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**PROBABILISTIC STRUCTURAL AND THERMAL
ANALYSIS OF A GASKETED FLANGE**

ATUL G. TANAWADE

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University of Mumbai

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This thesis has been approved
for the Department of MECHANICAL ENGINEERING
and the College of Graduate Studies by

Thesis Committee Chairperson, Dr. Rama S. R. Gorla

Department and Date

Dr. Majid Rashidi

Department and Date

Dr. Asuquo B. Ebiana

Department and Date

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PROBABILISTIC STRUCTURAL AND THERMAL ANALYSIS OF A GASKETED FLANGE

ATUL G. TANAWADE

ABSTRACT

Performance of a flange joint is characterized mainly by its ‘strength’ and ‘sealing capability’. A number of analytical and experimental studies have been conducted to study these characteristics under internal pressure loading. However, with the advent of new technological trends for high temperature and pressure applications, an increased demand for analysis is recognized. The effect of steady-state thermal loading makes the problem more complex as it leads to combined application of internal pressure and temperature.

Structural and thermal analysis of a gasketed flange was computationally simulated by a finite element method and probabilistically evaluated in view of the several uncertainties in the performance parameters. Cumulative distribution functions and sensitivity factors were computed for Maximum stresses and Von Mises Stresses due to the structural and thermodynamic random variables. These results can be used to quickly identify the most critical design variables in order to optimize the design and make it cost effective. The analysis leads to the selection of the appropriate measurements to be used in structural

and heat transfer analysis and to the identification of both the most critical measurements and parameters.

Conventional engineering design methods are generally deterministic. But in reality, many engineering systems are stochastic in nature where a probability assessment of the results becomes a necessity. This probabilistic engineering design analysis assumes probability distributions of design parameters, instead of mean values only. This enables the designer to design for a specific reliability and hence maximize safety, quality and cost.

In the present work, thermal and structural analysis on the flange was performed to obtain the areas of maximum stress under the given boundary conditions. The product was modeled and then simulated in Finite Element Analysis (FEA) software. The results obtained were probabilistically evaluated for optimum design.

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NOMENCLATURE

Material Properties:

Flange:

E_f : Modulus of Elasticity

ν_f : Poisson's Ratio

K_f : Thermal Conductivity

α_f : Coefficient of Thermal Expansion

ρ_f : Mass Density

C_{p_f} : Specific Heat

Bolt:

E_b : Modulus of Elasticity

ν_b : Poisson's Ratio

K_b : Thermal Conductivity

α_b : Coefficient of Thermal Expansion

ρ_b : Mass Density

C_{p_b} : Specific Heat

Gasket:

E_g : Modulus of Elasticity

ν_g : Poisson's Ratio

K_g : Thermal Conductivity

α_g : Coefficient of Thermal Expansion

ρ_g : Mass Density

C_{p_g} : Specific Heat

Pressure:

P_i : Internal Fluid Pressure

P_b : Bolt-up Pressure

Boundary Conditions:

h_i : Internal Heat Transfer Coefficient

h_∞ : Ambient Heat Transfer Coefficient

T_i : Internal Fluid Temperature

T_∞ : Ambient Temperature

Material Physical Properties:

t_p : Pipe Thickness

h_h : Hub Height

h_f : Flange Height

h_s : Shoulder Height

F_{id} : Flange Internal Diameter

h_p : Pipe Height

F_{od} : Flange Outside Diameter

S_{od} : Shoulder Outside Diameter

G_{id} : Gasket Internal Diameter

t_g : Gasket Thickness

CHAPTER I

INTRODUCTION AND LITERATURE REVIEW

Gasketed flange joints have been widely used in industry and have been the subject of detailed research for many centuries. The most significant contribution is by Waters et al. [1], for comprehensive flange design and became the basis of the well-known Taylor Forge method, which involve modeling of the joint elements using simplified plate and shell theory with known boundary conditions, and then combining the elements to derive stresses in various parts. Wide acceptance and the relative simplicity in its application has made the Taylor Forge method the most widely used flange design technique and is the basis of BS 5500 [2], ASME VIII [3] and many other codes. Murray and Stuart [4] performed flange analysis for taper hub flange, removing many of the assumptions of Water's model.

A number of numerical studies are available for the gasketed flange assembly for internal pressure loading [5, 6]. Extensive experimental studies for the flange assembly for

combined internal pressure, axial and bending loading were performed by Abid et al. [7] to observe a joint's behavior. From these studies, the failure of the gasketed flange joint is highlighted both in terms of its strength and sealing, even during bolt up conditions, which become worse for operating conditions.

To cost effectively accomplish the design task, we need to formally quantify the effect of uncertainties (variables) in the design. Probabilistic design is one effective method to formally quantify the effect of uncertainties. In the present study, a probabilistic analysis is presented for the influence of measurement accuracy on the random variables for Maximum Stress values and Von Mises stresses from a Flange Assembly. Small perturbation approach is used for the finite element methods to compute the sensitivity of the response to small fluctuations of the random variables present. The result is a parametric representation of the response in terms of a set of random variables with known statistical properties, which can be used to estimate the characteristics of the selected response variables such as stresses, heat transfer rate, pressure or temperature at a given point. Thanks to today's technological advancements in computers and software development, such engineering problems could be solved in a matter of hours. In this study, SolidWorks and SW Simulation were used for this analysis.

1.1. SOLIDWORKS

SolidWorks is a three dimensional mechanical CAD (computer-aided design) program used to design parts and assemblies. It runs on Microsoft Windows. SolidWorks is a Parasolid-based solid modeler, and utilizes a parametric feature-based approach to create models and assemblies.

1.2. SW SIMULATION

SolidWorks Simulation is a Finite Element Analysis software package, an addendum to SolidWorks Premium package that is a design validation tool used to show engineers how their designs will behave as physical objects. SolidWorks Simulation offers a wide spectrum of specialized analysis tools to help engineers virtually test and analyze complicated parts and assemblies. It is widely used to obtain solutions to large engineering problems in the static and dynamic structural analysis, heat transfer, and fluid flow.

1.3. NESTEM

NESTEM is a modular software program developed by NASA to perform structural and thermal probabilistic analyses on components and systems. It contains state of the art algorithms to compute probabilistic responses to engineering problems. Calculation and computations were performed in SW Simulation and results were entered into NESTEM. The program uses density functions and then propagates those functions on the model to yield uncertainty outputs, which could be related to the modes of failure of the model. The use of this program was possible through NASA Glenn Research Center in Cleveland, Ohio, and through Dr. R.S.R. Gorla.

CHAPTER II

MODEL AND PROBLEM APPROACH

The flange assembly in question is a conventional gasketed flange attached to a pipe of the same material and bolted to the other mirrored flange with a gasket in between them. The material for the flange and the pipe is ASTM A105, the bolt is ASTM A193 and the gasket is ASTM A182. The bolt-up pressure is 254 MPa. The flanges are free to move in either the axial or the radial direction. This provides flange rotation and the exact behavior of stress in the flange, bolt and gasket. Symmetry conditions are applied to the gasket lower portion, both sides of the gasket, bolt cross-sectional area, both sides of the flange ring and the attached pipe. Bolts are constrained in the radial and tangential directions. A nominal pre-load of about 35% (254 MPa) of the yield strength of the bolt (723 MPa) is chosen as per the achieved maximum strain in the bolt at the applied torque of 505 N.m, Abid et al. [5,7,13]. The associated ASME code does not specify the magnitude of pre-load for the bolts, only a minimum seating stress that relates to the

gasket style and composition. There is a heated fluid flowing inside the assembly with an internal pressure of 15.4 MPa. The temperature of the fluid is 100°C and the ambient temperature is 20°C. The internal heat transfer coefficient (h_i) is 150 W/m²per°C while the outside heat transfer coefficient (h_∞) is 20 W/m²per°C. The Young's modulus, Poisson ratio and other structural and thermal properties are used from Abid [7] for the materials specified.

Table I: Random Variable Mean Values

Variable	Mean Value
Material Properties:	
Flange:	
E_f : Modulus of Elasticity	1.73058E+05 N/mm ²
ν_f : Poisson's Ratio	0.3
K_f : Thermal Conductivity	47 W/m-°C
α_f : Coefficient of Thermal Expansion	1.25E-05 m/m-°C
ρ_f : Mass Density	7861 kg/m ³
C_{p_f} : Specific Heat	447.988 J/kg°C
Bolt:	
E_b : Modulus of Elasticity	1.68922E+05 N/mm ²
ν_b : Poisson's Ratio	0.3
K_b : Thermal Conductivity	37 W/m-°C
α_b : Coefficient of Thermal Expansion	1.41E-05 m/m-°C
ρ_b : Mass Density	7900 kg/m ³
C_{p_b} : Specific Heat	460 J/kg°C
Gasket:	
E_g : Modulus of Elasticity	1.64095E+05 N/mm ²
ν_g : Poisson's Ratio	0.3
K_g : Thermal Conductivity	20 W/m-°C
α_g : Coefficient of Thermal Expansion	3.00E-06 m/m-°C
ρ_g : Mass Density	7817 kg/m ³
C_{p_g} : Specific Heat	461 J/kg°C
Pressure:	
P_i : Internal Fluid Pressure	15.3 MPa
P_b : Bolt-up Pressure	254 MPa
Boundary Conditions:	
h_i : Internal Heat Transfer Coefficient	150 W/m ² per°C

h_{∞} : Ambient Heat Transfer Coefficient	20 W/m ² per°C
T_i : Internal Fluid Temperature	100°C
T_{∞} : Ambient Temperature	20°C
Material Physical Properties:	
t_p : Pipe Thickness	13.5 mm
h_h : Hub Height	69.9 mm
h_f : Flange Height	44.4 mm
h_s : Shoulder Height	6.4 mm
F_{id} : Flange Internal Diameter	87.3 mm
h_p : Pipe Height	271.67 mm
F_{od} : Flange Outside Diameter	292 mm
S_{od} : Shoulder Outside Diameter	157.2 mm
G_{id} : Gasket Internal Diameter	106.43 mm
t_g : Gasket Thickness	4.5 mm

Figure 1: Solid Model of the Flange Assembly

The objective of the present work is to analyze, test and verify the area of maximum stresses in the flange joint. The work was based on the information provided by Abid M, Nash DH [5]. Deterministic analysis was performed using Finite Element Analysis (FEA) code SW Simulation 2010 to find the locations of maximum stress. The results obtained were verified with Abid M. [14] and were found to be in excellent agreement (as shown in Table II). A probabilistic analysis was performed using the NESSUS analysis software to find the structural reliability and critical design parameters.

CHAPTER III

PROCEDURE FOR FINITE ELEMENT ANALYSIS

Let us consider a three dimensional partial differential equation of the form

$$\frac{\partial}{\partial x} \left(K_x \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(K_y \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(K_z \frac{\partial T}{\partial z} \right) + P(x,y,z)T + Q(x,y,z) = 0$$

The above equation is valid over a volume V.

We assume that on a portion of the boundary S_1 , $T = T_0$

On the remainder of the boundary, labeled S_2 , the general derivative boundary condition

is specified in the form

$$\left(K_x \frac{\partial T}{\partial x} \right) n_x + \left(K_y \frac{\partial T}{\partial y} \right) n_y + \left(K_z \frac{\partial T}{\partial z} \right) n_z + \alpha(x,y,z) + \beta(x,y,z) = 0 \dots\dots\dots (1)$$

Here, n_x , n_y , and n_z are direction cosines of the outward normal to S_2 .

The form of the functional may be written as

$$\iiint_v \left[\frac{1}{2} K_x \left(\frac{\partial T}{\partial x} \right)^2 + \frac{1}{2} K_y \left(\frac{\partial T}{\partial y} \right)^2 + \frac{1}{2} K_z \left(\frac{\partial T}{\partial z} \right)^2 - \frac{1}{2} P T^2 - Q T \right] dv + \iint_{s_3} \left[\frac{1}{2} \alpha T^2 + \beta T \right] ds$$

..... (2)

For an element e, the functional is obtained from equation (2) as

$$I^e = \iiint_{v^{(e)}} \left[\frac{1}{2} (T)^T (D_x)^T K_x (D_x) (T) + \frac{1}{2} (T)^T (D_y)^T K_y (D_y) (T) + \frac{1}{2} (T)^T (D_z)^T K_z (D_z) (T) \right. \\ \left. - \left[\frac{1}{2} (T)^T (N)^T P (N) (T) - (T)^T (N)^T Q \right]^e dV \right. \\ \left. + \iint_{s_2^{(e)}} \left[\frac{1}{2} (T)^T (N)^T \alpha (N) (T) + (T)^T (N)^T \beta \right]^{(e)} dS_2 \right] \dots \dots \dots (3)$$

The interpolation function for a three-dimensional simplex element is given by

$$T^e = (N)(T)$$

$$\text{where } (N) = (N_i \ N_j \ N_k \ N_l), (T) = \begin{pmatrix} T_i \\ T_j \\ T_k \\ T_l \end{pmatrix}$$

Also, the derivatives are given as

$$\frac{\partial T^e}{\partial x} = (D_x) = \frac{1}{6V} (b_i \ b_j \ b_k \ b_l)$$

$$\frac{\partial T^e}{\partial y} = (D_y) = \frac{1}{6V} (c_i \ c_j \ c_k \ c_l)$$

$$\frac{\partial T^e}{\partial z} = (D_z) = \frac{1}{6V} (d_i \ d_j \ d_k \ d_l)$$

For a simplex three dimensional element, we have extremized the above functional with respect to the unknown nodal temperatures. The resultant element matrices are then obtained from the following relation:

$$\begin{Bmatrix} \frac{\partial I}{\partial T_i} \\ \frac{\partial I}{\partial T_j} \\ \frac{\partial I}{\partial T_k} \\ \frac{\partial I}{\partial T_l} \end{Bmatrix} = [B]^{(e)} \begin{Bmatrix} T_i \\ T_j \\ T_k \\ T_l \end{Bmatrix} - (C)^e \dots\dots\dots (4)$$

The element matrix $[B]^{(e)}$ may be written as

$$[B]^{(e)} = \frac{K_x}{36V} \begin{bmatrix} b_i b_i & b_i b_j & b_i b_k & b_i b_l \\ & b_j b_j & b_j b_k & b_j b_l \\ & & b_k b_k & b_k b_l \\ & & & b_l b_l \end{bmatrix}^{(e)} \quad (K_x \text{ matrix})$$

$$+ \frac{K_y}{36V} \begin{bmatrix} c_i c_i & c_i c_j & c_i c_k & c_i c_l \\ & c_j c_j & c_j c_k & c_j c_l \\ & & c_k c_k & c_k c_l \\ & & & c_l c_l \end{bmatrix}^{(e)} \quad (K_y \text{ matrix})$$

$$+ \frac{K_z}{36V} \begin{bmatrix} d_i d_i & d_i d_j & d_i d_k & d_i d_l \\ & d_j d_j & d_j d_k & d_j d_l \\ & & d_k d_k & d_k d_l \\ & & & d_l d_l \end{bmatrix}^{(e)} \quad (K_z \text{ matrix})$$

$$- \frac{PV}{20} \begin{bmatrix} 2 & 1 & 1 & 1 \\ & 2 & 1 & 1 \\ & & 2 & 1 \\ & & & 2 \end{bmatrix}^{(e)} \quad (P \text{ matrix})$$

$$\begin{aligned}
& + \frac{\alpha A_{ijk}}{12} \begin{bmatrix} 2 & 1 & 1 & 0 \\ & 2 & 1 & 0 \\ & & 2 & 1 \\ & & & 2 \end{bmatrix}^{(e)} & (\alpha \text{ matrix } ijk) \\
& + \frac{\alpha A_{jkl}}{12} \begin{bmatrix} 0 & 0 & 0 & 0 \\ & 2 & 1 & 1 \\ & & 2 & 1 \\ & & & 2 \end{bmatrix}^{(e)} & (\alpha \text{ matrix } jkl) \\
& + \frac{\alpha A_{kli}}{12} \begin{bmatrix} 2 & 0 & 1 & 1 \\ & 0 & 0 & 0 \\ & & 2 & 1 \\ & & & 2 \end{bmatrix}^{(e)} & (\alpha \text{ matrix } kli) \\
& + \frac{\alpha A_{lij}}{12} \begin{bmatrix} 2 & 1 & 0 & 1 \\ & 2 & 0 & 1 \\ & & 0 & 0 \\ & & & 2 \end{bmatrix}^{(e)} & (\alpha \text{ matrix } lij) \dots\dots\dots (5)
\end{aligned}$$

Similarly, the element column $(C)^e$ can be written as

$$\begin{aligned}
(C)^e &= \frac{QV}{4} \begin{Bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{Bmatrix}^{(e)} & (\text{element } Q \text{ column}) \\
& - \frac{\beta A_{ijk}}{3} \begin{Bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{Bmatrix}^{(e)} & (\beta \text{ column } ijk) \\
& - \frac{\beta A_{jkl}}{3} \begin{Bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{Bmatrix}^{(e)} & (\beta \text{ column } jkl)
\end{aligned}$$

$$\begin{aligned}
& -\frac{\beta A_{kli}}{3} \begin{Bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{Bmatrix}^{(e)} && (\beta \text{ column } kli) \\
& -\frac{\beta A_{lij}}{3} \begin{Bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{Bmatrix}^{(e)} && (\beta \text{ column } lij) \dots\dots\dots (6)
\end{aligned}$$

Therefore, using equations (3), (4), (5), and (6),

The element matrix e in its general form may be written as

$$\begin{aligned}
[B]^{(e)} = & \iiint_{V^e} \left[(D_x)^T K_x (D_x) + (D_y)^T K_y (D_y) + (D_z)^T K_z (D_z) - (N)^T P(N) \right]^e dA \\
& + \iint_{S_2^{(e)}} \left[(G)^T \alpha(G) \right]^{(e)} dA \dots\dots\dots (7)
\end{aligned}$$

and the element column is

$$(C)^e = \iiint_{V^{(e)}} \left[(G)^T Q \right]^{(e)} dA - \iint_{S_2^{(e)}} \left[(G)^T \beta \right]^{(e)} dL_2 \dots\dots\dots (8)$$

THERMAL STRESS

If the distribution of the change in temperature $\Delta T(x, y)$ is known, the strain due to this change in temperature can be treated as an initial strain ε_0 . From the theory of mechanics of solids, ε_0 for plane stress can be represented by

$$\varepsilon_0 = (\alpha\Delta T, \alpha\Delta T, 0)^T \dots\dots\dots (1)$$

and the plane strain is given by

$$\varepsilon_0 = (1 + \nu) (\alpha\Delta T, \alpha\Delta T, 0)^T \dots\dots\dots (2)$$

The stresses and strains are related by

$$\sigma = D (\varepsilon - \varepsilon_0) \dots\dots\dots (3)$$

Where D is the symmetric (6X6) material matrix given by

$$D = \frac{E}{(1 + \nu)(1 - 2\nu)} \begin{bmatrix} 1 - \nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1 - \nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1 - \nu & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 - \nu & 0 & 0 \\ 0 & 0 & 0 & 0.5 - \nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5 - \nu \end{bmatrix} \dots\dots\dots (4)$$

The effect of temperature can be accounted for by considering the strain energy term.

$$\begin{aligned} U &= \frac{1}{2} \int (\varepsilon - \varepsilon_0)^T D (\varepsilon - \varepsilon_0) t dA \\ &= \frac{1}{2} \int (\varepsilon^T D \varepsilon - 2\varepsilon^T D \varepsilon_0 + \varepsilon_0^T D \varepsilon_0) t dA \dots\dots\dots (5) \end{aligned}$$

The first term in the previous expansion gives the stiffness matrix derived earlier. The last term is a constant, which has no effect on the minimization process. The middle term,

which yields the temperature load, is now considered in detail. Using the strain-displacement relationship $\varepsilon = Bq$,

$$\int_A \varepsilon^T D \varepsilon_0 t dA = \sum_e q^T (B^T D \varepsilon_0) t_e A_e \dots\dots\dots (6)$$

This step is obtained using the Galerkin approach where ε^T will be $\varepsilon^T(\phi)$ and q^T will be ψ^T . It is convenient to designate the element temperature load as

$$\theta^e = t_e A_e B^T D \varepsilon_0 \dots\dots\dots (7)$$

where,

$$\theta^e = [\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6]^T \dots\dots\dots (8)$$

The vector ε_0 is the strain in Equation (1) due to the average temperature change in the element. θ^e represents the element nodal load distributions that must be added to the global force vector. [10]

The stresses in an element are then obtained by using Equation (3) in the form

$$\sigma = D(Bq - \varepsilon_0) \dots\dots\dots (9)$$

CHAPTER IV

STRUCTURAL AND THERMAL ANALYSIS OF GASKETED FLANGE

Modeling and Analysis Procedure:

1. The geometry of the flange, gasket, and the bolts were modeled and assembled using SolidWorks 2010 as per the drawings provided by Abid M., Nash DH [5].
2. The FEA model of the assembly, as shown in Figure 3, was prepared in SW Simulation 2010.
3. The material, geometrical and thermal properties were obtained from Abid M. [9].
4. The boundary conditions were imposed.
5. The assembly was analyzed for the combined Von Mises Stress and the temperature distribution using the FEA Software.
6. Similarly, the assembly was analyzed for the combined Maximum stress and the temperature distribution.

Material properties, Input Pressures, Input Temperatures, Boundary Conditions, and Physical Dimensions were changed for each computational trial. Forty random variables were used to analyze the thermal stresses. These values were noted and given as input values in NESSUS to get the probabilistic results based on first order reliability method (FORM).

Figure 3: Meshed model of the Flange Assembly

Figure 4: Temperature distribution in the Flange Assembly

Figure 5: Sectional View of Temperature distribution

Figure 6: Sectional View of Von Mises Stress distribution

Figure 7: Sectional View of Maximum stress distribution

CHAPTER V

RESULTS AND DISCUSSION

The model was analyzed with appropriate set of boundary conditions and material properties to find the temperature distribution. This is required to evaluate the thermal stresses in structural analysis.

The temperature distribution is shown in Figures 4 and 5. The thermal stresses were analyzed from this temperature distribution (by linking it to the thermal analysis). Mechanical stresses were determined from the internal pressure and thermal effects. Maximum Von Mises Stress of 253.2 MPa and Maximum Stress of 282.4 MPa were found on the hub flange fillet and on the shoulder just below the flange-shoulder intersection. Table II represents the comparison between the deterministic analysis values with the literature data values for the temperature and stress distribution are they are in an excellent agreement.

Table II: Comparison of deterministic results with literature data

Temperature/Stress Value	Deterministic Analysis Value	Literature Data Value
Temperature Distribution	57.1°C to 88.5°C	57.2°C to 88.2°C
Maximum Stress Value	282.4 MPa	286 MPa

The probabilistic analysis was carried out based on the first order reliability method (FORM). Forty random variables were used to compute maximum stresses on various probability levels for each of the two stresses viz. Von Mises Stress and Maximum stress. Variation on Stresses due to the variation of each of the 40 random variables is shown in Table III & Table IV.

Cumulative distribution functions (CDF) and the sensitivity factors were developed by applying First order reliability method through NESSUS. Cumulative distribution functions for Maximum Von Mises Stress & Maximum stress are shown in Figure 8 & Figure 20. An important aspect of the FORM is the probabilistic sensitivity factors which help to identify the variables that contribute most of the reliability of the design. Probabilistic sensitivity factors include the sensitivity of p (Probability levels) with respect to a change in the mean value or the standard deviation of each random variable. Table I shows the random variables and their mean values used for the probabilistic analysis. All random variables were assumed to be independent. A scatter of $\pm 10\%$ was specified for all the random variables. This variation amounted for two standard deviations. Normal distribution was assumed for all random variables. Maximum stress location was determined from a pre-analysis of the flange. This location was used to evaluate the cumulative distribution functions (CDF) and the sensitivity factors for stress

response. Von Mises Stress & Maximum stress distributions in the flange are shown in Figure 6 & Figure 7. The normalized sensitivities for each of the Von Mises Stresses as a response are shown in the Figures from 9 to 19. Similarly, the normalized sensitivities for each of the Maximum stresses as a response are shown in the Figures from 21 to 31. We observe that the overall model thickness, flange height, shoulder height, gasket inner diameter and bolt pressure have a huge influence on the maximum stresses followed closely by flange outside diameter, shoulder outside diameter, thermal conductivity, Poisson's ratio, tensile strength and yield strength.

Table III: Maximum Von Mises Stress Values

Variable	Variation	Max. Von Mises Stress (MPa)
Material Properties:		
Flange:		
E_f : Modulus of Elasticity	(+)10%	251.3
	(-)10%	255.2
ν_f : Poisson's Ratio	(+)10%	248.4
	(-)10%	258.0
K_f : Thermal Conductivity	(+)10%	258.1
	(-)10%	248.4
α_f : Coefficient of Thermal Expansion	(+)10%	252.1
	(-)10%	254.3
ρ_f : Mass Density	(+)10%	253.2
	(-)10%	253.2
C_{p_f} : Specific Heat	(+)10%	252.0
	(-)10%	254.2
Bolt:		
E_b : Modulus of Elasticity	(+)10%	253.6
	(-)10%	252.8
ν_b : Poisson's Ratio	(+)10%	253.5
	(-)10%	253.0
K_b : Thermal Conductivity	(+)10%	253.9
	(-)10%	252.5
α_b : Coefficient of Thermal Expansion	(+)10%	253.7
	(-)10%	252.7
ρ_b : Mass Density	(+)10%	253.6
	(-)10%	252.8
C_{p_b} : Specific Heat	(+)10%	253.6
	(-)10%	252.9
Gasket:		
E_g : Modulus of Elasticity	(+)10%	253.9
	(-)10%	252.5

ν_g : Poisson's Ratio	(+)10%	254.4
	(-)10%	252.0
K_g : Thermal Conductivity	(+)10%	253.2
	(-)10%	253.2
α_g : Coefficient of Thermal Expansion	(+)10%	253.6
	(-)10%	252.8
ρ_g : Mass Density	(+)10%	253.2
	(-)10%	253.2
C_{p_g} : Specific Heat	(+)10%	253.2
	(-)10%	253.2
Pressure:		
P_i : Internal Fluid Pressure	(+)10%	251.0
	(-)10%	255.5
P_b : Bolt-up Pressure	(+)10%	277.4
	(-)10%	229.2
Boundary Conditions:		
h_i : Internal Heat Transfer Coefficient	(+)10%	250.4
	(-)10%	256.0
h_∞ : Ambient Heat Transfer Coefficient	(+)10%	256.1
	(-)10%	250.3
T_i : Internal Fluid Temperature	(+)10%	250.9
	(-)10%	255.6
T_∞ : Ambient Temperature	(+)10%	254.9
	(-)10%	251.6
Material Physical Properties:		
t_p : Pipe Thickness	(+)10%	241.6
	(-)10%	264.7
h_h : Hub Height	(+)10%	248.7
	(-)10%	259.0
h_f : Flange Height	(+)10%	236.9
	(-)10%	269.8

h_s : Shoulder Height	(+)10%	237.2
	(-)10%	269.3
F_{id} : Flange Internal Diameter	(+)10%	255.1
	(-)10%	251.3
h_p : Pipe Height	(+)10%	253.2
	(-)10%	253.2
F_{od} : Flange Outside Diameter	(+)10%	246.5
	(-)10%	260.0
S_{od} : Shoulder Outside Diameter	(+)10%	247.7
	(-)10%	258.7
G_{id} : Gasket Internal Diameter	(+)10%	264.3
	(-)10%	242.1
t_g : Gasket Thickness	(+)10%	251.2
	(-)10%	255.2

Table IV: Maximum Stress Values

Variable	Variation	Max. Stress (MPa)
Material Properties:		
Flange:		
E_f : Modulus of Elasticity	(+)10%	280.8
	(-)10%	284.0
ν_f : Poisson's Ratio	(+)10%	278.6
	(-)10%	286.2
K_f : Thermal Conductivity	(+)10%	286.7
	(-)10%	278.0
α_f : Coefficient of Thermal Expansion	(+)10%	279.0
	(-)10%	285.9
ρ_f : Mass Density	(+)10%	277.1
	(-)10%	289.1
C_{p_f} : Specific Heat	(+)10%	278.5
	(-)10%	286.3
Bolt:		
E_b : Modulus of Elasticity	(+)10%	282.6
	(-)10%	282.2
ν_b : Poisson's Ratio	(+)10%	282.6
	(-)10%	282.2
K_b : Thermal Conductivity	(+)10%	282.9
	(-)10%	281.9
α_b : Coefficient of Thermal Expansion	(+)10%	282.8
	(-)10%	282.1
ρ_b : Mass Density	(+)10%	282.6
	(-)10%	282.2
C_{p_b} : Specific Heat	(+)10%	282.7
	(-)10%	282.2
Gasket:		
E_g : Modulus of Elasticity	(+)10%	283.0
	(-)10%	281.8

v_g : Poisson's Ratio	(+)10%	283.6
	(-)10%	281.2
K_g : Thermal Conductivity	(+)10%	282.4
	(-)10%	282.4
α_g : Coefficient of Thermal Expansion	(+)10%	282.9
	(-)10%	281.8
ρ_g : Mass Density	(+)10%	282.4
	(-)10%	282.4
C_{p_g} : Specific Heat	(+)10%	282.4
	(-)10%	282.4
Pressure:		
P_i : Internal Fluid Pressure	(+)10%	280.0
	(-)10%	284.8
P_b : Bolt-up Pressure	(+)10%	309.7
	(-)10%	255.0
Boundary Conditions:		
h_i : Internal Heat Transfer Coefficient	(+)10%	279.9
	(-)10%	285.9
h_∞ : Ambient Heat Transfer Coefficient	(+)10%	285.0
	(-)10%	279.8
T_i : Internal Fluid Temperature	(+)10%	278.4
	(-)10%	286.4
T_∞ : Ambient Temperature	(+)10%	284.0
	(-)10%	280.7
Material Physical Properties:		
t_p : Pipe Thickness	(+)10%	273.3
	(-)10%	291.5
h_h : Hub Height	(+)10%	277.9
	(-)10%	287.9
h_f : Flange Height	(+)10%	263.7
	(-)10%	300.9

h_s : Shoulder Height	(+)10%	264.7
	(-)10%	309.1
F_{id} : Flange Internal Diameter	(+)10%	283.3
	(-)10%	281.5
h_p : Pipe Height	(+)10%	282.4
	(-)10%	282.4
F_{od} : Flange Outside Diameter	(+)10%	277.4
	(-)10%	287.4
S_{od} : Shoulder Outside Diameter	(+)10%	278.9
	(-)10%	285.9
G_{id} : Gasket Internal Diameter	(+)10%	291.6
	(-)10%	273.2
t_g : Gasket Thickness	(+)10%	279.9
	(-)10%	284.9

Figure 8: Cumulative Probability of Von Mises Stress

Figure 9: Sensitivity Factors for 0.001 Probability

Figure 10: Sensitivity Factors for 0.01 Probability

Figure 11: Sensitivity Factors for 0.1 Probability

Figure 12: Sensitivity Factors for 0.2 Probability

Figure 13: Sensitivity Factors for 0.4 Probability

Figure 14: Sensitivity Factors for 0.6 Probability

Figure 15: Sensitivity Factors for 0.8 Probability

Figure 16: Sensitivity Factors for 0.9 Probability

Figure 17: Sensitivity Factors for 0.95 Probability

Figure 18: Sensitivity Factors for 0.99 Probability

Figure 19: Sensitivity Factors for 0.999 Probability

Figure 20: Cumulative Probability of Maximum stress

Figure 21: Sensitivity Factors for 0.001 Probability

Figure 22: Sensitivity Factors for 0.01 Probability

Figure 23: Sensitivity Factors for 0.1 Probability

Figure 24: Sensitivity Factors for 0.2 Probability

Figure 25: Sensitivity Factors for 0.4 Probability

Figure 26: Sensitivity Factors for 0.6 Probability

Figure 27: Sensitivity Factors for 0.8 Probability

Figure 28: Sensitivity Factors for 0.9 Probability

Figure 29: Sensitivity Factors for 0.95 Probability

Figure 30: Sensitivity Factors for 0.99 Probability

Figure 31: Sensitivity Factors for 0.999 Probability

CHAPTER VI

CONCLUSION

A robust design is the one that has been created with a system of design tools that reduce product or process variability while guiding the performance towards an optimal setting. Robustness means achieving excellent performance under a wide range of operating conditions. All engineering systems function reasonably well under ideal conditions, but robust design continues to function well when the conditions are not ideal. Analytical robust design attempts to determine the values of design parameters, which maximize the reliability of the product without tightening the material or environmental tolerances. Probabilistic design and robust design go hand in hand. In order to determine the influence of scatter and uncertainties, the system has to be analyzed probabilistically.

In this thesis, deterministic method was supported by non-deterministic approach to find out the parameters responsible for the development of stresses. The novelty in this approach is the probabilistic evaluation of the finite element solution for the design

problem. Cumulative distribution functions (CDF) and sensitivity factors were computed for Von Mises Stresses and Maximum stresses generated due to random variables in the design.

In this thesis, our goals were to first check for the area of maximum stresses and then carry out the probabilistic analysis to investigate the effect of random variables on the stresses. Probabilistic analysis helps the designer to choose a suitable material and geometry from the available set of parameters. The cumulative distribution functions (CDF) completely describes the probability distribution of a real-valued random variable. In our case it is shown in Figure 8 & Figure 20 where X-axis shows the stress and Y-axis the cumulative probability in each case. Figures 9 to 19 and 21 to 31 show the effect of random variables on the design for various probability levels. The X-axis represents the random variable and the Y-axis represents the sensitivity factor. Sensitivity factors provide information about the relative importance of the variability or uncertainty of all of the input variables and their influence on the resulting fatigue life. These are the parameters the designer will be looking for designing an optimum reliability based design. Slight alteration those will have a significant impact on the performance of the design in practical conditions.

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