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Finite-time anti-synchronization of memristive stochastic BAM neural networks with probabilistic time-varying delays

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ABSTRACT

This paper investigates the drive-response finite-time anti-synchronization for memristive bidirectional associative memory neural networks (MBAMNNs). Firstly, a class of MBAMNNs with mixed probabilistic time-varying delays and stochastic perturbations is first formulated and analyzed in this paper. Secondly, an nonlinear control law is constructed and utilized to guarantee drive-response finite-time anti-synchronization of the neural networks. Thirdly, by employing some inequality technique and constructing an appropriate Lyapunov function, some anti-synchronization criteria are derived. Finally, a number simulation is provided to demonstrate the effectiveness of the proposed mechanism.

1. Introduction

In the past decades, a large amount of attention has been devoted to the bidirectional associative memory neural networks (BAMNNs) owing to their potential applications from signal processing, pattern recognition, associative memory, and so on [1–5]. Nowadays, with the increasing amount of data [6,7] and complex neural networks (NNs) [8,9], scientists conceive that we can get a new type of BAMNNs named MBAMNNs, which the self-feedback connection weights are implemented by memristors [10–16] rather than resistances. From the system theoretic point of view, MBAMNNs can be treated as a class of state-depend nonlinear system [17], and it is a challenging topic. With the research and development of memristor, plenty of researchers from various fields show skyscraping enthusiasm to this kind of memristive neural networks (MNNs) [18–22]. The MBAMNNs have remarkable properties such as satisfactory convergence rate, computer implementation, a large number of equilibrium points, and so on [23]. Therefore, this type of model can better emulate the human brain than the traditional neural networks.

Synchronization, which means the dynamical signals of chaotic coupled system achieve an identical behavior with time moving. In reality, it is significant to consider the synchronization of its different potential applications including biological systems, intelligent control, secure communication, and image protection. As a typical collective behavior, the stability and synchronization of MNNs have been widely discussed, including lag synchronization [24], exponential synchronization, anti-synchronization [25–28], finite time synchronization [29,30], and so on. Recently, chaotic synchronization of MBAMNNs has gained much attention due to its successful applications in various areas [31–34].

However, in addition to this result, there are few results on the anti-synchronization for MBAMNNs. Additionally, among these synchronization works of MBAMNNs, most are asymptotic, implying the stability or synchronization of chaotic systems can be accomplished only when time towards to infinity. But from the point of practical, owing to the life span of human and machine [35], it is more pressing to achieve synchronization within the finite time, that is, finite time synchronization [37–40]. Besides, finite-time synchronization can illustrates the faster synchronous rate after a finite time-interval named setting time. Therefore, it is more practical and valuable to investigate the finite-time anti-synchronization control of the MBAMNNs.

In the process of studying MBAMNNs, it is detected that the delays frequently appears owing to the limited transfer speed and the information processing. Actually, in the electronic implementation of analog NNs, time delay frequently occurring in the response and communication of neurons. And the time delay can lead to a series of questions, to a certain extent, affected the instability and oscillation to the NNs [41–44]. In the view of this, it is rewarding and reasonable to study the delayed NNs [45–49]. Nevertheless, time delays often occurring in a random way in the view of probabilistic reasons. This often occurs in real systems where the probability to taking vary large values of delay is very small [50].

Under these situations, probabilistic measurement delays would be regarded as a Bernoulli distributed white sequence [51] to depict more property of the real systems. Furthermore, it should not be neglected that in a real nervous system, there is a typical time delay named leakage delay which has tendency to impact on the dynamical behavior of NNs [52–54]. From the above discussions, it is significant and necessary to investigate some reasonable and practical systems for MBAMNNs. The actual communication between NNs is inevitably disturbed by a stochastic perturbation. The stochastic perturbation mainly comes from various uncertainties which probably results in package losses or influences the signal transmission. Hence, it is important to discuss the effect of probabilistic delays and stochastic perturbations. It should be mentioned that, the finite-time anti-synchronization results for stochastic MBAMNNs with probabilistic time-varying delays has not been studied yet, this motivates our present study.

Motivated by the aforementioned concerns, the aim of this paper is to investigate the finite-time anti-synchronization results for stochastic MBAMNNs with probabilistic time-varying delays. With the aid of the set-valued map, memristor mathematical model, differential inclusion, linear feedback controller, adaptive linear feedback controller, and the definition of anti- synchronization, two new sufficient criteria are derived to guarantee the finite-time anti-synchronization of MMAMNNs with mixed probabilistic time-varying delay. The main contributions of this paper can be summarized as follows.

1. We focus on the study of MBAMNNs models with stochastic perturbation and various time-varying delays, which including non-delay, discrete time-varying delays and a constant delay in the leakage term. Many other MBAMNNs models with delays are the special cases of our considered model.
2. We first attempt to address the finite-time anti-synchronization control problem for a class of proposed MBAMNNs models. By utilizing sign function and the definition of finite-time stability, a suitable nonlinear state feedback controller is designed. We consider and analysis the complex randomness of the time-varying delays rather than treat them as the same stability. Some main results are derived by utilizing the Lyapunov function, finite-time stability theorem, stochastic analysis theory and Wirtinger-type inequality.
3. Finally, we provided the numerical examples to illustrated the effectiveness and rationality of the proposed conclusions.

The rest of this paper is organized as follows. Some definitions, lemmas and assumptions about the proposed model are presented in Section 2. In Section 3 derives some sufficient conditions of finite time anti-synchronization based our considered MBAMNNs. Numerical simulations are demonstrated to verify the effectiveness of the obtained results in Section 4. Finally, the conclusion is given in Section 5.

Notations. For $r>0$, $\mathbf{C}([-r,0],\mathbb{R}^n)$ denotes the Banach space of all continuous functions mapping $[-r,0]$ into $\mathbb{R}^n$ with q-norm or ∞-norm by the following forms, respectively. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. The superscript T represents matrix or vector transposition. We define the norm of the vector as $||x_i||$ indicates the 2-norm of a vector x_i, i.e., $||x_i|| = (\sum_{i=1}^{n} x_i^2)^{\frac{1}{2}}$. $co[a, b]$ denotes the convex hull (closure) of $\{a, b\}$.

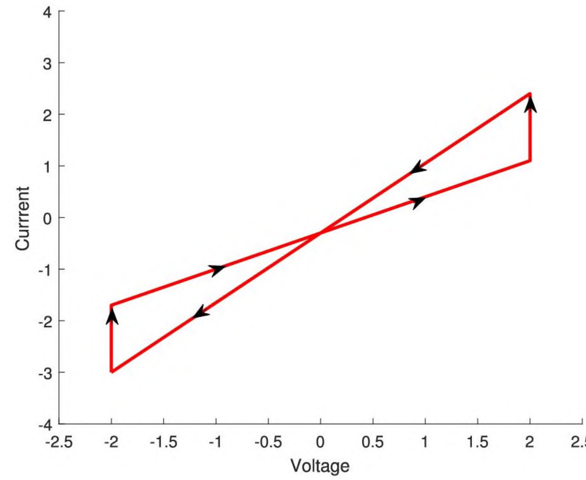


Fig. 1. Typical current-voltage characteristic of a memristor.

2. Model description and preliminaries

Based on the physical properties of memristor, a class of stochastic MBAMNNs with time-varying delays is described as follows

$$\dot{x}_i(t) = -a_i x_i(t-\tau) + \sum_{j=1}^{m} b_{ji}(x_i(t)) f_j(y_j(t)) + \sum_{j=1}^{m} c_{ji}(x_i(t-\tau(t))) f_j(y_j(t-\tau(t))), \tag{1}$$

$$\dot{y}_j(t) = -d_j y_j(t-\sigma) + \sum_{i=1}^{n} q_{ij}(y_j(t)) g_i(x_i(t)) + \sum_{i=1}^{n} p_{ij}(y_j(t-\sigma(t))) g_i(x_i(t-\sigma(t))),$$

where $x_i(t)$ and $y_j(t)$ denote the voltages of capacitor C_i and $\tilde{C}_j$ at time t, for $t\geq 0$ and $i=1,2,\ldots,n$ and $j=1,2,\ldots,m$. $a_i>0$ and $d_j>0$ represent the self-feedback connection weight. $f_j(y_j(t))$ and $g_i(x_i(t))$ are the feedback functions. $\tau(t)$ and $\sigma(t)$ are the time-varying delays which satisfied $0\leq\tau(t)\leq\tau_2$ and $0\leq\sigma(t)\leq\sigma_2$. $b_{ji}(x_i(t))$, $c_{ji}(x_i(t-\tau(t)))$, $q_{ij}(y_j(t))$ and $p_{ij}(y_j(t-\sigma(t)))$ represent memristor-based weights, and $b_{ji}(x_i(t)) = \frac{W_{(1)ji}}{\tilde{C}_j} \times \mathrm{sgin}_{ji}$, $c_{ji}(x_i(t-\tau(t))) = \frac{W_{(2)ji}}{\tilde{C}_j} \times \mathrm{sgin}_{ji}$, $q_{ij}(y_j(t)) = \frac{W_{(3)ij}}{C_i} \times \mathrm{sgin}_{ij}$, $p_{ij}(y_j(t-\sigma(t))) = \frac{W_{(4)ij}}{C_i} \times \mathrm{sgin}_{ij}$.

$$\mathrm{sgin}_{ij} = \begin{cases} 1, i \neq j, \\ -1, i = j, \end{cases} \quad \mathrm{sgin}_{ji} = \begin{cases} 1, j \neq i, \\ -1, j = i, \end{cases}$$

then $W_{(1)ji}$, $W_{(2)ji}$, $W_{(3)ij}$ and $W_{(4)ij}$ denote the memductance of memristors $R_{(1)ji}$, $R_{(2)ji}$, $R_{(3)ij}$ and $R_{(4)ij}$, respectively. In addition, $R_{(1)ji}$ presents the memristor between the feedback function $f_j(y_j(t))$ and $y_j(t)$, $R_{(2)ji}$ presents the memristor between the feedback function $f_j(y_j(t-\tau(t)))$ and $y_j(t-\tau(t))$, $R_{(3)ij}$ presents the memristor between the feedback function $g_i(x_i(t))$ and $x_i(t)$, $R_{(4)ij}$ presents the memristor between the feedback function $g_i(x_i(t-\sigma(t)))$ and $x_i(t-\sigma(t))$.

According to the feature of memristor and the current-voltage characteristic, Fig. 1 can illustrates the simplification current characteristic of a memristor. And we apply the state-dependent parameters of the system (1) are satisfy the following conditions

[51]:

$$b_{ji}(x_i(t)) = \begin{cases} \hat{b}_{ji}, & |x_i(t)| < \varpi_i, \\ \text{unchanged}, & |x_i(t)| = \varpi_i, \\ \check{b}_{ji}, & |x_i(t)| > \varpi_i, \end{cases}$$

$$q_{ij}(y_j(t)) = \begin{cases} \hat{q}_{ij}, & |y_j(t)| < \vartheta_j, \\ \text{unchanged}, & |y_j(t)| = \vartheta_j, \\ \check{q}_{ij}, & |y_j(t)| > \vartheta_j, \end{cases}$$

$$c_{ji}(x_i(t-\tau(t))) = \begin{cases} \hat{c}_{ji}, & |x_i(t-\tau(t))| < \varpi_i, \\ \text{unchanged}, & |x_i(t-\tau(t))| = \varpi_i, \\ \check{c}_{ji}, & |x_i(t-\tau(t))| > \varpi_i, \end{cases}$$

$$p_{ij}(y_j(t-\sigma(t))) = \begin{cases} \hat{p}_{ij}, & |y_j(t-\sigma(t))| < \vartheta_j, \\ \text{unchanged}, & |y_j(t-\sigma(t))| = \vartheta_j, \\ \check{p}_{ij}, & |y_j(t-\sigma(t))| > \vartheta_j, \end{cases}$$

where ϖ, ϑ are nonnegative constants denoting the memristive switching jumps, and unchanged means that the memristance keeps the current value. $\hat{b}_{ji}$, $\check{b}_{ji}$, $\hat{c}_{ji}$, $\check{c}_{ji}$, $\hat{q}_{ij}$, $\check{q}_{ij}$, $\hat{p}_{ij}$, $\check{p}_{ij}$, are known constants relating to memristances.

Assuming system (1) as the drive system then the response system is:

$$\begin{aligned} d\hat{x}_i(t) = \Big[& -a_i\hat{x}_i(t-\tau) + \sum_{j=1}^{m} b_{ji}(\hat{x}_i(t)) f_j(\hat{y}_j(t)) \\ & + \sum_{j=1}^{m} c_{ji}(\hat{x}_i(t-\tau(t))) f_j(\hat{y}_j(t-\tau(t))) + u_i(t)\Big]dt \\ & + \sum_{j=1}^{m} \beta_{ji}(t, e_j(t), e_j(t-\tau(t))) d\omega_j(t), \\ d\hat{y}_j(t) = \Big[& -d_j\hat{y}_j(t-\sigma) + \sum_{i=1}^{n} q_{ij}(\hat{y}_j(t)) g_i(\hat{x}_i(t)) \\ & + \sum_{i=1}^{n} p_{ij}(\hat{y}_j(t-\sigma(t))) g_i(\hat{x}_i(t-\sigma(t))) + u_j(t)\Big]dt \\ & + \sum_{i=1}^{n} \beta_{ij}(t, e_i(t), e_i(t-\sigma(t))) d\omega_i(t), \end{aligned} \tag{2}$$

where $u_i(t)$ and $u_j(t)$ are the appropriate control inputs that will be designed, $\omega_i(t)$ and $\omega_j(t)$ are the Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$.

Remark 1. According to the discussions above, the inner connection matrixes $b_{ji}(x_i(t))$, $q_{ij}(y_j(t))$, $c_{ji}(x_i(t-\tau(t)))$, $p_{ij}(y_j(t-\sigma(t)))$ of system (1) and $b_{ji}(\hat{x}_i(t))$, $q_{ij}(\hat{y}_j(t))$, $c_{ji}(\hat{x}_i(t-\tau(t)))$, $p_{ij}(\hat{y}_j(t-\sigma(t)))$ of system (2) are all with the change of the memristance. Therefore, the proposed MBAMNNs are considered as the time-varying systems which depend on the state switching. When the inner connection matrixes are all constants, systems (1) and (2) become a general class of BAMNNs.

In this paper, the time-varying delays $\tau(t)$ and $\sigma(t)$ in system (1) are bounded with $0 \le \tau(t) \le \tau_2$ and $0 \le \sigma(t) \le \sigma_2$. In practice, there exist constants τ_1 and σ_1 with $0 \le \tau_1 \le \tau_2$ and $0 \le \sigma_1 \le \sigma_2$. Furthermore, the probability distribution of $\tau(t)$ and $\sigma(t)$ taking values in $[0, \tau_1]$, $(\tau_1, \tau_2]$, $[0, \sigma_1]$ and $(\sigma_1, \sigma_2]$ with certain probability, respectively.

The information of probability distribution of $\tau(t)$ and $\sigma(t)$ are defined as

$$\mathbb{P}\{\tau(t) \in [0, \tau_1]\} = \delta_0, \quad \mathbb{P}\{\tau(t) \in (\tau_1, \tau_2]\} = 1 - \delta_0,$$
$$\mathbb{P}\{\sigma(t) \in [0, \sigma_1]\} = \rho_0, \quad \mathbb{P}\{\sigma(t) \in (\sigma_1, \sigma_2]\} = 1 - \rho_0,$$

where $0 \le \delta_0 \le 1$ and $0 \le \rho_0 \le 1$. Then

$$\tau(t) = \begin{cases} \tau_1(t), & \tau(t) \in [0, \tau_1], \\ \tau_2(t), & \tau(t) \in (\tau_1, \tau_2], \end{cases}$$

$$\sigma(t) = \begin{cases} \sigma_1(t), & \sigma(t) \in [0, \sigma_1], \\ \sigma_2(t), & \sigma(t) \in (\sigma_1, \sigma_2], \end{cases}$$

and $\delta(t)$ and $\sigma(t)$ are the Bernoulli distributed sequence with

$$\mathbb{P}\{\delta(t) = 1\} = \mathbb{P}\{\tau(t) \in [0, \tau_1]\} = \mathbb{E}\{\delta(t)\} = \delta_0,$$
$$\mathbb{P}\{\delta(t) = 0\} = \mathbb{P}\{\tau(t) \in (\tau_1, \tau_2]\} = \mathbb{E}\{1 - \delta(t)\} = 1 - \delta_0,$$
$$\mathbb{P}\{\rho(t) = 1\} = \mathbb{P}\{\sigma(t) \in [0, \sigma_1]\} = \mathbb{E}\{\rho(t)\} = \rho_0,$$
$$\mathbb{P}\{\rho(t) = 0\} = \mathbb{P}\{\sigma(t) \in (\sigma_1, \sigma_2]\} = \mathbb{E}\{1 - \rho(t)\} = 1 - \rho_0.$$

Furthermore, four time-varying delays $\tau_1(t)$, $\tau_2(t)$, $\sigma_1(t)$ and $\sigma_2(t)$ are introduced, such that

$$\tau(t) = \begin{cases} \tau_1(t), & \tau(t) \in [0, \tau_1], \\ \tau_2(t), & \tau(t) \in (\tau_1, \tau_2], \end{cases}$$

$$\sigma(t) = \begin{cases} \sigma_1(t), & \sigma(t) \in [0, \sigma_1], \\ \sigma_2(t), & \sigma(t) \in (\sigma_1, \sigma_2]. \end{cases}$$

By employing the new functions $\tau_1(t)$, $\tau_2(t)$, $\sigma_1(t)$, $\sigma_2(t)$ and the stochastic variables $\delta(t)$, $\rho(t)$. Systems (1) and (2) can be rewritten under the theories of set-valued maps and differential inclusions above,

$$\begin{aligned} \dot{x}_i(t) \in & -a_i x_i(t-\tau) + \sum_{j=1}^{m} co[b_{ji}(x_i(t))] f_j(y_j(t)) \\ & + \delta(t) \sum_{j=1}^{m} co[c_{ji}(x_i(t-\tau(t)))] f_j(y_j(t-\tau_1(t))) \\ & + (1-\delta(t)) \sum_{j=1}^{m} co[c_{ji}(x_i(t-\tau(t)))] f_j(y_j(t-\tau_2(t))) \\ \dot{y}_j(t) \in & -d_j y_j(t-\sigma) + \sum_{i=1}^{n} co[q_{ij}(y_j(t))] g_i(x_i(t)) \\ & + \rho(t) \sum_{i=1}^{n} co[p_{ij}(y_j(t-\sigma(t)))] g_i(x_i(t-\sigma_1(t))) \\ & + (1-\rho(t)) \sum_{i=1}^{n} co[p_{ij}(y_j(t-\sigma(t)))] g_i(x_i(t-\sigma_2(t))). \end{aligned} \tag{3}$$

Remark 2. In this paper, the core of finite-time stability is that the system converges to a stable condition within a finite time. At present, the research on finite-time stability of the MBAMNNs with probabilistic time-varying delays and stochastic perturbation is very few. Thus, the proposed model is more general than some exciting results. A lot of models about stability of MBAMNNs are special cases of our considered model [31–34]. Therefore, the obtained results are much general and practical.

The corresponding response system can be described as

$$\begin{aligned} d\hat{x}_i(t) \in \Big[& -a_i\hat{x}_i(t-\tau) + \sum_{j=1}^{m} co[b_{ji}(\hat{x}_i(t))] f_j(\hat{y}_j(t)) \\ & + \delta(t) \sum_{j=1}^{m} co[c_{ji}(\hat{x}_i(t-\tau(t)))] f_j(\hat{y}_j(t-\tau_1(t))) \\ & + (1-\delta(t)) \sum_{j=1}^{m} co[c_{ji}(\hat{x}_i(t-\tau(t)))] f_j(\hat{y}_j(t-\tau_2(t))) + u_i(t)\Big]dt \\ & + \sum_{j=1}^{m} \beta_{ji}(t, e_j(t), e_j(t-\tau_1(t))) d\omega_j(t) \end{aligned} \tag{4}$$

$$+\sum_{j=1}^{m}\beta_{ji}(t,e_j(t),e_j(t-\tau_2(t)))d\omega_j(t),$$

$$d\hat{y}_j(t)\in\Big[-d_j\hat{y}_j(t-\sigma)+\sum_{i=1}^{n}co[q_{ij}(\hat{y}_j(t))]g_i(\hat{x}_i(t))$$
$$+\rho(t)\sum_{i=1}^{n}co[p_{ij}(\hat{y}_j(t-\sigma(t)))]g_i(\hat{x}_i(t-\sigma_1(t)))$$
$$+(1-\rho(t))\sum_{i=1}^{n}co[p_{ij}(\hat{y}_j(t-\sigma(t)))]g_i(\hat{x}_i(t-\sigma_2(t)))+u_j(t)\Big]dt$$
$$+\sum_{i=1}^{n}\beta_{ij}(t,e_i(t),e_i(t-\sigma_1(t)))d\omega_i(t)$$
$$+\sum_{i=1}^{n}\beta_{ij}(t,e_i(t),e_i(t-\sigma_2(t)))d\omega_i(t),$$

where

$$b_{ji}(x_i(t))=\begin{cases}\hat{b}_{ji}, & |x_i(t)|<\varpi_i,\\ co[\underline{b}_{ji},\bar{b}_{ji}], & |x_i(t)|=\varpi_i,\\ \check{b}_{ji}, & |x_i(t)|>\varpi_i,\end{cases}$$

$$b_{ji}(\hat{x}_i(t))=\begin{cases}\hat{b}_{ji}, & |\hat{x}_i(t)|<\varpi_i,\\ co[\underline{b}_{ji},\bar{b}_{ji}], & |\hat{x}_i(t)|=\varpi_i,\\ \check{b}_{ji}, & |\hat{x}_i(t)|>\varpi_i,\end{cases}$$

$$c_{ji}(x_i(t-\tau(t)))=\begin{cases}\hat{c}_{ji}, & |x_i(t-\tau(t))|<\varpi_i,\\ co[\underline{c}_{ji},\bar{c}_{ji}], & |x_i(t-\tau(t))|=\varpi_i,\\ \check{c}_{ji}, & |x_i(t-\tau(t))|>\varpi_i,\end{cases}$$

$$c_{ji}(\hat{x}_i(t-\tau(t)))=\begin{cases}\hat{c}_{ji}, & |\hat{x}_i(t-\tau(t))|<\varpi_i,\\ co[\underline{c}_{ji},\bar{c}_{ji}], & |\hat{x}_i(t-\tau(t))|=\varpi_i,\\ \check{c}_{ji}, & |\hat{x}_i(t-\tau(t))|>\varpi_i,\end{cases}$$

$$q_{ij}(y_j(t))=\begin{cases}\hat{q}_{ij}, & |y_j(t)|<\vartheta_j,\\ co[\underline{q}_{ij},\bar{q}_{ij}], & |y_j(t)|=\vartheta_j,\\ \check{q}_{ij}, & |y_j(t)|>\vartheta_j,\end{cases}$$

$$q_{ij}(\hat{y}_j(t))=\begin{cases}\hat{q}_{ij}, & |\hat{y}_j(t)|<\vartheta_j,\\ co[\underline{q}_{ij},\bar{q}_{ij}], & |\hat{y}_j(t)|=\vartheta_j,\\ \check{q}_{ij}, & |\hat{y}_j(t)|>\vartheta_j,\end{cases}$$

$$p_{ij}(y_j(t-\sigma(t)))=\begin{cases}\hat{p}_{ij}, & |y_j(t-\sigma(t))|<\vartheta_j,\\ co[\underline{p}_{ij},\bar{p}_{ij}], & |y_j(t-\sigma(t))|=\vartheta_j,\\ \check{p}_{ij}, & |y_j(t-\sigma(t))|>\vartheta_j.\end{cases}$$

$$p_{ij}(\hat{y}_j(t-\sigma(t)))=\begin{cases}\hat{p}_{ij}, & |\hat{y}_j(t-\sigma(t))|<\vartheta_j,\\ co[\underline{p}_{ij},\bar{p}_{ij}], & |\hat{y}_j(t-\sigma(t))|=\vartheta_j,\\ \check{p}_{ij}, & |\hat{y}_j(t-\sigma(t))|>\vartheta_j,\end{cases}$$

and

$$\bar{b}_{ji}=\max\{\hat{b}_{ji},\check{b}_{ji}\},\quad \underline{b}_{ji}=\min\{\hat{b}_{ji},\check{b}_{ji}\},$$
$$\bar{c}_{ji}=\max\{\hat{c}_{ji},\check{c}_{ji}\},\quad \underline{c}_{ji}=\min\{\hat{c}_{ji},\check{c}_{ji}\},$$
$$\bar{q}_{ij}=\max\{\hat{q}_{ij},\check{q}_{ij}\},\quad \underline{q}_{ij}=\min\{\hat{q}_{ij},\check{q}_{ij}\},$$
$$\bar{p}_{ij}=\max\{\hat{p}_{ij},\check{p}_{ij}\}.\quad \underline{p}_{ij}=\min\{\hat{p}_{ij},\check{p}_{ij}\}.$$

Let $e_i(t)$ and $e_j(t)$ denote the error variables, where $e_i(t)=x_i(t)+\hat{x}_i(t)$ and $e_j(t)=y_j(t)+\hat{y}_j(t)$. From the Definitions 1 to 2, we obtain the error systems as follows

$$de_i(t)\in\Big[-a_ie_i(t-\tau)$$
$$+\sum_{j=1}^{m}\Big[co[b_{ji}(x_i(t))]f_j(y_j(t))+co[b_{ji}(\hat{x}_i(t))]f_j(\hat{y}_j(t))\Big]$$
$$+\delta(t)\sum_{j=1}^{m}\Big[co[c_{ji}(x_i(t-\tau(t)))]f_j(y_j(t-\tau_1(t)))$$
$$+co[c_{ji}(\hat{x}_i(t-\tau(t)))]f_j(\hat{y}_j(t-\tau_1(t)))\Big]$$
$$+(1-\delta(t))\sum_{j=1}^{m}\Big[co[c_{ji}(x_i(t-\tau(t)))]f_j(y_j(t-\tau_2(t)))$$
$$+co[c_{ji}(\hat{x}_i(t-\tau(t)))]f_j(\hat{y}_j(t-\tau_2(t)))\Big]+u_i(t)\Big]dt$$
$$+\sum_{j=1}^{m}\beta_{ji}(t,e_j(t),e_j(t-\tau_1(t)))d\omega_j(t)$$
$$+\sum_{j=1}^{m}\beta_{ji}(t,e_j(t),e_j(t-\tau_2(t)))d\omega_j(t),\tag{5}$$

$$de_j(t)\in\Big[-d_je_j(t-\sigma)$$
$$+\sum_{i=1}^{n}\Big[co[q_{ij}(y_j(t))]g_i(x_i(t))+co[q_{ij}(\hat{y}_j(t))]g_i(\hat{x}_i(t))\Big]$$
$$+\rho(t)\sum_{i=1}^{n}\Big[co[p_{ij}(y_j(t-\sigma(t)))]g_i(x_i(t-\sigma_1(t)))$$
$$+co[p_{ij}(\hat{y}_j(t-\sigma(t)))]g_i(\hat{x}_i(t-\sigma_1(t)))\Big]$$
$$+(1-\rho(t))\sum_{i=1}^{n}\Big[co[p_{ij}(\hat{y}_j(t-\sigma(t)))]g_i(\hat{x}_i(t-\sigma_2(t)))$$
$$+co[p_{ij}(y_j(t-\sigma(t)))]g_i(x_i(t-\sigma_2(t)))\Big]+u_j(t)\Big]dt$$
$$+\sum_{i=1}^{n}\beta_{ij}(t,e_i(t),e_i(t-\sigma_1(t)))d\omega_i(t)$$
$$+\sum_{i=1}^{n}\beta_{ij}(t,e_i(t),e_i(t-\sigma_2(t)))d\omega_i(t).\tag{6}$$

Throughout this paper, we have the following Assumptions and Definitions.

Definition 1 [37]. Suppose $B\subset\Re^n$. Then $d\mapsto F(d)$ is called as a *set-valued* map defined on B, if for each point d of B, there exists a corresponding nonempty set $F(d)\subset\Re^n$. A set-valued map F with nonempty values is considered to be *upper-semicontinuous* at $d_0\epsilon B$, if for any open set N containing $F(d_0)$, there exists a neighborhood K of d_0 such that $F(K)\subset N$. $F(d)$ is said to have a *closed image* if for each $d\epsilon B$, $F(d)$ is closed.

Definition 2 [27]. For the system $\dot{x}(t)=f(x)$, $x\epsilon\mathbb{R}^n$, with discontinuous right-hand sides, a set-valued map is defined as

$$F(t,x)=\bigcap_{\delta>0}\bigcap_{\mu(N)=0}co[f(B(x,\delta)\setminus N)],$$

where $co[E]$ is the closure of the convex hull of set E, $B(x,\delta)=\{y:\|y-x\|\le\delta\}$ and $\mu(N)$ is Lebesgue measure of set N. A solution in Filippov's sense of Cauchy problem for this system with initial condition $x(0)=x_0$ is an absolutely continuous function $x(t)$, which satisfies $x(0)=x_0$ and differential inclusion $\dot{x}(t)\epsilon F(t,x)$.

Definition 3 [38]. The trivial solution of system (1) or (2) is called stochastically finite-time stable, if there exist constants $t_1>0$ and $t_2>0$, t_1 and t_2 depend on the value of state variables $e_i(0)=(e_1(0),e_2(0),\dots,e_n(0))^{\mathrm{T}}$ and $e_j(0)=(e_1(0),e_2(0),\dots,e_m(0))^{\mathrm{T}}$, such that the following inequality holds:

$$\sum_{i=1}^{n}\mathbb{E}\left\|e_i(t)\right\|^2=0,\quad \sum_{j=1}^{m}\mathbb{E}\left\|e_j(t)\right\|^2=0.$$

and

$$\mathbb{E}\left\|e_i(t)\right\|^2\equiv 0,\quad \mathbb{E}\left\|e_j(t)\right\|^2\equiv 0.$$

under any initial condition.

$\phi(s)=(\phi_1(s),\phi_2(s),\dots,\phi_n(s))^{\mathrm{T}}$,

$\gamma(s)=(\gamma_1(s),\gamma_2(s),\dots,\gamma_n(s))^{\mathrm{T}}\in\mathcal{C}([-\nu,0],\mathbb{R}^n)$,

which denotes the Banach space of all continuous functions mapping $[-\nu,0]$ into $\mathbb{R}^n$ and $\nu=\max\{\tau,\tau_2\}$.

$\varphi(s)=(\varphi_1(s),\varphi_2(s),\dots,\varphi_m(s))^{\mathrm{T}}$,

$\psi(s)=(\psi_1(s),\psi_2(s),\dots,\psi_m(s))^{\mathrm{T}}\in\mathcal{C}([-\varsigma,0],\mathbb{R}^m)$.

which denotes the Banach space of all continuous functions mapping $[-\varsigma,0]$ into $\mathbb{R}^m$ and $\varsigma=\max\{\sigma,\sigma_2\}$.

Definition 4 [29]. Systems (1) and (2) are said to be stochastically finite-time stable for suitable feedback controller $u_i(t)$ and $u_j(t)$, there are exist constants $T_x>0$ and $T_y>0$ such that

$$\lim_{t\to T_x}||e_i(t)||=0,\, i=1,2,\dots,n,\ \lim_{t\to T_y}||e_j(t)||=0,\, j=1,2,\dots,m.$$

And $||e_i(t)||=0$ for $t>T_x$, $||e_j(t)||=0$ for $t>T_y$, where T_x and T_y are functions about the initial state vector values $e_i(t)=\gamma_i(t)$ for $t\in[-\nu,0]$ and $e_j(t)=\psi_j(t)$ for $t\in[-\varsigma,0]$. The functions T_x and T_y are named the setting-time functions and their values are called the setting times.

Assumption 1 [16]. There exist constants $R_1\geq 0$ and $R_2\geq 0$, such that

$$\mathrm{Trace}[\sigma^{\mathrm{T}}(t,x(t),x(t-\xi(t)))\sigma(t,x(t),x(t-\xi(t)))]$$
$$\leq x^T(t)R_1x(t)+x^{\mathrm{T}}(t-\xi(t))R_2x(t-\xi(t)).$$

For the stochastic system [38]:

$$dy(t)=g(t,y(t))dt+\sigma(t,y(t))d\omega(t), \tag{7}$$

where $\omega(t)$ is the Brownian motion and it is truely $\mathbb{E}\omega(t)=0$. $\mathscr{L}$ is the operator designed as following:

$$\mathscr{L}V(t,y)=V_t(t,y)+V_yg(t,y)+\frac{1}{2}\mathrm{Trace}[\sigma^{\mathrm{T}}(t,y(t))V_{yy}\sigma(t,y(t))], \tag{8}$$

where

$$V_t(t,y)=\frac{\partial V_t(t,y)}{\partial t},$$
$$V_{yy}(t,y)=\left(\frac{\partial^2 V_t(t,y)}{\partial y_i\partial y_j}\right),$$
$$V_y(t,y)=\left(\frac{\partial V_t(t,y)}{\partial y_1},\frac{\partial V_t(t,y)}{\partial y_2},\dots,\frac{\partial V_t(t,y)}{\partial y_n}\right)^{\mathrm{T}}.$$

Assumption 2. For $i=1,\dots,n,\ j=1,\dots,m,$

$$[\underline{b}_{ji},\overline{b}_{ji}]f_j(y_j(t))+[\underline{b}_{ji},\overline{b}_{ji}]f_j(\hat{y}_j(t))\subseteq[\underline{b}_{ji},\overline{b}_{ji}](f_j(y_j(t))+f_j(\hat{y}_j(t))),$$
$$[\underline{c}_{ji},\overline{c}_{ji}]f_j(y_j(t))+[\underline{c}_{ji},\overline{c}_{ji}]f_j(\hat{y}_j(t))\subseteq[\underline{c}_{ji},\overline{c}_{ji}](f_j(y_j(t))+f_j(\hat{y}_j(t))),$$
$$[\underline{p}_{ij},\overline{p}_{ij}]g_i(x_i(t))+[\underline{p}_{ij},\overline{p}_{ij}]g_i(\hat{x}_i(t))\subseteq[\underline{p}_{ij},\overline{p}_{ij}](g_i(x_i(t))+g_i(\hat{x}_i(t))),$$
$$[\underline{q}_{ij},\overline{q}_{ij}]g_i(x_i(t))+[\underline{q}_{ij},\overline{q}_{ij}]g_i(\hat{x}_i(t))\subseteq[\underline{q}_{ij},\overline{q}_{ij}](g_i(x_i(t))+g_i(\hat{x}_i(t))).$$

Remark 3. Some papers [55–57] of stability or synchronization of MNNs are derived from the assumption

$$co[\underline{a}_{ij},\overline{a}_{ij}]f_j(y_j(t))-co[\underline{a}_{ij},\overline{a}_{ij}]f_j(x_j(t))\subseteq co[\underline{a}_{ij},\overline{a}_{ij}](f_j(y_j(t))-f_j(x_j(t))),$$

but it should be mentioned that this assumption has been proved unreasonable in [36]. During the proof of Theorems 1 and 2 in the present paper, by making full use of Assumption 3 and activation functions, we put the knotty terms together to design the suitable error systems then get the inequalities Eqs. (10) and (11), successfully avoiding the assumption problem.

Assumption 3 [38]. The activation functions $f_j(\cdot)(j=1,2,\dots,m)$ and $g_i(\cdot)(i=1,2,\dots,n)$ are odd, bounded and globally Lipschitz continuous in $\mathbb{R}$, namely, there exist constants $\underline{L}_x$, $\overline{L}_x$, $\underline{L}_y$ and $\overline{L}_y$ for all $s_1,s_2\in\mathbb{R}$, $s_1\neq s_2$ such that

$$\underline{L}_x\leq\frac{f_j(s_1)-f_j(s_2)}{s_1-s_2}\leq\overline{L}_x,$$
$$\underline{L}_y\leq\frac{g_i(s_1)-g_i(s_2)}{s_1-s_2}\leq\overline{L}_y,$$

that the constants $\underline{L}_x$, $\overline{L}_x$, $\underline{L}_y$ and $\overline{L}_y$ can be positive numbers, negative numbers or zero.

Remark 3. According to Assumptions 2 and 3, the activations functions $f_j(t)$ and $g_i(t)$ are odd functions. Then for $e_i(t)\neq 0$ and $e_j(t)\neq 0$, we get $F_j(e_j(t))$ and $G_i(e_i(t))$ possess the following properties [25]:

$$\underline{L}_xe_j(t)\leq F_j(e_j(t))\leq\overline{L}_xe_j(t),$$
$$\underline{L}_ye_i(t)\leq G_i(e_i(t))\leq\overline{L}_ye_i(t).$$

and

$$F_j(0)=f_j(y_j(t))+f_j(-y_j(t)),\qquad G_i(0)=g_i(x_i(t))+g_i(-x_i(t)).$$

Therefore, the error systems admits zero solutions $e_i(t)\equiv 0$ and $e_j(t)\equiv 0$.

Lemma 1 *[24]. Give any real matrices X, Z, P of appropriate dimensions and scalars $\epsilon_0>0$, $P>0$, the following inequality holds:*

$$X^{\mathrm{T}}Z+Z^{\mathrm{T}}X\leq\epsilon_0X^{\mathrm{T}}PX+\epsilon_0^{-1}Z^{\mathrm{T}}PZ^{-1}Z.$$

In particular, if X and Z are vectors, $X^{\mathrm{T}}Z\leq\frac{1}{2}(X^{\mathrm{T}}X+Z^{\mathrm{T}}Z)$.

Lemma 2 *[38]. If there is a continuous, positive definite function $V(t)$ satisfied following differential inequality:*

$$\dot{V}(t)+\alpha V^{\eta}(t)\leq 0,\ \forall t\geq t_0,\ V(t_0)\geq 0,$$

where $\alpha>0$, $0<\eta<1$ are two constants. Then, for any provided t_0, $V(t)$ satisfies the following differential inequality:

$$V^{1-\eta}(t)\leq V^{1-\eta}(t_0)-\alpha(1-\eta)(t-t_0),\ t_0\leq t\leq t_1,$$

and

$$V(t)\equiv 0,\ \forall t\geq t_1,$$

with t_1 defined by

$$t_1=t_0+\frac{V^{1-\eta}(t_0)}{\alpha(1-\eta)}.$$

Lemma 3 *[40]. Let $x_1,x_2,\dots,x_n\in R^n$ are any vectors and $0<q<2$ is a real number satisfying:*

$$\|x_1\|^q+\|x_2\|^q+\dots+\|x_n\|^q\geq\left(\|x_1\|^2+\|x_2\|^2+\dots+\|x_n\|^2\right)^{q/2}.$$

Remark 4. Comparing with the exciting literatures for researching the finite synchronization of MBAMNNs [40], the considered systems contained not only non-delay, discrete time-varying delays but also distributed delays and leakage delays. Therefore, the obtained results are much practical and reasonable.

3. Main results

In this section, the finite-time anti-synchronization of MBAMNNs with probabilistic time-varying delays and stochastic perturbations are investigated.

Theorem 1. *Under Assumptions 1–3, the error system (5) of the stochastic MBAMNNs with probabilistic time-varying will globally stable in finite time via the following controllers*

$$u_i(t) = -\eta_x e_i(t) - k_x \mathrm{sign}(e_i(t))|e_i(t)|^{\alpha},$$
$$u_j(t) = -\eta_y e_j(t) - k_y \mathrm{sign}(e_j(t))|e_j(t)|^{\alpha}, \tag{9}$$

if there exists the gain constants η_x, η_y and k_x, k_y such that

$$\begin{cases} \sum_{i=1}^{n}\Big[|a_i| - \sum_{j=1}^{m}|\bar{b}_{ji}|\bar{L}_x - \sum_{j=1}^{m}|\bar{c}_{ji}|\bar{L}_x - R_1 - R_3 \\ +2\eta_x\Big] - \sum_{i=1}^{m}\sum_{j=1}^{n}|\bar{b}_{ij}|\bar{L}_x \geq 0, \\ |\bar{c}_{ji}|\bar{L}_x\delta_0 + \frac{1}{m}R_2 \leq 0, \\ |\bar{c}_{ji}|\bar{L}_x(1-\delta_0) + \frac{1}{m}R_4 \leq 0, \end{cases} \tag{10}$$

and

$$\begin{cases} \sum_{j=1}^{m}\Big[|d_j| - \sum_{i=1}^{n}|\bar{q}_{ij}|\bar{L}_y - \sum_{i=1}^{n}|\bar{p}_{ij}|\bar{L}_y - P_1 - P_3 \\ +2\eta_y\Big] - \sum_{i=1}^{m}\sum_{j=1}^{n}|\bar{q}_{ji}|\bar{L}_y \geq 0, \\ |\bar{p}_{ij}|\bar{L}_y\rho_0 + \frac{1}{n}P_2 \leq 0, \\ |\bar{p}_{ij}|\bar{L}_y(1-\rho_0) + \frac{1}{n}P_4 \leq 0, \end{cases} \tag{11}$$

where R_1, R_2, R_3, R_4 and P_1, P_2, P_3, P_4 are all positive constants determined in the later. Then, the synchronization time is estimated by

$$T_x = \frac{V^{\frac{1-\alpha}{2}}(0)}{k_x(1-\alpha)} = \frac{\|e_i(0)\|^{1-\alpha}}{k_x(1-\alpha)},$$
$$T_y = \frac{V^{\frac{1-\alpha}{2}}(0)}{k_y(1-\alpha)} = \frac{\|e_j(0)\|^{1-\alpha}}{k_y(1-\alpha)}.$$

Proof. *Construct the following Lyapunov function*

$$V(t, e_i(t), e_j(t)) = \sum_{i=1}^{n} e_i^T(t)e_i(t) + \sum_{j=1}^{m} e_j^T(t)e_j(t).$$

Let

$$V_1(t) = \sum_{i=1}^{n} e_i^T(t)e_i(t), V_2(t) = \sum_{j=1}^{m} e_j^T(t)e_j(t).$$

Then

$$V(t, e_i(t), e_j(t)) = V_1(t) + V_2(t).$$
$$\mathbb{E}\{\mathcal{L}V(t, e_i(t), e_j(t))\} = \mathbb{E}\{\mathcal{L}V_1(t)\} + \mathbb{E}\{\mathcal{L}V_2(t)\}.$$

And

$$\mathbb{E}\{\mathcal{L}V_1(t)\} = 2\sum_{i=1}^{n} e_i(t)\dot{e}_i(t),$$
$$\mathbb{E}\{\mathcal{L}V_2(t)\} = 2\sum_{j=1}^{m} e_j(t)\dot{e}_j(t). \tag{12}$$

Under the Assumption 2, the error systems be obtained as

$$\begin{aligned} de_i(t) \in \Big[&- a_i e_i(t-\tau) + \sum_{j=1}^{m}\Big[[\underline{b}_{ji}, \bar{b}_{ji}]\big(f_j(y_j(t)) + f_j(\hat{y}_j(t))\big)\Big] \\ &+ \delta(t)\sum_{j=1}^{m}\Big[[\underline{c}_{ji}, \bar{c}_{ji}]\big(f_j(y_j(t-\tau_1(t))) + f_j(\hat{y}_j(t-\tau_1(t)))\big)\Big] \\ &+ (1-\delta(t))\sum_{j=1}^{m}\Big[[\underline{c}_{ji}, \bar{c}_{ji}]\big(f_j(y_j(t-\tau_2(t))) \\ &+ f_j(\hat{y}_j(t-\tau_2(t)))\big)\Big] + u_i(t)\Big]dt \\ &+ \sum_{j=1}^{m}\beta_{ji}(t, e_j(t), e_j(t-\tau(t)))d\omega_j(t) \\ &+ \sum_{j=1}^{m}\beta_{ji}(t, e_j(t), e_j(t-\tau_2(t)))d\omega_j(t), \end{aligned} \tag{13}$$

$$\begin{aligned} de_j(t) \in \Big[&- d_j e_j(t-\sigma) + \sum_{i=1}^{n}\Big[[\underline{q}_{ij}, \bar{q}_{ij}]\big(g_i(x_i(t)) + g_i(\hat{x}_i(t))\big)\Big] \\ &+ \rho(t)\sum_{i=1}^{n}\Big[[\underline{p}_{ij}, \bar{p}_{ij}]\big(g_i(x_i(t-\sigma_1(t))) + g_i(\hat{x}_i(t-\sigma_1(t)))\big)\Big] \\ &+ (1-\rho(t))\sum_{i=1}^{n}\Big[[\underline{p}_{ij}, \bar{p}_{ij}]\big(g_i(\hat{x}_i(t-\sigma_2(t))) \\ &+ g_i(x_i(t-\sigma_2(t)))\big)\Big] + u_j(t)\Big]dt \\ &+ \sum_{i=1}^{n}\beta_{ij}(t, e_i(t), e_i(t-\sigma(t)))d\omega_i(t) \\ &+ \sum_{i=1}^{n}\beta_{ij}(t, e_i(t), e_i(t-\sigma_2(t)))d\omega_i(t). \end{aligned} \tag{14}$$

Then we get

$$\begin{aligned} de_i(t) \leq \Big[&\delta(t)\sum_{j=1}^{m}\Big[\bar{c}_{ji}\big[f_j(y_j(t-\tau_1(t))) + f_j(\hat{y}_j(t-\tau_1(t)))\big]\Big] \\ &- a_i e_i(t-\tau) + \sum_{j=1}^{m}\Big[\bar{b}_{ji}\big(f_j(y_j(t)) + f_j(\hat{y}_j(t))\big)\Big] \\ &+ (1-\delta(t))\sum_{j=1}^{m}\Big[\bar{c}_{ji}\big[f_j(y_j(t-\tau_2(t))) \\ &+ f_j(\hat{y}_j(t-\tau_2(t)))\big]\Big] + u_i(t)\Big]dt \\ &+ \sum_{j=1}^{m}\beta_{ji}(t, e_j(t), e_j(t-\tau(t)))d\omega_j(t) \\ &+ \sum_{j=1}^{m}\beta_{ji}(t, e_j(t), e_j(t-\tau_2(t)))d\omega_j(t). \end{aligned} \tag{15}$$

And

$$\begin{aligned} de_i(t) \leq \Big[&- a_i e_i(t-\tau) + \sum_{j=1}^{m}\bar{b}_{ji}F_j(e_j(t)) \\ &+ \delta(t)\sum_{j=1}^{m}\bar{c}_{ji}F_j(e_j(t-\tau_1(t))) \\ &+ (1-\delta(t))\sum_{j=1}^{m}\bar{c}_{ji}F_j(e_j(t-\tau_2(t))) + u_i(t)\Big]dt \\ &+ \sum_{j=1}^{m}\beta_{ji}(t, e_j(t), e_j(t-\tau(t)))d\omega_j(t) \\ &+ \sum_{j=1}^{m}\beta_{ji}(t, e_j(t), e_j(t-\tau_2(t)))d\omega_j(t), \end{aligned} \tag{16}$$

where $F_j(e_j(t)) = f_j(y_j(t)) + f_j(\hat{y}_j(t))$, $F_j(e_j(t-\tau_1(t))) = f_j(y_j(t-\tau_1(t))) + f_j(\hat{y}_j(t-\tau_1(t)))$, $F_j(e_j(t-\tau_2(t))) = f_j(y_j(t-\tau_2(t))) + f_j(\hat{y}_j(t-\tau_2(t)))$.

Then we have

$$\begin{aligned} \mathbb{E}\Big\{\mathcal{L}V_1(t)\Big\} = 2\sum_{i=1}^{n} e_i(t)\Big[&- a_i e_i(t-\tau) + \sum_{j=1}^{m}\bar{b}_{ji}F_j e_j(t) + u_i(t) \\ &+ \delta_0\sum_{j=1}^{m}\bar{c}_{ji}F_j e_j(t-\tau_1(t)) + (1-\delta_0)\sum_{j=1}^{m}\bar{c}_{ji}F_j e_j(t-\tau_2(t))\Big] \\ &+ \sum_{j=1}^{m}\mathrm{Trace}\Big[\rho_{ji}^T(t, y(t), y(t-\tau_1(t)))\rho_{ji}(t, y(t), \end{aligned}$$

$$y(t-\tau_1(t)))\Big]+\sum_{j=1}^{m}\mathrm{Trace}\Big[\rho_{ji}^{\mathrm{T}}(t,y(t),y(t-\tau_2(t)))\rho_{ji}(t,y(t),y(t-\tau_2(t)))\Big]. \tag{17}$$

According to the Assumption 1 and controller (9), we get

$$\begin{aligned}\mathbb{E}\Big\{\mathcal{L}V_1(t)\Big\} \le & -2\sum_{i=1}^{n}|a_i|e_i(t)e_i(t-\tau)+2\sum_{i=1}^{n}\sum_{j=1}^{m}|\bar{b}_{ji}|e_i(t)F_j(e_j(t))\\ &+2\sum_{i=1}^{n}\sum_{j=1}^{m}\delta_0|\bar{c}_{ji}|e_i(t)F_j(e_j(t-\tau_1(t)))-2\sum_{i=1}^{n}\eta_x e_i^2(t)\\ &+2\sum_{i=1}^{n}\sum_{j=1}^{m}(1-\delta_0)|\bar{c}_{ji}|e_i(t)F_j(e_j(t-\tau_2(t)))\\ &-2\sum_{i=1}^{n}k_x e_i(t)\mathrm{sign}(e_i(t))|e_i(t)|^{\alpha}+\sum_{j=1}^{m}e_j^{\mathrm{T}}(t)R_1e_j(t)\\ &+\sum_{j=1}^{m}e_j^{\mathrm{T}}(t-\tau_1(t))R_2e_j(t-\tau_1(t))+\sum_{j=1}^{m}e_j^{\mathrm{T}}(t)R_3e_j(t)\\ &+\sum_{j=1}^{m}e_j^{\mathrm{T}}(t-\tau_2(t))R_4e_j(t-\tau_2(t)).\end{aligned} \tag{18}$$

Based on the Assumption 3 and Lemma 1, we conclude

$$\begin{aligned}&2\sum_{i=1}^{n}|a_i|e_i(t)e_i(t-\tau)\le\sum_{i=1}^{n}|a_i|\Big[e_i^2(t)+e_i^2(t-\tau)\Big],\\ &2\sum_{i=1}^{n}\sum_{j=1}^{m}|\bar{b}_{ji}|e_i(t)F_je_j(t)\le 2\sum_{i=1}^{n}\sum_{j=1}^{m}|\bar{b}_{ji}|\bar{L}_ye_i(t)e_j(t)\\ &\qquad\le\sum_{i=1}^{n}\sum_{j=1}^{m}|\bar{b}_{ji}|\bar{L}_y\Big[e_i^2(t)+e_j^2(t)\Big],\end{aligned} \tag{19}$$

$$\begin{aligned}&2\delta_0\sum_{i=1}^{n}\sum_{j=1}^{m}|\bar{c}_{ji}|e_i(t)F_je_j(t-\tau_1(t))\\ &\le 2\delta_0\sum_{i=1}^{n}\sum_{j=1}^{m}|\bar{c}_{ji}|\bar{L}_ye_i(t)e_j(t-\tau_1(t))\\ &\le \delta_0\sum_{i=1}^{n}\sum_{j=1}^{m}|\bar{c}_{ji}|\bar{L}_y\Big[e_i^2(t)+e_j^2(t-\tau_1(t))\Big],\\ &2(1-\delta_0)\sum_{i=1}^{n}\sum_{j=1}^{m}|\bar{c}_{ji}|e_i(t)F_je_j(t-\tau_2(t))\\ &\le 2(1-\delta_0)\sum_{i=1}^{n}\sum_{j=1}^{m}|\bar{c}_{ji}|\bar{L}_ye_i(t)e_j(t-\tau_2(t))\\ &\le (1-\delta_0)\sum_{i=1}^{n}\sum_{j=1}^{m}|\bar{c}_{ji}|\bar{L}_y\Big[e_i^2(t)+e_j^2(t-\tau_2(t))\Big],\end{aligned}$$

By Eqs. (18) and (19), we have

$$\begin{aligned}\mathbb{E}\Big\{\mathcal{L}V_1(t)\Big\} \le & -\Bigg\{\sum_{i=1}^{n}\Bigg[|a_i|-\sum_{j=1}^{m}|\bar{b}_{ji}|\bar{L}_x\\ &-\sum_{j=1}^{m}|\bar{c}_{ji}|\bar{L}_x-R_1-R_3+2\eta_x\Bigg]-\sum_{i=1}^{m}\sum_{j=1}^{n}|\bar{b}_{ij}|\bar{L}_x\Bigg\}e_i^2(t)\\ &-\sum_{i=1}^{n}|a_i|e_i^2(t-\tau)+\sum_{i=1}^{n}\Bigg[\sum_{j=1}^{m}|\bar{c}_{ji}|\bar{L}_x\delta_0+R_2\Bigg]e_j^2(t-\tau_1(t))\\ &+\sum_{i=1}^{n}\Bigg[\sum_{j=1}^{m}|\bar{c}_{ji}|\bar{L}_x(1-\delta_0)+R_4\Bigg]e_j^2(t-\tau_2(t))\\ &-2\sum_{i=1}^{n}k_xe_i(t)\mathrm{sign}(e_i(t))|e_i(t)|^{\alpha}.\end{aligned} \tag{20}$$

Due to

$$\sum_{i=1}^{n}k_xe_i(t)\mathrm{sign}(e_i(t))|e_i(t)|^{\alpha}=\sum_{i=1}^{n}k_x|e_i(t)|^{\alpha+1},$$

From Lemma 3, We get

$$\Big(\sum_{i=1}^{n}|e_i(t)|^{\alpha+1}\Big)^{\frac{1}{\alpha+1}}=\Big(\sum_{i=1}^{n}|e_i(t)|^2\Big)^{\frac{1}{2}}.$$

Hence

$$\sum_{i=1}^{n}|e_i(t)|^{\alpha+1}=\Big(\sum_{i=1}^{n}|e_i(t)|^2\Big)^{\frac{\alpha+1}{2}}=\Big(\sum_{i=1}^{n}e_i^2(t)\Big)^{\frac{\alpha+1}{2}}.$$

Then we conclude

$$\begin{aligned}\mathbb{E}\Big\{\mathcal{L}V_1(t)\Big\} \le & -\Bigg\{\sum_{i=1}^{n}\Bigg[|a_i|-\sum_{j=1}^{m}|\bar{b}_{ji}|\bar{L}_x\\ &-\sum_{j=1}^{m}|\bar{c}_{ji}|\bar{L}_x-R_1-R_3+2\eta_x\Bigg]-\sum_{i=1}^{m}\sum_{j=m}^{n}|\bar{b}_{ij}|\bar{L}_x\Bigg\}e_i^2(t)\\ &-\sum_{i=1}^{n}|a_i|e_i^2(t-\tau)+\sum_{i=1}^{n}\Bigg[\sum_{j=1}^{m}|\bar{c}_{ji}|\bar{L}_x\delta_0+R_2\Bigg]e_j^2(t-\tau_1(t))\\ &+\sum_{i=1}^{n}\Bigg[\sum_{j=1}^{n}|\bar{c}_{ji}|\bar{L}_x(1-\delta_0)+R_4\Bigg]e_j^2(t-\tau_2(t))-2k_x\Big(\sum_{i=1}^{n}e_i^2(t)\Big)^{\frac{\alpha+1}{2}}.\end{aligned} \tag{21}$$

By Theorem 1 and Lemma 3, we get

$$\mathbb{E}\Big\{\mathcal{L}V_1(t)\Big\}\le\mathbb{E}\Big\{-2k_x\Big(\sum_{i=1}^{n}e_i^2(t)\Big)^{\frac{\alpha+1}{2}}\Big\}=-2k_x\mathbb{E}\Big\{V_1^{\frac{\alpha+1}{2}}(t)\Big\}. \tag{22}$$

With the similar process of $\mathcal{L}V_1(t)$, we also get

$$\mathbb{E}\Big\{\mathcal{L}V_2(t)\Big\}\le\mathbb{E}\Big\{-2k_y\Big(\sum_{j=1}^{m}e_j^2(t)\Big)^{\frac{\alpha+1}{2}}\Big\}=-2k_y\mathbb{E}\Big\{V_2^{\frac{\alpha+1}{2}}(t)\Big\}. \tag{23}$$

Therefore, according to Definitions 3 and 4, the finite-time anti-synchronization for system (5) can be achieved via the controller (9) with $0<\alpha<1$, and the finite times are given as:

$$\begin{aligned}T_x&=\frac{V^{\frac{1-\alpha}{2}}(0)}{k_i(1-\alpha)}=\frac{\|e_i(0)\|^{1-\alpha}}{k_i(1-\alpha)},\\ T_y&=\frac{V^{\frac{1-\alpha}{2}}(0)}{k_j(1-\alpha)}=\frac{\|e_j(0)\|^{1-\alpha}}{k_j(1-\alpha)}.\end{aligned} \tag{24}$$

The proof is completed. □

Remark 5. The systems (3) and (4) contain the case of $\delta(t)\equiv 1$, $(\delta_0=1)$, $\delta(t)\equiv 0$, $(\delta_0=0)$ and $\rho(t)\equiv 1$, $(\rho_0=1)$, $\rho(t)\equiv 0$, $(\rho_0=0)$. Thus, some previous study on BAMNNs can be treated as a special case of this paper.

Corollary 1. *Suppose Assumption 3 is satisfied, then the error systems (13) and (14) without leakage delay are exponentially stable in the mean square via the controller (9), if there exists the gain constants η_i, η_j and k_i, k_j such that*

$$\begin{cases}\sum_{i=1}^{n}\Big[|a_i|-\sum_{j=1}^{m}|\bar{b}_{ji}|\bar{L}_x-\sum_{j=1}^{m}|\bar{c}_{ji}|\bar{L}_x-R_1-R_3+2\eta_x\Big]\\ -\sum_{i=1}^{m}\sum_{j=1}^{n}|\bar{b}_{ij}|\bar{L}_x\ge 0,\\ |\bar{c}_{ji}|\bar{L}_x\delta_0+\frac{1}{m}R_2\le 0,\\ |\bar{c}_{ji}|\bar{L}_x(1-\delta_0)+\frac{1}{m}R_4\le 0.\end{cases} \tag{25}$$

and

$$\begin{cases}\sum_{j=1}^{m}\Big[|d_j|-\sum_{i=1}^{n}|\overline{q}_{ij}|\overline{L}_y-\sum_{i=1}^{n}|\overline{p}_{ij}|\overline{L}_y-P_1-P_3+2\eta_y\Big]\\ -\sum_{i=1}^{m}\sum_{j=1}^{n}|\overline{q}_{ji}|\overline{L}_y\geq 0,\\ |\overline{p}_{ij}|\overline{L}_y\rho_0+\frac{1}{n}P_2\leq 0,\\ |\overline{p}_{ij}|\overline{L}_y(1-\rho_0)+\frac{1}{n}P_4\leq 0.\end{cases} \tag{26}$$

where R_1, R_2, R_3, R_4 and P_1, P_2, P_3, P_4 are all positive constants determined in the later. Then, the synchronization times are estimated by

$$T_x=\frac{V^{\frac{1-\alpha}{2}}(0)}{k_x(1-\alpha)}=\frac{\|e_i(0)\|^{1-\alpha}}{k_x(1-\alpha)},\quad T_y=\frac{V^{\frac{1-\alpha}{2}}(0)}{k_y(1-\alpha)}=\frac{\|e_j(0)\|^{1-\alpha}}{k_y(1-\alpha)}. \tag{27}$$

Proof. Let the leakage time delay $\tau=0$ and $\sigma=0$ in Theorem 1. The proof can be followed, thus it is omitted here. □

Remark 6. At present, the research on finite-time stability of the MBAMNNs with time-varying delays and stochastic perturbations is very few. There are numerous models about stability of MBAMNNs within finite-time are special cases of our proposed model. Here we give Theorem 2 about the special case.

Throughout this paper, we consider system (4) without stochastic perturbations as the drive system and corresponding response system is as follows

$$\begin{aligned} d\hat{x}_i(t)\in\Big[&-a_i\hat{x}_i(t-\tau)+\sum_{j=1}^{m}co[b_{ji}(\hat{x}_i(t))]f_j(\hat{y}_j(t))\\ &+\delta(t)\sum_{j=1}^{m}co[c_{ji}(\hat{x}_i(t-\tau(t)))]f_j(\hat{y}_j(t-\tau_1(t)))\\ &+(1-\delta(t))\sum_{j=1}^{m}co[c_{ji}(\hat{x}_i(t-\tau(t)))]f_j(\hat{y}_j(t-\tau_2(t)))+u_i(t)\Big]dt,\\ d\hat{y}_j(t)\in\Big[&-d_j\hat{y}_j(t-\sigma)+\sum_{i=1}^{n}co[q_{ij}(\hat{y}_j(t))]g_i(\hat{x}_i(t))\\ &+\rho(t)\sum_{i=1}^{n}co[p_{ij}(\hat{y}_j(t-\sigma(t)))]g_i(\hat{x}_i(t-\sigma_1(t)))\\ &+(1-\rho(t))\sum_{i=1}^{n}co[p_{ij}(\hat{y}_j(t-\sigma(t)))]g_i(\hat{x}_i(t-\sigma_2(t)))+u_j(t)\Big]dt.\end{aligned} \tag{28}$$

Under this case, the error systems (13) and (14) reduce to

$$\begin{aligned} de_i(t)\leq\Big[&-a_ie_i(t-\tau)+\sum_{j=1}^{m}\overline{b}_{ji}F_j(e_j(t))\\ &+\delta(t)\sum_{j=1}^{m}\overline{c}_{ji}F_j(e_j(t-\tau_1(t)))\\ &+(1-\delta(t))\sum_{j=1}^{m}\overline{c}_{ji}F_j(e_j(t-\tau_2(t)))+u_i(t)\Big]dt. \end{aligned}\tag{29}$$

and

$$\begin{aligned} de_j(t)\leq\Big[&-d_je_j(t-\sigma)+\sum_{i=1}^{n}\overline{q}_{ij}G_i(e_i(t))\\ &+\rho(t)\sum_{i=1}^{n}\overline{p}_{ij}G_i(e_i(t-\sigma_1(t)))\\ &+(1-\rho(t))\sum_{i=1}^{n}\overline{p}_{ij}G_i(e_i(t-\sigma_2(t)))+u_j(t)\Big]dt. \end{aligned}\tag{30}$$

Thus, we have the following theorem.

Theorem 2. *Under Assumptions 1–3, the error systems (29) and (30) of the MBAMNNs with probabilistic time-varying will be globally stable in finite time via the controller (9), if there exists the gain constants η_x, η_y and k_x, k_y such that*

$$\begin{cases}\sum_{i=1}^{n}\Big[|a_i|-\sum_{j=1}^{m}|\overline{b}_{ji}|\overline{L}_x-\sum_{j=1}^{m}|\overline{c}_{ji}|\overline{L}_x+2\eta_x\Big]\\ -\sum_{i=1}^{m}\sum_{j=1}^{n}|\overline{b}_{ij}|\overline{L}_x\geq 0,\\ |\overline{c}_{ji}|\overline{L}_x\delta_0\leq 0,\quad |\overline{c}_{ji}|\overline{L}_x(1-\delta_0)\leq 0,\end{cases} \tag{31}$$

and

$$\begin{cases}\sum_{j=1}^{m}\Big[|d_j|-\sum_{i=1}^{n}|\overline{q}_{ij}|\overline{L}_y-\sum_{i=1}^{n}|\overline{p}_{ij}|L_y+2\eta_y\Big]\\ -\sum_{i=1}^{m}\sum_{j=1}^{n}|\overline{q}_{ji}|\overline{L}_y\geq 0,\\ |\overline{p}_{ij}|\overline{L}_y\rho_0\leq 0,\quad |\overline{p}_{ij}|\overline{L}_y(1-\rho_0)\leq 0.\end{cases} \tag{32}$$

Then, the synchronization time is estimated by

$$T_x=\frac{V^{\frac{1-\alpha}{2}}(0)}{k_x(1-\alpha)}=\frac{\|e_i(0)\|^{1-\alpha}}{k_x(1-\alpha)},\quad T_y=\frac{V^{\frac{1-\alpha}{2}}(0)}{k_y(1-\alpha)}=\frac{\|e_j(0)\|^{1-\alpha}}{k_y(1-\alpha)}. \tag{33}$$

Proof. *Construct the following Lyapunov function*

$$V(t,e_i(t),e_j(t))=\sum_{i=1}^{n}e_i^2(t)+\sum_{j=1}^{m}e_j^2(t).$$

Let

$$V_1(t)=\sum_{i=1}^{n}e_i^2(t),V_2(t)=\sum_{j=1}^{m}e_j^2(t).$$

Then $V(t,e_i(t),e_j(t))=V_1(t)+V_2(t)$.

$$\mathbb{E}\{\mathcal{L}V(t,e_i(t),e_j(t))\}=\mathbb{E}\{\mathcal{L}V_1(t)\}+\mathbb{E}\{\mathcal{L}V_2(t)\}.$$

By means of Assumptions 2 and 3 and Lemma 1, we get the following inequality And

$$\begin{aligned}\mathbb{E}\Big\{\mathcal{L}V_1(t)\Big\}=2\sum_{i=1}^{n}e_i(t)\Big[&-a_ie_i(t-\tau)+\sum_{j=1}^{m}\overline{b}_{ji}F_je_j(t)+u_i(t)\\ &+\delta_0\sum_{j=1}^{m}\overline{c}_{ji}F_je_j(t-\tau_1(t))\\ &+(1-\delta_0)\sum_{j=1}^{m}\overline{c}_{ji}F_je_j(t-\tau_2(t))\Big].\end{aligned} \tag{34}$$

Combine with the Assumption 1 and controller (9), we get

$$\begin{aligned}\mathbb{E}\Big\{\mathcal{L}V_1(t)\Big\}\leq&-2\sum_{i=1}^{n}|a_i|e_i(t)e_i(t-\tau)+2\sum_{i=1}^{n}\sum_{j=1}^{m}|\overline{b}_{ji}|e_i(t)F_j(e_j(t))\\ &+2\sum_{i=1}^{n}\sum_{j=1}^{m}\delta_0|\overline{c}_{ji}|e_i(t)F_j(e_j(t-\tau_1(t)))\\ &+2\sum_{i=1}^{n}\sum_{j=1}^{m}(1-\delta_0)|\overline{c}_{ji}|e_i(t)F_j(e_j(t-\tau_2(t)))\\ &-2\sum_{i=1}^{n}\eta_xe_i^2(t)-2\sum_{i=1}^{n}k_xe_i(t)\text{sign}(e_i(t))|e_i(t)|^{\alpha},\end{aligned} \tag{35}$$

By Eq. (13), we conclude

$$\mathbb{E}\Big\{\mathcal{L}V_1(t)\Big\} \le -\Bigg\{\sum_{i=1}^{n}\Big[|a_i| - \sum_{j=1}^{m}|\overline{b}_{ji}|\overline{L}_x - \sum_{j=1}^{m}|\overline{c}_{ji}|\overline{L}_x + 2\eta_x\Big] - \sum_{i=1}^{m}\sum_{j=1}^{n}|\overline{b}_{ij}|\overline{L}_x\Bigg\}e_i^2(t) - \sum_{i=1}^{n}|a_i|e_i^2(t-\tau) + \sum_{i=1}^{n}\Big[\sum_{j=1}^{m}|\overline{c}_{ji}|\overline{L}_x\delta_0\Big]e_j^2(t-\tau_1(t)) + \sum_{i=1}^{n}\Big[\sum_{j=1}^{m}|\overline{c}_{ji}|\overline{L}_x(1-\delta_0)\Big]e_j^2(t-\tau_2(t)) - 2\sum_{i=1}^{n}k_x e_i(t)\mathrm{sign}(e_i(t))|e_i(t)|^{\alpha}. \tag{36}$$

Due to

$$\sum_{i=1}^{n}k_x e_i(t)\mathrm{sign}(e_i(t))|e_i(t)|^{\alpha} = \sum_{i=1}^{n}k_x|e_i(t)|^{\alpha+1}.$$

From Lemma 3, We get

$$\Big(\sum_{i=1}^{n}|e_i(t)|^{\alpha+1}\Big)^{\frac{1}{\alpha+1}} = \Big(\sum_{i=1}^{n}|e_i(t)|^2\Big)^{\frac{1}{2}}.$$

Hence

$$\sum_{i=1}^{n}|e_i(t)|^{\alpha+1} = \Big(\sum_{i=1}^{n}|e_i(t)|^2\Big)^{\frac{\alpha+1}{2}} = \Big(\sum_{i=1}^{n}e_i^2(t)\Big)^{\frac{\alpha+1}{2}}.$$

Then we conclude

$$\mathbb{E}\Big\{\mathcal{L}V_1(t)\Big\} \le -\Bigg\{\sum_{i=1}^{n}\Big[|a_i| - \sum_{j=1}^{m}|\overline{b}_{ji}|\overline{L}_x - \sum_{j=1}^{m}|\overline{c}_{ji}|\overline{L}_x + 2\eta_x\Big] - \sum_{i=1}^{m}\sum_{j=1}^{n}|\overline{b}_{ij}|\overline{L}_x\Bigg\}e_i^2(t) - \sum_{i=1}^{n}|a_i|e_i^2(t-\tau) + \sum_{i=1}^{n}\Big[\sum_{j=1}^{n}|\overline{c}_{ji}|\overline{L}_x\delta_0\Big]e_j^2(t-\tau_1(t)) - 2k_x\Big(\sum_{i=1}^{n}e_i^2(t)\Big)^{\frac{\alpha+1}{2}} + \sum_{i=1}^{n}\Big[\sum_{j=1}^{n}|\overline{c}_{ji}|\overline{L}_x(1-\delta_0)\Big]e_j^2(t-\tau_2(t)). \tag{37}$$

So according to Theorem 1 and Lemma 3, we obtain

$$\mathbb{E}\Big\{\mathcal{L}V_1(t)\Big\} \le \mathbb{E}\Big\{-2k_x\Big(\sum_{i=1}^{n}e_i^2(t)\Big)^{\frac{\alpha+1}{2}}\Big\} = -2k_x\mathbb{E}\Big\{V_1^{\frac{\alpha+1}{2}}(t)\Big\}. \tag{38}$$

With the similar process of $\mathcal{L}V_1(t)$, we can have

$$\mathbb{E}\Big\{\mathcal{L}V_2(t)\Big\} \le \mathbb{E}\Big\{-2k_y\Big(\sum_{j=1}^{m}e_j^2(t)\Big)^{\frac{\alpha+1}{2}}\Big\} = -2k_y\mathbb{E}\Big\{V_2^{\frac{\alpha+1}{2}}(t)\Big\}. \tag{39}$$

Therefore, according to Definitions 3 and 4, the finite-time anti-synchronization for the systems (3) and (28) can be achieved via the controller (9) with $0<\alpha<1$, and the finite times are given as:

$$T_x = \frac{V^{\frac{1-\alpha}{2}}(0)}{k_x(1-\alpha)} = \frac{\|e_i(0)\|^{1-\alpha}}{k_x(1-\alpha)},\quad T_y = \frac{V^{\frac{1-\alpha}{2}}(0)}{k_y(1-\alpha)} = \frac{\|e_j(0)\|^{1-\alpha}}{k_y(1-\alpha)}. \tag{40}$$

The proof is completed. □

Corollary 2. *Suppose Assumption 3 is satisfied, then the error systems (29) and (30) without leakage delay are globally exponentially stable in the mean square via the controllers (9), if there exists the gain constants η_x, η_y and k_x, k_y such that*

$$\begin{cases}\sum_{i=1}^{n}\Big[|a_i| - \sum_{j=1}^{m}|\overline{b}_{ji}|\overline{L}_x - \sum_{j=1}^{m}|\overline{c}_{ji}|\overline{L}_x + 2\eta_x\Big] \\ -\sum_{i=1}^{m}\sum_{j=1}^{n}|\overline{b}_{ij}|\overline{L}_x \ge 0, \\ |\overline{c}_{ji}|\overline{L}_x\delta_0 \le 0, \quad |\overline{c}_{ji}|\overline{L}_x(1-\delta_0) \le 0,\end{cases} \tag{41}$$

and

$$\begin{cases}\sum_{j=1}^{m}\Big[|d_j| - \sum_{i=1}^{n}|\overline{q}_{ij}|\overline{L}_y - \sum_{i=1}^{n}|\overline{p}_{ij}|\overline{L}_y + 2\eta_y\Big] \\ -\sum_{i=1}^{m}\sum_{j=1}^{n}|\overline{q}_{ji}|\overline{L}_y \ge 0, \\ |\overline{p}_{ij}|\overline{L}_y\rho_0 \le 0, \quad |\overline{p}_{ij}|\overline{L}_y(1-\rho_0) \le 0.\end{cases} \tag{42}$$

Then, the synchronization time is estimated by

$$T_x = \frac{V^{\frac{1-\alpha}{2}}(0)}{k_x(1-\alpha)} = \frac{\|e_i(0)\|^{1-\alpha}}{k_x(1-\alpha)},\quad T_y = \frac{V^{\frac{1-\alpha}{2}}(0)}{k_y(1-\alpha)} = \frac{\|e_j(0)\|^{1-\alpha}}{k_y(1-\alpha)}. \tag{43}$$

Proof. *Let the leakage time delay $\tau = 0$ and $\sigma = 0$ in Theorem 2. The proof is omitted here.* □

Remark 7. Due to the condition of the time-varying delays $\tau(t)$, $\sigma(t)$, Theorem 2 provides a special case of Theorem 1. It should be mentioned that no complex numerical calculation such as computing redundance algebraic criterions [58] or solving linear matrix inequality (LMIs) [59,60] is needed in the anti-synchronization conditions. Thus, our anti-synchronization consequences have a wider adaptive capability and more successful application.

Remark 8. There is no extra demand on activation functions but requesting they are bounded and the time-varying delays are probabilistic. Furthermore, overall consideration of our obtained results with finite time anti-synchronization, it can be expected to have a powerful potential application in areas such as secret communication, image encryption, digital processing, and so on.

4. Numerical simulation

To show the effectiveness of the obtained results, several numerical simulations are presented as follows. There exist $b_{ji}(t) \in co[b_{ji}(x_i(t))]$, $c_{ji}(t) \in co[c_{ji}(x_i(t-\tau(t)))]$, $q_{ij}(t) \in co[q_{ij}(y_j(t))]$, $p_{ij}(t) \in co[p_{ij}(y_j(t-\sigma(t)))]$, so the drive system be considered as follows

$$\dot{x}_i(t) = -a_i x_i(t-\tau) + \sum_{j=1}^{2}b_{ji}(t)f_j(y_j(t)) + \delta_0\sum_{j=1}^{2}c_{ji}(t)g_j(y_j(t-\tau_1(t))) + (1-\delta_0)\sum_{j=1}^{2}c_{ji}(t)g_j(y_j(t-\tau_2(t))),$$
$$\dot{y}_j(t) = -d_j y_j(t-\sigma) + \sum_{i=1}^{2}q_{ij}(t)f_i(x_i(t)) + \rho_0\sum_{i=1}^{2}p_{ij}(t)g_i(x_i(t-\sigma_1(t))) + (1-\rho_0)\sum_{i=1}^{2}p_{ij}(t)g_i(x_i(t-\sigma_2(t))). \tag{44}$$

And the response system is defined as

$$d\hat{x}_i(t) = \Big[-a_i\hat{x}_i(t-\tau) + \sum_{j=1}^{2} b_{ji}(t) f_j(\hat{y}_j(t))$$
$$+ \delta_0 \sum_{j=1}^{2} c_{ji}(t) g_j(\hat{y}_j(t-\tau_1(t)))$$
$$+ (1-\delta_0)\sum_{j=1}^{2} c_{ji}(t) g_j(\hat{y}_j(t-\tau_2(t))) + u_i(t)\Big]dt$$
$$+ \sum_{j=1}^{2} \beta_{ji}(t, e_j(t), e_j(t-\tau_1(t)))d\omega_j(t)$$
$$+ \sum_{j=1}^{2} \beta_{ji}(t, e_j(t), e_j(t-\tau_2(t)))d\omega_j(t), \quad (45)$$

$$d\hat{y}_j(t) = \Big[-d_j\hat{y}_j(t-\sigma) + \sum_{i=1}^{2} q_{ij}(t) f_i(\hat{x}_i(t))$$
$$+ \rho_0 \sum_{i=1}^{2} p_{ij}(t) g_i(\hat{x}_i(t-\sigma_1(t)))$$
$$+ (1-\rho_0)\sum_{i=1}^{2} p_{ij}(t) g_i(\hat{x}_i(t-\sigma_2(t))) + u_j(t)\Big]dt$$
$$+ \sum_{i=1}^{2} \beta_{ij}(t, e_i(t), e_i(t-\sigma_1(t)))d\omega_i(t)$$
$$+ \sum_{i=1}^{2} \beta_{ij}(t, e_i(t), e_i(t-\sigma_2(t)))d\omega_i(t).$$

Taking the activation function as $g(\cdot) = \tanh(|\cdot|-1)$ and $f(\cdot) = \sin(|\cdot|-1)$. $\tau_1(t) = \sigma_1(t) = 0.75 - 0.25\cos(t)$, $\tau_2(t) = \sigma_2(t) = 0.75 - 0.25\sin(t)$. We have $\tau = \sigma = 0.15$. $\delta_0 = \rho_0 = 0.2$.

$$\mathbf{a} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{d} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$b_{11}(x_1(t)) = \begin{cases} 1.0, & |x_1(t)| \le 1, \\ 1.0, & |x_1(t)| > 1, \end{cases}$$

$$b_{12}(x_1(t)) = \begin{cases} 7.0, & |x_1(t)| \le 1, \\ 5.0, & |x_1(t)| > 1, \end{cases}$$

$$b_{21}(x_2(t)) = \begin{cases} 1.8, & |x_2(t)| \le 1, \\ 1.8, & |x_2(t)| > 1, \end{cases}$$

$$b_{22}(x_2(t)) = \begin{cases} 0.8, & |x_2(t)| \le 1, \\ 1.0, & |x_2(t)| > 1, \end{cases}$$

$$q_{11}(y_1(t)) = \begin{cases} 1.1, & |y_1(t)| \le 2, \\ 2.0, & |y_1(t)| > 2, \end{cases}$$

$$q_{12}(y_1(t)) = \begin{cases} 6.8, & |y_1(t)| \le 2, \\ 4.6, & |y_1(t)| > 2, \end{cases}$$

$$q_{21}(y_2(t)) = \begin{cases} 1.5, & |y_2(t)| \le 2, \\ 1.5, & |y_2(t)| > 2, \end{cases}$$

$$q_{22}(y_2(t)) = \begin{cases} 0.9, & |y_2(t)| \le 2, \\ 1.2, & |y_2(t)| > 2, \end{cases}$$

$$c_{11}(x_1(t-\tau(t))) = \begin{cases} -1.5, & |x_1(t-\tau(t))| \le 1, \\ -1.2, & |x_1(t-\tau(t))| > 1, \end{cases}$$

$$c_{12}(x_1(t-\tau(t))) = \begin{cases} 1.0, & |x_1(t-\tau(t))| \le 1, \\ 0.8, & |x_1(t-\tau(t))| > 1, \end{cases}$$

$$c_{21}(x_2(t-\tau(t))) = \begin{cases} 0.8, & |x_2(t-\tau(t))| \le 1, \\ 1.0, & |x_2(t-\tau(t))| > 1, \end{cases}$$

$$c_{22}(x_2(t-\tau(t))) = \begin{cases} -1.4, & |x_2(t-\tau(t))| \le 1, \\ -1.6, & |x_2(t-\tau(t))| > 1, \end{cases}$$

$$p_{11}(y_1(t-\sigma(t))) = \begin{cases} -1.3, & |y_1(t-\sigma(t))| \le 2, \\ -1.1, & |y_1(t-\sigma(t))| > 2, \end{cases}$$

$$p_{12}(y_1(t-\sigma(t))) = \begin{cases} 0.7, & |y_1(t-\sigma(t))| \le 2, \\ 0.9, & |y_1(t-\sigma(t))| > 2, \end{cases}$$

$$p_{21}(y_2(t-\sigma(t))) = \begin{cases} 0.9, & |y_2(t-\sigma(t))| \le 2, \\ 1.2, & |y_2(t-\sigma(t))| > 2, \end{cases}$$

$$p_{22}(y_2(t-\sigma(t))) = \begin{cases} -1.3, & |y_2(t-\sigma(t))| \le 2, \\ -1.7, & |y_2(t-\sigma(t))| > 2. \end{cases}$$

For the controllers, we let $k_x = \text{diag}\{1, 1\}$, $k_y = \text{diag}\{1, 1\}$ and $\eta_x = \text{diag}\{115, 160\}$, $\eta_y = \text{diag}\{175, 177\}$ We choose $\alpha = 0.6$, and the initial values of the state variables as $[x_1(t), x_2(t)] = [-1.45, 1.6]$, $[\hat{x}_1(t), \hat{x}_2(t)] = [-1.35, 1.5]$, $[y_1(t), y_2(t)] = [-1.95, 1.2]$ and $[\hat{y}_1(t), \hat{y}_2(t)] = [-1.85, 1.3]$.

The Brownian motion satisfies $E\omega(t) = 0$, $D\omega(t) = 1$.

$\beta_{ji}(\mathbf{t}, \mathbf{e_j}(\mathbf{t}), \mathbf{e_j}(\mathbf{t}-\tau_1(\mathbf{t})) = \text{diag}\{0.4e_1(t) + 0.3e_1(t-\tau_1(t)), -0.5e_2(t) + 0.2e_2(t-\tau_1(t))\}$,
$\beta_{ji}(\mathbf{t}, \mathbf{e_j}(\mathbf{t}), \mathbf{e_j}(\mathbf{t}-\tau_2(\mathbf{t})) = \text{diag}\{0.4e_1(t) + 0.4e_1(t-\tau_2(t)), -0.5e_2(t) + 0.2e_2(t-\tau_2(t))\}$,
$\beta_{ij}(\mathbf{t}, \mathbf{e_i}(\mathbf{t}), \mathbf{e_i}(\mathbf{t}-\sigma_1(\mathbf{t})) = \text{diag}\{0.4e_1(t) + 0.3e_1(t-\sigma_1(t)), -0.5e_2(t) + 0.2e_2(t-\sigma_1(t))\}$,
$\beta_{ij}(\mathbf{t}, \mathbf{e_i}(\mathbf{t}), \mathbf{e_i}(\mathbf{t}-\sigma_2(\mathbf{t})) = \text{diag}\{0.4e_1(t) + 0.3e_1(t-\sigma_2(t)), -0.5e_2(t) + 0.2e_2(t-\sigma_2(t))\}$.

Fig. 2 shows that the drive system (3) has a limit cycle in the case of the above-mentioned parameters. It is clear that drive system (3) and the corresponding response system (4) are globally anti-synchronized in the mean square. Figs. 3 and 4 depict the state trajectories and anti-synchronization error of the state variables between systems (3) and (4), respectively.

Get together the above mentioned parameters with the condition (27), we calculate the setting times $T_{x1} = T_{x2} = T_{y1} = T_{y2} = 0.995$. It is easy to see that the states are quickly converging to stable according to Fig. 4 within the setting time. Hence, it can be concluded that, according to Theorem 1, the considered system (3) can be globally anti-synchronized with system (4) in the mean square.

The drive system (3) without leakage delays has chaotic attractors under the initial values which can be seen in Fig. 5. It follows from Corollary 1, that drive system (3) and the corresponding response system (4) without leakage delays are global anti-synchronized in the mean square. Fig. 6 depict the state trajectories and anti-synchronization error of the state variables $x_1(t) + \hat{x}_1(t)$, $x_2(t) + \hat{x}_2(t)$, $y_1(t) + \hat{y}_1(t)$, and $y_2(t) + \hat{y}_2(t)$ between systems (3) and (4) without leakage delays, respectively. From Fig. 7, the results shown the feedback control inputs contribute to the chaos anti-synchronization in the mean square.

It is shown that drive system (3) and the corresponding response system (28) are exponentially anti-synchronized in the

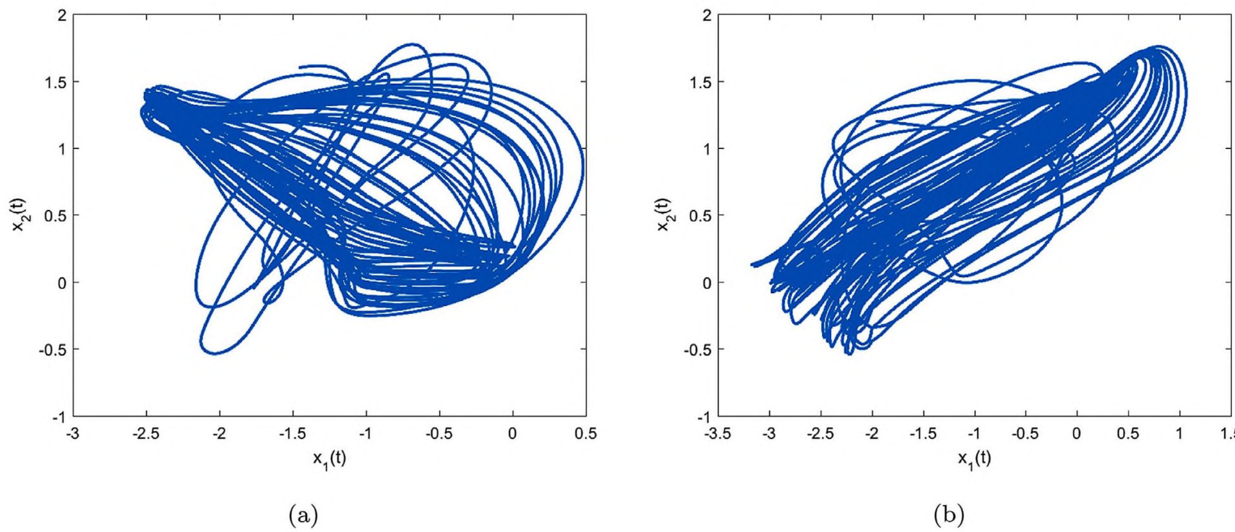


Fig. 2. Phase trajectories of the drive system (3).

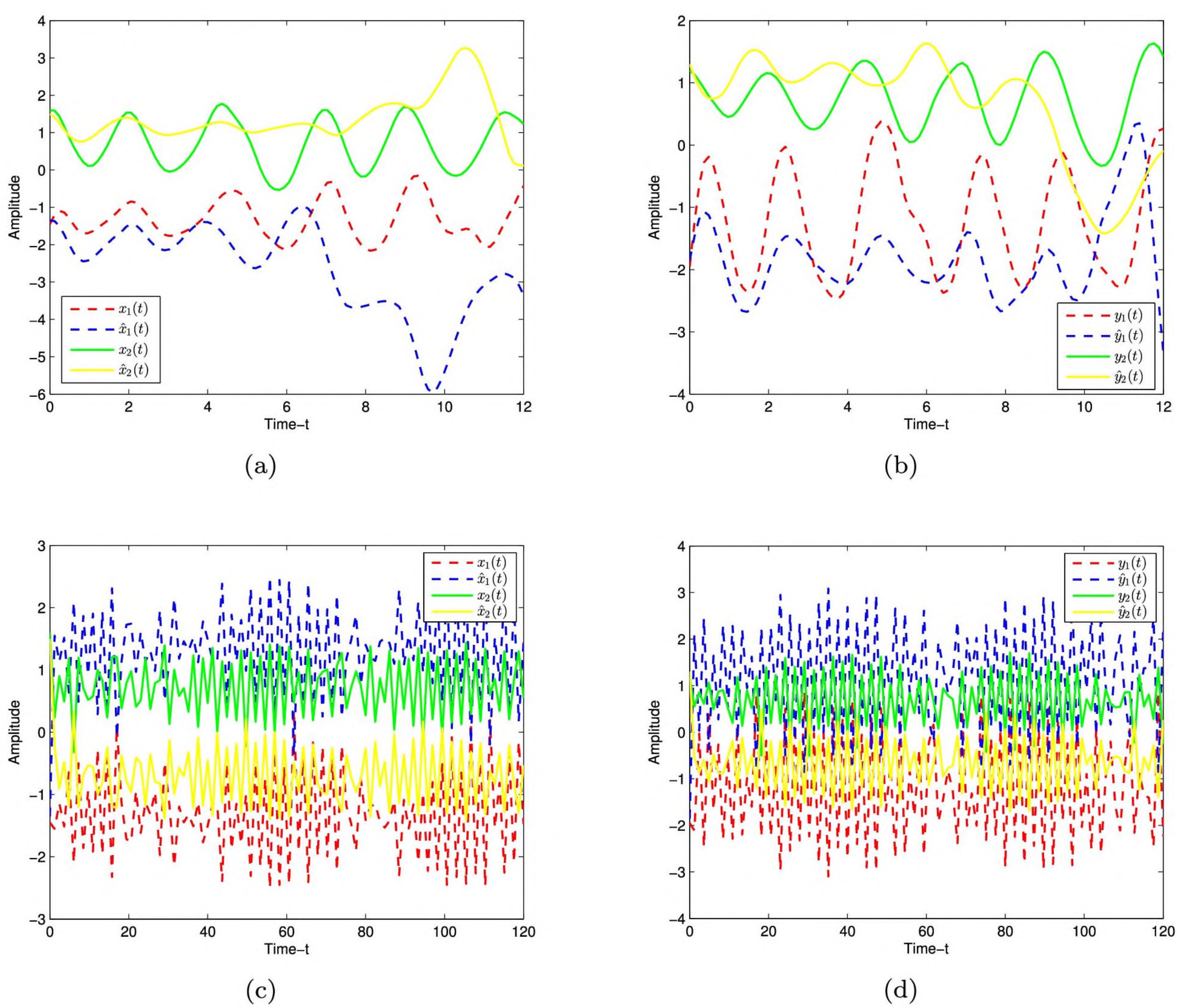
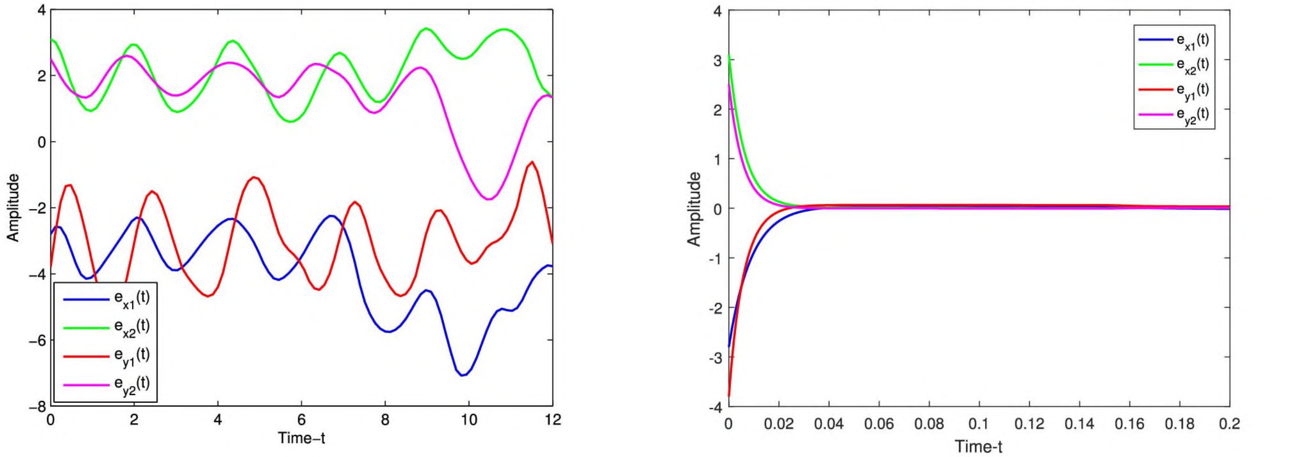
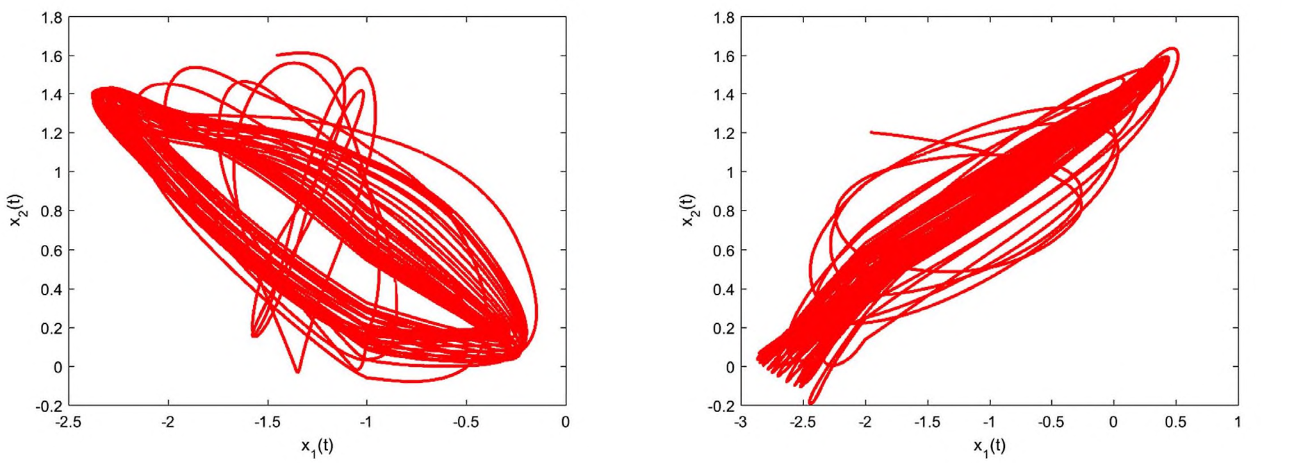


Fig. 3. (*a*) and (*b*) State trajectories of the systems (3) and (4) without control; (*c*) and (*d*) State trajectories of the systems (3) and (4) under control, respectively.



(a) (b)

Fig. 4. (*a*) The anti-synchronization error of the systems (3) and (4) without control ; (*b*) The anti-synchronization error of the systems (3) and (4) under control.



(a) (b)

Fig. 5. Phase trajectories of the drive system (3) without leakage delays.

mean square. Figs. 8 and 9 illustrated the state trajectories and anti-synchronization error of the state variables between systems (3) and (28), respectively. It is easy to see that the states are quickly converging to stable according to Fig. 9. Hence, it can be concluded that, according to Theorem 2, the considered BAM system (3) can be globally anti-synchronized with system (28) in the mean square.

The drive system (3) without leakage delays has chaotic attractors with the initial values which can be seen in Fig. 6. From Figs. 10 to 11, it is observed that the state trajectories of the considered MBAMNNs converge to zero globally in the mean square. Therefore it follows, from Corollary 2, the error systems (29) and (30) are stable in the mean square. This implies that the system (9) is a proper feedback controller to the chaos anti-synchronization.

5. Conclusion

We have investigated the finite-time anti-synchronization of stochastic MBAMNNs with probabilistic and leakage time-varying delays. The mixed time-varying delays involve discrete time delays, probabilistic time-varying delays and a constant delay in the leakage term. We also considered the stochastic perturbations corresponding to the probabilistic time-varying delays. Based on an effective Lyapunov function, finite-time stability theorem, stochastic analysis theory and Wirtinger-type inequality, sufficient conditions that depend on the time-varying delays are derived to ensure that the MBAMNNs achieve anti-synchronization in the mean square within finite-time. Numerical examples were provided to demonstrate the usefulness and effectiveness of the proposed control strategy. The obtained results extend and improve some previous works on conventional BAMNNs.

The future work mainly includes the following aspects: (1) MBAMNNs is a new and interesting topic, searching for more practical and complex MBAMNNs model is our further work. Since the MBAMNNs can be treated as a discontinuous switched system, it is significant to address a more preferable mathematical method to study; (2) So far, we have completed the application of MNNs to secure communication and image encryption. How to apply the results in practice, such as the associative memory and forget of brain-like, mass storage, machine learning, and so on. In summary,

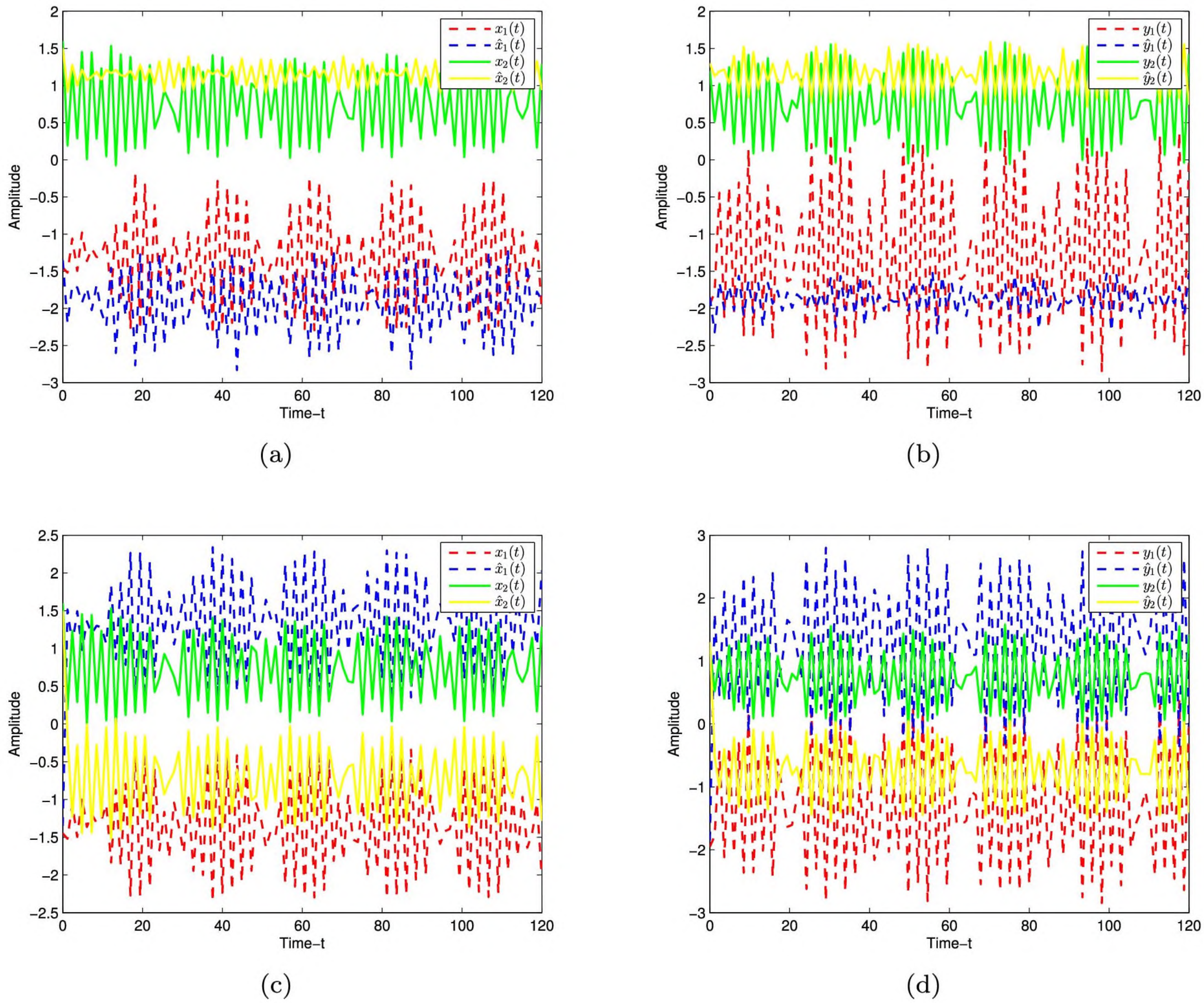


Fig. 6. (*a*) and (*b*) State trajectories of the systems (3) and (4) without leakage delays and control; (*c*) and (*d*) State trajectories of the systems (3) and (4) without leakage delays but under control, respectively.

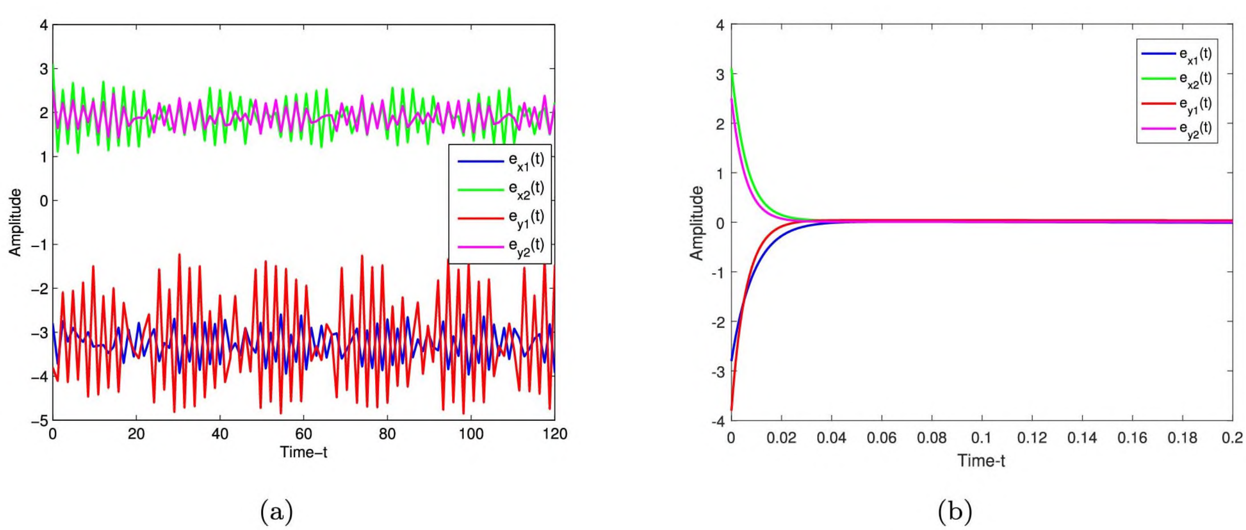


Fig. 7. (*a*) The anti-synchronization error of the systems (3) and (4) without leakage delays and control; (*b*) The anti-synchronization error of the systems (3) and (4) without leakage delays but under control.

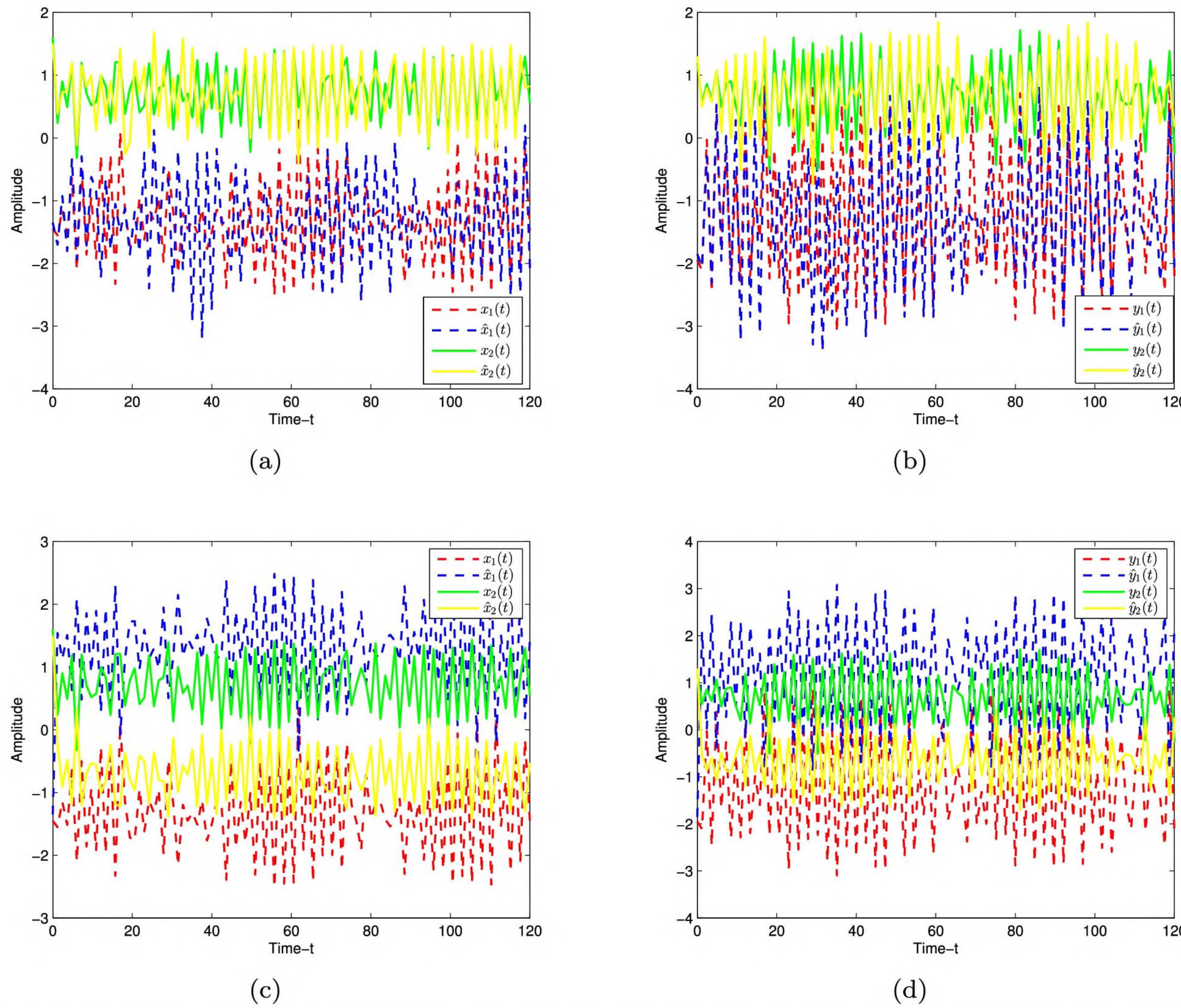


Fig. 8. (*a*) and (*b*) State trajectories of the systems (3) and (28) without control; (*c*) and (*d*) State trajectories of the systems (3) and (28) under control, respectively.

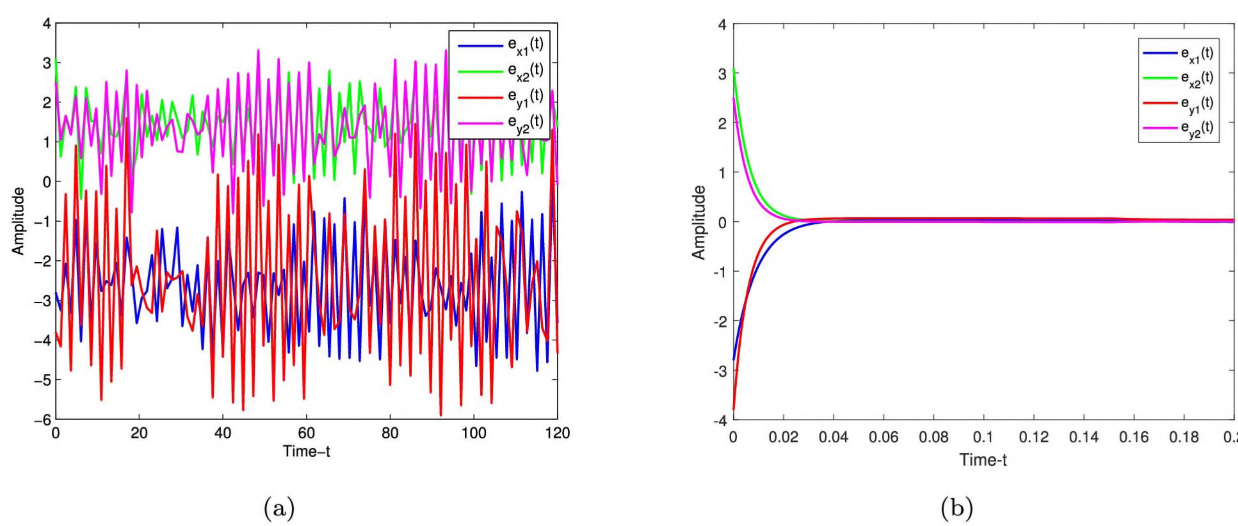


Fig. 9. (*a*) The anti-synchronization error of the systems (3) and (28) without control; (*b*) The anti-synchronization error of the systems (3) and (28) under control.

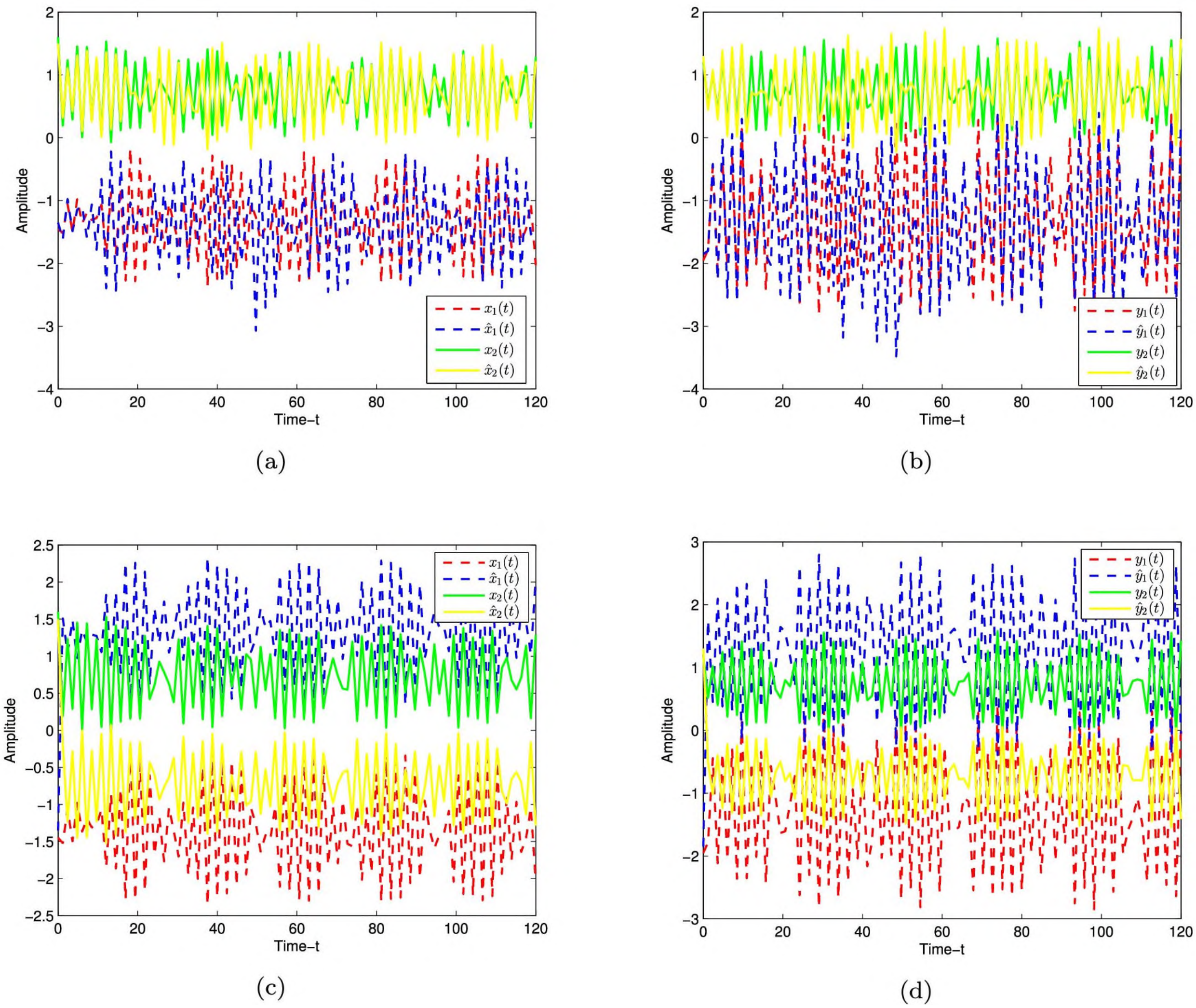


Fig. 10. (*a*) and (*b*) State trajectories of the systems (3)–(28) without leakage delays and control; (*c*) and (*d*) State trajectories of the systems (3) and (28) without leakage delays but under control, respectively.

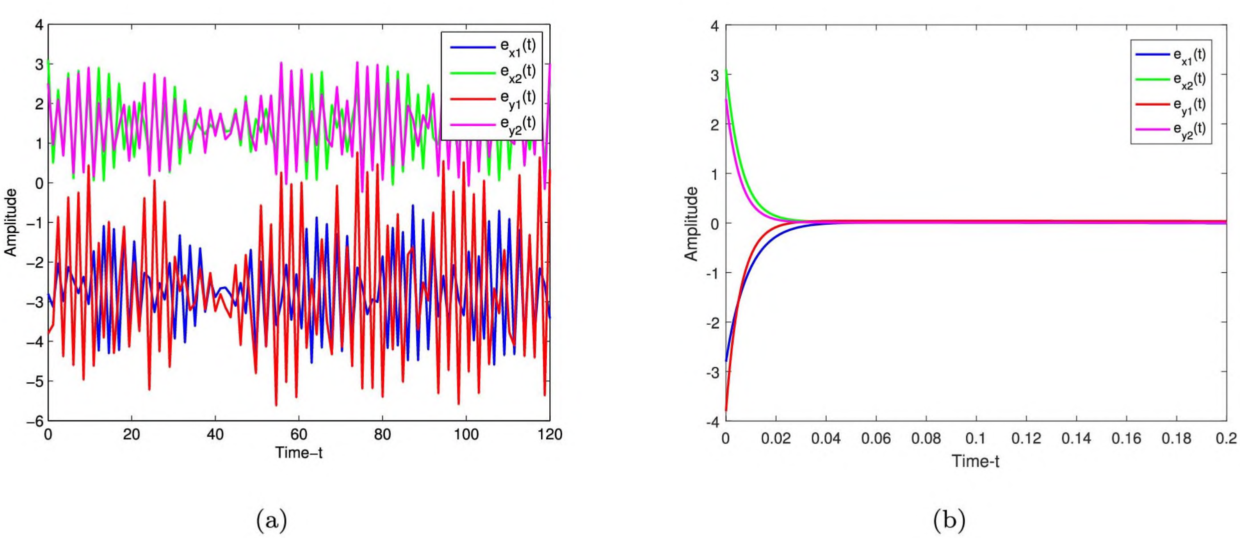


Fig. 11. (*a*) The anti-synchronization error of the systems (3) and (4) without leakage delays and control; (*b*) The anti-synchronization error of the systems (3) and (4) without leakage delays but under control.

the memristive associative memory neural networks still have a lot of problems worthy of further study.

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