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NONINTERACTIVE MACROSCOPIC RELIABILITY MODEL FOR CERAMIC MATRIX COMPOSITES WITH ORTHOTROPIC MATERIAL SYMMETRY

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ABSTRACT

A macroscopic noninteractive reliability model for ceramic matrix composites is presented. The model is multiaxial and applicable to composites that can be characterized as orthotropic. Tensorial invariant theory is used to create an integrity basis with invariants that correspond to physical mechanisms related to fracture. This integrity basis is then used to construct a failure function per unit volume (or area) of material. It is assumed that the overall strength of the composite is governed by weakest link theory. This leads to a Weibull type model similar in nature to the principle of independent action (PIA) model for isotropic monolithic ceramics. An experimental program to obtain model parameters is briefly discussed. In addition, qualitative features of the model are illustrated by presenting reliability surfaces for various model parameters.

INTRODUCTION

The potential advantages of ceramic matrix composites include increased fracture toughness as well as creep and corrosion resistance at very high service temperatures. The primary applications under consideration are advanced turbine engine components, cutting tool bits, heat exchangers and aerospace components (specifically those of the national aerospace plane). Considering that these composites will be produced from nonstrategic materials, it is not surprising that concerted research efforts are underway both in the field of materials science to advance processing techniques and in the field of engineering mechanics to develop design methodologies for these material systems.

The material system of interest in this paper is the whisker-toughened ceramic matrix composite. With this media the reliability analysis must account for material symmetry imposed by whisker orientation. Duffy and Arnold (1989) presented a macroscopic model that accounted for the transversely isotropic material symmetry often encountered in hot-pressed and injection molded whisker-toughened ceramics. A similar approach is used herein to develop a noninteractive reliability model for a material with orthotropic symmetry. This continuum approach excludes any consideration of the microstructural events that involve interactions between individual whiskers and the matrix. Other authors have addressed fracture of ceramic matrix composites on a more local scale. Wetherhold (1989) developed a model based on probabilistic principles to compute an increased energy absorption during fracture due to whisker pull-out. Faber and Evans (1983) have addressed the process of crack deflection, and Lange (1970) has modeled crack pinning. The latter two approaches are founded in deterministic fracture mechanics. Knowing that these crack mitigation processes strongly interact, it is a seemingly intractable task to experimentally detect or analytically predict the sequence of mechanisms leading to failure. A more feasible approach is the continuum based criterion whereby reliability is computed in terms of macrovariables.

As pointed out by Leckie (1981), the difference between the materials scientist and the engineer is one of scale. He notes that the materials scientist is interested in mechanisms of failure at the microstructural level and the engineer focuses on this issue at the component level. We adopt the engineer's viewpoint and present a model of practical utility that macroscopically captures the probabilistic failure phenomenon of whisker-toughened ceramics. This point of view implies that the material element under consideration is small enough to be homogeneous in stress and temperature, yet large enough to contain a sufficient number of whiskers such that the element is a statistically homogeneous continuum. This does not imply that the microscopic and macroscopic levels of focus are mutually exclusive. Indeed a close relationship must exist between the materials scientist and engineer so as to develop better failure models.

NONINTERACTIVE RELIABILITY MODEL

Here we consider a continuum to be a chain comprised of links connected in series. Therefore, the overall strength of the continuum is governed by the strength of its weakest link. We further assume that

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the events leading to failure of an individual link are not influenced by any other link in the chain. Defining f as the failure of an individual link, then

$$f = \psi \, \Delta V \, , \qquad (1)$$

where ΔV denotes an increment in volume and ψ is a failure function per unit volume of material. Taking r as the reliability of an individual link, then

$$r = 1 - \psi \, \Delta V \, . \qquad (2)$$

If the failure of an individual link is considered a statistical event, and we assume these events are independent, then the reliability of the continuum, denoted as R is

$$R = \lim_{N \to \infty} \left[\prod_{\lambda=1}^{N} r_\lambda \right]$$

$$R = \lim_{N \to \infty} \left\{ \prod_{\lambda=1}^{N} \left[1 - \psi(x_i) \Delta V \right]_\lambda \right\} \, . \qquad (3)$$

Here $\psi(x_i)$ is the failure function per unit volume at position x_i within the continuum. Lower case Roman letter subscripts denote tensor indices with an implied range from 1 to 3. Greek letter subscripts are associated with products or summations with ranges that are explicit in each expression. Adopting an argument used by Cassenti (1984), the reliability of the continuum is given by the following expression

$$R = \exp \left[- \int_V \psi \, dV \right] \, . \qquad (4)$$

For orthotropic composites the failure function must also reflect the appropriate material symmetry. This requires

$$\psi = \psi(\sigma_{ij}, a_i, b_i) \qquad (5)$$

where a_i and b_i are unit vectors that identify local material orientations, and σ_{ij} represents the Cauchy stress tensor. These orientations are depicted in Fig. 1.

The sense of a_i and b_i is immaterial, thus their influence is taken through the products $a_i a_j$ and $b_i b_j$, i.e.

$$\psi = \psi(\sigma_{ij}, a_i a_j, b_i b_j) \, . \qquad (6)$$

Note that $a_i a_j$ and $b_i b_j$ are symmetric second order tensors that satisfy the identities

$$a_i a_i = 1 \qquad (7)$$

and

$$b_i b_i = 1 \qquad (8)$$

Furthermore, the stress and local preferred directions may vary from point to point in the continuum. Thus, Eq. (6) implies that the stress field and unit vector fields, i.e. $\sigma_{ij}(x_k)$, $a_i(x_k)$ and $b_i(x_k)$, must be specified to define ψ.

As ψ is a scalar valued function, it must remain form invariant under arbitrary proper orthogonal transformations. Work by Reiner (1945), Rivlin and Smith (1969), Spencer (1971) and others demonstrate that through the application of the Cayley-Hamilton theorem and elementary properties of tensors, a finite set of invariants (as opposed to the work of Tsai and Wu (1971), where an infinite number of invariants are allowed) known as an integrity basis can be developed. Form invariance of ψ is ensured if dependence is taken on invariants that constitute the integrity basis, or any subset thereof. Adapting the above mentioned work to ψ results in an integrity basis composed of 28 tensor products. Following arguments similar to Spencer (1984) several of these tensor products are equal and others are trivial identities such that the final integrity basis for ψ contains only the invariants

$$I_1 = \sigma_{ii} \, , \qquad (9)$$

$$I_2 = \sigma_{ij}\sigma_{ji} \, , \qquad (10)$$

$$I_3 = \sigma_{ij}\sigma_{jk}\sigma_{ki} \, , \qquad (11)$$

$$I_4 = a_i a_j \sigma_{ji} \, , \qquad (12)$$

$$I_5 = a_i a_j \sigma_{jk}\sigma_{ki} \, , \qquad (13)$$

$$I_6 = b_i b_j \sigma_{ji} \qquad (14)$$

and

$$I_7 = b_i b_j \sigma_{jk}\sigma_{ki} \, . \qquad (15)$$

A slightly different set of invariants that corresponds to physical mechanisms related to fracture can be constructed from the above integrity basis. This new set of invariants includes

$$\hat{I}_1 = I_4 \, , \qquad (16)$$

$$\hat{I}_2 = \left[I_5 - (I_4)^2 \right]^{1/2} \, , \qquad (17)$$

$$\hat{I}_3 = I_6 \, , \qquad (18)$$

$$\hat{I}_4 = \left[I_7 - (I_6)^2 \right]^{1/2} \qquad (19)$$

and

$$\hat{I}_5 = I_1 - I_4 - I_6 \, . \qquad (20)$$

Considering a uniformly stressed volume, or in the context of Weibull analysis a single link, the invariant $\hat{I}_1$ corresponds to the magnitude of the stress component in the direction of a_i, as shown in Fig. 1. $\hat{I}_2$ corresponds to the shear stress on the face normal to a_i. Similar interpretations can be made for invariants $\hat{I}_3$ and $\hat{I}_4$ and the direction b_i. Invariant $\hat{I}_5$ is the normal stress in the direction defined by the cross product of vectors a_j and b_k, i.e.

$$d_i = e_{ijk} a_j b_k \, , \qquad (21)$$

where e_{ijk} is the permutation tensor. As proposed, these physical mechanisms are independent of I_2 and I_3. Taking

$$\psi = \psi\left(\hat{I}_1, \hat{I}_2, \hat{I}_3, \hat{I}_4, \hat{I}_5\right) \tag{22}$$

ensures ψ is form invariant.

It is assumed that compressive stresses associated with $\hat{I}_1$, $\hat{I}_3$ and $\hat{I}_5$ do not contribute to failure so that

$$\langle \hat{I}_1 \rangle = \begin{cases} \hat{I}_1 & \hat{I}_1 > 0 \\ 0 & \hat{I}_1 \le 0 \end{cases}, \tag{23}$$

$$\langle \hat{I}_3 \rangle = \begin{cases} \hat{I}_3 & \hat{I}_3 > 0 \\ 0 & \hat{I}_3 \le 0 \end{cases} \tag{24}$$

and

$$\langle \hat{I}_5 \rangle = \begin{cases} \hat{I}_5 & \hat{I}_5 > 0 \\ 0 & \hat{I}_5 \le 0 \end{cases}. \tag{25}$$

In addition,

$$\langle \hat{I}_2 \rangle = |\hat{I}_2| \tag{26}$$

and

$$\langle \hat{I}_4 \rangle = |\hat{I}_4| \tag{27}$$

for all values of $\hat{I}_2$ and $\hat{I}_4$. At this point we assume that the stress components identified by the invariants above act independently (i.e., a noninteractive theory) in producing failure. Following reasoning similar to Wetherhold (1983), ψ takes the form

$$\psi = \left[\frac{\langle \hat{I}_1 \rangle}{\beta_1}\right]^{\alpha_1} + \left[\frac{\langle \hat{I}_2 \rangle}{\beta_2}\right]^{\alpha_2} + \left[\frac{\langle \hat{I}_3 \rangle}{\beta_3}\right]^{\alpha_3} + \left[\frac{\langle \hat{I}_4 \rangle}{\beta_4}\right]^{\alpha_4} + \left[\frac{\langle \hat{I}_5 \rangle}{\beta_5}\right]^{\alpha_5} \tag{28}$$

Insertion of Eq. (28) into the volume integration given by Eq. (4) along with Eqs. (23) to (27) yields a reliability model for a three-dimensional state of stress in an orthotropic ceramic composite. This model is similar in nature, yet different in form, to the PIA theory for monolithic ceramics.

In association with each invariant, the α's correspond to the Weibull shape parameters and the β's correspond to Weibull scale parameters. A variety of test methods could be used to determine these model parameters. One approach is to obtain the requisite tensile data from fast fracture of simple bend test specimens, often referred to as modulus of rupture (MOR) bars. The parameters α_1 and β_1 could be obtained from MOR bar tests conducted on specimens machined from a billet of hot pressed whisker-toughened ceramics. These specimens would be oriented along material direction a_i, shown as orientation 1 in Fig. 2. MOR bar specimens machined with orientation 2 (i.e., along b_i) could be used to determine α_3 and β_3, and similarly, orientation 3 would be used to determine α_5 and β_5. The Weibull parameters associated with shear tractions across a_i and b_i could be obtained from shear tests such as Iosipescu tests (Walrath and Adams, 1983). The parameters α_2 and β_2 could be determined from this type of test using specimens with orientation 1 in Fig. 2. Similarly, the final two parameters, α_4 and β_4, could be obtained from shear tests conducted on specimens with orientation 2.

In comparison to the transversely isotropic model (Duffy and Arnold, 1989), for which three sets of Weibull parameters are necessary, the orthotropic model requires five. However it should be noted here that, as with monolithic ceramics, it is quite possible that the surface and volume of the material will fail due to distinctly different flaw populations. These populations differ for several reasons:

(1) the as-fired condition of the surface may be different due to the formation of a reaction layer;

(2) grinding will change the surface and may impart subsurface damage; and

(3) even if the surface flaw distribution is solely due to the intersection of internal flaws with the surface, the presence of the free surface will reduce the applied load necessary for fracture.

The surface would then have different sets of Weibull parameters than the volume and Eq. (4) would also have to be independently evaluated over the surface area.

IMPLICATIONS OF THE MODEL

Subsequent to the determination of the Weibull parameters, multiaxial experiments should be conducted to assess the model's accuracy. One method is tension and/or torsion loadings applied to thin-walled tube specimens. The thin-walled tube ensures homogeneous, biaxial states of stress. The tubular specimen would be highly appropriate for planar applications such as heat exchangers where one of the material orientations (radial) can be ignored. Unfortunately, at the present time no data base exists to estimate model parameters, although efforts (Shaw and Bubsey, 1987) are underway to accomplish this goal. Thus an assessment of the model in comparison to experimental data is reserved for a later date, and for the examples that follow, model parameters are arbitrarily chosen for the purpose of illustration.

The calculations for the reliability contours shown herein are representative of homogeneously stressed continuum elements (or links) of unit volume. For dimensionless R, the Weibull parameter β has units of stress$\cdot$(volume)$^{1/\alpha}$. Consider the material symmetry where $a_i = (1,0,0)$ and $b_i = (0,1,0)$. The uniaxial loading $\sigma_{11} \neq 0$ ($\sigma_{12} = \sigma_{13} = \sigma_{22} = \sigma_{23} = \sigma_{33} = 0$) for this orientation yields the following invariants for ψ

$$\hat{I}_1 = \sigma_{11} \tag{29}$$

$$\hat{I}_2 = \hat{I}_3 = \hat{I}_4 = \hat{I}_5 = 0 \tag{30}$$

Hence Eq. (4) becomes

$$R = \exp\left[-\left(\frac{\sigma_{11}}{\beta_1}\right)^{\alpha_1}\right] \tag{31}$$

Taking $\sigma_{11} = \beta_1$ in this expression yields $R = 0.3679$. Therefore, any contour $R = 0.3679$, in a stress space containing σ_{11}, yields β_1 as the intercept along the σ_{11}-axis. This can be seen in Fig. 3(a) where this reliability contour is plotted in the σ_{11}-σ_{22} stress space, and $\beta_1 = 750$. Also note that the intercept along the σ_{22}-axis, which represents a uniaxial loading in the direction b_i, yields the Weibull parameter $\beta_3 = 500$. In a similar fashion the pure shear loading $\sigma_{13} \neq 0(\sigma_{11} = \sigma_{12} = \sigma_{22} = \sigma_{23} = \sigma_{33} = 0)$ would yield β_2 as an intercept along the σ_{13}-axis, and $\sigma_{23} \neq 0(\sigma_{11} = \sigma_{12} = \sigma_{13} = \sigma_{22} = \sigma_{33} = 0)$ would yield β_4 as an intercept along the σ_{23}-axis. This can be seen in Fig. 3(b) where $\beta_2 = 400$ and $\beta_4 = 300$.

Figure 4(a) depicts the intersection of level surfaces of R with the σ_{11}-σ_{22} stress plane for the previously mentioned material orientation. Here $\alpha_1 = 15$, $\alpha_3 = 10.5$, and as before, $\beta_1 = 750$ and $\beta_3 = 500$. The three surfaces correspond to $R = 0.95$, 0.50 and 0.05. Note that a decrease in the α's increases the spacing of the contours which indicates a higher scatter in fracture strength. This can be seen in Fig. 4(b) where $\alpha_3 = 5$. The spacing between contours increases in the σ_{22} direction, however the contours in the σ_{11} direction remain unchanged because there is no variation in α_1 or β_1. If α_3 had been increased, the spacing between contours would diminish, and the corners in the first quadrant would sharpen. In general, as the α's increase, eventually the reliability contours would not be distinct from each other and they would effectively map a deterministic maximum stress failure surface.

We next consider the effects of off-axis (relative to loading) material orientation. Figure 5(a) represents level surfaces projected onto the σ_{11}-σ_{33} stress plane for the material orientation defined by $a_i = (1,0,0)$ and $b_i = (0,0,1)$. Here $\alpha_1 = 15$, $\beta_1 = 750$, $\alpha_3 = 10.5$ and $\beta_3 = 500$, and once again three contours of reliability are depicted: $R = 0.95$, 0.5 and 0.05. Rotation of the material orientation vectors such that $a_i = (1/\sqrt{2},0,1/\sqrt{2})$ and $b_i = (-1/\sqrt{2},0,1/\sqrt{2})$, for the conditions identical to those associated with Fig. 5(a) results in the contours found in Fig. 5(b). Due to the new material orientation, compressive components of the stress tensor σ_{ij} contribute to failure. This does not contradict the assumption that compressive stresses along the material direction a_i (identified by $\hat{I}_1$), and along the material direction b_i (identified by $\hat{I}_3$) do not contribute to a reduction in reliability. In this case σ_{11} and σ_{33} are no longer coincident with the stresses associated with $\hat{I}_1$ and $\hat{I}_3$. Expansion of the invariants demonstrates that compressive values of σ_{11} and σ_{33} contribute to both $\hat{I}_2$ and $\hat{I}_4$ (the shear stresses across the material orientation vectors a_i and b_i, respectively) and hence cause a reduction in reliability.

Figure 6(a) represents level surfaces of reliability projected onto σ_{13}-σ_{11} stress plane. Here a uniaxial compressive stress (σ_{11}) maintains a reliability of unity and the contours are symmetric with respect to the σ_{11}-axis. This results from the assumptions associated with Eqs. (23), (26) and (27). Rotation of the material orientation such that $a_i = (1/\sqrt{2},0,1/\sqrt{2})$ and $b_i = (-1/\sqrt{2},0,1/\sqrt{2})$ results in the contours found in Fig. 6(b). Because the contours are closed, any state of stress that has nonzero σ_{11} or σ_{13} components would result in a nonzero probability of failure.

CONCLUDING REMARKS

In this paper we have developed a reliability model for whisker-toughened ceramic composites with orthotropic material symmetry. The model was constructed in a rational manner using an invariant formulation. Such an approach indicates the maximum number and form of the stress invariants necessary in defining the failure function ψ. A subset of the integrity basis for ψ was constructed with invariants that correspond to the macrovariables assumed to be directly related to fracture. The whiskers in each continuum element were assumed to be distributed in such a manner that three orthogonal material orientations could be identified. These material orientations are not restricted to be the same at each point in the continuum and could vary along a family of curves within a component. Thus the material is locally orthotropic with respect to the local material orientation of each element (or link) of the component. This offers flexibility when modeling a component by a finite element method.

Relative to conducting experiments to determine model parameters, it was pointed out that while tubular specimens yield homogeneous states of stress, not all of the parameters could be found. However a combination of MOR and shear tests would yield all the parameters. Furthermore, several figures were presented that depicted reliability contours under different loading conditions. These figures point to behavior that requires verification in multiaxial experiments. Hence, a significant amount of experimental data must be generated before a final assessment of the theory can be made.

REFERENCES

Cassenti, B.N., 1984, "Probabilistic Static Failure of Composite Material," AIAA Journal, Vol. 22, pp. 103-110.

Duffy, S.F., and Arnold, S.M., 1989, "Noninteractive Reliability Model for Whisker-Reinforced Ceramic Composites," Journal of Composite Materials, in review.

Faber, K.T., and Evans, A.G., 1983, "Crack Deflection Processes-I. Theory," Acta Metallurgica, Vol. 31, pp. 565-576.

Lange, F.F., 1970, "The Interaction of a Crack Front with a Second-Phase Dispersion," Philosophical Magazine, Vol. 22, pp. 983-992.

Leckie, F.A., 1981, "Advances in Creep Mechanics," Creep in Structures, A.R.S. Ponter and D. R. Hayhurst, eds., Springer-Verlag, New York, pp. 13-47.

Reiner, M., 1945, "A Mathematical Theory of Dilatancy," American Journal of Mathematics, Vol. 67, pp. 350-362.

Rivlin, R.S. and Smith, G.F., 1969, "Orthogonal Integrity Basis for N Symmetric Matrices," Contributions to Mechanics, D. Abir, ed., Oxford.

Shaw, N.J. and Bubsey, R.T., 1987, "Toughened Ceramics Life Prediction," proposal submitted to Oak Ridge National Laboratory - Ceramic Technology for Advanced Heat Engines Project.

Spencer, A.J.M., 1971, "Theory of Invariants," Continuum Physics - Volume I, Mathematics, A.C. Eringen, ed., Academic Press, New York, pp. 239-255.

Spencer, A.J.M., 1984, "Constitutive Theory for Strongly Anisotropic Solids," Continuum Theory of the Mechanics of Fibre-Reinforced Composites, A.J.M. Spencer, ed., Springer-Verlag, New York, pp. 1-32.

Tsai, S.W. and Wu, E.M., 1971, "A General Theory of Strength for Anisotropic Materials," Journal of Composite Materials, Vol. 5, pp. 58-80.

Walrath, D.E. and Adams, D.F., 1983, "The Iosipescu Shear Test as Applied to Composite Materials," Experimental Mechanics, Vol. 23, pp. 105-110.

Wetherhold, R.C., 1983, "Statistics of Fracture of Composite Material Under Multiaxial Loading," Ph.D. Thesis, University of Delaware.

Wetherhold, R.C., 1989, "Energy of Fracture for Short Brittle Fiber/Brittle Matrix Composites with Planar Orientations," Materials Science and Engineering, in review.

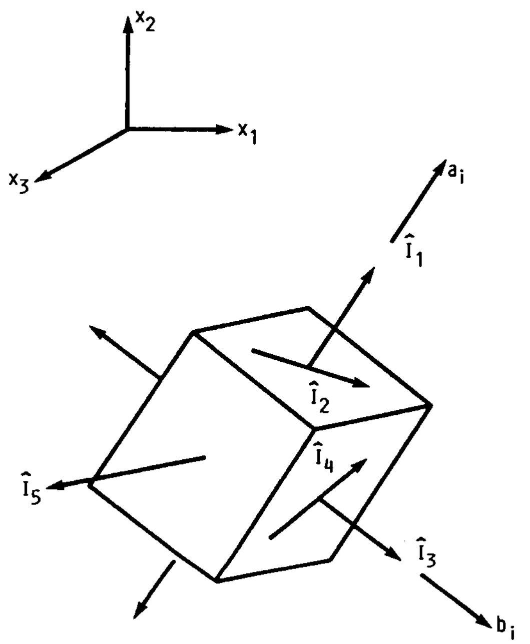


FIGURE 1. - PHYSICAL INTERPRETATIONS OF INVARIANTS CORRESPONDING TO FRACTURE MECHANISMS.

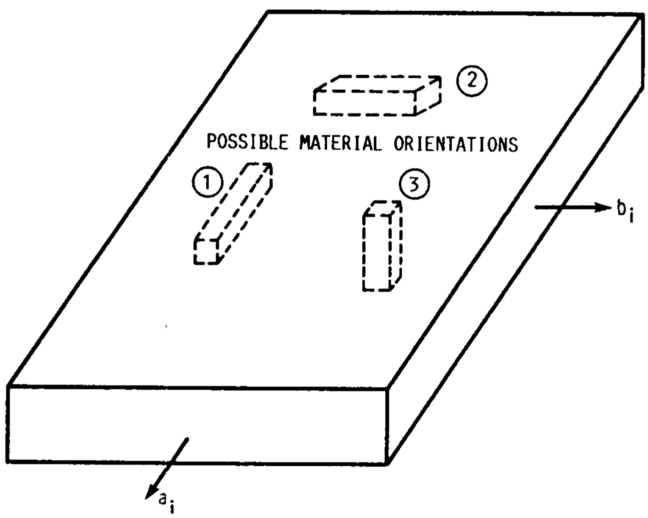
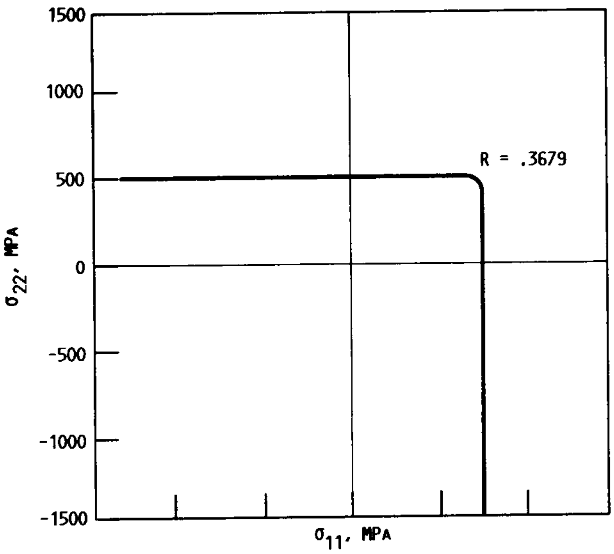
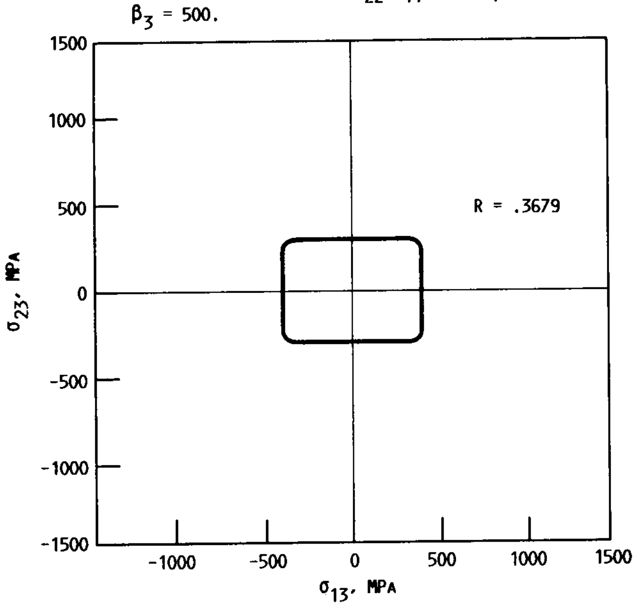


FIGURE 2. - ORIENTATIONS OF SPECIMENS FOR MOR BAR AND SHEAR TESTS FOR FULL THREE-DIMENSIONAL CHARACTERIZATION OF THE WEIBULL PARAMETERS.

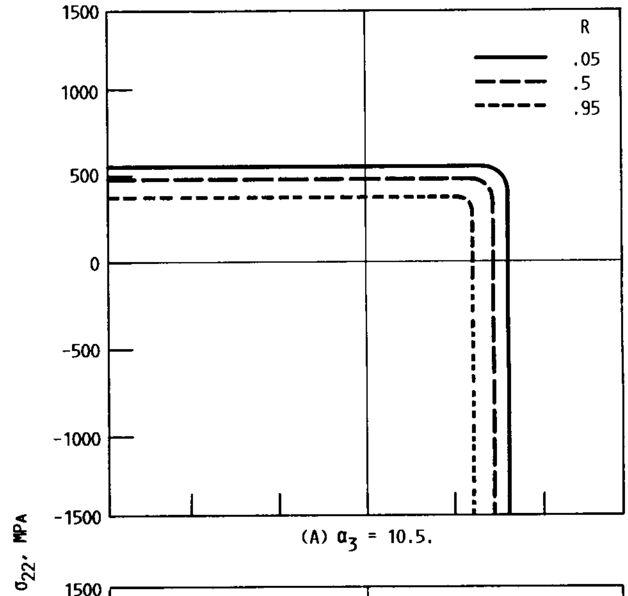


(A) BIAXIAL STRESS SPACE σ_{22}-σ_{11} WITH β_1 = 750 AND β_3 = 500.

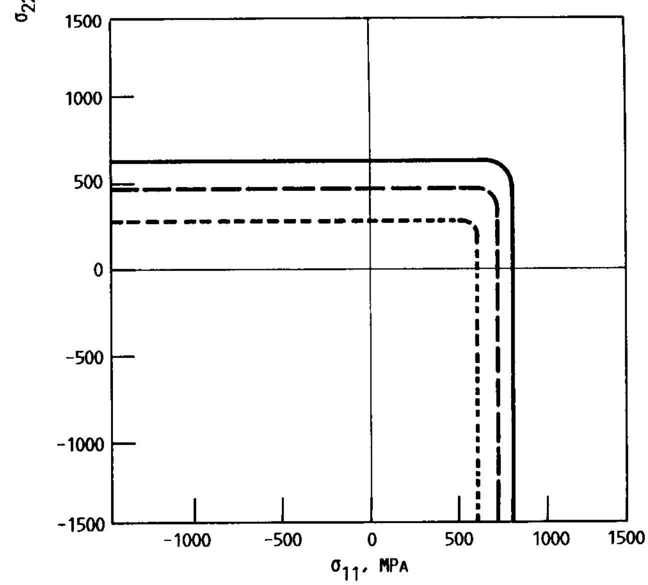


(B) SHEAR STRESS SPACE σ_{13}-σ_{23} WITH β_2 = 400 AND β_4 = 300.

FIGURE 3. - RELIABILITY CONTOURS OF AN ORTHOTROPIC CONTINUUM OF UNIT VOLUME WITH MATERIAL DIRECTIONS a_i AND b_i COINCIDENT WITH THE 1- AND 2-DIRECTIONS RESPECTIVELY. FAILURE FUNCTION, ψ, EQUAL TO UNITY.

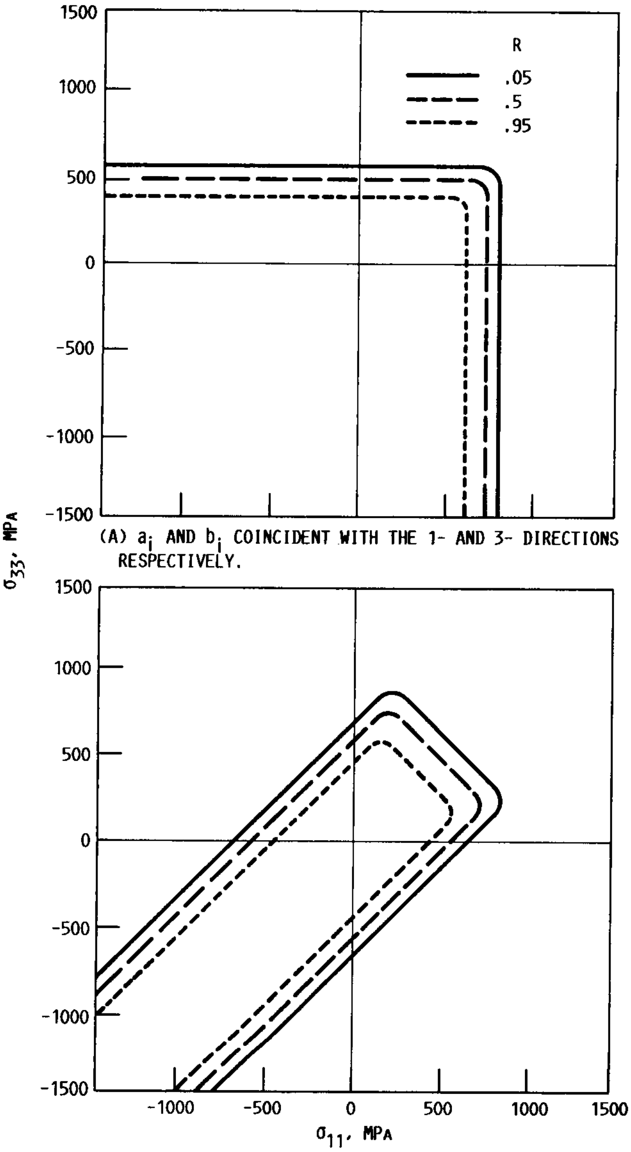


(A) α_3 = 10.5.



(B) α_3 = 5.

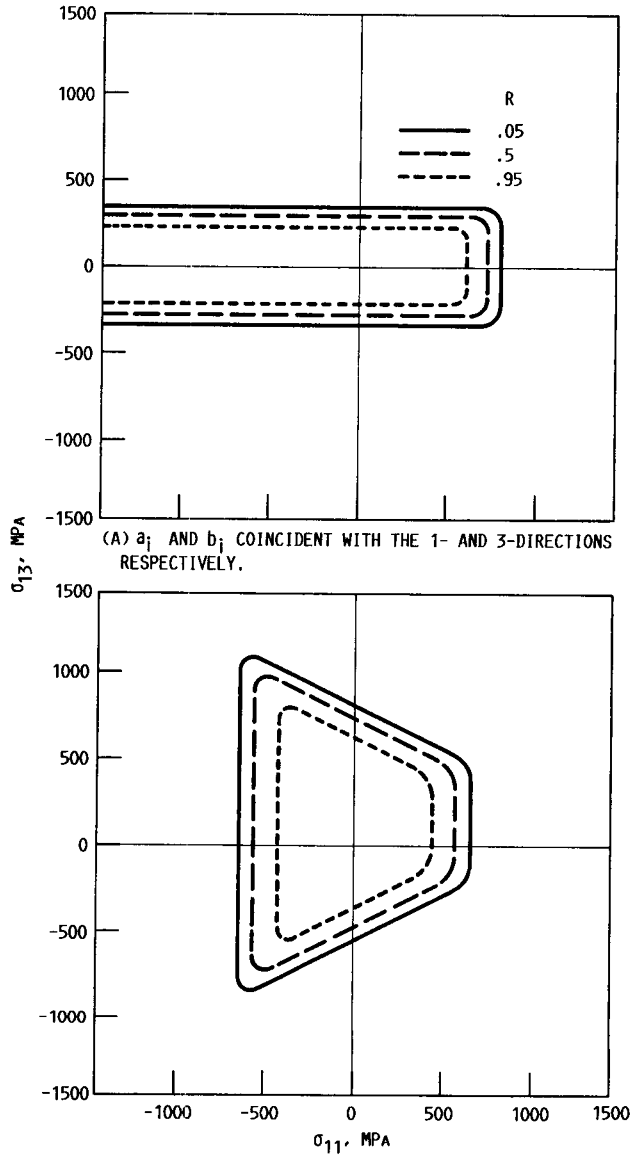
FIGURE 4. - FAMILIES OF RELIABILITY CONTOURS DEPICTING THE EFFECT OF REDUCING THE WEIBULL SHAPE PARAMETER α_3. MATERIAL DIRECTIONS ARE THE SAME AS IN FIGURE 3 WITH α_1 = 15, β_1 = 750 AND β_3 = 500.



(A) a_i AND b_i COINCIDENT WITH THE 1- AND 3- DIRECTIONS RESPECTIVELY.

(B) MATERIAL DIRECTIONS a_i AND b_i ROTATED 45^0 IN THE 1-3 PLANE.

FIGURE 5. - LEVEL SURFACES OF RELIABILITY ILLUSTRATING THE RESULT OF OFF-AXIS LOADING IN A NORMAL STRESS SPACE. WEIBULL PARAMETERS FOR INVARIANTS $\hat{I}_1$ AND $\hat{I}_3$ ARE THE SAME AS IN FIGURE 3a, ALONG WITH $\alpha_2 = 12$, $\beta_2 = 400$, $\alpha_4 = 10$ AND $\beta_4 = 300$.



(A) a_i AND b_i COINCIDENT WITH THE 1- AND 3-DIRECTIONS RESPECTIVELY.

(B) MATERIAL DIRECTIONS ROTATED 45^0 IN THE 1-3 PLANE.

FIGURE 6. - FAMILIES OF RELIABILITY CONTOURS COMPARING ON- AND OFF-AXIS ORIENTATIONS OF THE MATERIAL DIRECTIONS a_i AND b_i IN THE σ_{11}-σ_{13} STRESS SPACE. THE WEIBULL PARAMETERS FOR $\hat{I}_1$, $\hat{I}_2$, $\hat{I}_3$ AND $\hat{I}_4$ ARE THE SAME AS IN FIGURE 5.

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16. Abstract

A macroscopic noninteractive reliability model for ceramic matrix composites is presented. The model is multiaxial and applicable to composites that can be characterized as orthotropic. Tensorial invariant theory is used to create an integrity basis with invariants that correspond to physical mechanisms related to fracture. This integrity basis is then used to construct a failure function per unit volume (or area) of material. It is assumed that the overall strength of the composite is governed by weakest link theory. This leads to a Weibull type model similar in nature to the principle of independent action (PIA) model for isotropic monolithic ceramics. An experimental program to obtain model parameters is briefly discussed. In addition, qualitative features of the model are illustrated by presenting reliability surfaces for various model parameters.

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