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Visco-Elastic Properties of Duct Tape

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Viscoelastic properties of duct tape

Ashley Brown and Ulrich Zürcher

Abstract
A simple experiment for determining the nonlinear stress–strain relation of duct tape is described. After weights are added and subsequently removed, the tape does not return to its original state and is no longer taut. The tape exhibits hysteresis, which implies the loss of work during the cyclical process. The exponent describing the nonlinearity is related to the fractional work loss.

Introduction

Key quantities in the theory of elasticity are the force per unit area (stress), $\sigma = F/A$, and the fractional length (strain), $\epsilon = \delta l/l$. We write

$$\sigma = Y\epsilon, \tag{1}$$

where Y is Young's modulus [1]. The strain is dimensionless and the stress and Young's modulus have a unit of pressure (pascal). We assume that the cross-sectional area is constant $A = A_0$, and find $F = (YA_0/l)\delta l$, or

$$F = k\delta l, \tag{2}$$

where we introduce the spring constant [2]

$$k = \frac{YA_0}{l}, \tag{3}$$

with unit $[k] = \mathrm{N\,m^{-1}}$. For $k = \mathrm{const}$, equation (2) is known as Hooke's law.

Hooke's law is only an approximation, however, and many materials show nonlinear behavior. For engineering materials the stress–strain relation is a *concave* function. For sufficiently large deformation, a power-law behavior is observed:

$$\frac{\sigma}{\sigma_E} = \left(\frac{Y\epsilon}{\sigma_E}\right)^n, \tag{4}$$

where $0 < n < 1$ is often referred to as the 'work-hardening exponent' [3]. The quantity σ_E is the *yield strength* of the material. The power-law dependence is referred to as Ramberg–Osgood relationship.

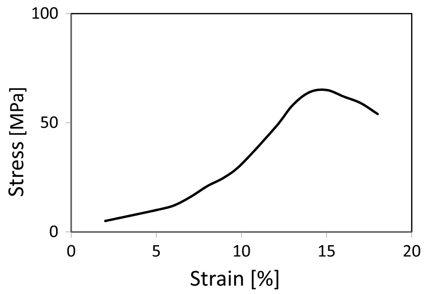

Figure 1. The typical behavior of the stress–strain relationship for biological material.

The typical behavior for biological systems is shown in figure 1. The ligament breaks for strains $\epsilon > 0.18$. The range $0 < \epsilon < 0.18$ is the *plastic region*, which varies from material to material. The stress–strain relation is generally a *convex* function corresponding to values $n > 1$ for the exponent in equation (4). We refer the reader to [4] for a comprehensive discussion of the elastic properties of biological materials.

It is very beneficial when introductory physics teaching is connected to the scientific disciplines of those students enrolled on the course. For the algebra-based sequence, the majority of students are majoring in biology and health-related subjects (including pre-medical/dental/veterinary), so that biomedical applications are the most relevant [5], and books [6] and supplemental materials [7, 8] have been published. An exposure to cross-disciplinary topics prepares students for advanced topics at the (under-)graduate level [9], and hands-on experience is particularly helpful in this respect. Teaching laboratories in physics departments are usually not equipped to deal with biological specimens; it is therefore necessary to find a non-biological substitute that mimics the elastic behavior of ligaments and tendons.

In this paper, we show that the elastic behavior of duct tape shows the convex stress–strain behavior of ligaments and tendons. This paper summarizes an inquiry by one of us (AB) for the *Research Methods* course that is part of the *UTeach* curriculum [11]. The outline of the paper is as follows. In section 2, we describe the experiment and characterize the nonlinear elastic behavior. We then relate in section 3 the exponent n of the power-law behavior to the fractional work loss during the cyclical loading of the tape. We summarize our results in section 4.

Stress–strain relation

A sketch of the experiment is shown in figure 1. Duct tape [10] is secured diagonally between two laboratory tables (height $H_0 = 78\,\mathrm{cm}$). The length of the tape is $L_0 = 4.2\,\mathrm{m}$ and $L_0 = 2.1\,\mathrm{m}$, respectively. The tape is initially *taut*, with no noticeable vertical displacement ('sag'). Short strings are placed near the middle of the tape and weights are added one by one (see figure 2). We stop the process before the hangers touch the floor and then remove the weights. The addition and subsequent removal of masses describes a cyclical process $F = 0 \to F_{\mathrm{max}} \to 0$. The tape sags after all weights have been removed so that the tape does not return to the initial 'state', i.e., with the tape taut. This phenomena is called hysteresis, which is common for nonlinear systems [12].

Simple geometry gives the strain:

$$\epsilon = \frac{2\sqrt{(L_0/2)^2 + h^2} - L_0}{L_0}. \tag{5}$$

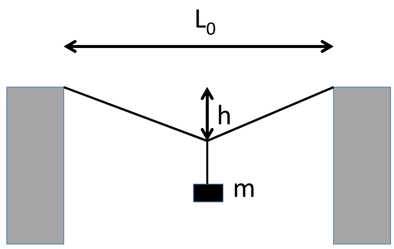

Figure 2. Duct tape with length L_0 is taped between two tables. Variable masses are added to the hanger secured at the center of the tape. The vertical displacement of the hanger is h.

Table 1. The strain ϵ, see equation (5), for the various weights F of the hanging masses.

$L_0 = 4.2\,\mathrm{m}$			$L_0 = 2.1\,\mathrm{m}$		
F (N)	ϵ^+ (10^{-3})	ϵ^- (10^{-3})	F (N)	ϵ^+ (10^{-3})	ϵ^- (10^{-3})
0	0	0.017	0	0	0.743
1.0	0.014	0.102	1.0	0.093	1.481
2.0	0.040	0.192	1.5	0.232	2.016
6.9	0.213	0.487	6.4	0.928	3.332
11.8	0.392	0.706	11.3	1.672	4.322
16.7	0.591	0.874	16.2	2.314	4.977
21.6	0.787	1.035	21.1	2.972	5.680
26.5	0.964	1.159	26.0	3.712	6.427
31.4	1.159	1.290	30.9	4.535	7.086
36.3	1.372		35.8	5.206	7.636
			40.7	5.923	8.062
			45.6	6.687	8.649
			50.5	7.497	8.949
			55.4	8.062	9.255
			60.3	8.649	9.566
			65.2	9.566	9.883
			70.1	10.204	

When the vertical displacement is much shorter than the length of the tape, $h \ll L_0$, we use $\sqrt{1 + (2h/L_0)^2} \simeq 1 + 2(h/L_0)^2$ so that $\epsilon \simeq 2(h/L_0)^2$. The experimental setup allows us to explore only a relatively small range of strain:

$$\epsilon_{\max} = \begin{cases} 0.137\% & L_0 = 4.2\,\mathrm{m} \\ 1.02\% & L_0 = 2.1\,\mathrm{m}. \end{cases} \tag{6}$$

We use the notation ϵ^+ (ϵ^-) when weight is added to (removed from) the tape. The data are shown in table 1. The tape has 'slack' when the weights are removed so that $\epsilon^+ > \epsilon^-$ for the same weight. We use $\epsilon_0^- > 0$ for the 'remnant' strain at the end of the cyclical process. Since the tape is initially taut, we define $\epsilon_0^+ = 0$ to simplify the notation.

We start from equation (4) and replace the stress σ by the force F, and identify σ_E with the maximum force $F_{\max}$ so that the LHS of the Ramberg–Osgood relation is written $F/F_{\max}$. For the RHS, we note that the ratio of the yield strength and Young's modulus is a strain, which we identify with the maximum strain $\epsilon_{\max} - \epsilon_0$. We thus write equation (4) in the form

$$\frac{F}{F_{\max}} = \left(\frac{\epsilon - \epsilon_0}{\epsilon_{\max} - \epsilon_0}\right)^n. \tag{7}$$

That is, the scaled stress–strain relation is characterized by the exponent n, and the strain offset ϵ_0. In the experiment, the weight is the *independent* variable and the strain is the *dependent*

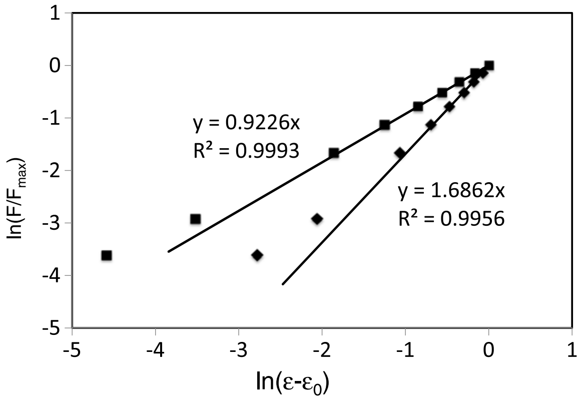

Figure 3. The scaled force F/F_{max} versus the strain ϵ^+ (squares) and ϵ^- (diamonds) in a double-logarithmic plot for $L_0 = 4.2$ m.

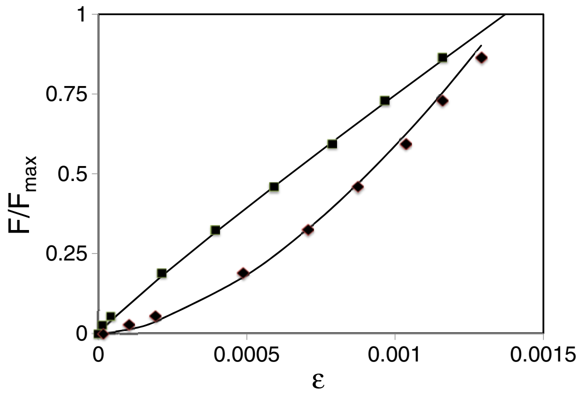

Figure 4. The scaled force F/F_{max} versus the strain ϵ^+ (squares) and ϵ^- (diamonds) for $L_0 = 4.2$ m and the power-law fit.

Table 2. Fit parameters for stress–strain relation, see equation (7), for $L_0 = 4.2$ m and $L_0 = 2.1$ m.

	$L_0 = 4.2$ m	$L_0 = 2.1$ m
n^+	0.92	0.99
n^-	1.68	1.66
$\epsilon_0^-/\epsilon_{\mathrm{max}}$	0.012	0.073

variable. However, we follow convention and plot stress (strain) along the vertical (horizontal) axis. We determine the exponent of the power law from the slope in the double-logarithmic plot (figures 3 and 5). The stress–strain relation is shown in figures 4 and 6.

The fit parameters are summarized in table 2. The addition of weights is consistent with linear behavior, while removing weights exhibits nonlinearity

$$n = \begin{cases} n^+ = 1 & \text{adding weights,} \\ n^- = 5/3 & \text{removing weights.} \end{cases} \tag{8}$$

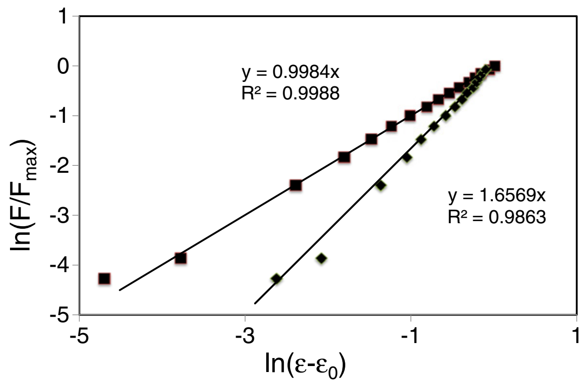

Figure 5. The scaled force $F/F_{\rm max}$ versus the strain ϵ^+ (squares) and ϵ^- (diamonds) in a double-logarithmic plot for $L_0 = 2.1$ m.

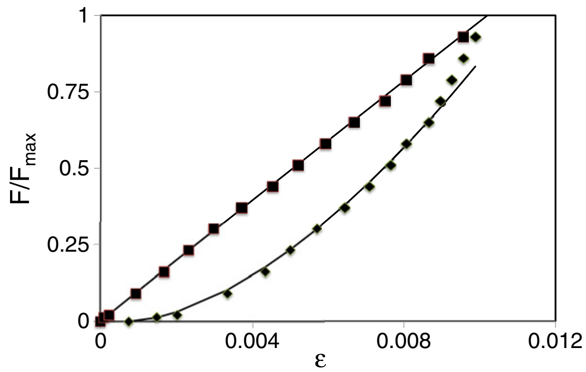

Figure 6. The scaled force $F/F_{\rm max}$ versus the strain ϵ^+ (squares) and ϵ^- (diamonds) for $L_0 = 2.1$ m and the power-law fit.

Here we removed the two data points corresponding to the smallest strain values. The deviation from simple power-law behavior may attributed to either material properties or the difficulty of measuring the smallest displacements of the weight with sufficient accuracy. There is no initial slack of the tape so that $\epsilon_0^+ = 0$, and we conclude that the functional form of the stress–strain relationship is independent of the length of the duct tape.

Elastic work—hysteresis

We use a simplified form for the stress–strain relationship. We have for adding weights:

$$\frac{F^+}{F_{\rm max}} = \frac{\epsilon}{\epsilon_{\rm max}}, \tag{9}$$

and for removing weights ($\epsilon_0^- < \epsilon < \epsilon_{\rm max}$):

$$\frac{F^-}{F_{\rm max}} = \left(\frac{\epsilon - \epsilon_0^-}{\epsilon_{\rm max} - \epsilon_0^-}\right)^{5/3}. \tag{10}$$

The maximum force $F_{\max}$ and strain define the maximum work,

$$W_{\max} = F_{\max}\epsilon_{\max}L_0. \tag{11}$$

We find the work done on the tape by the addition of weights,

$$\frac{W^+}{W_{\max}} = \frac{1}{2}. \tag{12}$$

Since $\epsilon_{\max}L_0 = x$ and $F_{\max} = kx$, equations (10) and (11) are equivalent to the familiar expression $W = kx^2/2$. The recovered work when removing the weights follows

$$\frac{W^-}{W_{\max}} = \int_{\epsilon_0^-}^{\epsilon_{\max}} \left(\frac{\epsilon - \epsilon_0^-}{\epsilon_{\max} - \epsilon_0^-}\right)^{n^-} \mathrm{d}\epsilon = \frac{1}{1+n^-}. \tag{13}$$

Alternatively, the work W^- can be found using numerical methods. If the stress–strain relationship is convex $n^- > n^+ = 1$, we find that more work is necessary to stretch the tape than is recovered when the tape contracts, $W^+ > W^-$. We conclude that energy is 'dissipated' during the cyclical process of first adding and then removing weights. This loss of work associated with stress–strain is referred to as viscoelastic behavior.

We find the fractional work loss,

$$\frac{\delta W}{W^+} = \frac{W^+ - W^-}{W^+} = \frac{n^- - 1}{n^- + 1}. \tag{14}$$

For duct tape $n^- = 5/3$ and we find,

$$\frac{\delta W}{W^+} = \frac{(5/3) - 1}{(5/3) + 1} = \frac{1}{4}. \tag{15}$$

Thus, the exponent of the stress–strain relationship (Ramberg–Osgood) reflects the fractional energy loss of the nonlinear material. Our result suggests that the fractional energy loss depends solely on the intrinsic properties of the tape, but is independent of the length of the tape and the maximum added weight.

Discussion

We describe a simple experiment to determine the viscoelastic behavior of duct tape, which can be implemented easily in a laboratory accompanying an introductory physics course. The tape shows the linear stress–strain relationship when weights are added; however, nonlinear behavior is observed when weights are removed. The tape is no longer taut at the end of the cyclical process and thus shows hysteresis. We simplify the stress–strain relation and use a power-law behavior. We calculate the fractional loss of work $\delta W/W_{\max}$ associated with the cyclical process and find a simple relationship, see equation (14). We write for $n^- > 1$,

$$\eta = \ln n^- \tag{16}$$

so that $\eta = 0$ for $n^- = 1$. We find the fractional energy loss

$$\frac{\delta W}{W^+} = \tanh\frac{\eta}{2}. \tag{17}$$

It would be interesting to relate the exponent of the stress–strain relation behavior to the underlying microscopic properties of the system.

Despite the small strain of duct tape in this experiment, the observed nonlinear behavior resembles that of tendons [13]. This experiment makes it possible to introduce the concept of hysteresis into the introductory physics sequence. In textbooks for the algebra-based course, hysteresis is not mentioned at all [2], or is mentioned only in the context of magnetism [14]. Thus, most premedical students are never exposed to nonlinear stress–strain and the phenomenon of hysteresis, despite its importance in the physiology of breathing [15].

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