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Probabilistic Finite Element Heat Transfer and Structural Analysis of a Cone-Cylinder Pressure Vessel

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PROBABILISTIC FINITE ELEMENT HEAT TRANSFER AND STRUCTURAL ANALYSIS OF A CONE-CYLINDER PRESSURE VESSEL

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I dedicate this thesis to my father; Issam, my mother; Soha, and my sister; Rana. There is absolutely no way to thank them enough for their support, direction, love, and all their sacrifices until this day. A dedication also goes to my friend and “big brother” George Kanaan for all his help and continuous support especially when I first relocated to Ohio from Lebanon in 1999. You will always be my first employer!

Finally, I would like to dedicate my work to all the Lebanese youth, who relocate to the United States in search of education and opportunities, leaving their families, friends and loved ones behind.

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ABSTRACT

Stress analysis of a cone-cylinder pressure vessel was computationally simulated by a finite element method and probabilistically evaluated in view of the several uncertainties in the performance parameters. Cumulative distribution functions and sensitivity factors were computed for overall Von Mises stresses due to the structural and thermodynamic random variables. These results can be used to quickly identify the most critical design variables in order to optimize the design and make it cost effective. The analysis leads to the selection of the appropriate measurements to be used in structural and heat transfer analysis and to the identification of both the most critical measurements and parameters.

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NOMENCLATURE

T_∞: Ambient Temperature

Ti: Initial Temperature

Kx: Thermal Conductivity in the x Direction

Ky: Thermal Conductivity in the y Direction

n_x: Direction Cosine in the x Direction

n_y: Direction Cosine in the y Direction

L1: Boundary Length

L2: Boundary Length Remainder

α: Thermal Coefficient of Expansion in the x Direction

β: Thermal Coefficient of Expansion in the x Direction

A: Area

Pi: Internal Pressure

Pe: External Pressure

Q: Internal Heat Source

Di: Inner Cylinder Diameter

t: Cylinder Thickness

E: Modulus of Elasticity

υ: Poisson's Ratio

h_i: Internal Convection Coefficient

h_e: External Convection Coefficient

H: Cylinder Height

Md: Mass Density

x: x Co-ordinate

y: y Co-ordinate

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CHAPTER I

INTRODUCTION

The cone-cylinder shells are used extensively as pressure vessels in several industrial applications. Under the influence of hydrostatic pressure, the local stresses in the vicinity of the con-cylinder intersection will be much higher than those in other parts of the shell. The high local stresses are caused by the discontinuous variation of the tangent slope of the shell meridian at the joint. Teng [1-4] investigated the collapse and buckling behavior of uniform thickness cone-cylinder intersections under internal pressure. Jones [5] analyzed the buckling failure of a cone-cylinder intersection. Gabriel [6] conducted experiments on internally pressurized cone-cylinder intersections. Anwen [7] studied the stresses and stability for the cone-cylinder shells with toroidal transition.

To cost effectively accomplish the design task, we need to formally quantify the effect of uncertainties (variables) in the design. Probabilistic design is one effective method to formally quantify the effect of uncertainties. In the present study, a probabilistic analysis is presented for the influence of measurement accuracy on the random variables for Von Mises stresses from a cone-cylinder pressure vessel. Small perturbation approach is used for the finite element methods to compute the sensitivity of the response to small fluctuations of the random variables present. The result is a parametric representation of the response in terms of a set of random variables with known statistical properties, which can be used to estimate the characteristics of the selected response variables such as stresses, heat transfer rate or temperature at a given point. Thanks to today's technological advancements in computers and software development, such engineering problems could be solved in a matter of hours. In this study, SolidWorks and Algor were used for this analysis.

1.1 SOLIDWORKS

SolidWorks is a three dimensional computer aided design software package, used to design parts and assemblies. It is a Windows based program.

1.2 ALGOR

Algor is a finite element analysis package used to obtain solutions to large engineering problems in the static and dynamic structural analysis, heat transfer, and fluid flow. Algor is completely compatible with SolidWorks; hence the transfer of three dimensional models into the finite element program is a possible.

1.3 NESTEM

NESTEM is a modular software program developed by NASA to perform structural and thermal probabilistic analyses on components and systems. It contains state of the art algorithms to compute probabilistic responses to engineering problems. Calculation and computations were performed in Algor and results were inputted into

NESTEM. The program uses density functions and then propagates those functions on the model to yield uncertainty outputs, which could be related to the modes of failure of the model. The use of this program was possible through NASA Glenn Research Center in Cleveland, Ohio, and through Dr. Rama Gorla.

CHAPTER II

MODEL AND PROBLEM APPROACH

The history of the iterative algorithm is illustrated by means of an example involving hydrostatic pressure loading, conduction, and convection in a cone-cylinder pressure vessel. The inner surface of the pressure vessel was exposed to an environment maintained at 1100°C, while the outer surface exposed to ambient temperature at 15°C. The pressure vessel was filled with a fluid at a pressure of 10 MPa. Steel 4130 was selected for the material of the cone cylinder. Table 1 shows the random variables and their mean values. All random variables were assumed to be independent. A scatter of ±10 % was specified for all the variables. This variation amounted to two standard deviations. Normal distribution was assumed for all random variable scatters.

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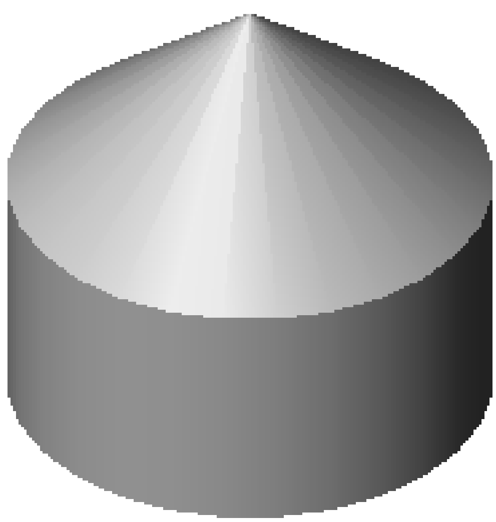
Table I: Random Variable Mean Values

Variable	**Mean Value**
Internal Pressure (Pi)	10 MPa
Internal Temperature(Ti)	1100 ^{o}C
Inner Diameter(Di)	1 m
Wall Thickness(t)	4 mm
Height of Cylinder(H)	0.5m
Thermal Conductivity(K)	0.043235 J/(s*mm* ^{o}C)
Thermal Coefficient of Expansion(α)	0.0000135 /^{o}C
Modulus of Elasticity(E)	203395 N/mm^2
Poisson's Ratio(υ)	0.3
Internal Heat Transfer Coefficient(hi)	525 W/m^{2} *^{o}C
External Heat Transfer Coefficient(he)	75 W/m^{2} *^{o}C
Ambient Temperature(T∞)	15 ^{o}C
Mass Density(Md)	0.0004 Kg/m^3

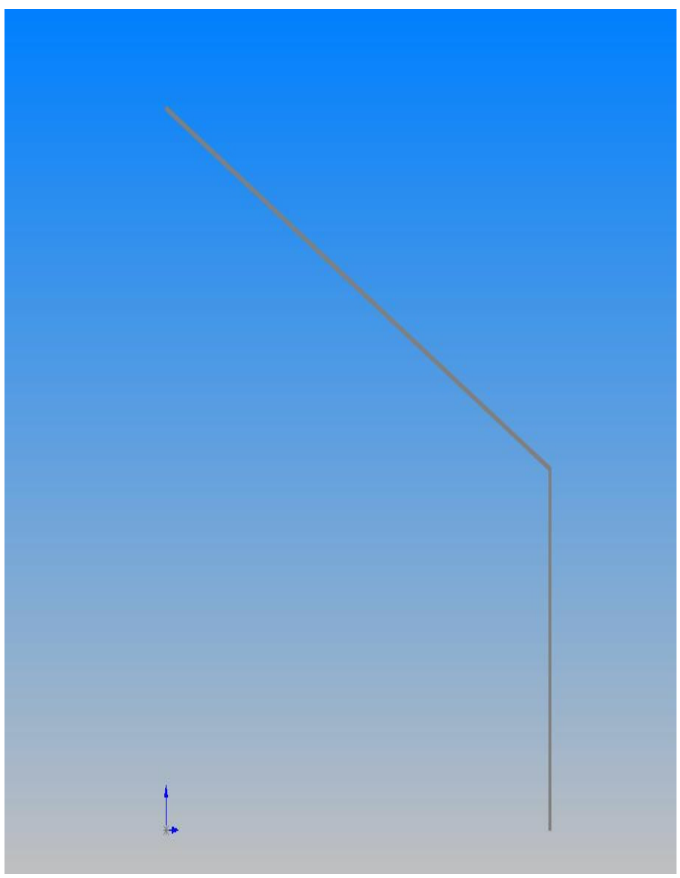
Figure 1: Solid Model of Cylinder



2.1 AXISYMMETRIC MODEL

This cone-head cylinder pressure vessel model exhibits symmetry about the z-axis of rotation, or in other words it is a solid of revolution. The boundary and loading conditions are also symmetric to the axis of rotation; hence they are independent of the circumferential coordinate θ. Material properties used in this study are isotropic. In this study the analysis was performed using a 2-D axisymmetric model of the cone cylinder. Figure 2 shows the axisymmetric model.

Figure 2: Axisymmetric Model



CHAPTER III

ANALYSIS

3.1 FINITE ELEMENT EQUATIONS

Let us consider a two-dimensional partial differential equation of the form

$$\frac{\partial}{\partial x}\left[K_x(x,y)\frac{\partial T}{\partial x}\right]+\frac{\partial}{\partial y}\left[K_y(x,y)\frac{\partial T}{\partial y}\right]+P(x,y)T+Q(x,y)=0. \qquad (4)$$

The above equation is valid over an area A. We assume that on a portion of the boundary L_1, $T = T_i(x,y)$.

On the remainder of the boundary, labeled L_2, the general derivative boundary condition is specified in the form

$$K_x(x,y)\frac{\partial T}{\partial X}n_x + K_y(x,y)\frac{\partial T}{\partial y}n_y + \alpha(x,y)T + \beta(x,y) = 0. \qquad (5)$$

Here, n_x and n_y are direction cosines of the outward normal to L_2. The form of the functional may be written as

$$I(T)= \iint_A \left[\frac{1}{2} K_x \left(\frac{\partial T}{\partial x} \right)^2 + \frac{1}{2} K_y \left(\frac{\partial T}{\partial y} \right)^2 - \frac{1}{2} P T^2 - Q T \right] dA + \int_{L_2} \left(\frac{\alpha T^2}{2} + \beta T \right) d \qquad (6)$$

.

For a simplex two-dimensional element, we have extremized the above functional with respect to the unknown nodal temperatures. The resultant element matrices are then obtained from the following relation:

$$\begin{Bmatrix} \dfrac{\partial I}{\partial T_i} \\ \dfrac{\partial I}{\partial T_j} \\ \dfrac{\partial I}{\partial T_k} \end{Bmatrix}^{(e)} = [B]^{(e)} [T]^{(e)} - [C]^{(e)} \qquad (7)$$

The element matrix $[B]^{(e)}$ and the element column $[C]^{(e)}$ may be written as

$$[B]^{(e)} = \begin{bmatrix} B_{ii} B_{ij} B_{ik} \\ B_{ji} B_{jj} B_{jk} \\ B_{ki} B_{kj} B_{kk} \end{bmatrix}, \; [C]^{(e)} = \begin{Bmatrix} C_i \\ C_j \\ C_k \end{Bmatrix}^{(e)} \qquad (8)$$

Where

$$B_{ii} = \frac{K}{4A}(b_i^2 + c_i^2) - \frac{PA}{6} + \frac{(\alpha L_{ij})_{sideij}}{3} + \frac{(\alpha L_{ki})_{sideki}}{3},$$

$$B_{ij} = \frac{K}{4A}(b_i b_j + c_i c_j) - \frac{PA}{12} + \frac{(\alpha L_{ij})_{sideij}}{6},$$

$$B_{ik} = \frac{K}{4A}(b_i b_k + c_i c_k) - \frac{PA}{12} + \frac{(\alpha L_{ki})_{sideki}}{3},$$

$$B_{jj} = \frac{K}{4A}(b_j^2 + c_j^2) - \frac{PA}{6} + \frac{(\alpha L_{jk})_{sidejk}}{3} + \frac{(\alpha L_{ij})_{sideij}}{3},$$

$$B_{jk} = \frac{K}{4A}(b_j b_k + c_j c_k) - \frac{PA}{12} + \frac{(\alpha L_{jk})_{sidejk}}{6},$$

$$B_{kk} = \frac{K}{4A}(b_k^2 + c_k^2) - \frac{PA}{6} + \frac{(\alpha L_{ki})_{sideki}}{3} + \frac{(\alpha L_{jk})_{sidejk}}{3},$$

$$C_i = \frac{QA}{3} - \frac{\beta L_{ij}}{2} - \frac{\beta L_{ki}}{2},$$

$$C_j = \frac{QA}{3} - \frac{\beta L_{jk}}{2} - \frac{\beta L_{ij}}{2},$$

$$C_k = \frac{QA}{3} - \frac{\beta L_{ki}}{2} - \frac{\beta L_{jk}}{2},$$

The element matrices were then assembled into the global matrices and vectors. The prescribed boundary conditions were implemented at the appropriate nodal points. The algebraic equations in the global assembled form were solved by the Gauss elimination procedure. These details may be found in reference [8].

3.2 PERTURBATION OF THE HEAT TRANSFER PROBLEM

The finite element solution for the heat transfer problem may be reduced to the following equation in the unperturbed state:

$$[B][T] = [C] \qquad (9)$$

The perturbed problem involving small fluctuations of the random variables may be written as

$$[\hat{B}]\ [\hat{T}] = [\hat{C}] \qquad (10)$$

where

$$[\hat{B}] = [B] + d[B]$$

$$[\hat{T}] = [T] + d[T]$$

$$[\hat{C}] = [C] + d[C] \qquad (11)$$

Therefore, we may write equation (9) as

$$[B]d[T] = [C] - [\hat{B}][\hat{T}] - d[B]\ d[T]$$

$$\cong dx_i \ \frac{\partial[C]}{\partial x_i} - dx_i \frac{\partial[B]}{\partial x_i}[T] \qquad (12)$$

In the last step in equation (12), we ignored the second order term d [B]. d [T]. Here, x_i are the random variables. A simple form of the iterative algorithm is given by:

$$[B]d[\hat{T}]^{n+1} = [\hat{C}] - [\hat{B}][\hat{T}]^{n} \qquad (13)$$

$$[\hat{T}]^{n+1} = [\hat{T}]^{n} + d[\hat{T}]^{n+1} \qquad (14)$$

In order to start the iteration, we may use

$$[\hat{T}]^{0} = [T]$$

The effect of temperature-dependent properties may be included in equation (13). From equation (13), we may write:

$$[B]d[\hat{T}]^{n} = [\hat{C}] - [\hat{B}][\hat{T}]^{n-1} \qquad (15)$$

From equations (13) and (15) we may write

$$[B]d[\hat{T}]^{n+1} = [B]d[\hat{T}]^{n} - [\hat{B}]\ d[\hat{T}]^{n} \qquad (16)$$

From equation (16), we may write

$$d[\hat{T}]^{n+1} = [A]\ d[\hat{T}]^{n} \qquad (17)$$

where $[A]=[I]-[B]^{-1}\ [\hat{B}]$ is the amplification matrix. The iterative process will remain stable if the spectral radius of the amplification matrix [A] is less than unity. This will be true when the imposed perturbations on the original element matrix are small.

3.3 RESULTS AND DISCUSSION

Maximum stress location was determined from a pre-analysis of the pressure vessel. This location was used to evaluate the cumulative distribution functions (CDF) and the sensitivity factors for thermal stress response. Temperature distribution in the pressure vessel is shown in Figure 3. A typical stress distribution in the pressure vessel is shown in Figure 4. CDF for the stress is shown in Figure 17. The sensitivity factors for the stress versus the random variables are shown in Figures 5 through 17. We observe that the modulus of elasticity, Poisson's ratio, coefficient of thermal expansion of the tank material and inside fluid temperature have a lot of influence on the stresses.

Figure 3:   Temperature Distribution

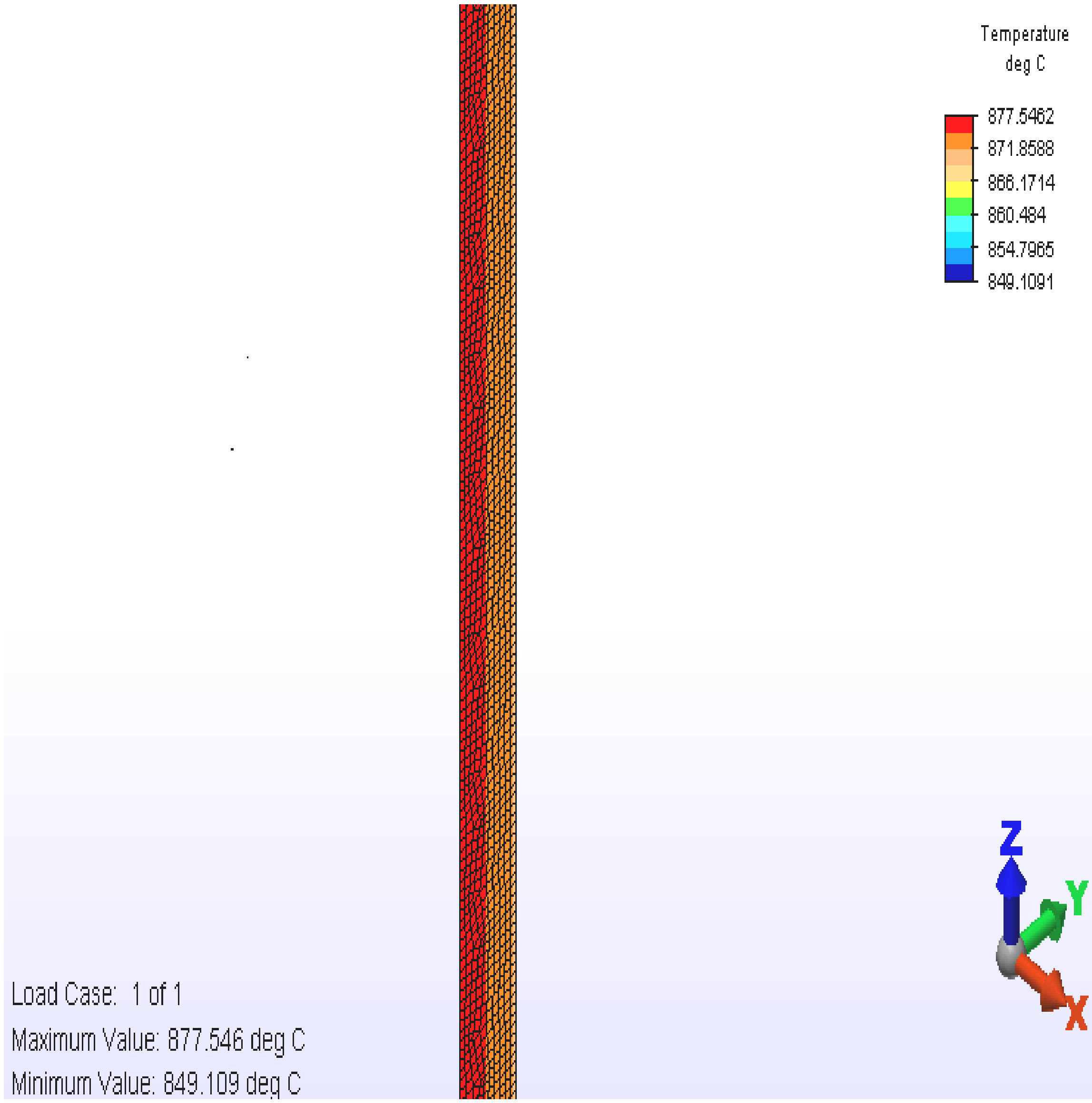


Figure 4: Stress Distribution

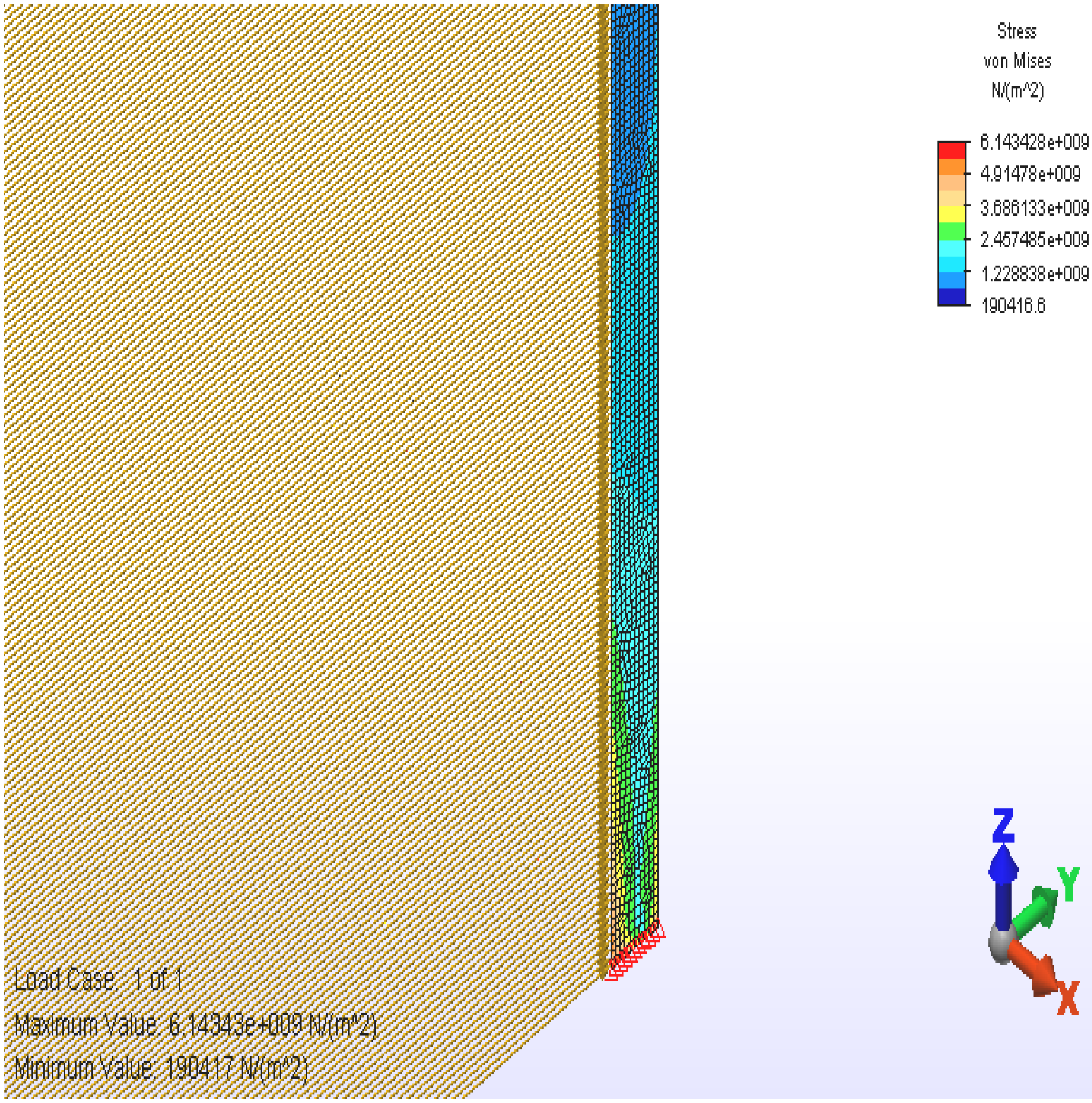


Table II: Maximum Von Mises Stress Values

Variable	**Variation**	**Maximum Stress Value**
Internal Pressure(Pi)	+ 10%	6.1312530E+09
	-10%	6.1310620E+09
Internal Temperature(Ti)	+ 10%	6.7423490E+09
	-10%	5.5199670E+09
Ambient Temperature ($T\infty$)	+ 10%	6.1329870E+09
	-10%	6.1293290E+09
Internal Heat Transfer Coefficient (hi)	+ 10%	6.1983800E+09
	-10%	6.0470370E+09
External Heat Transfer Coefficient (he)	+ 10%	6.0542980E+09
	-10%	6.2100610E+09
Thermal Coefficient of Expansion (α)	+ 10%	6.7441780E+09
	-10%	5.5181380E+09
Poisson's Ratio (υ)	+ 10%	6.3248840E+09
	-10%	5.9875590E+09
Modulus of Elasticity (E)	+ 10%	6.6760070E+09
	-10%	5.5580290E+09
Mass Density (Md)	+ 10%	6.1311580E+09
	-10%	6.1311580E+09
Cylinder Height (H)	+ 10%	6.1336580E+09
	-10%	6.1255090E+09
Cylinder Thickness (t)	+ 10%	6.4221680E+09
	-10%	5.9563780E+09

Figure 5: Sensitivity Factor for 0.001 Probability

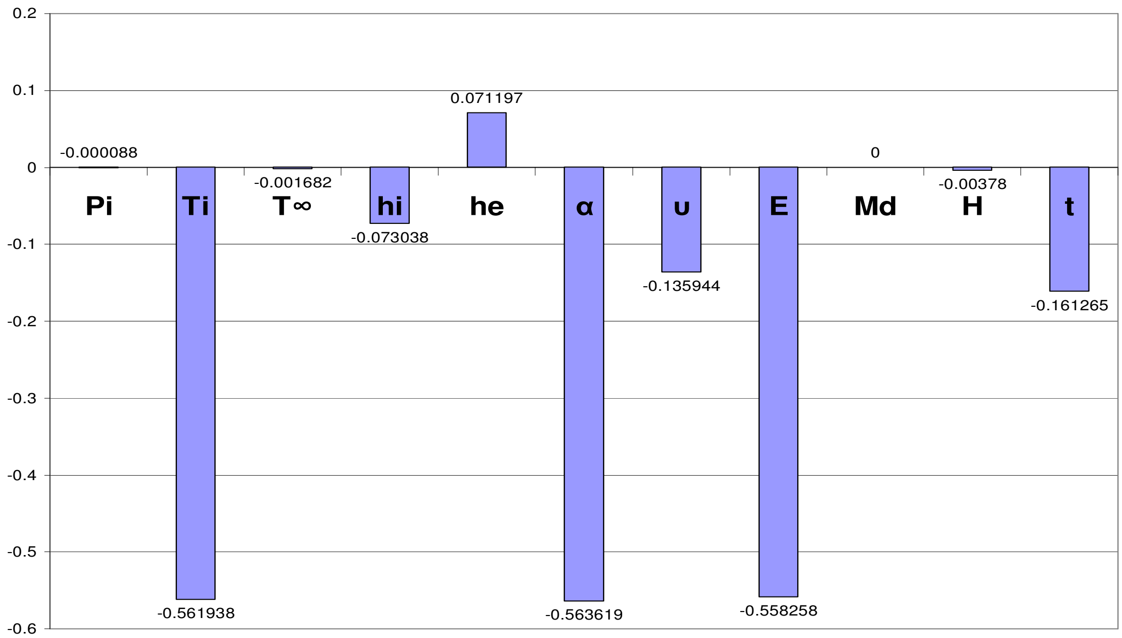


Figure 6: Sensitivity Factor for 0.01 Probability

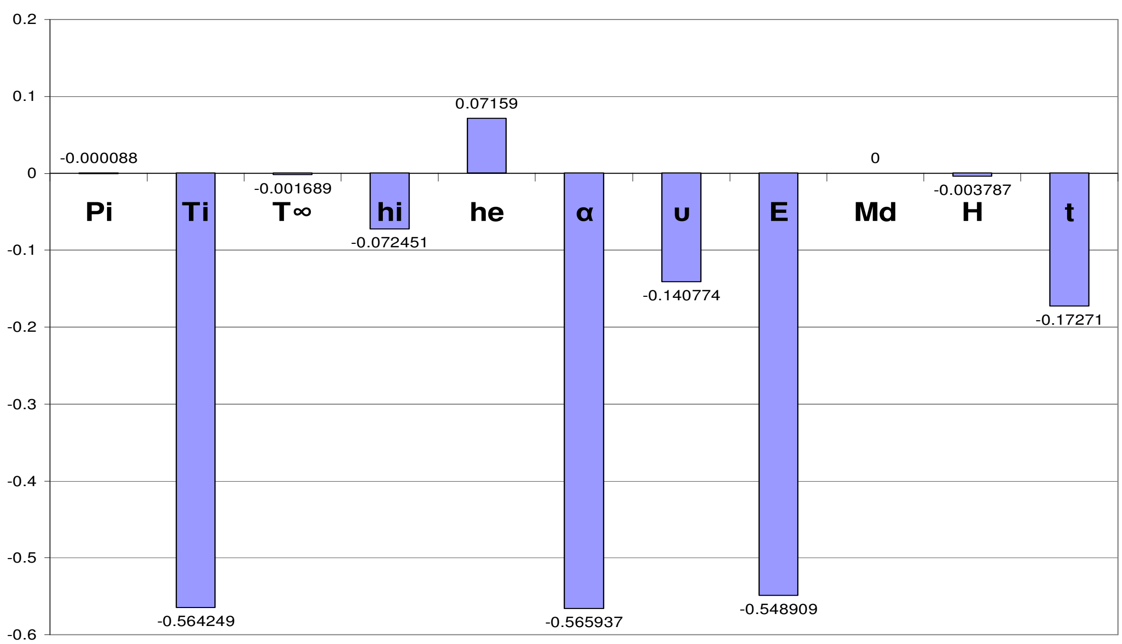


Figure 7: Sensitivity Factor for 0.1 Probability

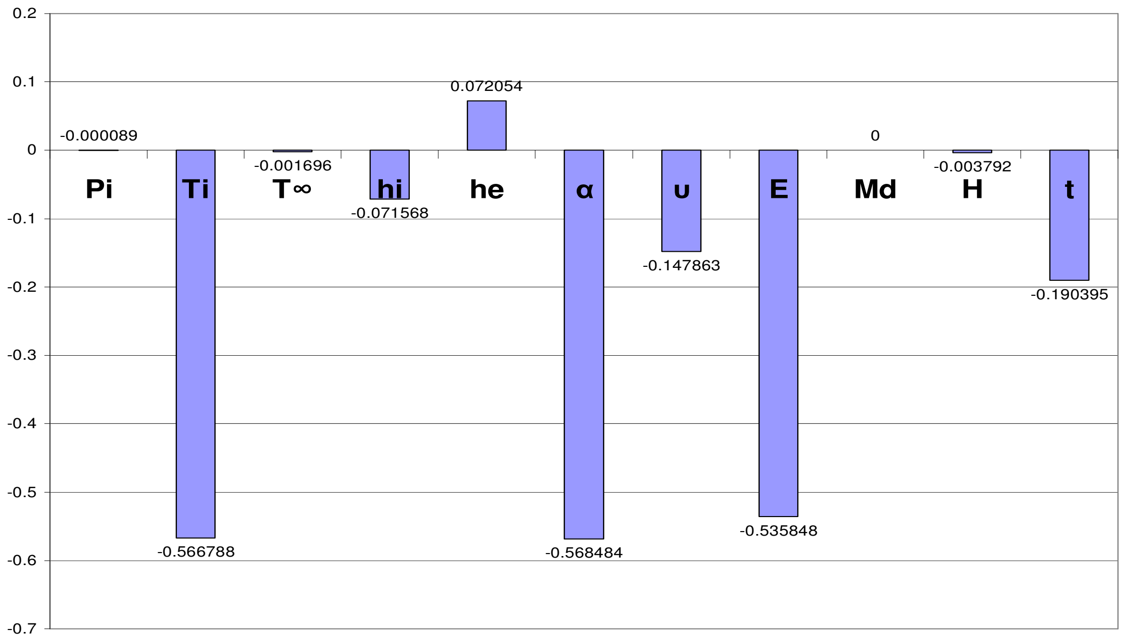


Figure 8: Sensitivity Factor for 0.2 Probability

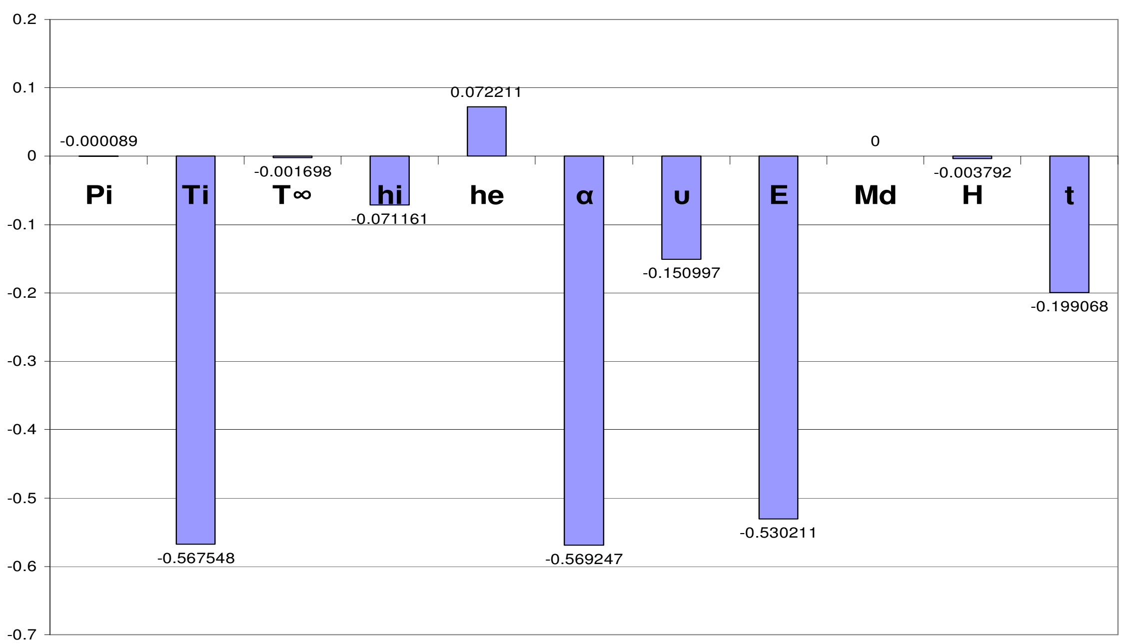


Figure 9:   Sensitivity Factor for 0.4 Probability

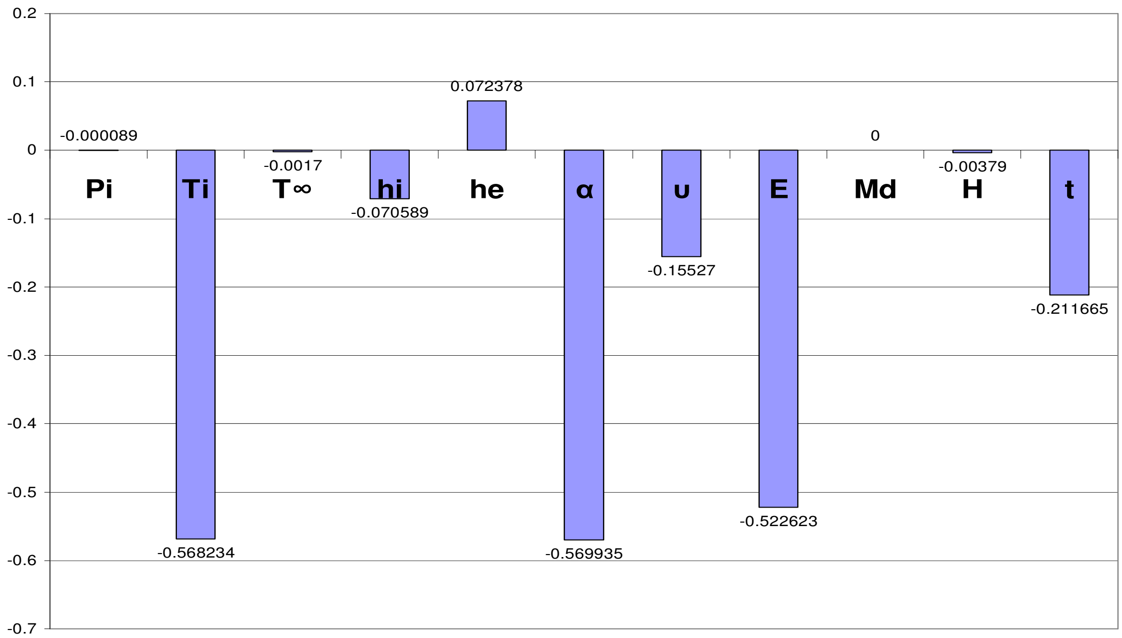


Figure 10: Sensitivity Factor for 0.5 Probability

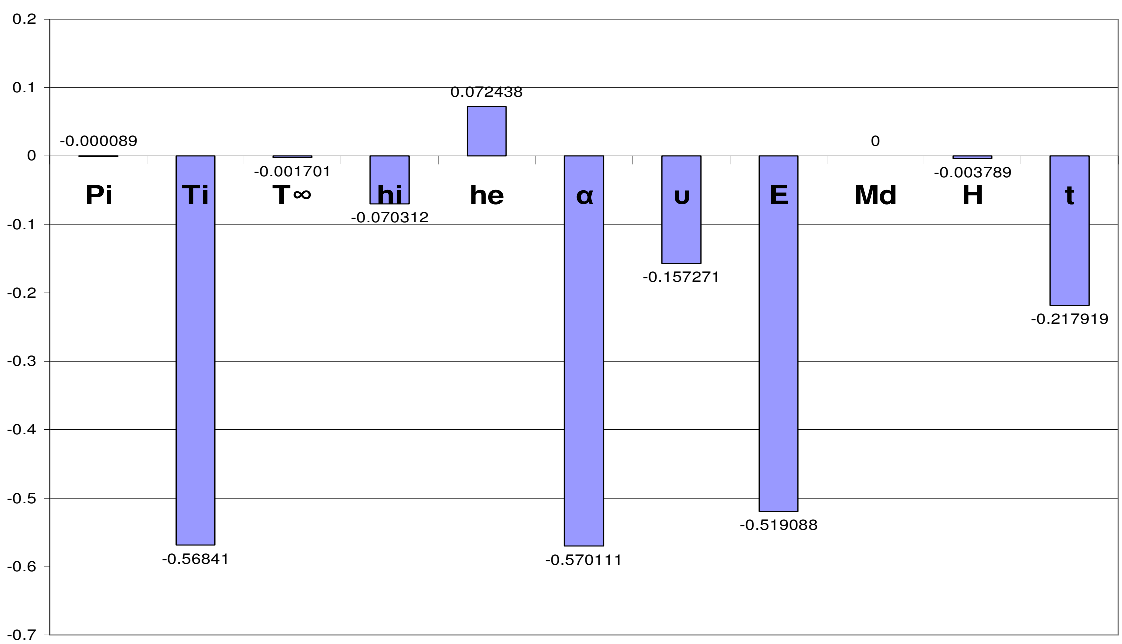


Figure 11: Sensitivity Factor for 0.6 Probability

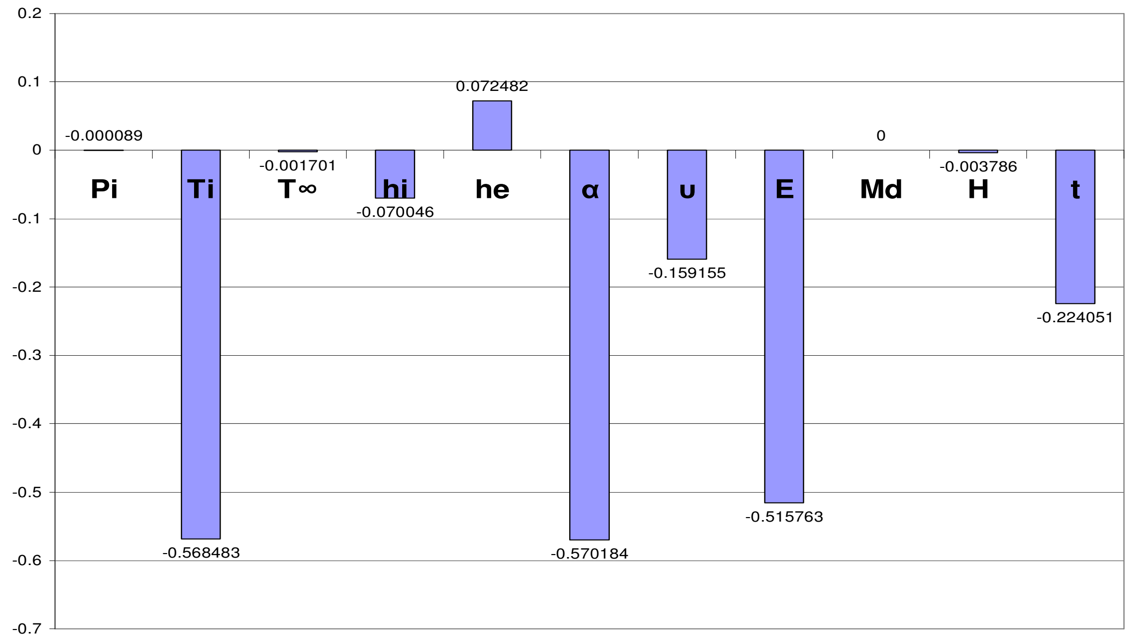


Figure 12: Sensitivity Factor for 0.8 Probability

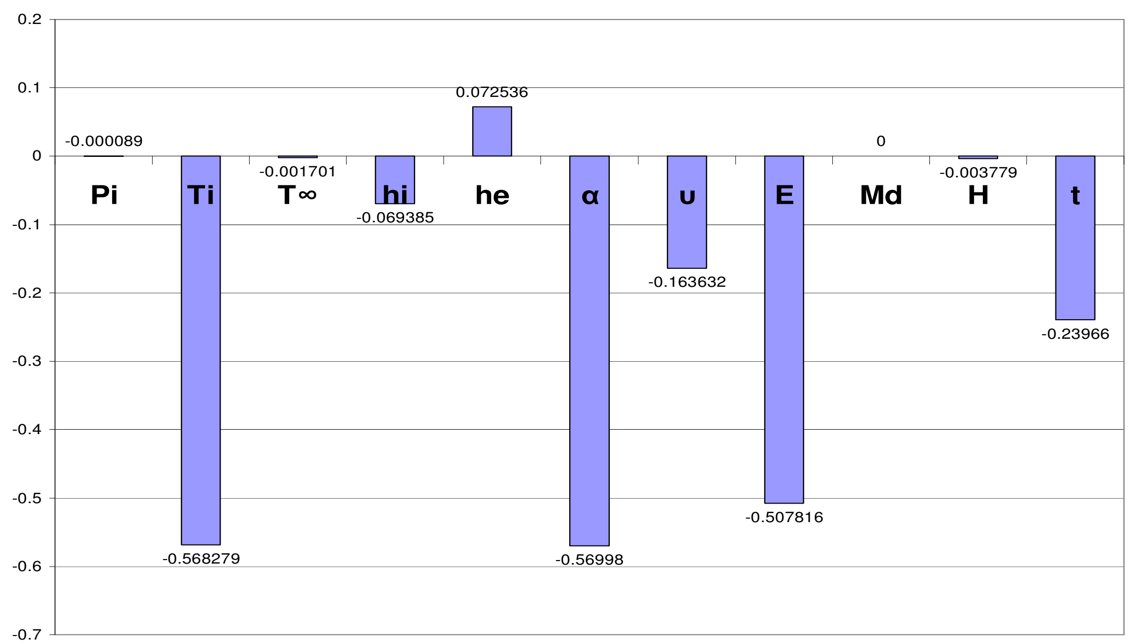


Figure 13: Sensitivity Factor for 0.9 Probability

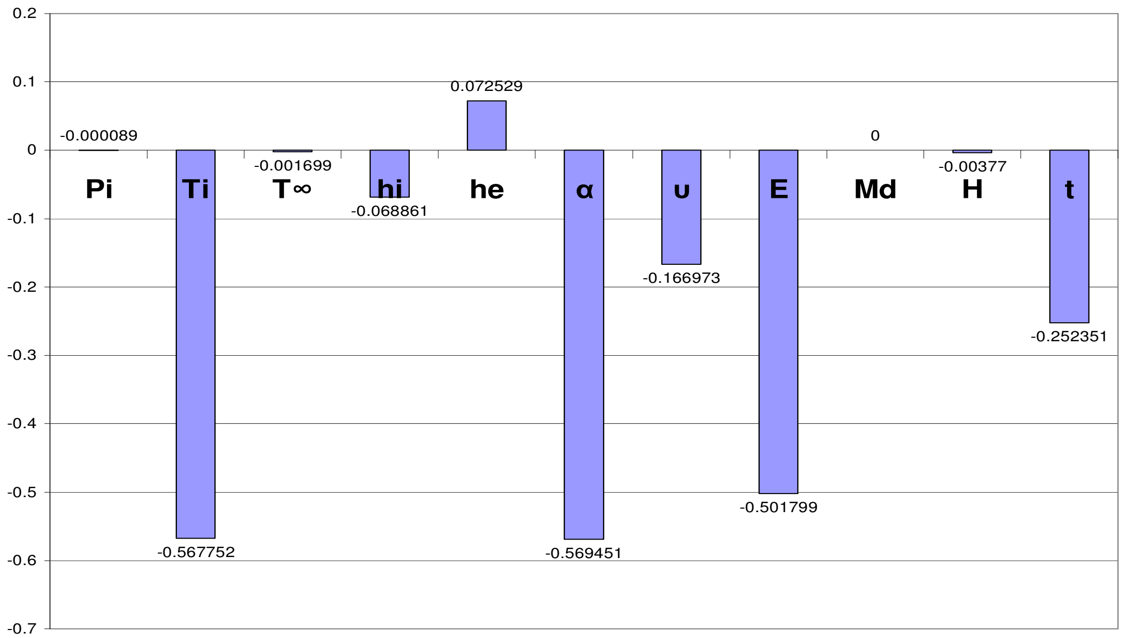


Figure 14: Sensitivity Factor for 0.95 Probability

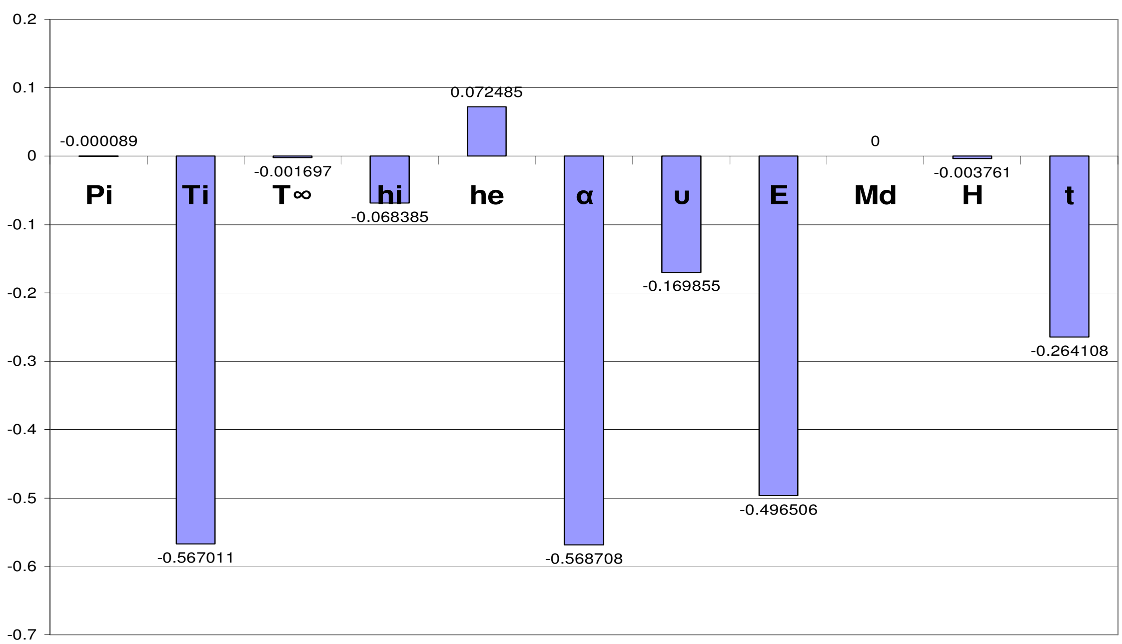


Figure 15: Sensitivity Factor for 0.99 Probability

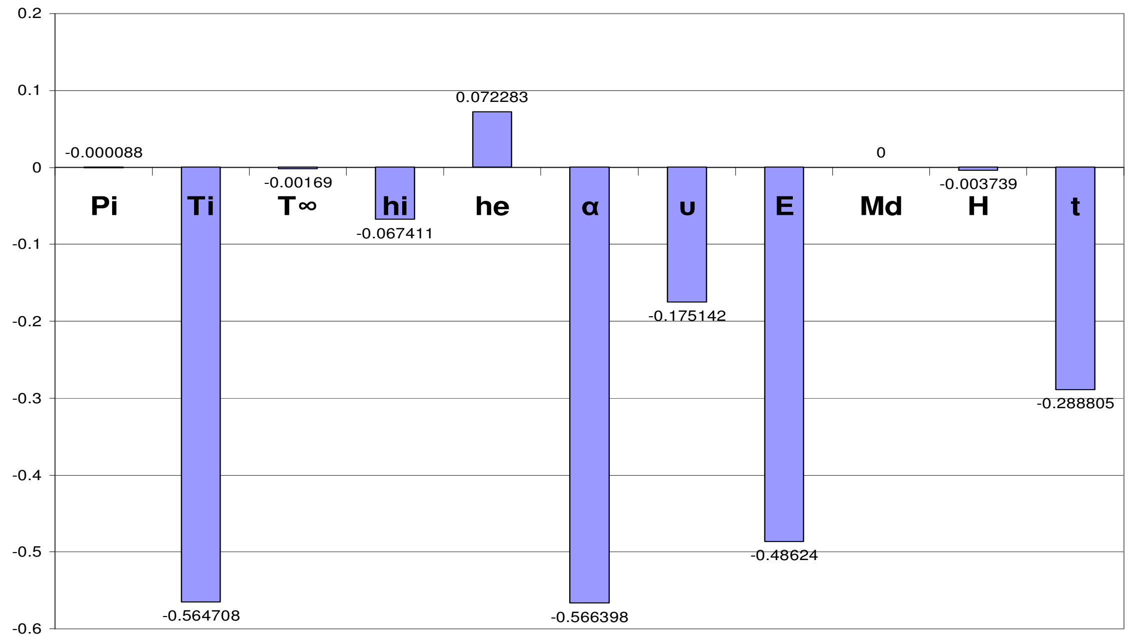


Figure 16: Sensitivity Factor for 0.999 Probability

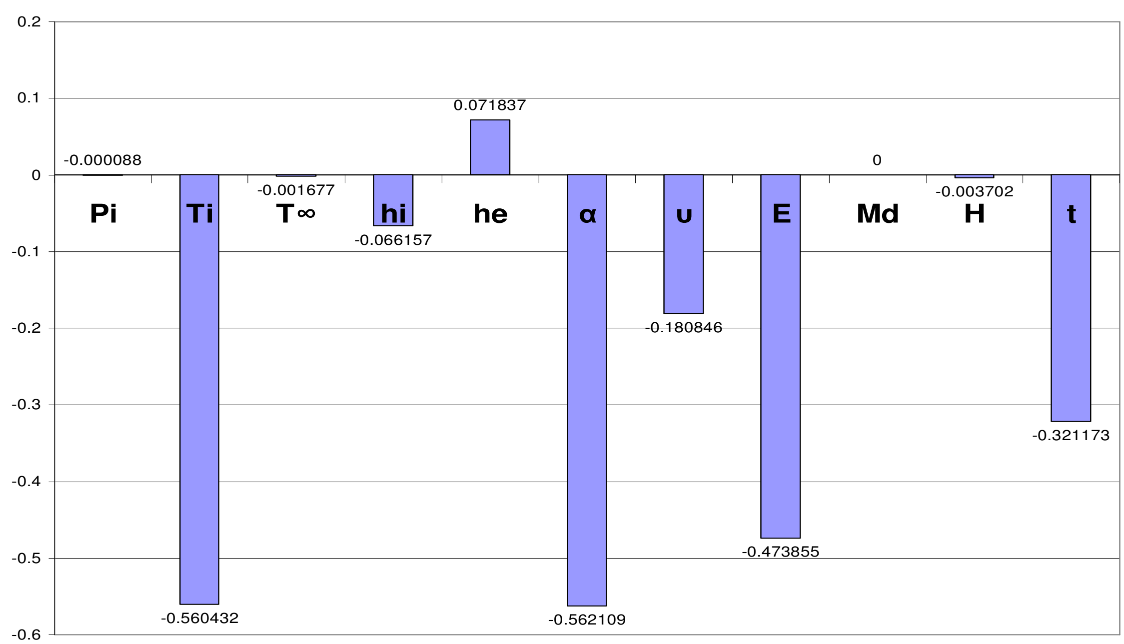
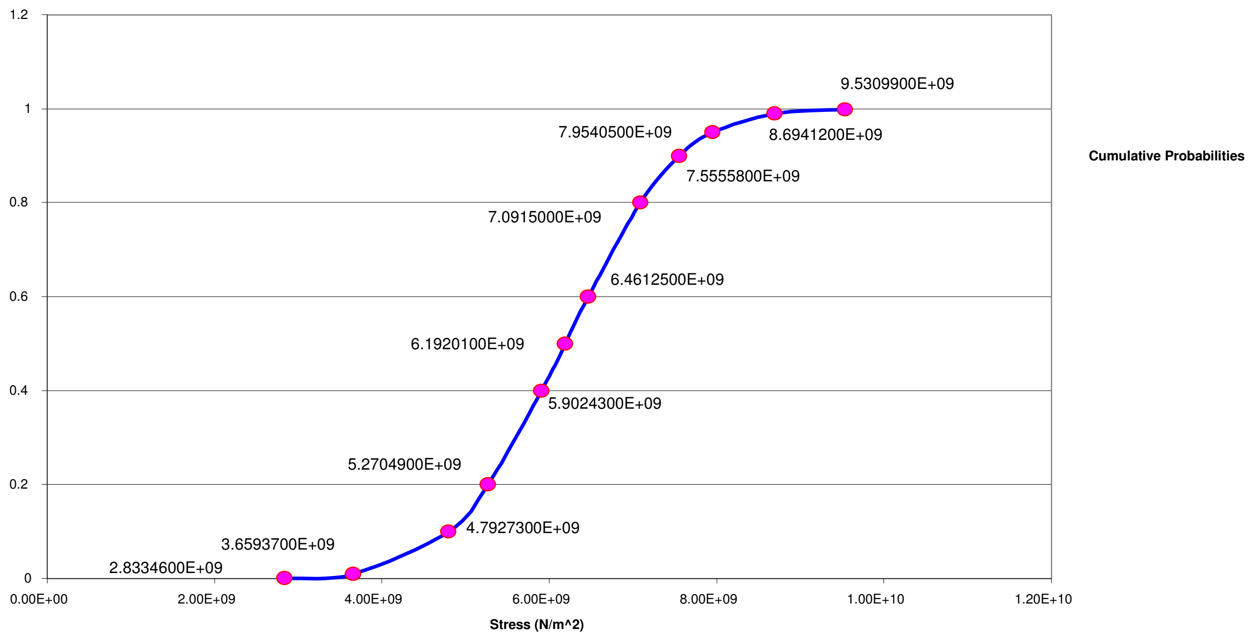


Figure 17: Cumulative Probabilities Versus Maximum Stresses



CHAPTER IV

CONCLUSION

A robust design is one that has been created with a system of design tools that reduce product or process variability while guiding the performance toward an optimal setting. Robustness means achieving excellent performance under a wide range of operating conditions. All engineering systems function reasonably well under ideal conditions, but robust designs continues to function well when the conditions are non-ideal. Over the years, the traditional way to account for uncertainties and variations in design parameters was to apply design safety factors. In today's medical device market, the need for complex and reliable products has increased dramatically. Depending only on applying safety factors to such designs can accumulate to cause over design of the product and result in an uncertain reliability. Analytical robust design attempts to determine the values of design parameters, which maximize the reliability, quality, and safety of the product without tightening the material or environmental tolerances and without adding the cost of overdesign. Probabilistic design and robust design go hand in hand. In order to determine the domains of stability, the system has to be analyzed probabilistically.

In the present work, a pressurized cone head cylinder was analyzed probabilistically to determine which of the variables has the most effect on the product. It was evident from maximum Von Mises stress results presented in Table 2 that the internal temperature Ti, the coefficient of thermal expansion (α), Poisson's ratio (υ), the material modulus of elasticity (E), and the vessel wall thickness (t) have the most effect on the product performance. This observation was validated using the probabilistic approach.

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