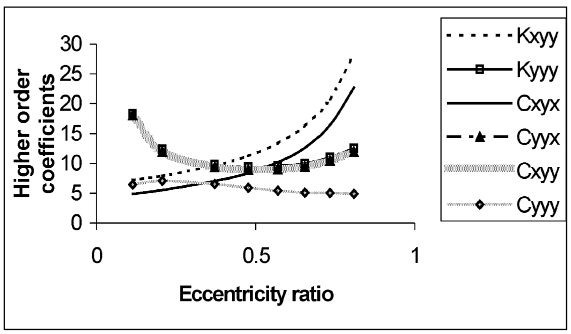
(c) L/D =1.5

FIGURE 2
Higher order coefficients for displacement perturbation in x-direction.

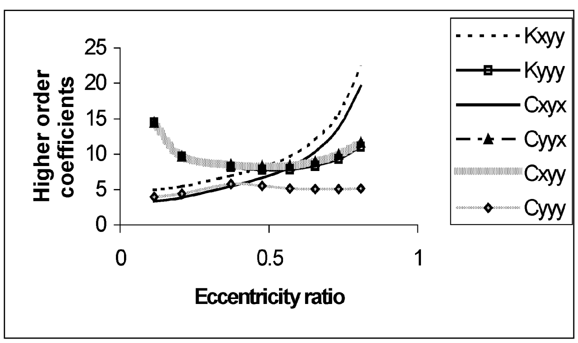
The higher order nondimensional coefficients for displacement perturbation in the y-direction (perpendicular to the direction of radial load) for the L/D ratios under study are shown in Figure 3. These higher order coefficients are included in the first order Taylor series expansion of nonlinear stiffness coefficients K_{xy} and K_{yy}. It is shown that all the higher order coefficients K_{xyy}, K_{yyy}, C_{xyx}, C_{yyx}, C_{xyy}, C_{yyy} decrease in the range of their values with increase in L/D ratios from 0.5 to 1.5, which is similar to the variation of higher order coefficients contained in the expressions for coefficients K_{xx} and K_{yx}.

TABLE 2
Bearing Parameters and Operating Conditions
(Muller-Karger and Granados, 1997)

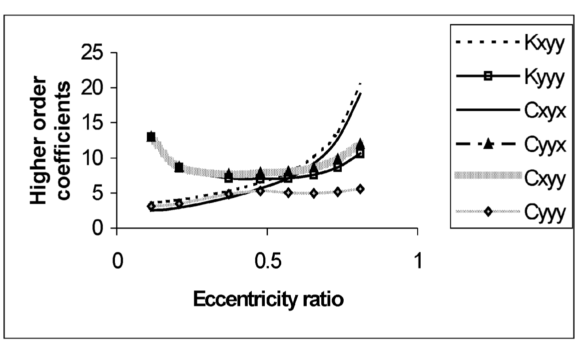
Sommerfeld number	0.2813, 0.1643 and 1.1252
Bearing type	Two-axial groove
Pad arc	150°
Groove angle	30°
Preload	0.0
Offset factor	0.5
L/D	0.5
Dynamic force orientation	0° and 90°
Force amplitude	0.60
Excitation frequency	Synchronous



(a) L/D =0.5



(b) L/D =1.0

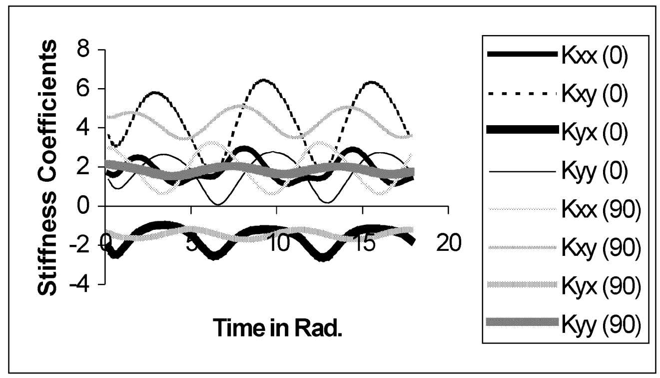


(c) L/D =1.5

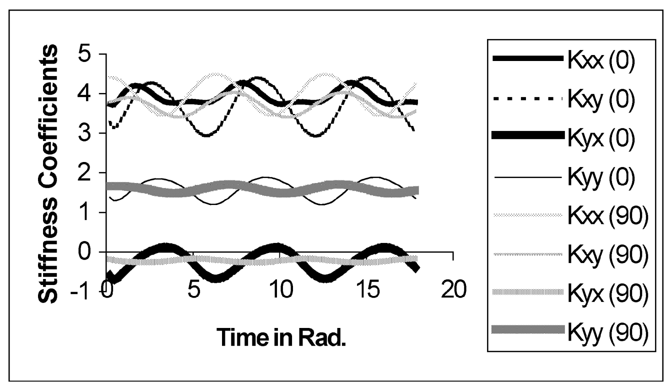
FIGURE 3
Higher order coefficients for displacement perturbation in *y*-direction.

The variation of the stiffness and damping coefficients along the journal orbit is considered for various journal operating conditions, similar to those studied by Muller-Karger and Granados (1997). The bearing parameters, operating conditions and the type of external dynamic load are given in Table 2.

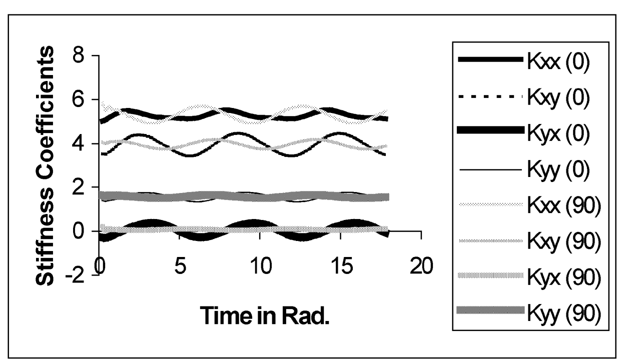
The variation of the stiffness coefficients along the journal orbit obtained at three Sommerfeld numbers ($S = 1.1252$, 0.2813 and 0.1643) for a dynamic load ($F_d = 0.6$) applied along and perpendicular to the radial load ($\alpha = 90°$ and 0°) is shown in Figure 4. For high Sommerfeld number ($S = 1.1252$), the



(a) S=1.1252 and M=0.578

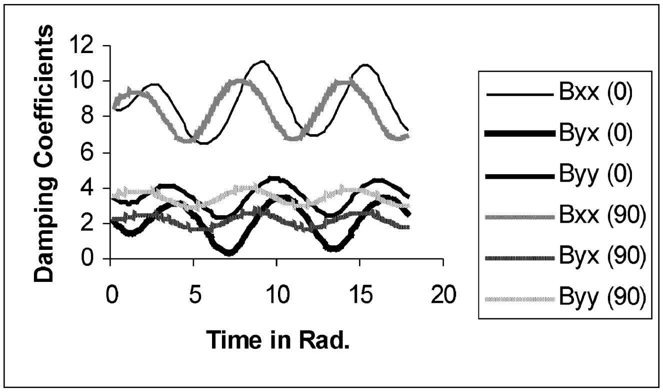


(b) S=0.2813 and M=0.145

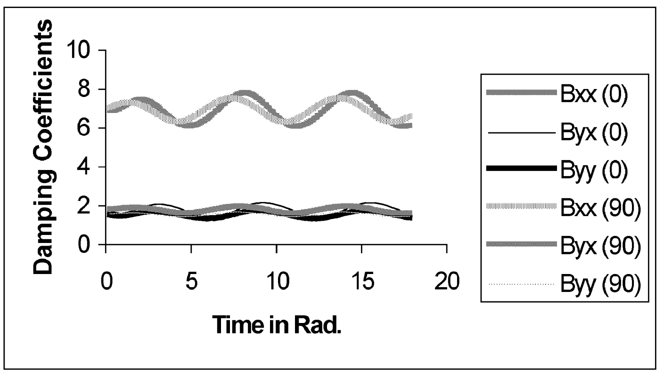


(c) S=0.1643 and M=0.085

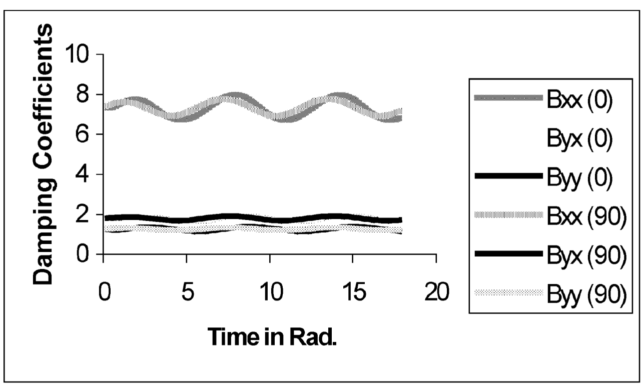
FIGURE 4
Variation of stiffness coefficients along the journal orbit.



(a) S=1.1252 and M=0.578



(b) S=0.2813 and M=0.145



(c) S=0.1643 and M=0.085

FIGURE 5
Variation of damping coefficients along the journal orbit.

variation of four stiffness coefficients at $\alpha = 90°$ and $0°$ is higher as compared to lower values of Sommerfeld numbers ($S = 0.2813$ and 0.1643), i.e., higher eccentricity ratios.

Figure 5 depicts the variation of damping coefficients along the orbital response for three Sommerfeld numbers ($S = 1.1252$, 0.2813 and 0.1643) under the dynamic load ($F_d = 0.6$) applied along ($\alpha = 0°$) and perpendicular ($\alpha = 90°$) to the radial load. It is observed from the results obtained that the variation of damping coefficients at three Sommerfeld numbers is very similar to that predicted for stiffness coefficients. Alos, for a given journal transient response obtained for all the Sommerfeld numbers, the variation of dynamic coefficients (both stiffness and damping coefficients shown in Figures 4 and 5) for $\alpha = 0°$ is higher as compared to those obtained for $\alpha = 90°$.

The purpose of calculating the stiffness and damping coefficients variation along the journal locus for the given journal orbit at a given journal operating position (or for a given Sommerfeld number) is to understand the degree of nonlinearity for the given orbit. The nonlinearities in fluid film are larger at higher eccentricity ratios than at lower eccentricity ratios. However, the change in nonlinearity with perturbation amplitudes is higher at lower eccentricity ratios (higher Sommerfeld numbers) than at larger eccentricity ratios (lower Sommerfeld numbers).

CONCLUSIONS

A methodology to predict the degree of nonlinearity in the orbital response for a two axial groove bearing is presented for three Sommerfeld numbers. The orbital response is obtained for a dynamic load along and perpendicular to the radial load, and the variation of nonlinear dynamic coefficients is analyzed along the locus of the journal response. Nonlinear dynamic coefficients are obtained by the infinitesimal perturbation of unsteady pressure gradients at the unsteady journal position (at any point on the journal locus) with respect to the equilibrium position. These nonlinear dynamic coefficients are functions of first order and higher order dynamic coefficients evaluated at the journal equilibrium position. Higher order dynamic coefficients are evaluated for various eccentricity ratios and slenderness ratios.

Based on the results obtained using the present methodology it is concluded that the variation of dynamic coefficients along the journal locus is larger at high Sommerfeld numbers (low eccentricity ratios). When the dynamic load is perpendicular to the radial load, the journal orbits are slender as compared to those orbits obtained for dynamic load parallel to the radial load. For all Sommerfeld numbers the variation of dynamic coefficients along slender orbits is larger than that for round orbits. The degree of nonlinearity introduced into the nonlinear dynamic coefficients by the journal orbit (nonlinear analysis) is larger at low eccentricity ratios than at high eccentricity ratios.

The present methodology can be used to predict the variation of dynamic coefficients along the journal center locus to determine the nonlinearity of "stable" journal orbits.

NOMENCLATURE

b_{ijo}, B_{ijo} — Damping coefficients evaluated at equilibrium position, Ns/m; $B_{ijo} = b_{ijo} C\omega/w$, $i, j = x, y$

b_{ij}, B_{ij} — Damping coefficients evaluated for the journal orbit, Ns/m; $B_{ij} = b_{ij} C\omega/w$, $i, j = x, y$

b_{ijk}, B_{ijk} — Higher order damping coefficients evaluated at equilibrium position, Ns/m^2; $B_{ijk} = b_{ijk} C^2\omega/w$; where $i, j, k = x, y$ denote the direction of force, first order perturbation amplitude,

	and higher order perturbation displacement, respectively
C	Radial clearance, m
c_{ijk}, C_{ijk}	Higher order cross coefficients evaluated at equilibrium position, Ns/m^2; $C_{ijk} = c_{ijk}C^2\omega/w$; where $i, j, k = x, y$ denote the direction of force, first order perturbation amplitude, and higher order perturbation velocity, respectively
D	Journal diameter, m
f_x, f_y, F_x, F_y	Bearing forces along vertical and horizontal directions, N; $F_x = f_x/\eta\omega(R/C)^2RL$, $F_x = f_x/\eta\omega(R/C)^2RL$
f_d, F_d	Dynamic force of excitation, N; $F_d = f_d/mC\omega^2$
h, H	Oil film thickness, m; $H = h/C$
k_{ijo}, K_{ijo}	Stiffness coefficients evaluated at the equilibrium position, N/m; $K_{ijo} = k_{ijo}C/w, i, j = x, y$
k_{ij}, K_{ij}	Stiffness coefficients evaluated for the journal orbit, N/m; $K_{ij} = k_{ij}C/w, i, j = x, y$
k_{ijk}, K_{ijk}	Higher order stiffness coefficients evaluated at equilibrium position, N/m^2; $K_{ij} = k_{ijk}C^2/w$; $i, j, k = x, y$ denote the direction of force, first order perturbation amplitude, and higher order perturbation displacement, respectively
L	Width of the bearing, m
m, M	Rotor mass, kg; $M = mC\omega^2$/w
O_b, O_j	Bearing and journal centers respectively
p, P	Pressure in the oil film, N/m^2; $P = p/\eta\omega(R/C)^2$
P_o	Non-dimensional steady state pressure for a finite bearing
P_j	Non-dimensional pressure gradients for finite bearing; $j = x, y, \dot{x}, \dot{y}$
P_{ij}	Non-dimensional higher order pressure gradients for finite bearing; $i, j = x, y, \dot{x}, \dot{y}$
R	Journal radius, m
t, T	Time, sec; $T = t\omega_p$
w, W	Static load, N; $W = w/\eta\omega(R/C)^2RL$
x, y, X, Y	Vertical and horizontal coordinates with respect to bearing center, m; $X = x/C, Y = y/C$
$X, \dot{X}$	Non-dimensional journal center displacement and velocity in x-direction
$Y, \dot{Y}$	Non-dimensional journal center displacement and velocity in y-direction
z, Z	Coordinate along the axial direction, m; $Z = z/L$
α	Angle of orientation of dynamic force with respect to horizontal direction
η	Oil viscosity, Ns/m^2
θ	Angular coordinate measured from the vertical load direction
ω	Journal angular velocity, rad/s
ω_s	Speed parameter; $\omega_s = \omega\sqrt{\frac{mC}{W}}$
ω_p	Angular velocity of whirl
Ω	Whirl ratio, ω_p/ω

Subscript

o	Pressures, forces, and dynamic coefficients calculated with reference to equilibrium position

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