5-20-1992

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Rayleigh–Brillouin scattering to determine one-dimensional temperature and number density profiles of a gas flow field

James A. Lock, Richard G. Seasholtz, and W. Trevor John

Rayleigh–Brillouin spectra for heated nitrogen gas were measured by imaging the output of a Fabry–Perot interferometer onto a CCD array. The spectra were compared with the theoretical 6-moment model of Rayleigh–Brillouin scattering convolved with the Fabry–Perot instrument function. Estimates of the temperature and a dimensionless parameter proportional to the number density of the gas as functions of position in the laser beam were calculated by least-squares deviation fits between theory and experiment.

I. Introduction

The quasi-elastic scattering of light by an atomic or molecular gas is known as Rayleigh–Brillouin scattering.1-3 At a low gas density the frequency of the scattered light is Doppler shifted by the thermal motions of the individual gas molecules. At a high gas density the frequency shift is produced by the interaction with various collective motions of the molecules, specifically sound waves and thermal diffusion modes. If \( \lambda_0 \) and \( \omega_0 \) are the wavelength and angular frequency of the incident light, the frequency shift of the scattered light and the gas density may be parameterized by the dimensionless quantities:

\[
\begin{align*}
\chi &= \frac{\Delta \omega}{\kappa} \left( \frac{m}{2kT} \right)^{1/2}, \\
y &= \frac{\eta}{\kappa} \left( \frac{mkT^{1/2}}{2} \right),
\end{align*}
\]

where

\[
\Delta \omega = \omega_{\text{scattered}} - \omega_0;
\]

the wave number associated with the scattering momentum transfer is

\[
\kappa = \frac{4\pi}{\lambda_0} \sin \frac{\alpha}{2},
\]

where \( \alpha \) is the scattering angle; \( k \) is Boltzmann’s constant, and \( \eta, \eta, m, \) and \( T \) are the number density, shear viscosity, molecular mass, and temperature of the gas, respectively.

For gases containing a single species, it has been shown that the normalized scattered intensity per molecule \( S \) is a function of \( \chi \) and \( y \) alone.4 A number of kinetic theory models for \( S(\chi, y) \) have been developed for single-species atomic5-8 and molecular9-12 gases and compared with experimental data.13-19 The 6-moment model of Ref. 11 is the best model that is currently available for the description of Rayleigh–Brillouin scattering by a single-species gas.20,21 By using various kinetic theory models, Rayleigh–Brillouin scattering has now begun to be employed as a diagnostic technique to determine the temperature and number density of gases in combustion22,23 and remote sensing situations.24-27

With this technique the temperature of a gas sample is determined by scaling the parameter \( \chi \) of Eq. (1) to the measured frequency spectrum. The number density of the gas is determined from the shape of the frequency spectrum by the density parameter \( y \) of Eq. (2). This technique represents a novel way of measuring the gas density, since the absolute magnitude of the scattered light intensity (which is usually employed in Rayleigh scattering determinations of density) is not measured. When using this technique, great care must be taken in

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Received 27 December 1990.
0003-6935/92/152839-10$05.00/0.
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measuring and analyzing the scattered spectrum since the temperature is a rapidly varying function of \( x \) (i.e., \( \propto x^2 \)). This rapid variation is potentially capable of producing a large uncertainty in the computed temperature if even a moderate uncertainty exists in the experimental frequency spectrum. The best results for obtaining the density should occur where the spectral shape is most sensitive to changes in \( y \). This occurs in the so-called kinetic regime for which \( y \sim 1 \).

A Fabry–Perot interferometer may be employed either in the scanning mode or in the static mode to measure the Rayleigh–Brillouin spectrum. In the scanning mode the frequency spectrum is obtained by varying the separation between the interferometer’s etalon plates. By positioning a single detector in the focal plane of the fringe-forming lens at the center of the interference pattern, the spectrum is obtained for one location within the gas sample. If the instrument is used in the static mode with a detector array at the focal plane, spectra can be measured throughout a plane section of the gas sample. The frequency spectrum in this case is given by the radial dependence of the intensity of the Fabry–Perot fringe rings.

The Fabry–Perot imaging configuration has been used to obtain temperature and velocity profiles of thermospheric winds during geomagnetic storms.28–30 We describe an experiment that measures one-dimensional temperature and number density profiles of a gas flow field by using Rayleigh–Brillouin scattering and the Fabry–Perot interferometer in the imaging configuration. The ultimate goal of this work is the development of a two-dimensional single-pulse combustion diagnostic.

The present experiment, by assessing the practicality of the imaging Rayleigh–Brillouin scattering technique, serves as an initial step toward this goal. Statistical errors in temperature measurements for this application are discussed in Appendix A.

II. Fabry–Perot Interferometer in the Imaging Configuration

Consider the geometry of Fig. 1. A ribbon laser beam of thickness \( t \) is formed by a cylindrical lens and passes through a gas sample. An element of illuminated volume \( dV_i = t(dA_i) \) centered at the coordinate \( r_i \) is in the focal plane of a lens that is a distance \( f_1 \) away. Molecules within the illuminated volume scatter light through the angle \( \alpha \) into the lens with the angular frequency shifted by \( \Delta \omega \). The light passes through the lens and into the Fabry–Perot interferometer whose intensity transmission function is \( T(\omega, \theta) \) where

\[
\tan \theta = r_i/f_1. \tag{5}
\]

Light leaving the interferometer passes through a second lens of focal length \( f_2 \) and is imaged onto a CCD array. The volume element \( dV_i \) is imaged onto a single CCD pixel of area \( dA_0 \) centered at the position

\[
r_0 = M r_i, \tag{6}
\]

where the magnification of the imaging system is

\[
M = \frac{f_2}{f_1}. \tag{7}
\]

If the counting efficiency of the CCD pixel is \( \epsilon \), the number of photons per second of all frequencies that are counted by the CCD element is

\[
\rho(r_0) = I_{\text{laser}}(r_i) \left[ \frac{d\sigma}{d\Omega}(\alpha) \right] \mathcal{S}(r_i)(\Delta \Omega_i) \frac{t(dA_0)}{M^2} \frac{\epsilon}{\omega \hbar} \times \int_{\omega}^{\omega + \Delta \omega} d\omega S(x, y) T \left( \frac{\omega}{f_2}, \frac{\omega_0}{f_2} \right), \tag{8}
\]

where \( I_{\text{laser}}(r_i) \) is the laser intensity at \( r_i \), \( (d\sigma/d\Omega)(\alpha) \) is the Rayleigh scattering differential cross section per molecule at the scattering angle \( \alpha \), \( \mathcal{S}(r_i) \) is the number density of molecules at \( r_i \), \( \Delta \Omega_i \) is the solid angle of lens 1 subtended at the scattering volume \( dV_i \), \( h \) is Planck’s constant divided by \( 2\pi \), and \( S(x, y) \) is the fraction of the total scattered intensity between the frequencies corresponding to the parameters \( x \) and \( x + dx \). In Eq. (8) we have assumed that \( I_{\text{laser}} \) and \( \mathcal{N} \) are constant throughout the volume \( dV_i \), \( (d\sigma/d\Omega)(\alpha) \) is constant throughout the solid angle \( \Delta \Omega_i \), the counting efficiency of the CCD element is frequency independent, \( \Delta \omega \) is small compared with \( \omega_0 \), and \( r_0 \ll f_2 \). Collecting all the constants together, we may rewrite Eq. (8) as

\[
\rho(r_0) = K \int_{\omega}^{\omega + \Delta \omega} d\omega S(x, y) T \left( \frac{\omega}{f_2}, \frac{\omega_0}{f_2} \right), \tag{9}
\]

The transmission function of the Fabry–Perot interferometer is

\[
T(\omega, \theta) = \left[ 1 + \frac{\sin^2 \left( \frac{\pi c}{2 \beta} \right)}{\sin^2 \left( \frac{\theta}{2} \right)} \right]^{-1}, \tag{10}
\]

where the magnification of the imaging system is

\[
M = \frac{f_2}{f_1}. \tag{7}
\]
where $c$ is the velocity of light, $d$ is the etalon plate spacing, and $\mathcal{F}$ is the instrument’s finesse.

Consider the transmission function for the laser frequency $\omega_0$ for small $\theta$ where

$$\cos \theta = 1 - \frac{r_0^2}{2f_2^2}. \quad (11)$$

The nonlinearity of the cosine function implies that each instrument function fringe is asymmetric with respect to its center; the frequency shift scale parameterized by the quantity $x$ of Eq. (1) is a nonlinear function of pixel position, and Eq. (9) does not reduce to a convolution integral. However, to assess the importance of the nonlinearity in the present experiment we let $R_0$ be the position of the center of a particular Fabry–Perot fringe on the CCD array with respect to the center of the Fabry–Perot fringe pattern for the laser frequency $\omega_0$, and we measure positions on the CCD array with respect to $R_0$ (i.e., $r = R_0 + u$). In the conditions of the present experiment $\omega_0 = 4 \times 10^{12}$ rad/s, $\Delta \omega = 6 \times 10^9$ rad/s (i.e., the thermal half-width at half-maximum of the Rayleigh–Brillouin spectrum), $R_0 = 4$ mm at a typical fringe, $\delta = 0.02$ mm is the half-width at half-maximum of the instrument function fringe, $d = 20$ mm, $f_2 = 500$ mm, and $u < 0.09$ mm for the gas spectrum corresponding to that fringe. For these conditions, although there is substantial nonlinearity from fringe to fringe in the Fabry–Perot pattern, the nonlinearity across a single fringe is found to be only a small effect, and the instrument function may be approximated to within $\sim 2\%$ accuracy for a single fringe by

$$T\left(\frac{\omega_0 + \Delta \omega}{f_2}\right) = \left[1 + \frac{\sin^2\left(\frac{\omega_0 dR_0 u}{f_2^2 c} - \frac{d \Delta \omega}{c}\right)}{\sin^2\left(\frac{\omega_0 dR_0 \delta}{f_2^2 c}\right)}\right]^{-1}. \quad (12)$$

With this approximation to the instrument function, Eq. (9) for a single fringe may be written as the convolution integral

$$\mathcal{R}(\xi) = K \int_{-\infty}^{\infty} \frac{dxS(x, y)}{1 + \sin^2\left(w\xi - x\right) \sin^2\left(w\beta\xi\right)}, \quad (13)$$

where

$$\beta = \frac{\omega_0 R_0}{\sqrt{2kT}} \left| \frac{m}{\omega f_2^2} \right|^{1/2}, \quad (14)$$

$$w = \frac{\omega d}{c} \left(\frac{2kT}{T}\right)^{1/2}, \quad (15)$$

$$\xi = \beta u. \quad (16)$$

This convolution approximation applied to only one fringe at a time considerably simplifies the analysis of the single fringe.

III. Experimental Procedure

A schematic diagram of the apparatus is shown in Fig. 2. An étalon was used with a Model 95 Lexel argon-ion laser to provide the single axial mode ($\lambda_0 = 514.5$ nm) that is needed for spectral measurements. The laser output was 0.4 W, and the beam was vertically polarized. A cylindrical lens system was used to focus the beam into the center of the 38-cm diam stainless-steel scattering chamber. The resulting ribbon beam had a thickness of 100 $\mu$m and a height of 1 mm. The optics that were used to form the ribbon beam consisted of one spherical lens and two cylindrical lenses. The spherical lens and one cylindrical lens (oriented with its axis vertical) were used to set the beam thickness to 100 $\mu$m at the center of the field of view inside the scattering chamber. The spherical lens and second cylindrical lens (axis horizontal) were used to set the beam height to 1 mm. Parasitic light was controlled with baffles located in the chamber, and the beam was terminated in a light trap.

A portion of the laser beam was coupled into an optical fiber through a manually controlled shutter. The output of the fiber was directed into the chamber and illuminated the field of view of the light collection system. This provided an approximately spatially uniform light source at the laser frequency and was used to measure the instrument function of the interferometer.

Light scattered at $90^\circ$ was collected through a port in the chamber and collimated by an f/5.6, 500-mm focal-length telephoto lens. The input aperture of the interferometer was 70 mm, which resulted in an effective f-number of 7.1 for the collection of the Rayleigh scattered light. After passing through the Fabry–Perot interferometer (Burleigh Model RC-110, mirror reflectance of 90%), the light was focused by a second 500-mm focal length lens onto the 384 x 576 pixel array of a Photometrics Star I cooled CCD camera. The light scattered from the ribbon beam in the scattering chamber was thus imaged onto the CCD array with the image modulated by the transmis-
sion function of the Fabry–Perot interferometer. Each pixel was 23 μm × 23 μm square. The exposure time could be set (0.1 s to hours); thus the scattered light could be integrated for sufficient time to achieve a satisfactory signal-to-noise ratio. The CCD images were digitized to 12 bits/pixel by the camera controller and transferred to a Compaq 386/33 computer for storage.

A gas flow field was set up in the scattering chamber as shown in Fig. 3. A length of stainless-steel tubing of 2.1-mm inner diameter and 3.2-mm outer diameter was connected to an external tank of nitrogen. When nitrogen gas was blown gently through the tubing, the end of the tubing in the scattering chamber acted as a nitrogen gas jet. The tubing could be resistively heated so that the nitrogen leaving the jet had a temperature of up to 600 K. The jet was 7 mm below the ribbon laser beam whose height was 1 mm. An exit port 19 mm above the gas jet was fitted with a thermocouple to monitor the gas temperature and was connected to a vacuum pump to collect and remove the heat that was generated by the hot gas. The portion of the flow field sampled by the laser beam thus consisted of the gas in the chamber at ambient temperature and pressure and the localized region of heated gas immediately above the gas jet. With the imaging system of unit magnification, the CCD camera recorded 13.25 mm of this flow field along the length of the laser beam. A second thermocouple was placed 46 mm downbeam from the gas jet below the laser beam to measure the ambient temperature of the scattering chamber. The gas pressure within the chamber was also monitored, and the scattered spectrum was taken for both heated and unheated nitrogen leaving the gas jet.

Because the temperature is a rapidly varying function of the scaling of the frequency shift $\chi$ through Eq. (1), the instrument function fringes must be extremely stable and the fringe half-width at half-maximum $\delta$ must be known accurately. As a test of the stability of both the interferometer and the counting statistics of the CCD camera, exposure times from 10 to 300 s were examined for both the Fabry–Perot instrument function and the Rayleigh–Brillouin frequency spectrum for unheated nitrogen. On a time scale of ~10 s there were small temperature fluctuations within the scattering chamber that produced shifts in the fitted locations of the centers of the Fabry–Perot instrument function fringes of approximately ±0.1 pixel. It was found that integrating over a number of these shifts for longer exposure times did not widen the Fabry–Perot instrument function fringes seriously or overestimate the temperatures calculated from the gas spectra. It was also found that the temperatures derived from the gas spectra at atmospheric pressure showed a certain degree of variation until the exposure time was ~50 s. In addition the étalon plate alignment also had a small drift for exposures over 300 s. As a result, for this first test of the feasibility of the Fabry–Perot imaging system, a compromise exposure time of 100 s (incident energy ≈ 40 J) was chosen. The interferometer, imaging lens, and CCD camera were aligned so that the counting rate for each Fabry–Perot fringe was the same to both the left and right of the center of the fringe pattern. The center of the fringe pattern was positioned so that there were five or six usable fringes to each side of center. The central fringe was usually sufficiently wide and sufficiently nonlinear so as to be unusable for temperature and gas density determinations. An experimental run consisted of exposing the instrument function once and the scattering from the heated gas twice for each of seven separations of the interferometer étalon plates. Typical spectra are shown in Fig. 4.

**Fig. 3.** Flow-field geometry inside the scattering chamber. The two thermocouples are at location T.
Fig. 4. Typical spectra for (a) the Fabry-Perot interferometer instrument function, (b) Rayleigh-Brillouin scattering from room-temperature nitrogen gas, and (c) Rayleigh-Brillouin scattering from the elevated temperature flow field.
The carrier transform of the instrument function was normalized to unity, the theoretical spectrum was normalized so as to have the same total integrated power as the experimental data, and the sampled values of \( S(x, y) \) were registered with respect to the experimental data so that the locations of the centers of gravity of the theoretical spectrum and the experimental data were identical. The least-squares difference between the convolved theoretical spectrum and the experimental data was then calculated, and the values of \( T \) and the density parameter \( y \) producing the best two-parameter fit were determined by iteration. The 6-moment fits to the experimental data reproduce the actual temperature to within 10% and the actual density parameter to within 20%. As a check of the internal consistency of the best-fit parameters, the value of \( y \) derived from the best-fit temperature by employing the ideal gas law with constant pressure \( y(T_{\text{fit}}) \) was compared with the best-fit value of \( y \). Ideally the ratio \( y(T_{\text{fit}})/y(T_{\text{fit}}) \) should be unity. The actual values of this ratio for the 35 sets of data ranged between 0.24 and 1.07 with an average value of 0.82. The value of the ratio was generally smaller at lower pressures (0.2 atm) and was generally closer to unity at higher pressures (1.0 atm). These small discrepancies in the fitted values of \( T \) and \( y \) are not altogether surprising, since the 6-moment model gives only an approximation to the actual light-scattering spectrum, since the temperature is a rapidly varying function of \( x \), since \( S(x, y) \) is a slowly varying function of \( y \), and since the convolution with the instrument function washes out small differences in \( S \) for differing \( y \) values. It is currently unknown whether these discrepancies result from the approximations inherent in kinetic models for Rayleigh–Brillouin scattering themselves, or represent difficulty in obtaining detailed information about the shape of the Rayleigh–Brillouin spectrum from the discretely recorded experimental spectrum, or represent an uncertainty in the determination of the half-width of the instrument function. It is also not known how noise in the experimental data affects the deconvolution process. The fitted temperatures are expected to be more accurate than the fitted densities since the density depends on the details of the shape of the Rayleigh–Brillouin spectrum, while the temperature depends to a large extent only on the width of the spectrum, both in the \( y \ll 1 \) and \( y \sim 1 \) regions.

Returning to the fitting of the gas spectra of the static mode Fabry–Perot (i.e., imaging) experiment, we again performed the convolution of Eq. (13) in Fourier transform space, using the fast Fourier transform algorithm for only one fringe at a time with a 64-element zero-padded array for the single fringe after normalizing the instrument function to unity, normalizing the total integrated power within the theoretical scattering spectrum to the total integrated power in the experimental data for the single fringe, and registering the centers of gravity of the theoretical spectrum and the experimental data with respect to each other. The resulting least-squares

![Fig. 5. Gas temperature from the least squares deviation fit as a function of pixel position. The T symbols are the readings of the two thermocouples of Fig. 3.](image-url)
Fig. 6. Density parameter $y$ from the least-squares deviation fit as a function of pixel position.

The results in Figs. 5 and 6 indicate that, by using a Fabry–Perot interferometer in an imaging mode, Rayleigh–Brillouin scattering has the potential to be useful as a two-dimensional combustion diagnostic, especially in the mapping of the temperature field. Of major concern, however, is the fact that the calculated temperature is a rapidly varying function of the $x$ parameter and thus is sensitive to the measured width of the instrument function. This, coupled with the requirement of imaging, represents a trade-off situation. For a detector array of fixed size with a fixed number of pixels, broadening the instrument function fringes and the gas spectra to improve their resolution results in fewer spectra appearing on the detector array and thus fewer temperature and number density data points. This also produces data points that are averaged over large spatial intervals. On the other hand, obtaining more spectra on the detector array results in narrower fringes that are...
difficult to resolve. We believe that our choice of \( \sim 11 \) usable spectra on the 576-element array is approximately the largest number that one can have and yet maintain reasonable resolvability of each spectrum. When the full width at half-maximum (FWHM) of the instrument function falls below 1 pixel, its deconvolution from the corresponding gas spectrum becomes imprecise and causes a large uncertainty in the fitted temperature. This was beginning to be the case for the data points that were near the left-hand end of the scan at pixels 15–20.

To be useful as a single-pulse combustion diagnostic, statistically reliable data must be obtained from the incident energy of a single laser pulse, i.e., \( \sim 1 \) J. As we mentioned above, the data of this experiment employed an incident energy of 40 J. However, in addition to the 100-s data reported here, we also obtained reasonably stable data with integration times as low as 50 s. We also measured the light scattered from only the central one-third of the laser beam. Better laser focusing could have resulted in the use of more of the incident laser power. Last, the vignetting resulting from the entrance and exit ports of the Fabry–Perot interferometer was more severe than the \( f/5.6 \) aperture of the collection and imaging lenses, which caused an additional \( \sim 5 \)% in usable light. These potential areas of improvement suggest that we could have obtained reasonable data with substantially less incident energy. This matter is pursued further in Appendix A. If additional refinements of the apparatus decrease the incident energy requirement further, measurement of Rayleigh–Brillouin scattering by using a Fabry–Perot interferometer in an imaging mode could serve as a feasible combustion diagnostic, especially for temperature measurements.

Appendix A: Error Estimation for Temperature Measurement by Using Two-Dimensional Rayleigh Imaging

In this appendix we obtain an estimate of the error in the temperature that is derived from the measured Fabry–Perot fringe pattern detected with a two-dimensional square pixel imaging device. For simplicity a Gaussian Rayleigh scattering spectrum, i.e., \( y \ll 1 \), is assumed. It is assumed further that the error in the actual Rayleigh spectral width is equal to the error in the measured width. Only errors that result from the assumed Poisson statistics of the detected light are considered. Thus these results represent a lower bound for the measurement error.

A single-pulse ribbon laser beam uniformly illuminates a region in the system object plane corresponding to the image of \( N \) rows of detector pixels measured perpendicular to the beam propagation direction. The detector pixel size is \( L_x \times L_y \) and the system has 1:1 imaging. The number of detected photons per pixel can be written as

\[
P = \frac{E_p}{N} \left( \frac{d\sigma}{d\Omega} \right) \Delta \Omega L_x L_y \frac{\lambda_0}{hc}.
\]

where \( E_p \) is the laser pulse energy, \( \lambda_0 \) is the laser wavelength, \( (d\sigma/d\Omega) \) is the differential Rayleigh scattering cross section per molecule, \( \Delta \Omega \) is the molecular number density, \( \Delta \Omega \) is the solid angle of the detected light, \( h \) is Planck’s constant, \( c \) is the velocity of light, and \( \epsilon \) is the overall detection efficiency including detector quantum efficiency and system losses. As an example, consider nitrogen at 1 atm and 300 K giving \( \Delta \Omega = 2.45 \times 10^{-35} \text{ m}^2 \text{ sr} \), a single pulse from a frequency-doubled Nd:YAG pulsed laser having \( \lambda_0 = 532 \text{ nm} \) and \( E_p = 1 \text{ J} \), and the parameters of the experiment described in Section III, \( L_x = 23 \mu \text{ m} \), \( N = 384 \), \( \Delta \Omega = 0.016 \text{ sr} \). If we define the angular size of a single pixel as \( \phi_2 = L_x/L_0 \), the width in pixels of a spectral feature of width \( \Delta \omega \) in rad/s is

\[
\Delta \phi = \frac{\lambda \Delta \omega}{2 \pi \phi_2}.
\]

Assume that the measured spectrum of the Rayleigh scattered light has the form

\[
I_i = a_i \exp\left[-\left(\frac{\phi_1 - \phi_3}{a_3}\right)^2\right],
\]

where \( \phi_1 \) is the pixel index and where the peak amplitude \( a_i \), the spectral width \( a_2 \), and peak location \( a_3 \) are unknown parameters. If we assume that the data \( I_i \) have Poisson statistics and that there is no error in the \( \rho_i \), prediction analysis techniques give the fractional uncertainty in the width \( a_2 \):

\[
\frac{\sigma_{a_2}}{a_2} = \frac{\psi_2}{n_p a_3^{1/2}},
\]

where \( n_p \) is the number of data points (pixels) in the data set, and \( \psi_2 \) is a constant that is related to the fraction of the total spectral width included in the \( n_p \) data set. Since the temperature is proportional to \( a_2^2 \), the predicted fractional error in the temperature measurement is

\[
\frac{\sigma_T}{T} = 2 \frac{\sigma_{a_2}}{a_2}.
\]
If the data set to be analyzed consists of three FWHM's of the spectrum, $\Delta \phi_2$ attains its minimum value of $\sim 1.2$. Using a large number of data points, i.e., a wider frequency interval, does not appreciably decrease the error. For the Gaussian spectrum given by Eq. (A4), the parameter $\alpha_2$ is related to the FWHM of the spectrum. This width, measured in pixels or in frequency interval, is denoted by $\Delta_{\text{FWHM}}$ and $\Delta \phi_{\text{FWHM}}$, respectively. We have

$$\Delta_{\text{FWHM}} = 2\alpha_2 (\ln 2)^{1/2}. \quad (A7)$$

The parameter $\alpha_1$ is related to the number of counts in the pixel at the spectral peak by

$$\alpha_1 = \left( \frac{\ln 2}{\pi} \right)^{1/2} \frac{4 \pi B P}{\Delta \phi_{\text{FWHM}}}, \quad (A8)$$

where $B$ is the instrumental bandwidth, i.e., the ratio of the interferometer free spectral range to the finesse. The number of pixels corresponding to $3 \times \Delta_{\text{FWHM}}$ is thus given by Eq. (A3) as

$$\Delta_{\text{FWHM}} = \frac{\lambda B \Delta \phi_{\text{FWHM}}}{2 \pi C \Delta \phi_x}. \quad (A9)$$

The predicted uncertainty in the temperature is then given by Eqs. (A5), (A6), (A8), and (A9):

$$\frac{\sigma_T}{T} = 2\alpha_2 \left( \frac{\pi}{\ln 2} \right)^{1/4} \left( \frac{c \Phi_0}{8 \pi B P} \right)^{1/2}. \quad (A10)$$

As an example consider $L_0 = 23 \mu \text{m}$ and $f_2 = 500 \text{ mm}$ giving $\Phi_x = 0.046 \text{ mrad}$, a mirror spacing of $d = 20 \text{ m}$, a free spectral range of $7.5 \text{ GHz}$, and a finesse of $20$ giving $B = 375 \text{ MHz}$ and $P = 38 \text{ counts/pixel}$. The values for the fringe radii (assuming a bright central fringe), the FWHM of the Gaussian spectrum expressed in pixels, and the predicted uncertainty in the temperature measurement are given in Table I. It is important to note that these results are for a single row of pixels (i.e., using only 1/384 of the total 1-J laser energy).

A reasonable approach would be to use an approximately square region (say, 16 rows of pixels for our example), which would decrease the estimated error by a factor of 4 (e.g., to 4.6% for the third fringe). The measurement then represents an average over this area. An additional way to reduce the uncertainty includes using higher laser pulse power and concentrating the laser power into a narrower strip (e.g., focusing the entire pulse energy into the 16 rows of pixels instead of the full frame of 384 rows would decrease the uncertainty by an additional factor of $384/(16) = 24$). Where feasible, multipassing a narrow beam through the field of view could greatly increase the available power per pixel. The energy could also be increased by using multiple laser pulses, although the instantaneous nature of the measurement would be lost.

This work was carried out while J. A. Lock was a Case/NASA and NASA/American Society for Engineering Education Summer Faculty Fellow in the Optical Measurement Systems Branch of the NASA Lewis Research Center. The authors thank G. Tenti for kindly providing us with the computer code for the 6-moment model.

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34. Reference 32, Section 3.7.


