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Probabilistic Stress Analysis of Circular Fins of Different Profiles

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PROBABILISTIC STRESS ANALYSIS OF CIRCULAR FINS OF DIFFERENT PROFILES

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SRIDHAR MADHIRA VENKATA

ABSTRACT

The temperature distribution and thermal Stresses induced by a temperature difference for steady state heat transfer in silicon carbide (SiC) ceramic tube heat exchanger with circular fins was computationally simulated by a finite element method and probabilistically evaluated in view of the several uncertainties in the performance parameters. Cumulative distribution functions and sensitivity factors were computed for Hoop and Radial stresses due to the structural and thermodynamic random variables. These results are used to identify the most critical design variables in order to optimize the design and make it cost effective. The probabilistic analysis leads to the selection of the appropriate measurements to be used in structural and heat transfer analysis and to the identification of both the most critical measurements and parameters.

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NOMENCLATURE

K_x	Thermal Conductivity in the x Direction
K_y	Thermal Conductivity in the y Direction
n_x	Direction Cosine in the x Direction
n_y	Direction Cosine in the y Direction
L_1	Boundary Length
L_2	Boundary Length Remainder
a	Thermal Coefficient of Expansion in the x Direction
b	Thermal Coefficient of Expansion in the x Direction
A	Area
h_i	Inside Convective heat transfer coefficient
h_o	Outer Convective heat transfer coefficient
T_{b_i}	Inner Fluid Bulk temperature
T_{b_o}	Outer Fluid Bulk temperature/Ambient temperature
k	Thermal conductivity
Cp	Specific heat capacity of material
E	Young's modulus of the material
α	Thermal expansion coefficient
t	Fin thickness
L	Fin height
r_i	Inner radius of tube

T	tube thickness
ρ	Density
ν	Poisson's ratio
SX	Radial Stress
SY	Hoop Stress
$F_X(x)$	CDF of X
$f_X(x)$	PDF OF X
$g(.)$	Limit state function
X	Random Variable
n	Number of random variables
p_f	Probability of Failure
u	Standardized Normal (Gaussian) Variable
$Z(.)$	Performance or Response Function (Z-function)
α	Probabilistic Sensitivity Factor
β	Minimum Distance
$\Phi(u)$	CDF of u
$\phi(u)$	PDF of u
μ	Mean Value
σ	Standard deviation

CHAPTER I

INTRODUCTION

1.1 Overview of Finned Tube Heat Exchangers

The properties such as high temperature capability and resistance to oxidation make ceramics preferable for high temperature heat exchangers and due to this they can be used for process heat exchange, power generation, and industrial heat recovery. Ceramic materials such silicon carbide exhibit excellent high temperature and strength above 1100 deg centigrade and are perfect candidates for high temperature heat exchangers.The majority of very high temperature heat recovery applications involve gas-to-gas heat transfer, and the most commonly used types of heat exchanger construction involve tubular geometries. In gas-to-gas heat exchangers the heat transfer coefficients are low, the use of enhanced surface is necessary to improve the performance and reduce the size of the heat exchangers and annular fins find numerous applications. However, not many research papers have been published in connection with temperature distributions and thermal stresses in SiC ceramic heat exchanger tubes with annular fins. Therefore, in this work, the temperature distribution and thermal stresses are calculated numerically using the COSMOS Works finite element software, taking into account the effect of the profile of the circular fin.

1.2 Overview of NESSUS

A stochastic process or a random process in probability is the counterpart to a deterministic process. Instead of dealing with only one possible outcome of how the process might evolve under time, in a stochastic or random process there is some indeterminacy in its future evolution described by probability distributions. This means that even if the initial condition is known, there are many possibilities the process might go to, but some paths may be more probable and others less so.

NESSUS (Numerical Evaluation of Stochastic Structures under Stress) developed by NASA Glenn Research Center. The code combines state of the art probabilistic algorithms with general purpose structural analysis methods to compute the probabilistic response and the reliability of engineering structures. Uncertainty in loading, material properties, geometry, boundary conditions and initial conditions can be simulated. The structural analysis methods include nonlinear finite element methods and boundary element methods. Several probabilistic algorithms are available such as the advanced mean value method and the adaptive importance sampling method. The application of the code includes probabilistic structural response, component and system reliability and risk analysis of structures considering cost of failure. The basic heat transfer variables are included as random variables along with the mechanical random variables to quantify risk using probabilistic methods to perform sensitivity analysis.

1.3 Model and Problem Approach

The goal of the reported work is to study temperature distribution of steady-state heat transfer and the thermal stresses induced in a 3D model of annular finned tube heat exchanger of different profiles in COSMOS Works, a finite element code for structural and heat transfer analysis. The design variables considered to be as random variables are then used to perform a Probabilistic Analysis using NESSUS to account for uncertainties as they affect structural reliability.

CHAPTER II

FINITE ELEMENT ANALYSIS

2.1 Finite Element Analysis of Heat Transfer

The three dimensional problems with geometric symmetry about a reference axis, in the present case Z-Axis can be solved by two dimensional elements where boundary conditions and field functions are independent of circumferential direction(θ).

Let us consider a two-dimensional partial differential equation of the form

$$\frac{1}{r}\frac{\partial}{\partial r}\left[K_r r\frac{\partial T}{\partial r}\right]+\frac{\partial}{\partial z}\left[K_z\frac{\partial T}{\partial z}\right]+PT+Q=0 \quad \text{in Area, A} \tag{2.1}$$

Where

$$T=T_0 \quad \text{on} \quad L_1 \tag{2.2}$$

or

$$K_r\frac{\partial T}{\partial r}n_r+K_\theta\frac{\partial T}{\partial z}n_z+\alpha T+\beta=0 \quad \text{on} \quad L_2 \tag{2.3}$$

are the boundary conditions and the corresponding functional is given by

$$I=\iint_A \left\{\frac{1}{2}K_r\left(\frac{\partial T}{\partial r}\right)^2+\frac{1}{2}K_z\left(\frac{\partial T}{\partial z}\right)^2-\frac{1}{2}PT^2-QT\right\}2\pi r\ dA \tag{2.4}$$

where, n_r and n_z are direction cosines of the outward normal to L_2.

The temperature distribution is:

$$T = N_i T_i + N_j T_j + N_k T_k \tag{2.5}$$

This is similar to linear triangular plane element where the shape functions are

$$N_i = \frac{1}{2A}\left(a_i + b_i r + c_i z\right)$$

$$N_j = \frac{1}{2A}\left(a_j + b_j r + c_j z\right)$$

$$N_k = \frac{1}{2A}\left(a_k + b_k r + c_k z\right)$$

The constants in the above are defined as:

$$\begin{array}{lll} a_i = r_j z_k - r_k z_j & a_j = r_k z_i - r_i z_k & a_k = r_i z_j - r_j z_i \\ b_i = z_j - z_k & b_j = z_k - z_i & b_k = z_i - z_j \\ c_i = r_k - r_j & c_j = r_i - r_k & c_k = r_j - r_i \end{array}$$

and

$$2A = b_i c_j + b_j c_i$$

Here, r_i and z_i are the coordinates of the node i.

The element minimization equations are given by:

$$\begin{Bmatrix} \dfrac{\partial I}{\partial T_i} \\ \dfrac{\partial I}{\partial T_j} \\ \dfrac{\partial I}{\partial T_k} \end{Bmatrix}^{(e)} = \left[B\right]^{(e)} \left\{T\right\}^{(e)} - \left\{C\right\}^{(e)} \tag{2.6}$$

where the element matrix $[B]$ is

$$[B]^{(e)} = \iint_{A^{(e)}} (2\pi r)\Big[[D]^T [K][D] - P\{N\}^T \{N\}\Big] dA$$

$$+ \int_{L^{(e)}} (2\pi r)\alpha \{N\}^T \{N\} \, dL_2 \tag{2.7}$$

and the element column $\{C\}$ is

$$[C]^{(e)} = \iint_{A^{(e)}} (2\pi r) Q [N]^T \, dA + \int_{L_2^{(e)}} (2\pi r)\beta \{N\}^T \, dL_2 \tag{2.8}$$

In the case of simplex ring element with a centroidal radial approximation, the radial term $2\pi\bar{r}$ comes outside the element integrals resulting the above equations in

$$[B]^{(e)} = 2\pi\bar{r} \iint_{A^{(e)}} \Big[[D]^T [K][D] - P\{N\}^T \{N\}\Big] dA + 2\pi\bar{r}_s \int_{L^{(e)}} \alpha \{N\}^T \{N\} \, dL_2 \tag{2.9}$$

and

$$[C]^{(e)} = 2\pi\bar{r} \iint_{A^{(e)}} Q [N]^T \, dA + 2\pi\bar{r}_s \int_{L_2^{(e)}} \beta \{N\}^T \, dL_2 \tag{2.10}$$

Here r_s denotes the centroid of the side and the integrals are the same as those in Cartesian coordinates.

For constant element properties, the element matrix $[B]$ becomes (2.11)

$$[B]^{(e)} = \frac{2\pi\bar{r}K_{rr}}{4A} \begin{bmatrix} b_i b_i & b_i b_j & b_i b_k \\ b_i b_j & b_j b_j & b_j b_k \\ b_i b_k & b_j b_k & b_k b_k \end{bmatrix}^{(e)} + \frac{2\pi\bar{r}K_{rr}}{4A} \begin{bmatrix} c_i c_i & c_i c_j & c_i c_k \\ c_i c_j & c_j c_j & c_j c_k \\ c_i c_k & c_j c_k & c_k c_k \end{bmatrix}^{(e)}$$

$$-\frac{2\pi\ \bar{r}\ \ PA}{12}\begin{bmatrix}2 & 1 & 1\\1 & 2 & 1\\1 & 1 & 2\end{bmatrix}^{(e)} \qquad +\frac{2\pi}{6}\left(\alpha\bar{r}L\right)_{ij}\begin{bmatrix}2 & 1 & 0\\1 & 2 & 0\\0 & 0 & 0\end{bmatrix}^{(e)}_{on\ side\ ij}$$

$$+\frac{2\pi}{6}\left(\alpha\bar{r}L\right)_{jk}\begin{bmatrix}0 & 0 & 0\\0 & 2 & 1\\0 & 1 & 2\end{bmatrix}^{(e)}_{on\ side\ jk} \qquad +\frac{2\pi}{6}\left(\alpha\bar{r}L\right)_{ki}\begin{bmatrix}2 & 0 & 1\\0 & 0 & 0\\1 & 0 & 2\end{bmatrix}^{(e)}_{on\ side\ ki}$$

and the element column $\{C\}$ is (2.12)

$$\{C\}^{(e)} = \frac{2\pi\bar{r}QA}{3}\begin{Bmatrix}1\\1\\1\end{Bmatrix}^{(e)} - \frac{2\pi\left(\beta\bar{r}L\right)_{ij}}{2}\begin{Bmatrix}1\\1\\0\end{Bmatrix}^{(e)} - \frac{2\pi\left(\beta\bar{r}L\right)_{jk}}{2}\begin{Bmatrix}0\\1\\1\end{Bmatrix}^{(e)} - \frac{2\pi\left(\beta\bar{r}L\right)_{ki}}{2}\begin{Bmatrix}1\\0\\1\end{Bmatrix}^{(e)}$$

On each side, the term $\bar{r}$ denotes the centroid of that side. As in 2-D problem, the element matrix $[B]$ is a 3 * 3 matrix, and the element column $\{C\}$ is a three component column.

For the terms evaluated along the side of elements, β is considered to be a constant within the element. The side lengths are given by (2.13)

$$L_{ij} = \left[\left(r_i - r_j\right)^2 + \left(z_i - z_j\right)^2\right]^{1/2}$$

$$L_{jk} = \left[\left(r_j - r_k\right)^2 + \left(z_j - z_k\right)^2\right]^{1/2}$$

$$L_{ki} = \left[\left(r_k - r_i\right)^2 + \left(z_k - z_k\right)^2\right]^{1/2}$$

The derivative boundary conditions to be imposed on a certain side are the derivative boundary matrix and column included in the appropriate element matrix and column. The element matrices were assembled into global matrices and vectors and the prescribed boundary conditions were applied at the appropriate nodal points. The algebraic equations in the global assembled form were then solved by the Gauss elimination procedure.

2.2 Thermal Stress Evaluation Procedure

If the change in temperature $\Delta T(x, y)$ is known, the strain due to this change in temperature can be treated as an initial strain ε_0. From the theory of solid mechanics, plane stress can be given by

$$\varepsilon_0 = (\alpha\Delta T, \alpha\Delta T, 0)^T \tag{2.14}$$

and the plane strain is given by

$$\varepsilon_0 = (1 + \nu)(\alpha\Delta T, \alpha\Delta T, 0)^T \tag{2.15}$$

The stresses and strains are related by

$$\sigma = D(\varepsilon - \varepsilon_0) \tag{2.16}$$

Where D is the symmetric (6 X6) material matrix given by

$$D = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5-\nu & 0 & 0 \\ 0 & 0 & 0 & 0.5-\nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5-\nu \end{bmatrix} \tag{2.17}$$

The effect of temperature can be accounted by considering the strain energy term.

$$\mathrm{U} = \frac{1}{2}\int (\varepsilon - \varepsilon_0)^T D(\varepsilon - \varepsilon_0)tdA$$

$$= \frac{1}{2}\int (\varepsilon^T D\varepsilon - 2\varepsilon^T D\varepsilon_0 + \varepsilon_0^T D\varepsilon_0)tdA \tag{2.18}$$

The first term in the previous expansion gives the stiffness matrix derived earlier. The last term is a constant and has no effect on the minimization process. The middle term, which yields the temperature load, is now considered in detail. Using the strain-displacement relationship $\varepsilon = Bq$,

$$\int_A \varepsilon^T D\varepsilon_0 t dA = \sum_e q^T (B^T D\varepsilon_0) t_e A_e \tag{2.19}$$

This is obtained using the Galerkin approach where ε^T will be $\varepsilon^T(\phi)$ and q^T will be ψ^T The symbol ϕ defines the shape function and ψ defines the weight function. It is convenient to designate the element temperature load as

$$\theta^e = t_e A_e B^T D\varepsilon_0 \tag{2.20}$$

where,

$$\theta^e = [\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6]^T \tag{2.21}$$

The vector ε_0 is the strain due to the average temperature change in the element. θ^e represents the element nodal load distributions that must be added to the global force vector.

The stresses in an element are then obtained in the form

$$\sigma = D(Bq - \varepsilon_0) \tag{2.22}$$

2.3 Analysis in Cosmos Works

Steady State Heat Transfer Analysis

The heat transfer analysis is based on the following assumptions:

- Steady-state heat flow,
- There is no heat source,

- The temperature of the surrounding fluid is uniform,
- The thermal conductivity of the material is constant,
- The convection heat transfer coefficient is the same all over the surface,
- The materials are homogeneous and isotropic.

The representative models of the finned-tube considered in the present study are drawn in the sketch mode of SOLIDWORKS to build the solid 3D model of heat exchanger tube with the fin and are shown in Fig.1 and Fig.2.A new material is defined as silicon carbide with silicon as base material by applying the material properties shown in Table 1. The loads are applied as boundary conditions at the inner and outer sides (Fin surface) convective heat transfer coefficients are 1000 and 100 W/m^2K, respectively, and for those sides the fluid bulk temperatures are kept at 1200 and 300 K. Each element of the tube is no less than 1/10 the length to diameter R of the tube to provide resistance to bending [6]. The displacement and heat flux were kept at zero at both tube edges, horizontal lines (y-direction).

Since the model is Axi-symmertric we can perform the simulation by considering only one quarter of the model to save computing time by saving the memory required for computing.

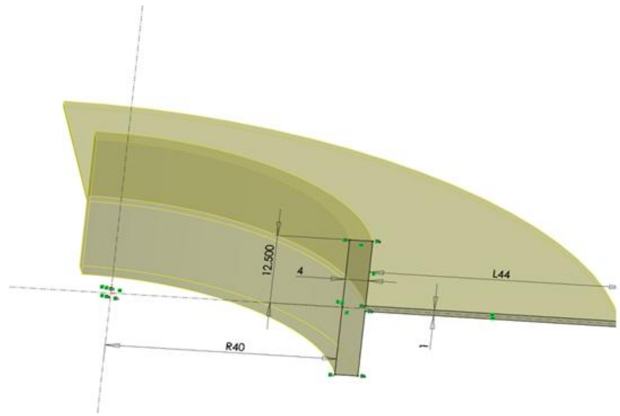

Fig.1.Axi-symmetric circular fin of Rectangular profile

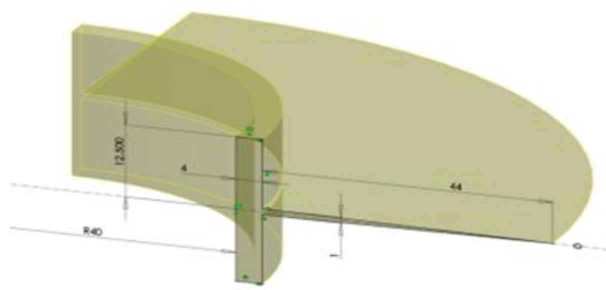

Fig.2.Axi-symmetric circular fin of Triangular profile

Material properties of SiC ceramics for thermal analysis	
Thermal conductivity, k (W/mk)	42
Specific heat capacity of material, Cp (J/kgK)	2540
Young’s modulus of the material, E (GPa)	427
Thermal expansion coefficient, α (10-6 oC-1)	4.8
Density, ρ (kg/m3)	3210
Poisson’s ratio, ν	0.17
Tensile strength, Mpa	950

Table 1.Material properties of Silicon Carbide

After applying material properties and defining the loads the model has been discretized in to finite number of elements with nodes by meshing it. The mesh properties are shown in table 3.By changing the mesh quality one can specify different sizes of elements for components, faces, edges, and vertices. The present case of meshing considered is of 3D tetrahedral solid elements. The mesh is of uniform elements unless specified as a mixed mesh.

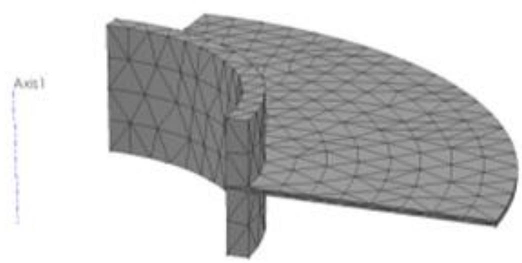

Fig.3.Mesh model of Rectangular fin

Mesh type	Solid Mesh
Mesher Used	Curvature based mesh
Max Element Size	6.3146 mm
Min Element Size	6.3146 mm
Mesh quality	High
Total nodes	2624
Total elements	1228

Table.2.Mesh details for Rectangular and Triangular profile

The type of study chosen for heat transfer analysis is steady state heat transfer analysis by considering the assumptions and the temperature results obtained are plotted as shown in fig.6.Since COSMOS Works cannot perform heat transfer and static stress simultaneously, the heat transfer analysis was performed first and then the results of the thermal analysis were imported to the static stress analysis.

Once the temperature results are calculated, the mode of the analysis is changed from steady-state heat transfer to linear static stress.

Thermal Stress Analysis

All the constraints and the loads from the heat transfer analysis are applied to the model and the steady state static stress analysis is performed and stresses are calculated at each node. In the present case solid works simulation uses FFE Plus which is an iterative solver. The iterative solver uses Direct Stiffness method to compute member forces and displacements in structures. The force-displacement relationship for each element is determined separately and then each individual is combined to contribute to the whole structure to form in to a Global Structural Stiffness Matrix. The matrix is then solved for all unknown displacements and there by the forces by Gaussian elimination by substituting boundary conditions. From these nodal stress values, the maximum Hoop and Radial stresses can be found. The temperature and stress profiles of the fin have been shown.

The above simulation procedures are applied for both rectangular and triangular profiles. All the temperature and stress results obtained and their respective profiles have been shown in figures.

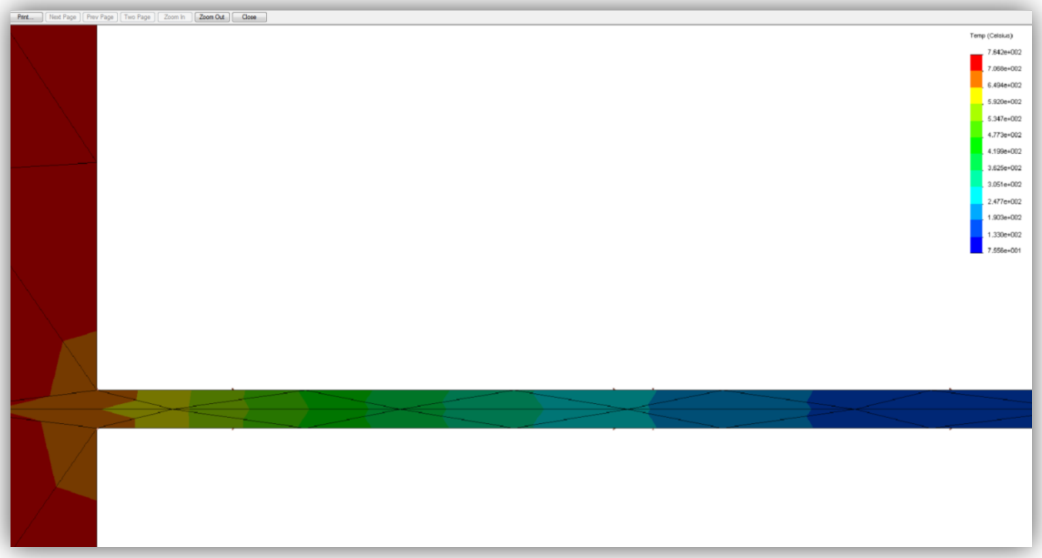

Fig.4.Temperature distribution of Rectangular fin

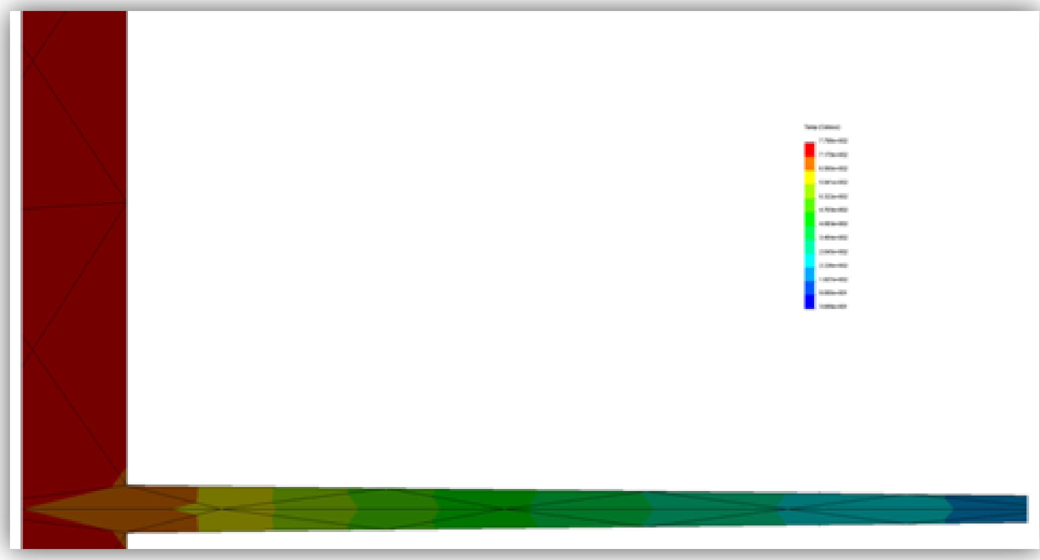

Fig.5.Temperature distribution of Triangular fin

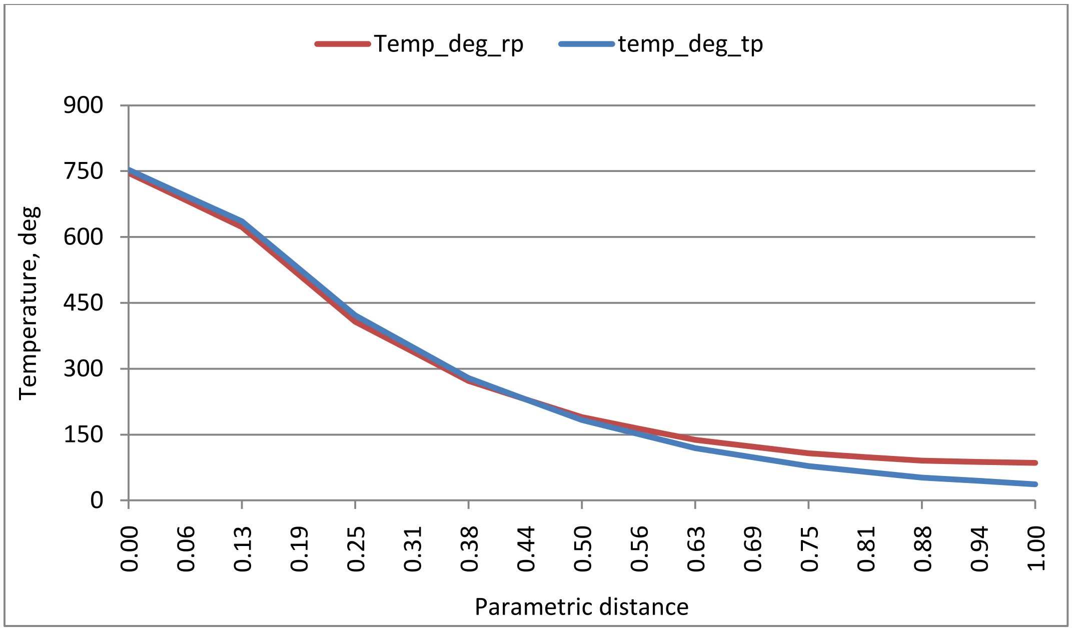

Fig.6.Temperature distribution along mid-plane of rectangular and triangular fins

Fig.7. Hoop stress distribution of Rectangular fin

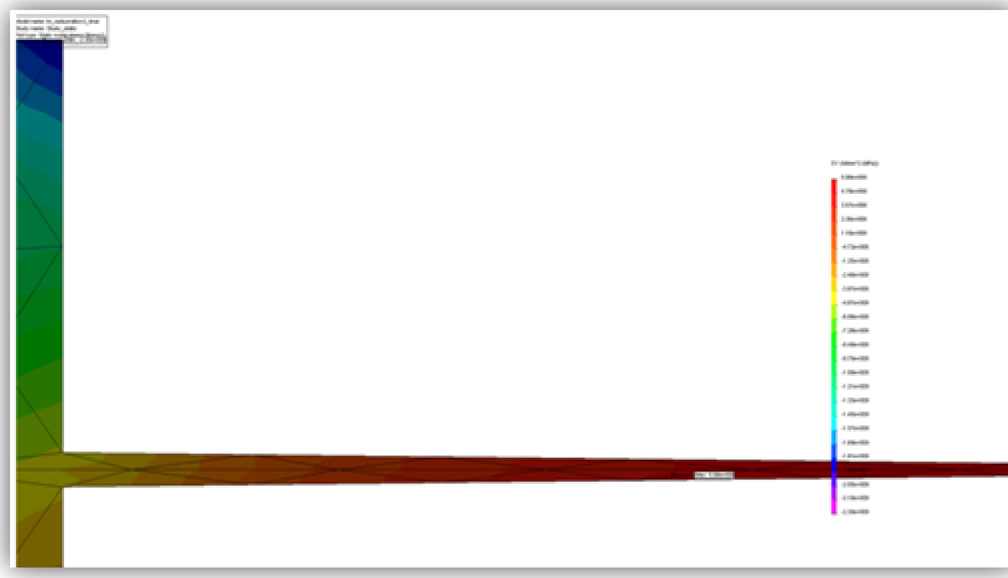

Fig.8. Hoop stress distribution of Triangular fin

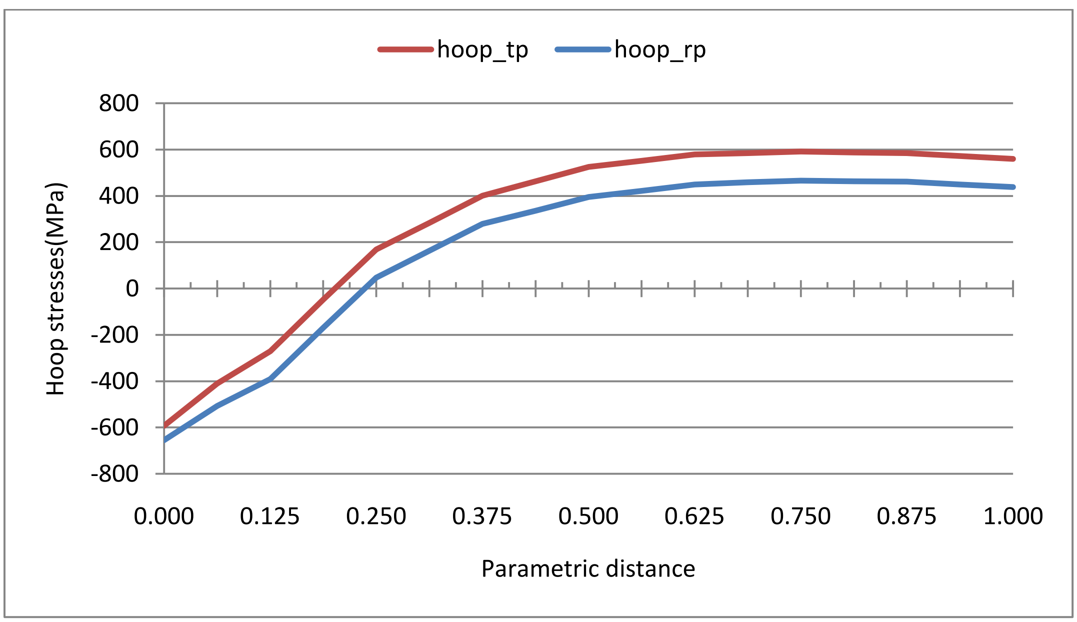

Fig.9.Hoop stress distribution along mid-plane of rectangular and triangular fins

Fig.10.Radial Stress distribution of Rectangular fin.

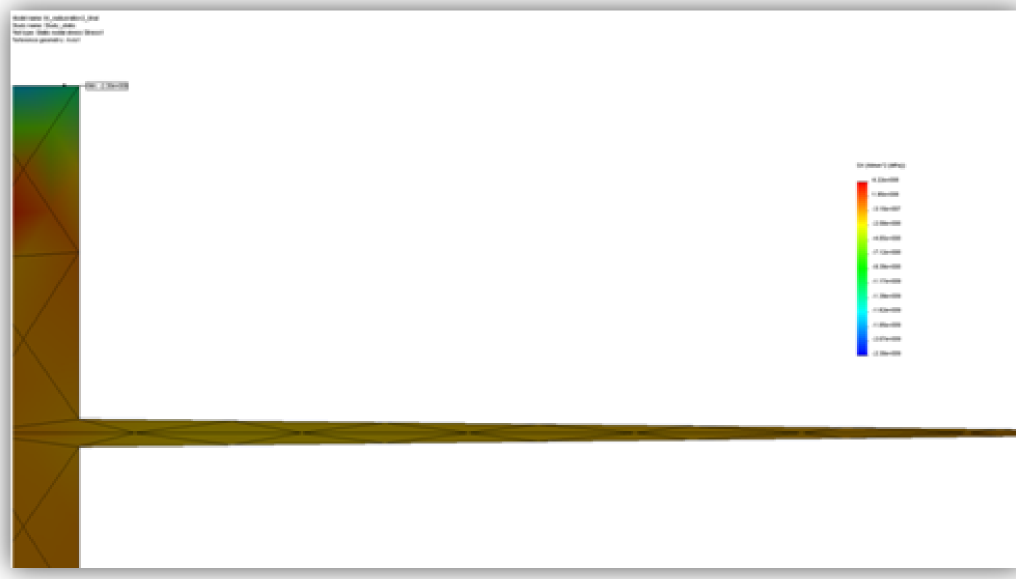

Fig.11.Radial Stress distribution of Triangular fin.

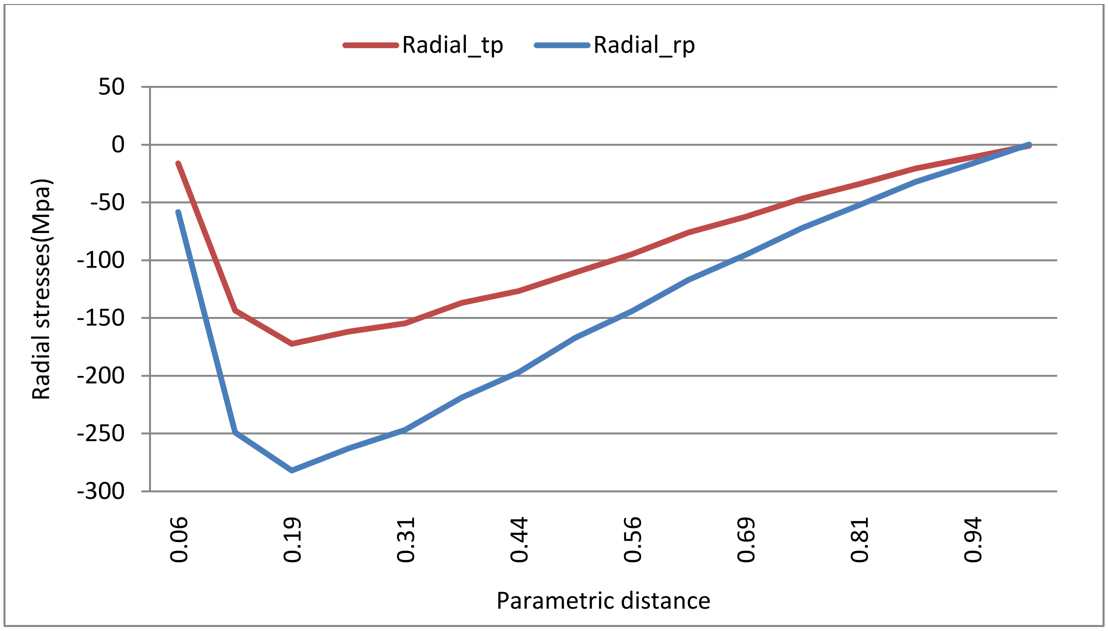

Fig.12.Radial stress distribution along mid-plane of rectangular and triangular fins.

The maximum stress values obtained in global coordinate system have been changed to local cylindrical coordinate system by taking the axis as reference in order to get hoop and radial stresses. Figures 4, 5 and 6 shows the temperature distribution along the mid plane of the fin. Similarly Hoop stresses in fig's 7, 8, 9 and radial stresses in fig's 10, 11 and 12 are shown along the radial direction at the mid plane of the fin.

The hoop stresses for both rectangular and triangular profiles are compressive near the base of the fins but changes to tensile towards the tip of the fins.

The radial stresses along that plane are compressive and reaches to zero close to tip of the fins. As seen from the figures the radial stresses are maximum near the base of the fins.

The previous work [19] shows that the temperature distribution of triangular profile is higher than for a rectangular profile.

The same is the case for the present case i.e temperature distribution of triangular profile is higher than for a rectangular profile.

Table 3.Comparison of Thermal Stress Analysis Results

	Present Work	**Previous work[19]**
	Finite Element Analysis in COSMOS Works	Finite Element Analysis in ANSYS
Temperatures	Distribution in Triangular fin > Rectangular Fin	Distribution in Triangular fin > Rectangular Fin
Hoop Stress	Contour of rectangular profile < Contour of triangular profile	Hoop stress contour is smaller than radial stress for both profiles
Radial Stress	Contour of Rectangular profile >Contour of triangular profile	High and tensile close to the base of rectangular fin which may cause failure

Now Considering 15 design variables (includes boundary conditions, material properties and dimensions) as random variables as shown in Table.4. for input to probabilistic analysis, 60 simulation runs have been carried out by varying random variables by +/-10%, one at a time keeping others constant. Max hoop and radial stresses of rectangular and triangular profiles for each run have been recorded and tabulated in tables 4 and 5 for probabilistic analysis.

Table 4.Random Variables

No.	Random Variable	Mean Value
1	Convective heat transfer coefficient, hi (W/m2k)	1000
2	Convective heat transfer coefficient, ho (W/m2k)	100
3	Fluid Bulk temperature, Tbi	1200
4	Fluid Bulk temperature, Tbo	300
5	Thermal conductivity, k (W/mk)	42
6	Specific heat capacity of material, Cp (J/kgK)	2540
7	Young’s modulus of the material, E (GPa)	427
8	Thermal expansion coefficient, α (10-6 oC-1)	4.8
9	Fin thickness, t (mm)	1
10	Fin height, L (mm)	44
11	Inner radius of tube, ri (mm)	40
12	Tube thickness, T	4
13	Density, ρ (kg/m3)	3210
14	Poisson’s ratio, ν	0.17
15	Tensile strength , Mpa	950

Table 5.Max Hoop and radial stresses of Rectangular fin for various runs in Cosmos

Rectangular Profile	values	variation from mean	Max SX, Mpa	Max SY, Mpa
Boundary conditions, model dimensions, material properties			Max radial	Max hoop
Mean values of the design		0%	7.43257E+08	4.85220E+08
Convective heat transfer coefficient, hi (W/m2k)	1100	+10%	7.56404E+08	4.93920E+08
Convective heat transfer coefficient, hi (W/m2k)	900	-10%	7.27783E+08	4.74997E+08
Convective heat transfer coefficient, ho (W/m2k)	110	+10%	7.33804E+08	4.82403E+08
Convective heat transfer coefficient, ho (W/m2k)	90	-10%	7.33804E+08	4.82403E+08
Fluid Bulk temperature, Tbi	1320	+10%	8.42085E+08	5.49968E+08
Fluid Bulk temperature, Tbi	1080	-10%	6.44428E+08	4.20471E+08
Fluid Bulk temperature, Tbo	330	+10%	7.49215E+08	4.63133E+08
Fluid Bulk temperature, Tbo	270	-10%	7.37299E+08	5.07307E+08
Thermal conductivity, k (W/mk)	46.2	+10%	7.38829E+08	4.77853E+08
Thermal conductivity, k (W/mk)	37.8	-10%	7.47905E+08	4.92113E+08
Specific heat capacity of material, Cp (J/kgK)	2790	+10%	7.43257E+08	4.85220E+08
Specific heat capacity of material, Cp (J/kgK)	2286	-10%	7.43257E+08	4.85220E+08
Young's modulus of the material, E (GPa)	469.7	+10%	8.17582E+08	5.33742E+08
Young's modulus of the material, E (GPa)	384.3	-10%	6.68931E+08	4.36698E+08
Thermal expansion coefficient, α (10-6 oC-1)	5.28	+10%	8.17582E+08	5.33742E+08
Thermal expansion coefficient, α (10-6 oC-1)	4.32	-10%	6.68931E+08	4.36698E+08
Fin thickness, t (mm)	1.1	+10%	7.08677E+08	4.73860E+08
Fin thickness, t (mm)	0.9	-10%	6.63423E+08	4.95779E+08
Fin height, L (mm)	48.4	+10%	6.33836E+08	4.71796E+08
Fin height, L (mm)	39.6	-10%	6.30534E+08	4.90457E+08

Inner radius of tube, ri (mm)	44	+10%	6.36152E+08	4.89186E+08
Inner radius of tube, ri (mm)	36	-10%	7.99266E+08	4.76536E+08
tube thickness, mm	4.4	+10%	7.03697E+08	4.81112E+08
tube thickness, mm	3.6	-10%	6.04604E+08	4.86115E+08
Density, ρ (kg/m3)	3531	+10%	7.43257E+08	4.85220E+08
Density, ρ (kg/m3)	2889	-10%	7.43257E+08	4.85220E+08
Poisson's ratio, ν	0.187	+10%	7.63027E+08	4.83438E+08
Poisson's ratio, ν	0.153	-10%	7.24411E+08	4.86983E+08
tensile strength, Mpa	1045	+10%	7.43257E+08	4.85220E+08
tensile strength, Mpa	855	-10%	7.43257E+08	4.85220E+08

Table.6.Max Hoop and Radial stresses of Triangular fin for various runs in Cosmos

Triangular profile	**values**	**Variation From mean**	**Max SX, Mpa**	**Max SY, Mpa**
Boundary conditions, model dimensions, material properties			**Max Radial**	**Max hoop**
Mean values of the design		0%	3.90937E+08	6.06117E+08
Convective heat transfer coefficient, hi (W/m2k)	1100	+10%	3.97567E+08	6.16526E+08
Convective heat transfer coefficient, hi (W/m2k)	900	-10%	3.83122E+08	5.93868E+08
Convective heat transfer coefficient, ho(W/m2k)	110	+10%	3.85854E+08	5.99036E+08
Convective heat transfer coefficient, ho(W/m2k)	90	-10%	3.96268E+08	6.13929E+08
Fluid Bulk temperature, Tbi	1320	+10%	4.42919E+08	6.86999E+08
Fluid Bulk temperature, Tbi	1080	-10%	3.38954E+08	5.25235E+08
Fluid Bulk temperature, Tbo	330	+10%	3.93929E+08	5.78389E+08
Fluid Bulk temperature, Tbo	270	-10%	3.87944E+08	6.33845E+08
Thermal conductivity, k (W/mk)	46.2	+10%	4.25661E+08	6.02988E+08

Thermal conductivity, k (W/mk)	37.8	-10%	4.30041E+08	6.09592E+08
Specific heat capacity of material, Cp (J/kgK)	2790	+10%	4.27797E+08	6.06463E+08
Specific heat capacity of material, Cp (J/kgK)	2286	-10%	4.27797E+08	6.06463E+08
Young’s modulus of the material, E (GPa)	470	+10%	4.70577E+08	6.67110E+08
Young’s modulus of the material, E (GPa)	384	-10%	3.85017E+08	5.45817E+08
Thermal expansion coefficient, α (10-6 oC-1)	5.28	+10%	4.70577E+08	6.67110E+08
Thermal expansion coefficient, α (10-6 oC-1)	4.32	-10%	3.85017E+08	5.45817E+08
Fin thickness, t (mm)	1.1	+10%	3.89966E+08	5.99345E+08
Fin thickness, t (mm)	0.9	-10%	3.92817E+08	6.12492E+08
Fin height, L (mm)	48.4	+10%	6.23382E+08	5.88211E+08
Fin height, L (mm)	39.6	-10%	3.88146E+08	6.24923E+08
Inner radius of tube, ri (mm)	44	+10%	4.67860E+08	6.08612E+08
Inner radius of tube, ri (mm)	36	-10%	4.89794E+08	6.01666E+08
tube thickness, mm	4.4	+10%	4.59195E+08	5.97552E+08
tube thickness, mm	3.6	-10%	4.58790E+08	6.15519E+08
Density, ρ (kg/m3)	3531	+10%	4.27797E+08	6.06463E+08
Density, ρ (kg/m3)	2889	-10%	4.27797E+08	6.06463E+08
Poisson’s ratio, ν	0.19	+10%	4.44021E+08	6.03836E+08
Poisson’s ratio, ν	0.15	-10%	4.12296E+08	6.09063E+08
tensile strength, Mpa	1045	+10%	4.27797E+08	6.06463E+08
tensile strength, Mpa	855	-10%	4.27797E+08	6.06463E+08

CHAPTER III

PROBABILISTIC ANALYSIS

The ability to quantify the uncertainty of complex engineered systems subject to inherent randomness in loading, environment, material properties, and geometric parameters is becoming increasingly important in design and certification efforts. Traditional design approaches typically use worst case assumptions and safety factors to certify a design. This approach is overly conservative, does not quantify the reliability; nor does it identify critical parameters or failure modes affecting the system performance.

A probabilistic analysis approach characterizes input variability using probability density functions and then propagates these density functions through the performance model to yield uncertain model outputs, which can be related to failure metrics such as fatigue life, rupture, or stress intensity. The approach quantifies the reliability, can reduce over-conservatism, and identifies critical parameters and failure modes driving the reliability of the system.

The programmers and researchers try to achieve the following in the development of the analysis algorithm.

- Identifying sources of errors and uncertainties
- Developing probability distributions for input variables

- Determining spatial and temporal variations
- Developing probabilistic load modeling
- Tailoring failure models for modeling uncertainty and obtaining appropriate system performance measure
- Creating system models (multiple failure mode and components)

Numerical evaluation of stochastic structures under stress (NESSUS) is a tool for computing the probabilistic response or reliability of engineered systems. NESSUS can be used to simulate uncertainties in loads, geometry, material behavior, and other user-defined random variables to predict the probabilistic response, reliability and probabilistic sensitivity measures of engineered systems.

NESSUS is a software system that integrates advanced reliability methods with finite element and boundary element methods and probabilistic algorithms in order to model uncertainties in loads, material properties, and geometries with random variables. Probabilistic performance models implemented in it include stress, strain, displacement, vibration, fatigue, fracture, and creep. It can perform reliability analyses for multiple components and failure modes, and identify critical random variables and failure modes to support structural design, certification, and risk assessment.

Fast Probability Integration is a software designed to aid in in probabilistic engineering analysis. It was developed at the Southwest Research Institute for the NASA Glenn Research Center.

Most Probable Point (MPP) is the concept on which it is based on and uses a response function that depends on the probabilistic distributions of the input variables.

For every combination of input variable values there has a particular probability of occurrence, which is based on the input distributions and therefore, it follows that the probability of getting a given response is equal to the probability of obtaining the input combination.

This information is then applied to determine two quantities of interest:

- The probability of meeting or exceeding a requirement , and
- The most likely combinations of input variables which produce satisfactory response values.

Analysis Type

NESSUS categorizes probabilistic analysis into three analysis types.

Specified Probability Levels

Sometimes referred to as ‘inverse reliability analysis’, the input is one or more probabilities, and the program computes the corresponding response levels. For a response Z, the probability is interpreted as P [z < Z], where z is a particular response level.

Specified Performance Levels

Sometimes referred to as ‘forward reliability analysis’, in which the input is to provide performance levels, and the program computes the probability of the response which is less than the specified values.

Full cumulative distribution

Selection and computation of multiple probability levels by the program so that the entire CDF of the response can be visualized.

In the present work Specified probability levels have been considered which requires the input as different probabilities (0.001 to 0.999) for which performances will be calculated.

3.1 Fast Probability Integration

The FPI concept was originated from structural reliability analysis where failure conditions must be pre-defined using limit states. To expand the concept to CDF (cumulative distribution function) analysis, the following definitions and notations are adopted that distinguish response functions from limit state functions.

A Z-function is a response function or a performance function such as stress, displacement, natural frequency, fatigue life, etc.

$$Z(X) = Z(X_1, X_2, X_{3,} ..., X_n)$$

where X_i (i = 1,n) are the random variables

A g-function is a limit state (also called performance function) defined as:

$$g = Z(X) - z_0$$

where z_0 is a particular value of Z. The g-function is defined such that g(X) = 0 is a boundary that divides the random variable space into two regions: failure [g≤0] and safe [g >0]. Because the CDF of Z at $z_0$equals the probability that [g≤ 0], the CDF can be computed by varying $z_0$and computing the point probability.

A component reliability problem has only one g-function whereas a system reliability problem involves multiple g-functions.

Probability Integration

If a g-function and the joint probability density function, PDF, $f_X(x)$, is given then the probability of failure is the probability in the failure domain Ω and is given by:

$$p_f = \int_{\Omega} \dots \int f_x(x) dx$$

This integral can be computed using a straightforward standard Monte Carlo procedure. However, when the g-function is complicated, requiring an intensive numerical calculation for each sample of X, and p_f is small, this random sampling procedure becomes impractical for engineering analysis and design. For practical purposes, efficient and approximate analysis tools are needed.

3.2 Most Probable Point

Several methods in FPI are based on the Most Probable Point (MPP) concept.

In the structural reliability, the MPP is also known as the Design Point, which is defined in a coordinate system of an independent, standardized normal vector u. In the u-space, the joint probability density function (PDF) is rotationally symmetric around the origin and decays exponentially with the square of the distance from the origin. For a two-variable case, the joint PDF has a bell-shaped surface.

By transforming g(X) to g(u) using a distribution transformation, the MPP ($x*$ is a point that defines the minimum distance, β , from the origin u = 0) to the limit-state surface g(u) = 0. This minimum-distance point is a most-probable-point on g(u) = 0 (in the u-space) because the joint PDF at a point (u_1, u_2, . . . , u_n) in the u-space is proportional to $\exp[-0.5(u_1^2 + u_2^2 + \dots + u_n^2)]$ where the sum of squares defines the distance. Therefore the density is a maximum when the distance is a minimum.

Because of its unique probability properties, the MPP is a key point for FPI.

3.3 Distribution Transformation

Non-normal dependent variables X can be transformed to standardized normal variables u using Rosenblatt

$$u_1 = \mathrm{F}^{-1}[F_1(x_1)]$$

$$u_2 = \mathrm{F}^{-1}[F_2(x_2|x_1)]$$

$$.$$

$$.$$

$$u_n = \mathrm{F}^{-1}[F_n(x_n|x_1, x_2,\ldots,x_{n-1})]$$

where $F_i(x_i)$ is the CDF of X_i, $F_n(X_n | \ldots)$ is the conditional CDF, and Φ^{-1} is the inverse CDF of a standardized normal random variable.

When the variables are mutually independent, this transformation reduces to:

$$u = \mathrm{F}^{-1}[F_x(x)]$$

The inverse transformation is:

$$x = F_x^{-1}[\mathrm{F}(u)]$$

Using the above transformation, the entire g(X)-function can be transformed to g(u) and allow the probabilistic analysis to be performed in the u-space. Numerically, however, the X-to-u or u-to-X transformations are needed only at points required to find the MPP, construct polynomials, and perform importance sampling. The advantage for transforming to the u-space is that probabilistic analysis becomes mathematically more tractable. The drawback is that the involved transformation may significantly distort the g-function such that an originally flat surface becomes highly curved. For engineering applications, the Rosenblatt transformation for dependent random variables may be impractical because the available data is often insufficient to establish the joint and the conditional probability distributions. A more realistic model transforms each correlated, non-normal random variable into a normal variable and generates a new set of correlation coefficients for the transformed normals. The generated normal variables are then assumed to have a joint normal density function (which is generally not true) and the correlation coefficients are used to generate a set of independent normal random variables. The inputs required for the second option include only the marginal distributions and the correlation coefficients. This option gives exact solutions for correlated normal random variables. Consider two random variables X_i and X_j with correlation coefficient R. The correlation coefficient of the transformed normal variables U_i and U_j, denoted as r, can be found by solving the following equation.

$$R = \int_{-\infty}^{\infty}\int_{\infty}^{\infty}\left(\frac{x_i - \mu_i}{\sigma_i}\right)\left(\frac{x_j - \mu_j}{\sigma_j}\right)\phi_{ij}\, du_i\, du_j$$

$$\phi_{ij} = \frac{1}{2\pi\sqrt{1-r^2}}\exp\left(\frac{-u_i^{\,2} - 2ru_iu_j + u_j^{\,2}}{2(1-r^2)}\right)$$

In general, the calculation of r requires iteratively solving the above equation. A convenient approach relating R and r was implemented in FPI.

3.4 Probability Sensitivity Analysis

In deterministic analysis, sensitivity is defined as $\partial Z/\partial X_i$, which measures the change in the performance due to the change in a design parameter. In probabilistic analysis the sensitivity measure is $\partial p/\partial \theta_i$, which measures the change in the probability relative to the change in a distribution parameter (e.g., mean and standard deviation).

Another useful probability sensitivity analysis is the determination of the relative importance of the random variables. This can be done by performing several probabilistic analyses in which one of the random variables is treated as a deterministic variable (i.e., by reducing the standard deviation to zero) for each analysis. Based on the resulting probability changes, the relative importance of the random variables can be determined. Repeated analyses, however, may be very time consuming for large numbers of random variables.

A more efficient way of evaluating the relative importance of the random variables is based on the location of the MPP.

At the MPP, $u* = (u*_1, u*_2 \ldots u*_n)$, the first-order probability estimate is $\Phi(-\beta)$ where,

$$\beta^2 = u_1^{*2} + u_2^{*2} + \ldots u_n^{*2}$$

The unit normal vector at the MPP of the g=0 surface is defined as:

$$\alpha = -\frac{\nabla g}{|\nabla g|}$$

The α vector is positive towards the direction of decreasing g (i.e., to the failure region). The sensitivity factors are the projections of α vector to the u-axes. Thus, they are the directional cosines of the α vector, and can be written as:

$$\alpha_i = \frac{u_i^*}{\beta}$$

The directional cosines satisfy the following rule:

$$\alpha_1^2 + \alpha_2^2 + \ldots \alpha_n^2 = 1$$

which implies that each $\alpha^2 i$ is a measure of the contribution to the probability (since the probability is related to β); higher α (in magnitude) indicates higher contribution. Thus, the sensitivity factors provide first-order information on the importance of the individual random variable.

From the definition of Sensitivity Factors

It can be shown that in the u-space,

$$\alpha_i \left(\frac{\partial g}{\partial u_i} \right)_{u*}$$

and in X-space,

$$\alpha_i \left(\frac{\partial g}{\partial X_i} \right)_{x*} \sigma_i$$

where σ_i is the normal (or approximate normal for non-normal distribution) standard deviation. It can be concluded that the sensitivity factors are functions of both the deterministic sensitivity and the uncertainty (characterized by the standard deviation).

In general, the sensitivity factors depend on the g-function as well as the input probability distributions. In a CDF analysis, the sensitivity factors will usually be different for different response or probability levels. This is because the performance sensitivity or the approximate standard deviation may be different for different response or probability levels. Because the above probabilistic sensitivity analysis is based on the first-order reliability method, α is a good probability sensitivity measure only if $\Phi(-\beta)$ is a good approximation to the true probability.

Based on MPP, other sensitivity measures with respect to a distribution parameter (mean or standard deviation) or a limit-state function parameter can be computed based on the sensitivity factors and the distribution.

CHAPTER IV

RESULTS AND DISCUSSION

The effect of adding circular fins to a heat exchanger tube on Max hoop and radial stress was examined for rectangular and triangular fin profiles. The hoop stresses for both rectangular and triangular profiles are compressive near the base of the fins but changes to tensile towards the tip of the fins. The radial stresses along that plane are compressive and reaches to zero close to tip of the fins. As seen from the figures the radial stresses are maximum near the base of the fins. In a comparison of stress contours the radial stress distribution resulted in lesser compressive characteristics close to the base of the fin for triangular profile. The iterative program of probabilistic analysis is carried out by considering the thermal stresses developed by pre-analysis of steady state convection of circular fins. The maximum hoop and radial stresses obtained in COSMOS works by changing all the random variables from their mean values by +/- 10%, one at a time by keeping others unchanged was used as input to NESSUS statistical analysis program. The program then calculates Cumulative distribution functions (CDF), Sensitivity factors of each random variable for different probabilities from .001 to 0.999.

The cumulative distribution functions and sensitivity factors for various levels of probability for rectangular and triangular profile are shown in figures.

Following observations for both fin profiles have been made from the results.

Rectangular profile

Hoop stress

Inside bulk temperature, elastic modulus, thermal expansion coefficient and outer bulk temperature has a lot of influence on thermal stresses and can be seen from fig's 1.1.0-1.1.11

Radial stress

Fin height, tube thickness, inner radius of tube and inside bulk temperature has a lot of influence on thermal stresses and can be seen from fig's 2.1.1-2.1.11

Triangular profile

Hoop stress

Inside bulk temperature, elastic modulus, thermal expansion coefficient, outer bulk temperatures have a lot of influence on thermal stresses and can be seen from fig's 3.1.1-3.1.11

Radial stress

Fin height, inside bulk temperature, elastic modulus, and thermal expansion coefficient have a lot of influence on thermal stresses and can be seen from fig's 4.1.1-4.1.11

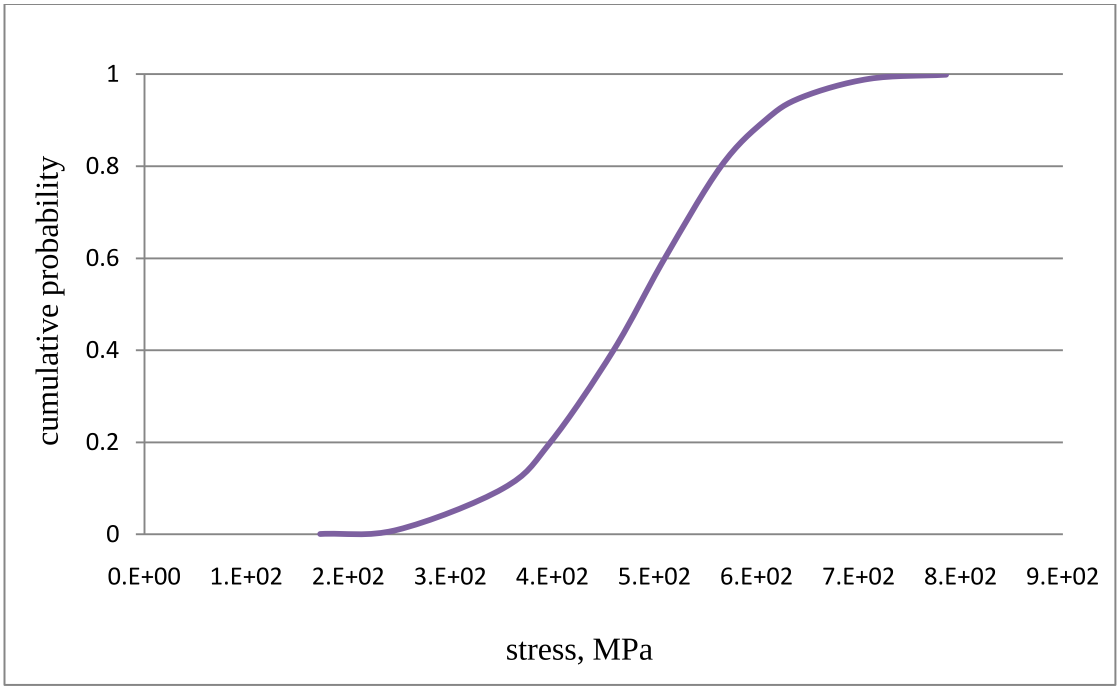

FIG.1.1.0.Cumulative probability Vs Maximum hoop stress of Rectangular fin.

Sensitivity factors Vs Random variables for hoop stress of rectangular Fin profile.

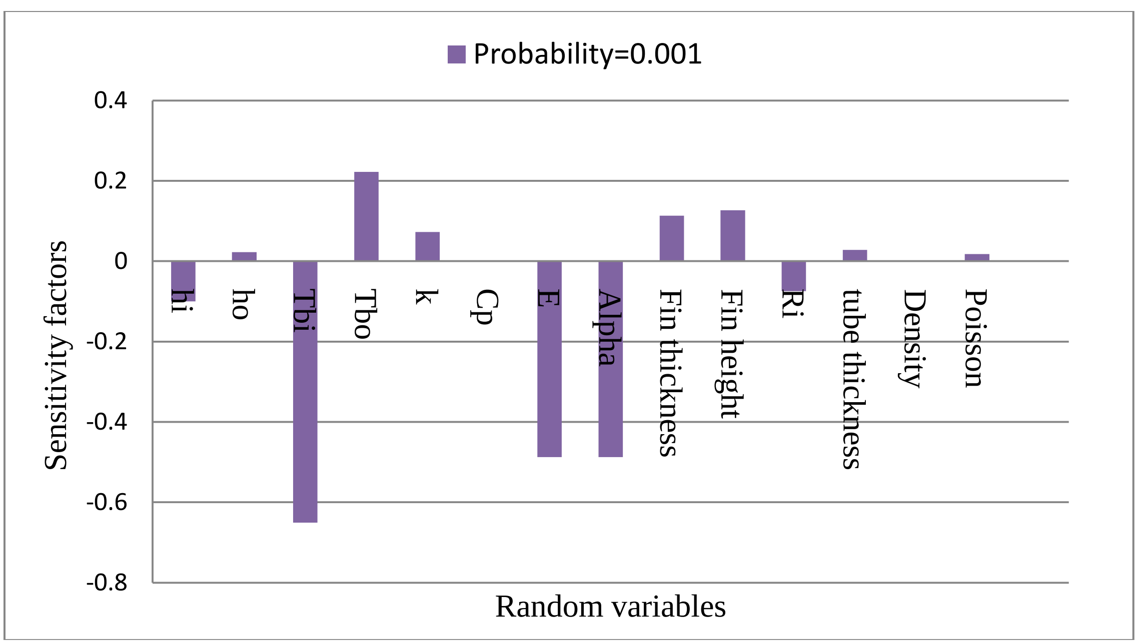

FIG.1.1.1 Sensitivity Factor for 0.001 Probability

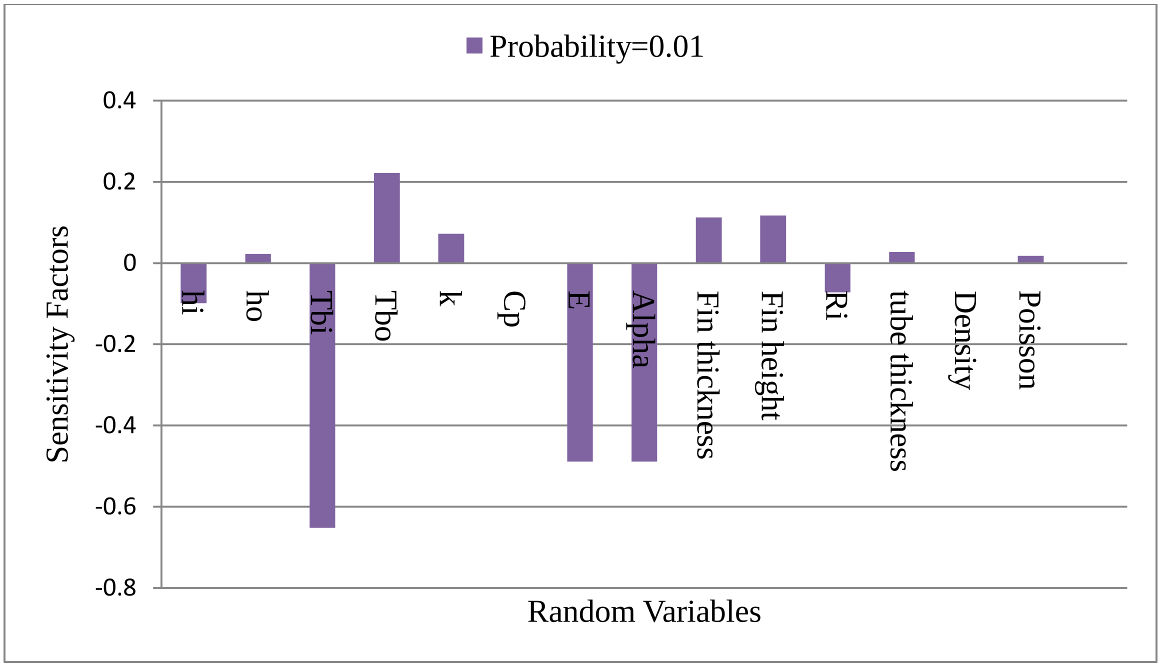

FIG.1.1.2 Sensitivity Factor for 0.01 Probability

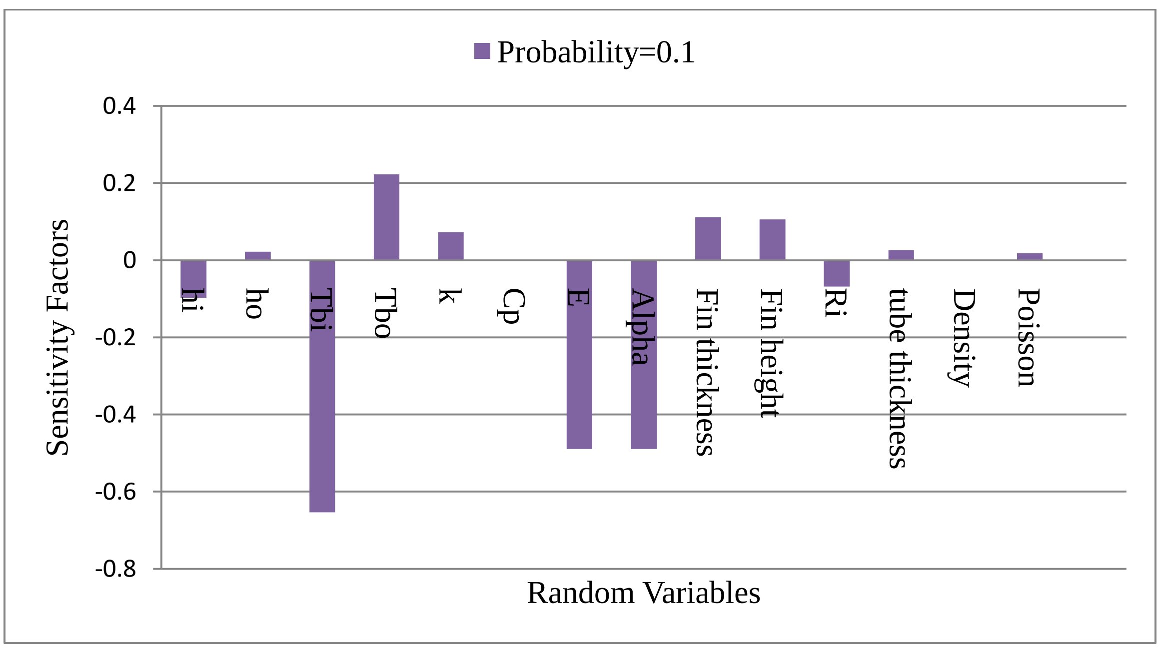

FIG.1.1.3 Sensitivity Factor for 0.1 Probability

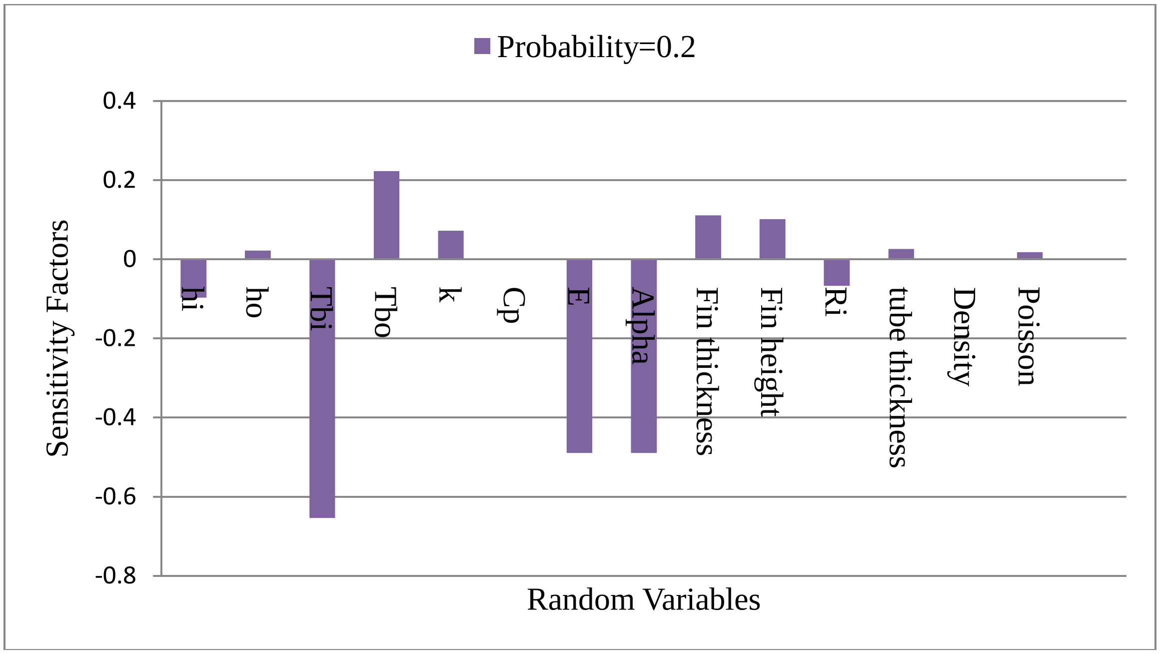

FIG.1.1.4 Sensitivity Factor for 0.2 Probability

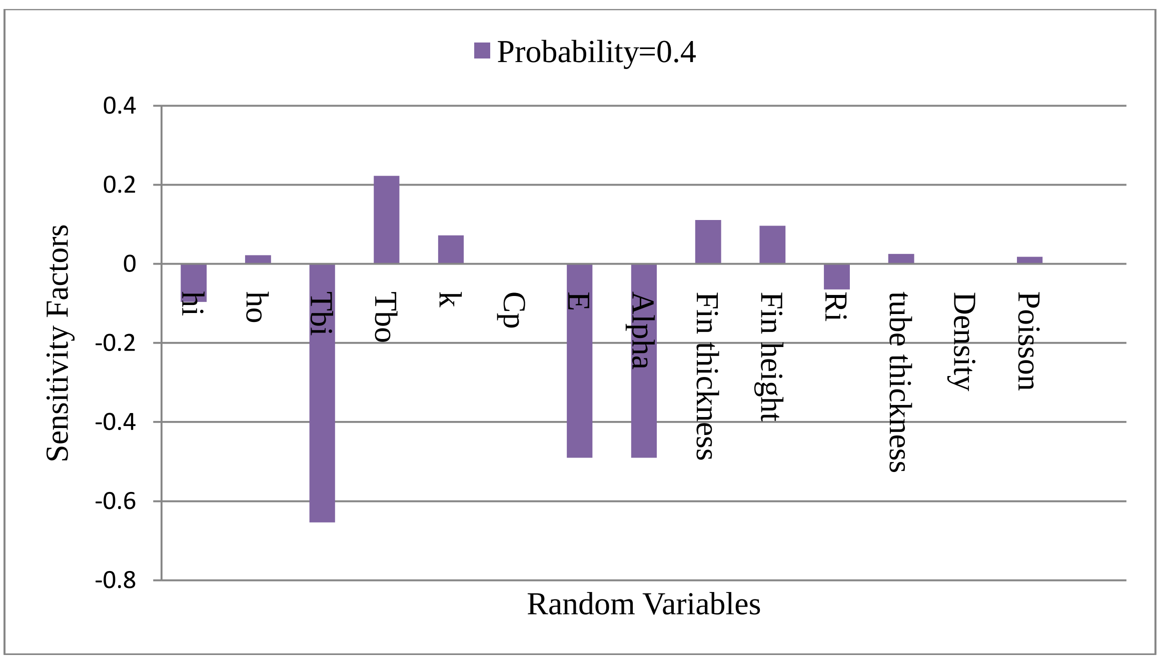

FIG.1.1.5 Sensitivity Factor for 0.4 Probability

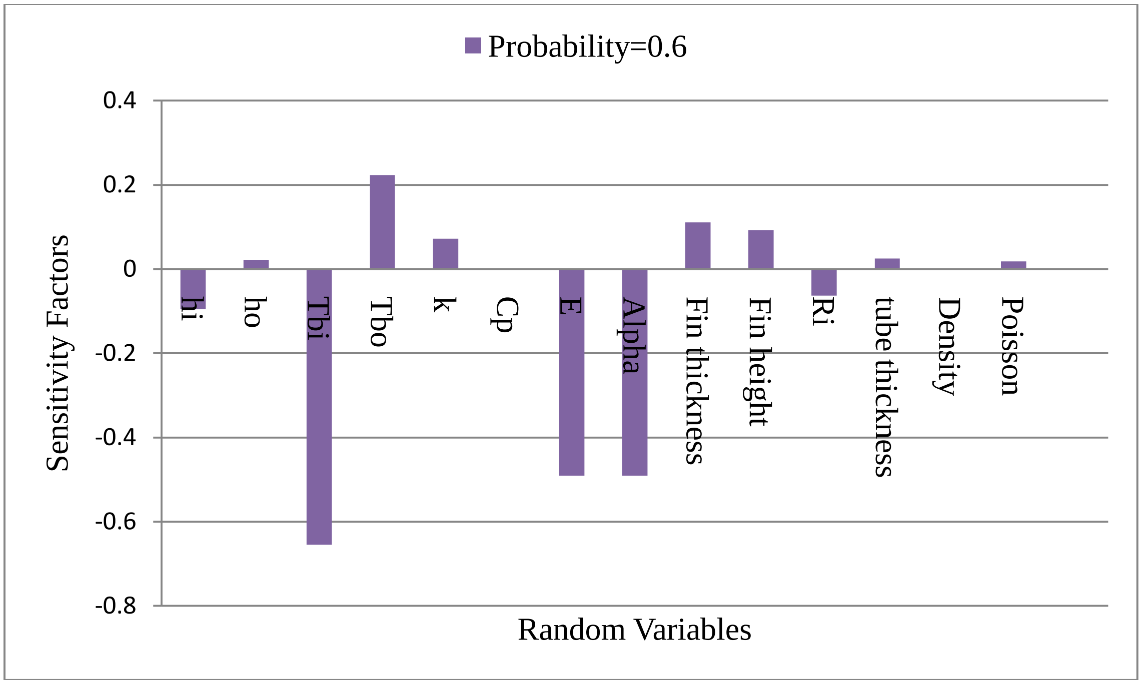

FIG.1.1.6 Sensitivity Factor for 0.6 Probability

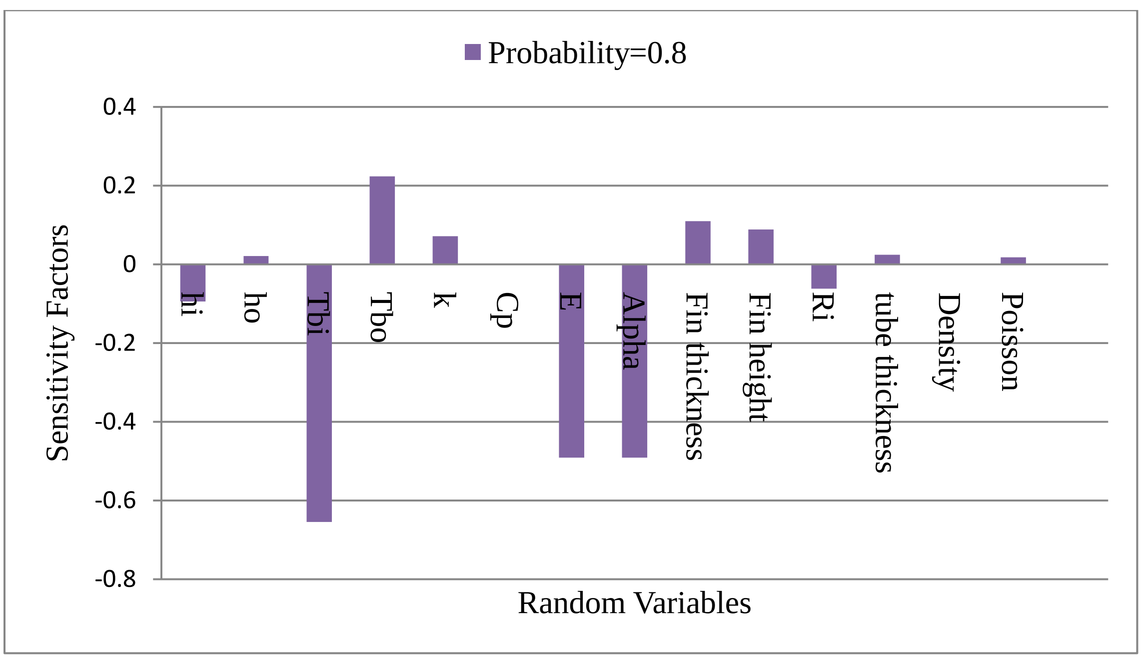

FIG.1.1.7 Sensitivity Factor for 0.8 Probability

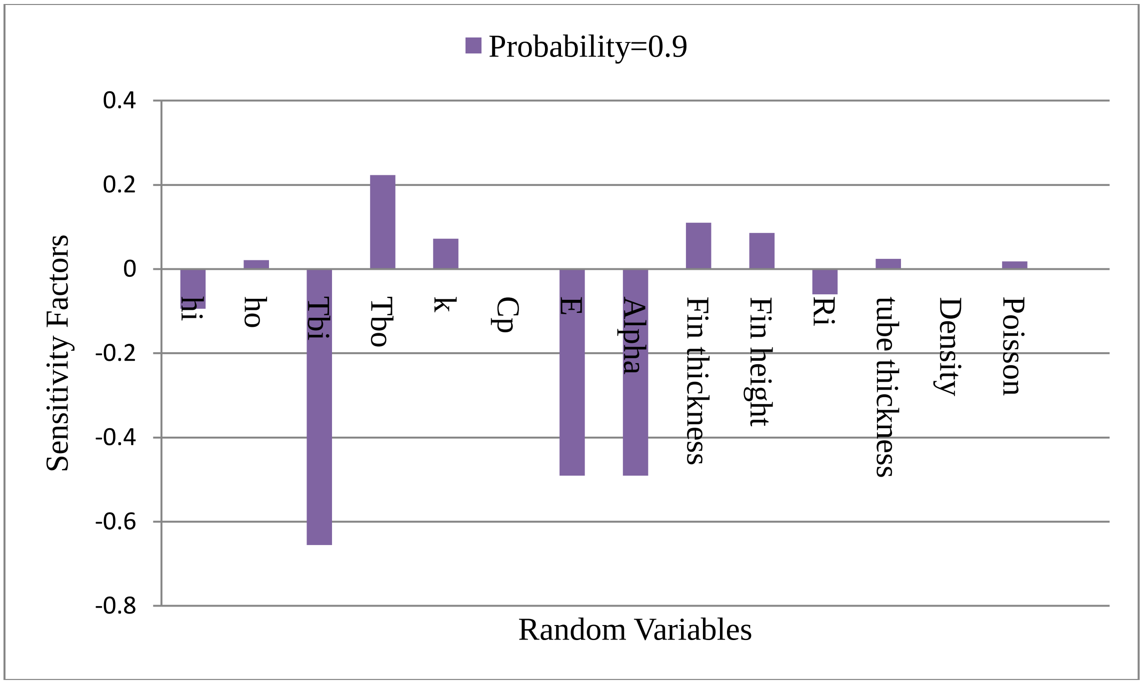

FIG.1.1.8 Sensitivity Factor for 0.9 Probability

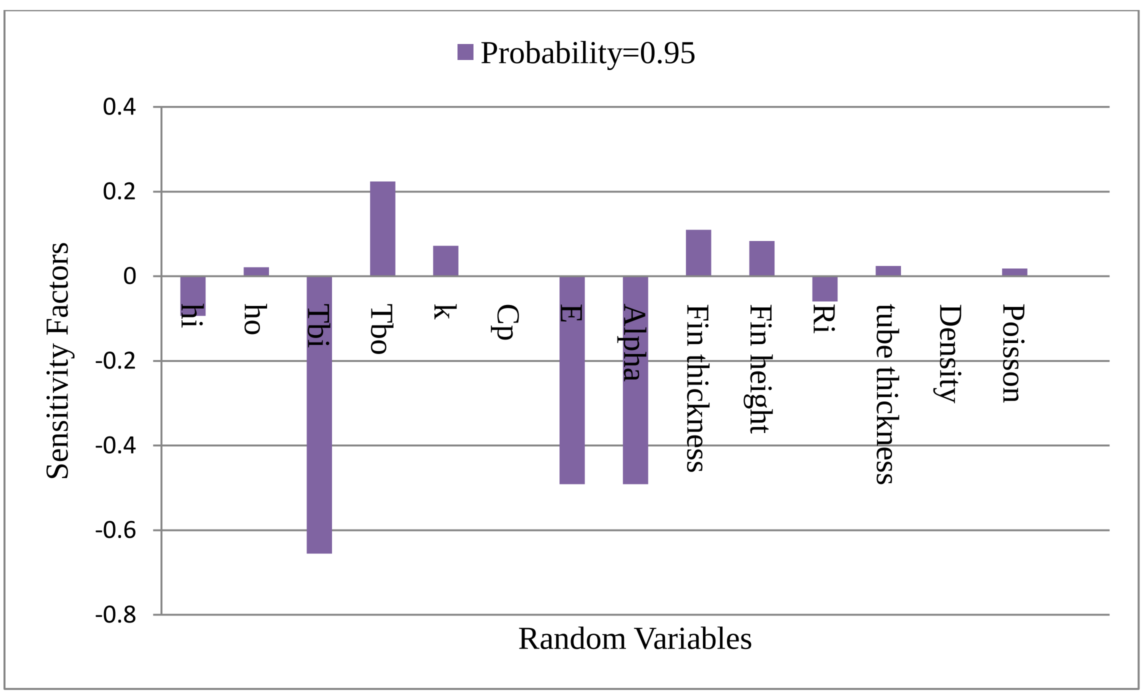

FIG.1.1.9 Sensitivity Factor for 0.95 Probability

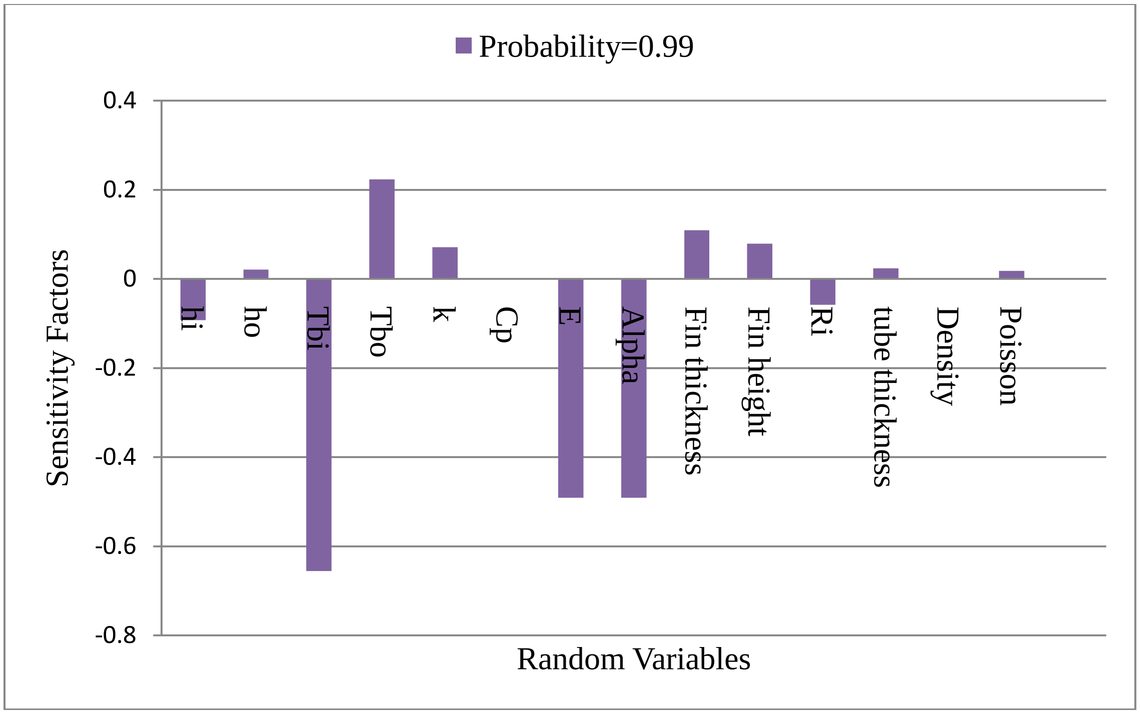

FIG.1.1.10 Sensitivity Factor for 0.99 Probability

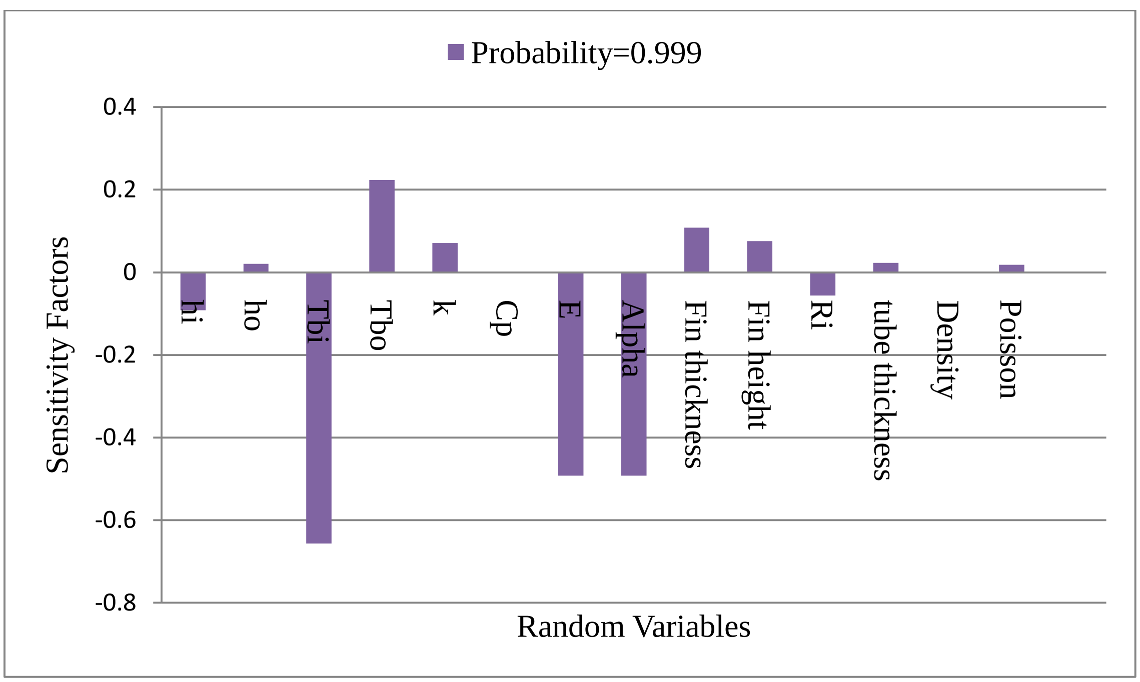

FIG.1.1.11 Sensitivity Factor for 0.999 Probability

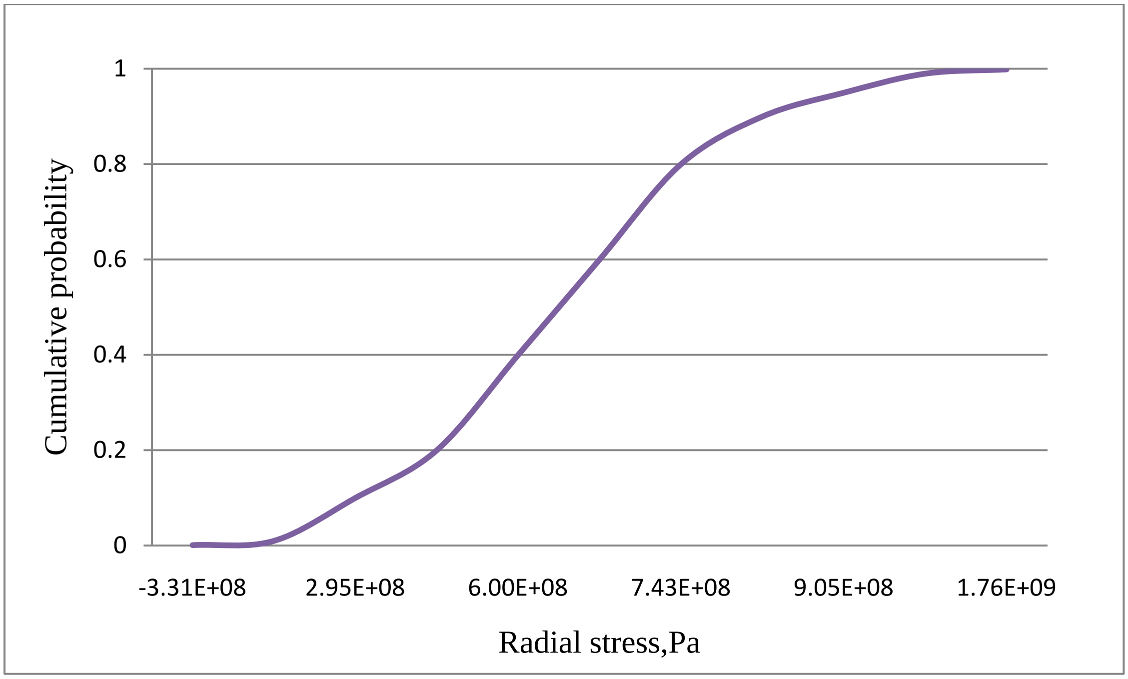

Fig.2.1.0.Cumulative probability Vs Maximum Radial stress of Rectangular Fin

Sensitivity factors Vs Random variables for radial stress of Rectangular Fin profile.

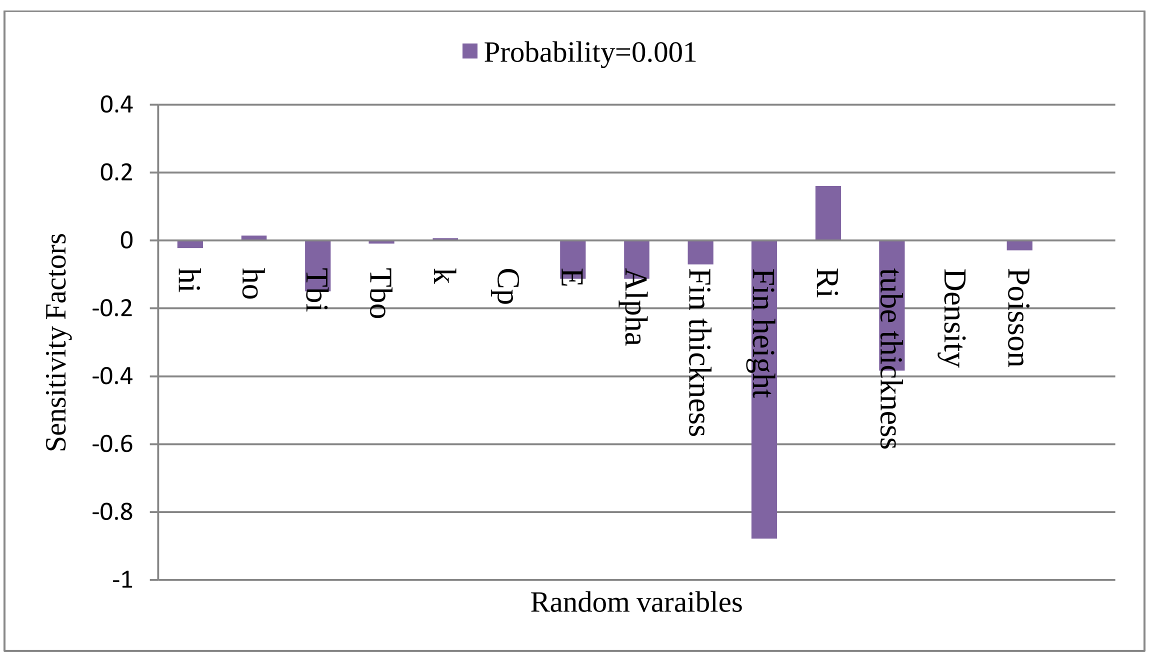

FIG.2.1.1 Sensitivity Factor for 0.001 Probability

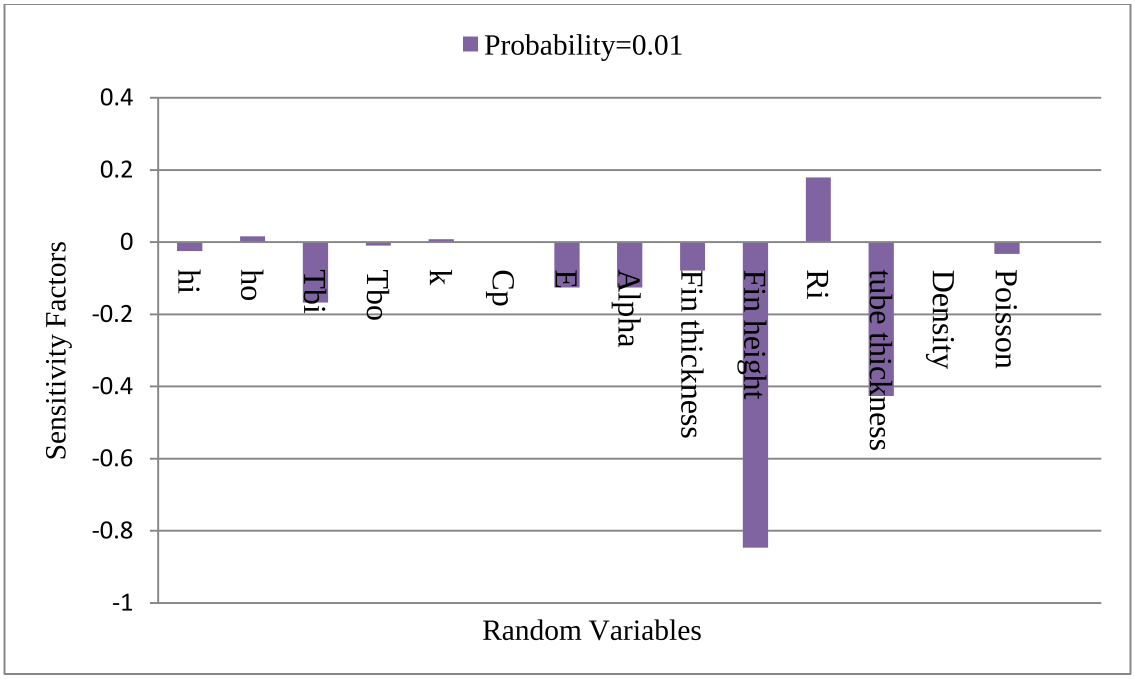

FIG.2.1.2 Sensitivity Factor for 0.01 Probability

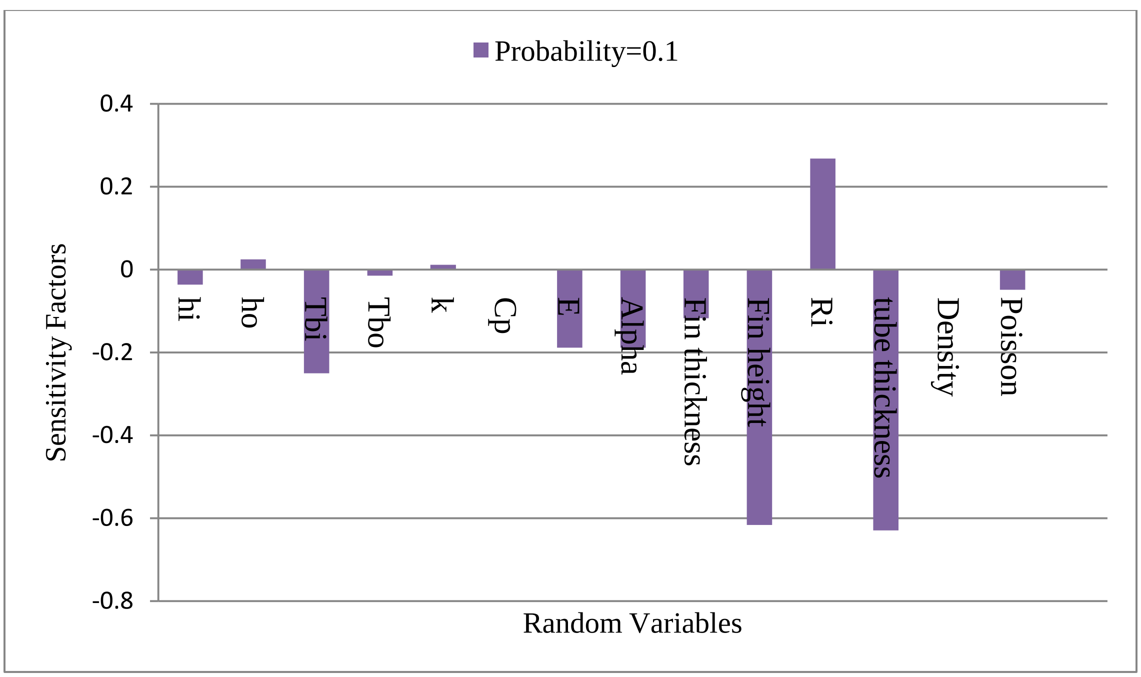

FIG.2.1.3 Sensitivity Factor for 0.1 Probability

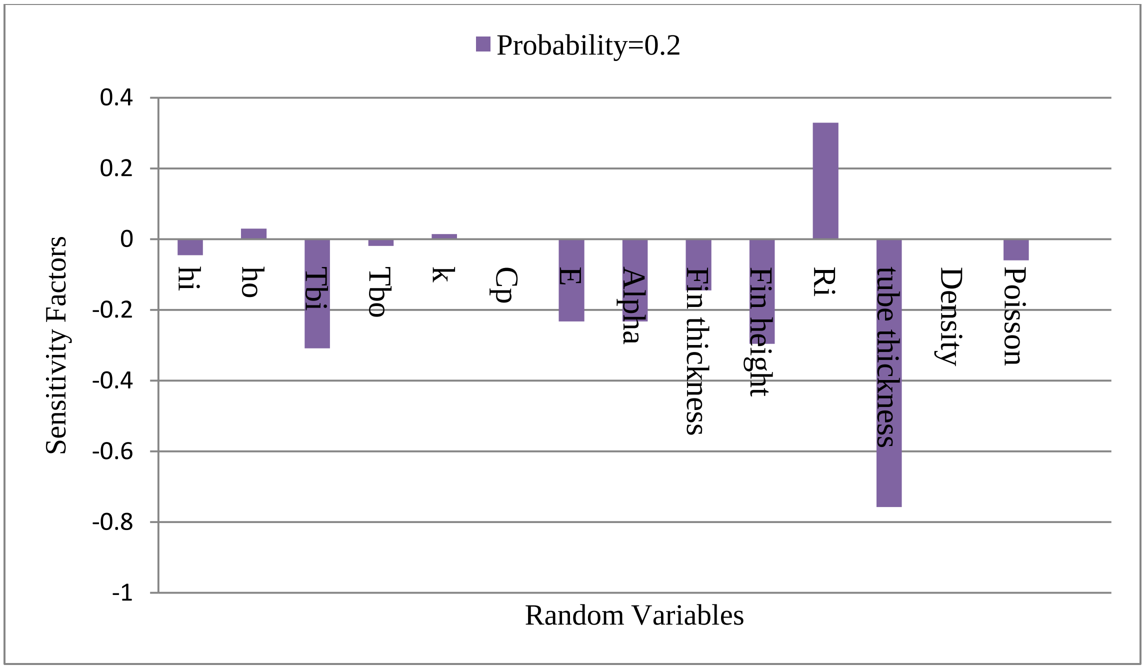

FIG.2.1.4 Sensitivity Factor for 0.2 Probability

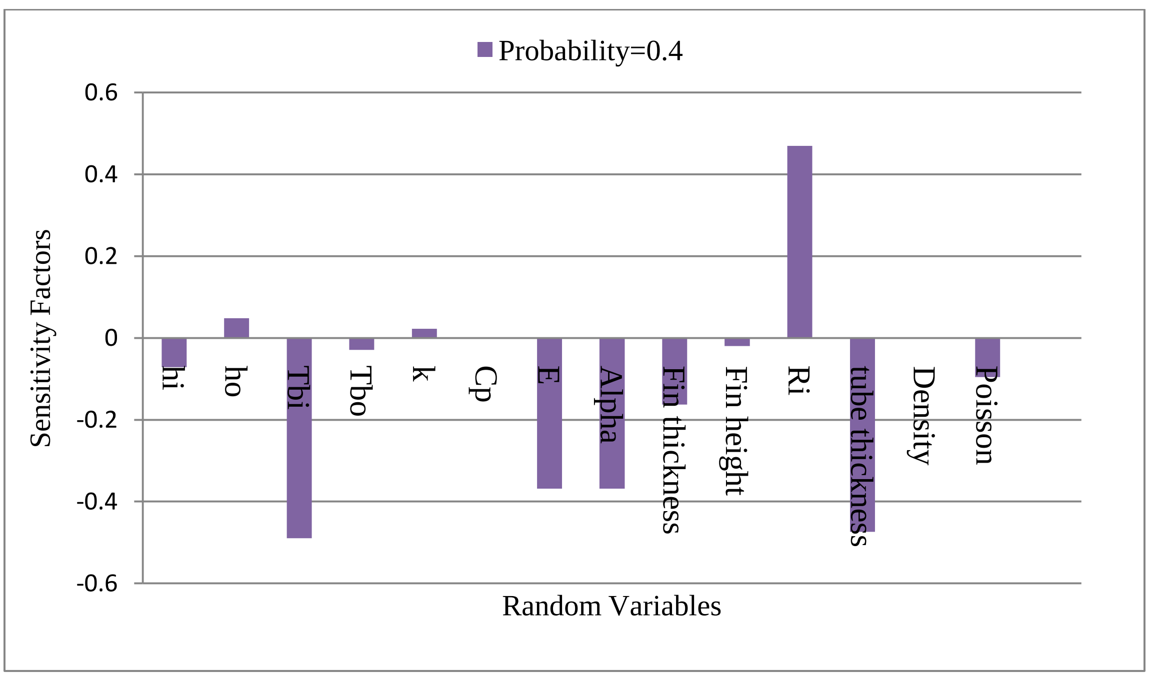

FIG.2.1.5 Sensitivity Factor for 0.4 Probability

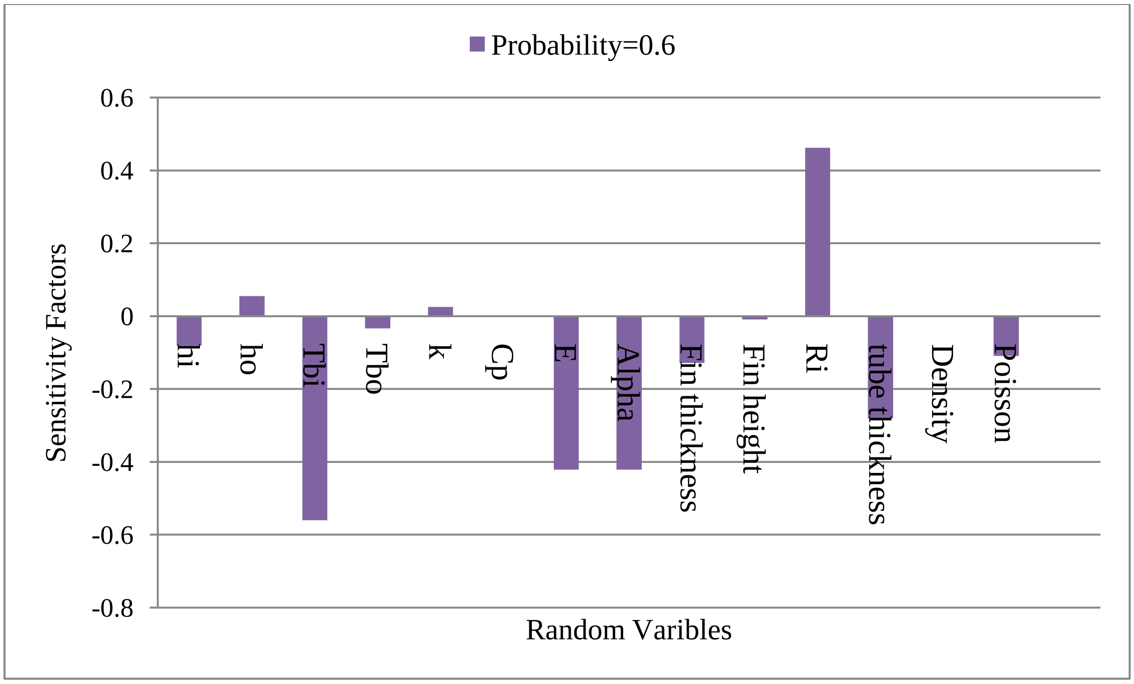

FIG.2.1.6 Sensitivity Factor for 0.6 Probability

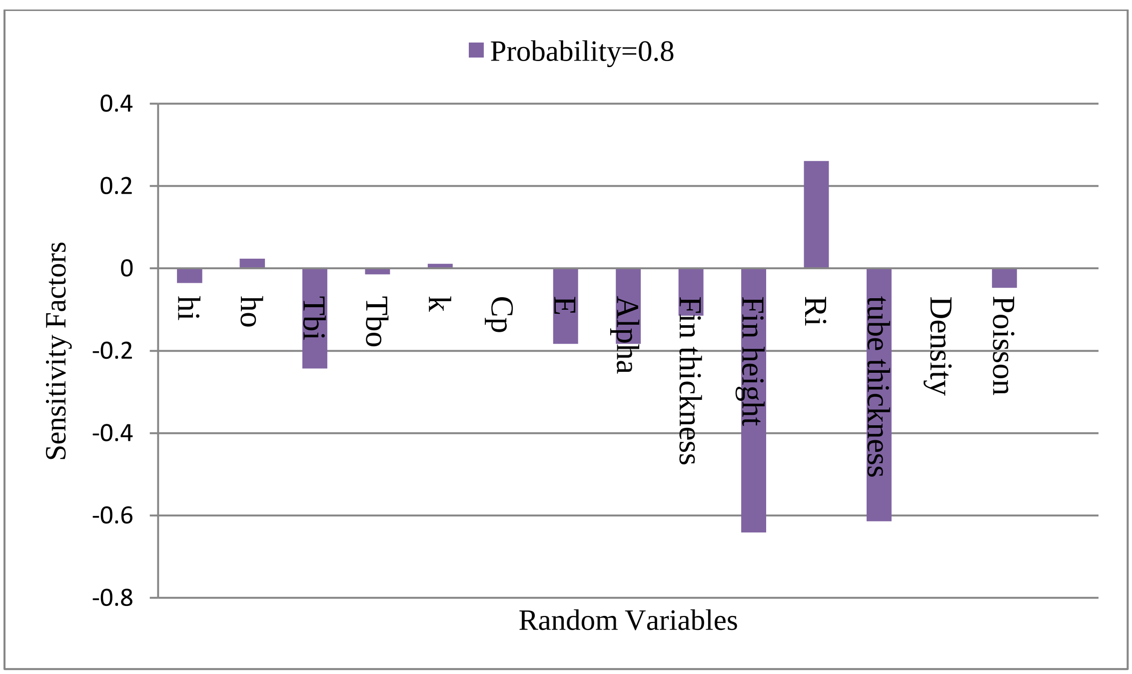

FIG.2.1.7 Sensitivity Factor for 0.8 Probability

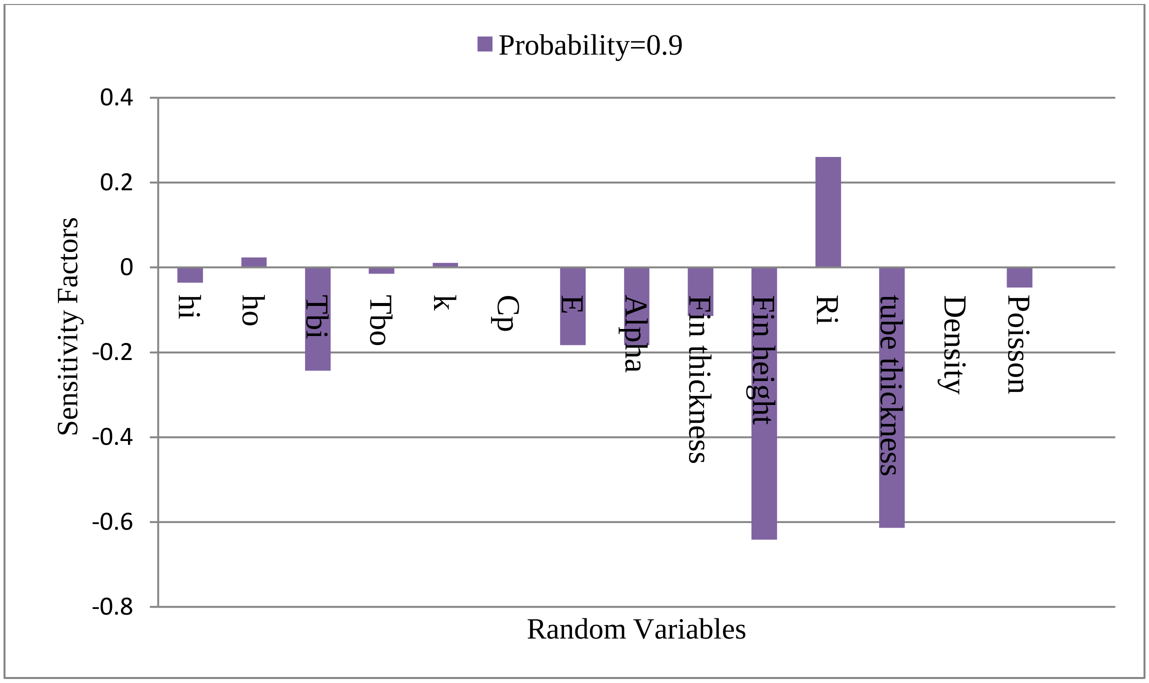

FIG.2.1.8 Sensitivity Factor for 0.9 Probability

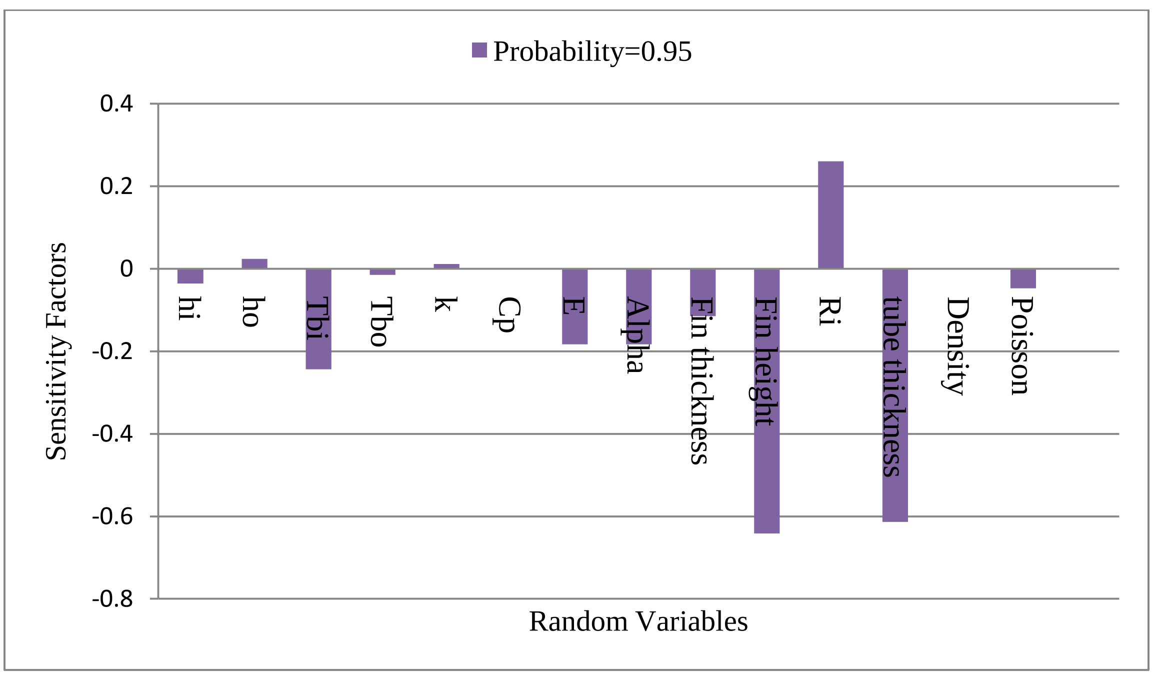

FIG.2.1.9 Sensitivity Factor for 0.95 Probability

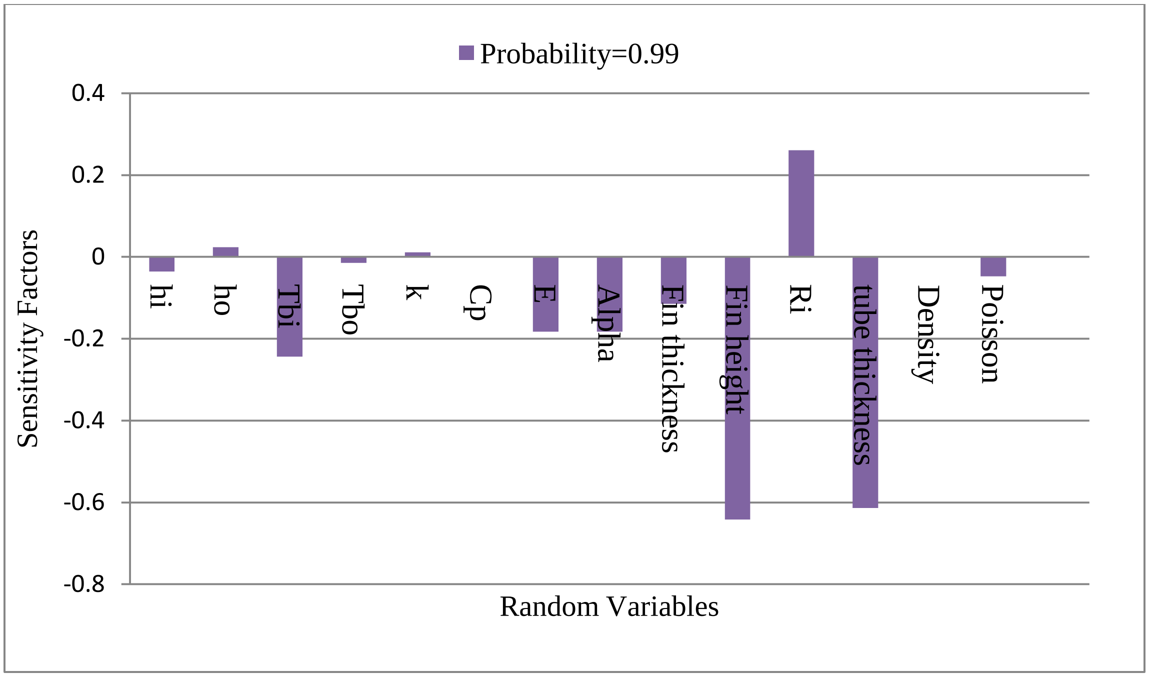

FIG.2.1.10 Sensitivity Factor for 0.99 Probability

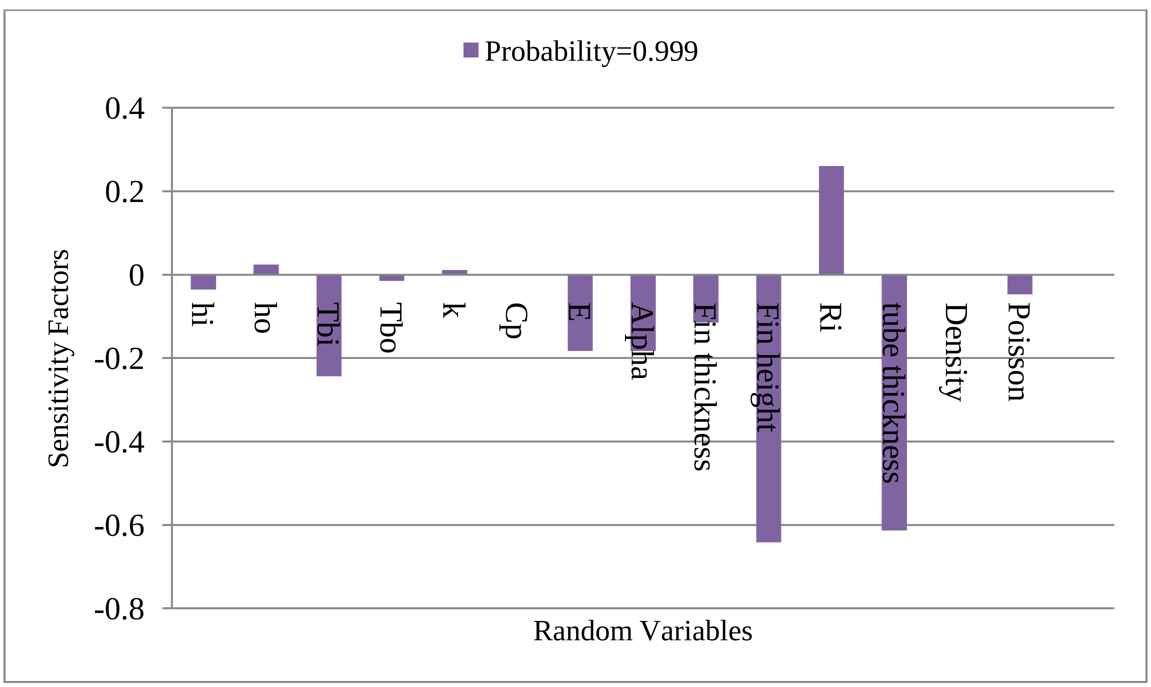

FIG.2.1.11 Sensitivity Factor for 0.999 Probability

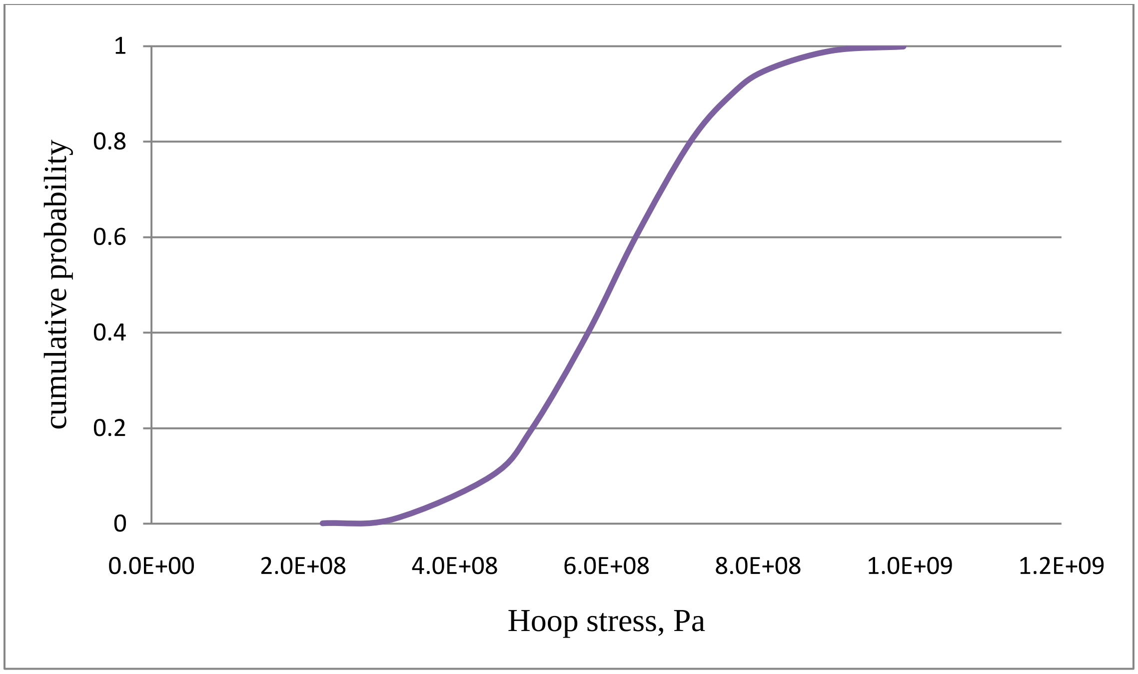

Fig.3.1.0. Cumulative probability Vs Maximum Hoop stress of Triangular Fin

Sensitivity factors Vs Random variables for hoop stress of Triangular Fin profile.

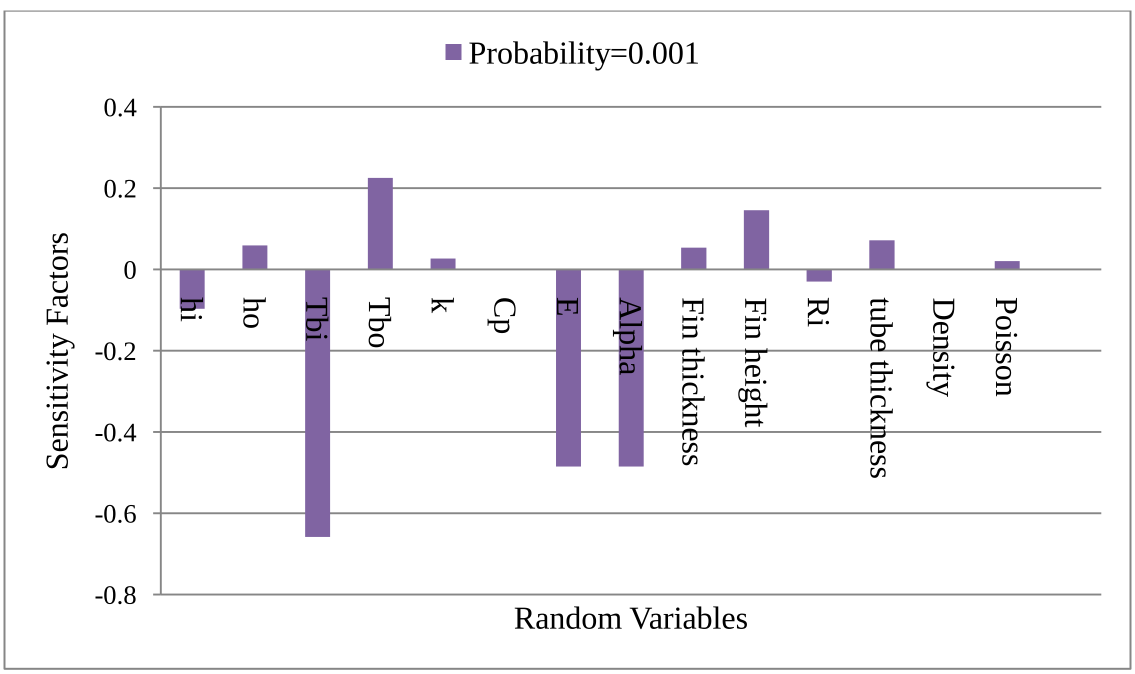

FIG.3.1.1 Sensitivity Factor for 0.001 Probability

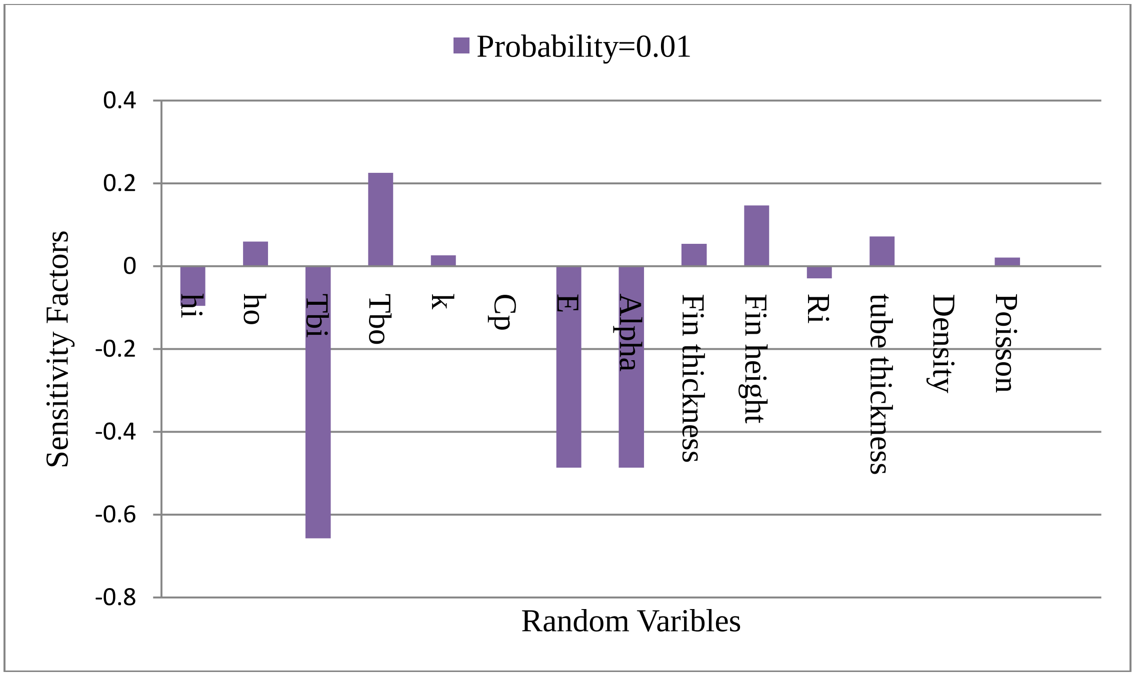

FIG.3.1.2 Sensitivity Factor for 0.01 Probability

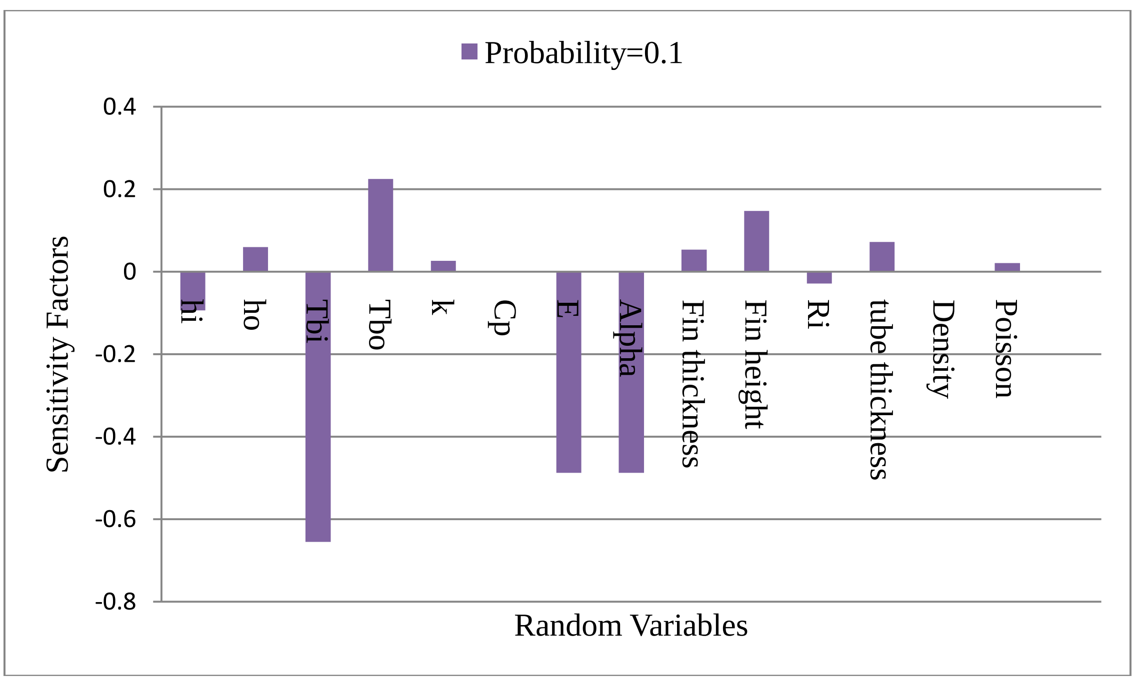

FIG.3.1.3 Sensitivity Factor for 0.1 Probability

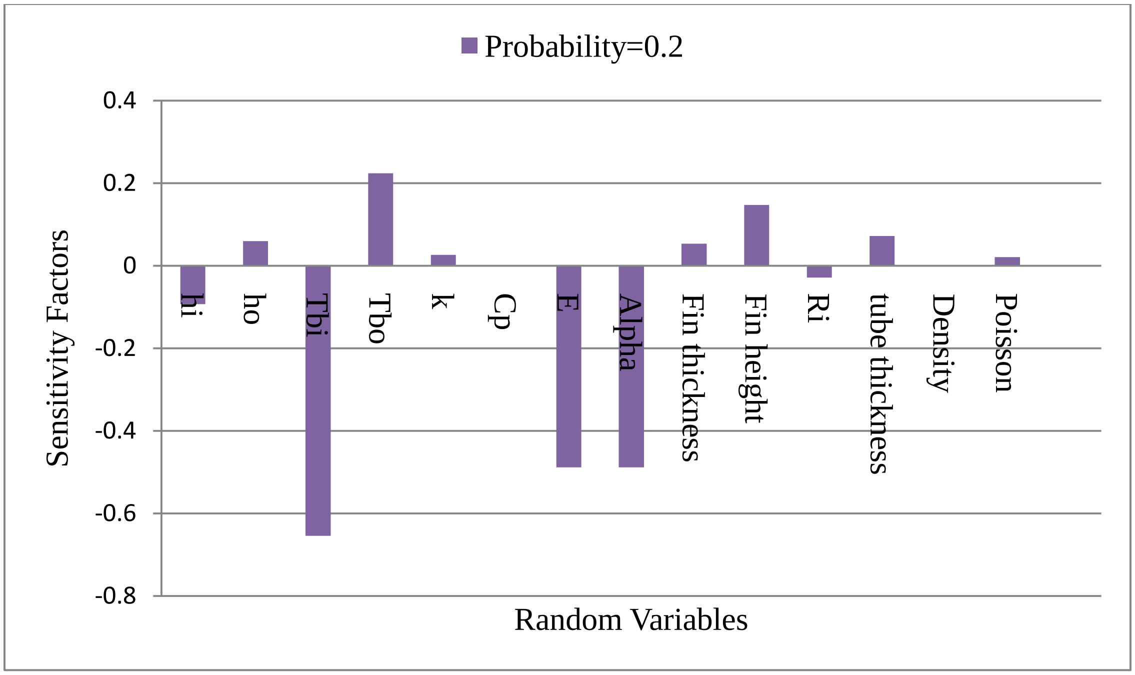

FIG.3.1.4 Sensitivity Factor for 0.2 Probability

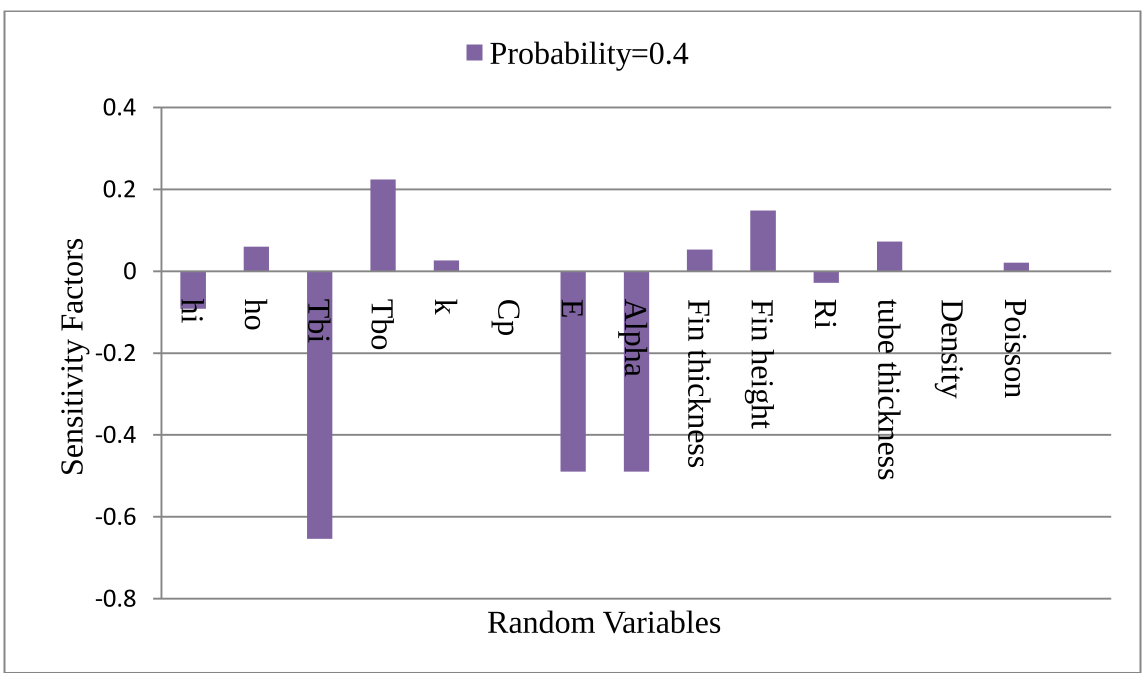

FIG.3.1.5 Sensitivity Factor for 0.4 Probability

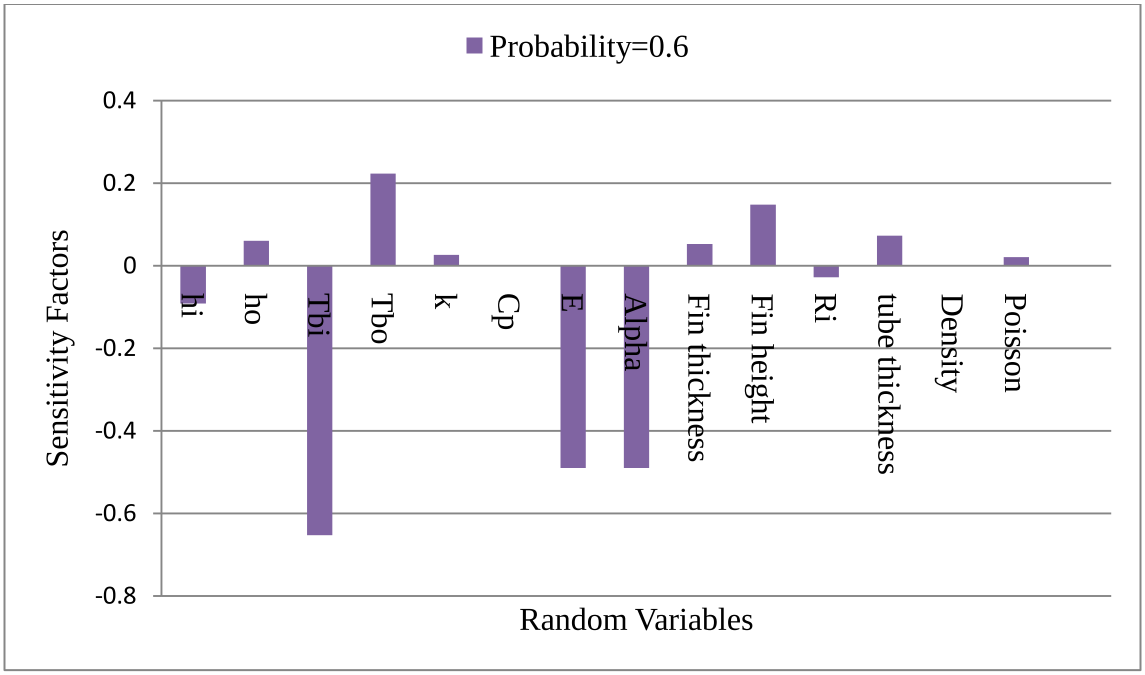

FIG.3.1.6 Sensitivity Factor for 0.6 Probability

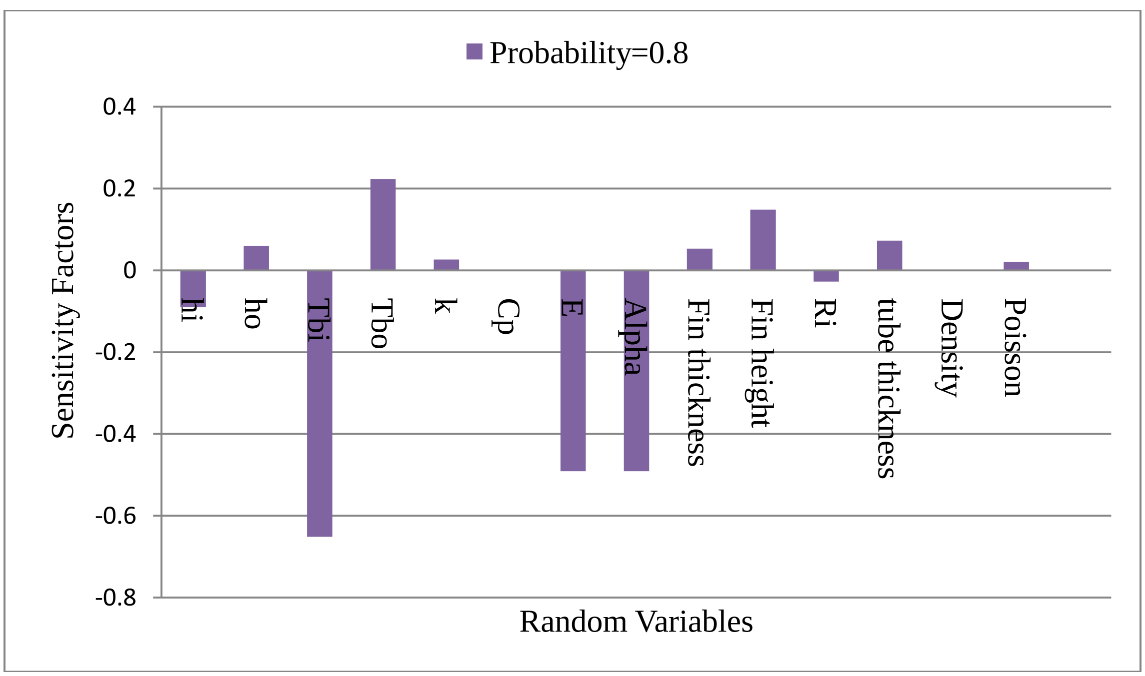

FIG.3.1.7 Sensitivity Factor for 0.8 Probability

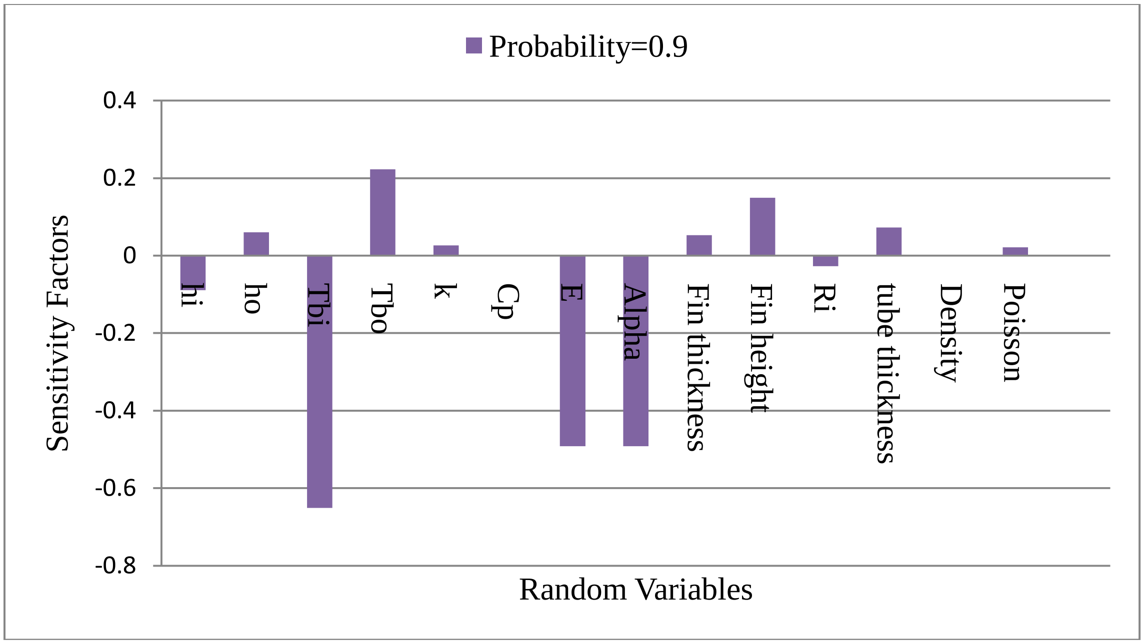

FIG.3.1.8 Sensitivity Factor for 0.9 Probability

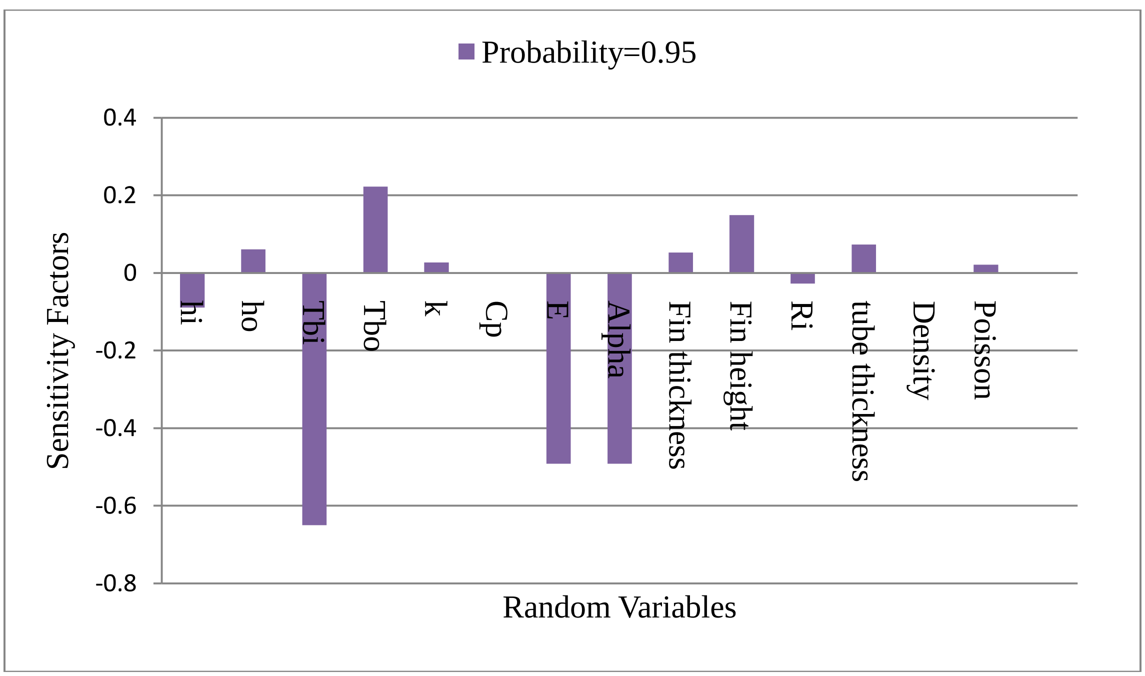

FIG.3.1.9 Sensitivity Factor for 0.95 Probability

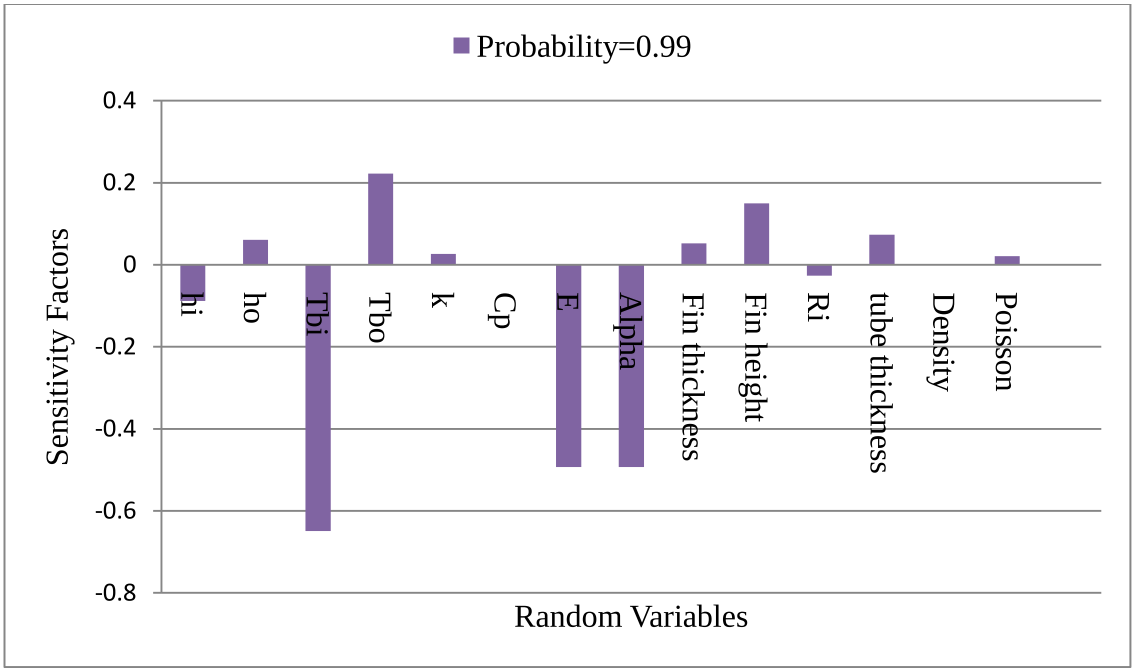

FIG.3.1.10 Sensitivity Factor for 0.99 Probability

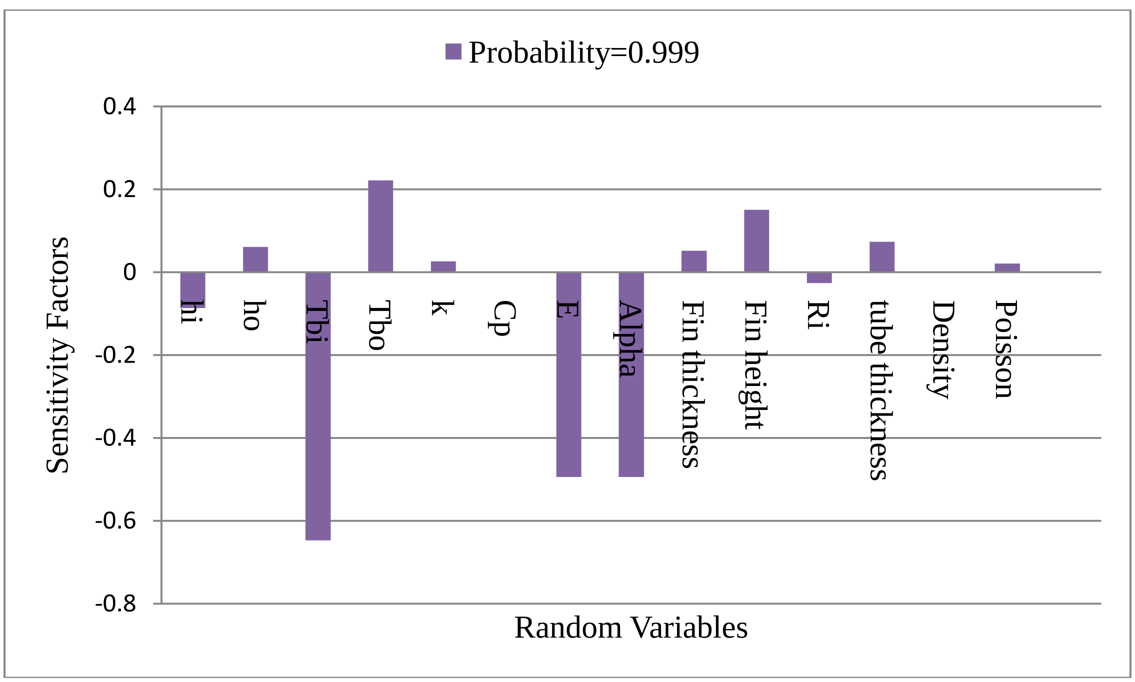

FIG.3.1.11 Sensitivity Factor for 0.999 Probability

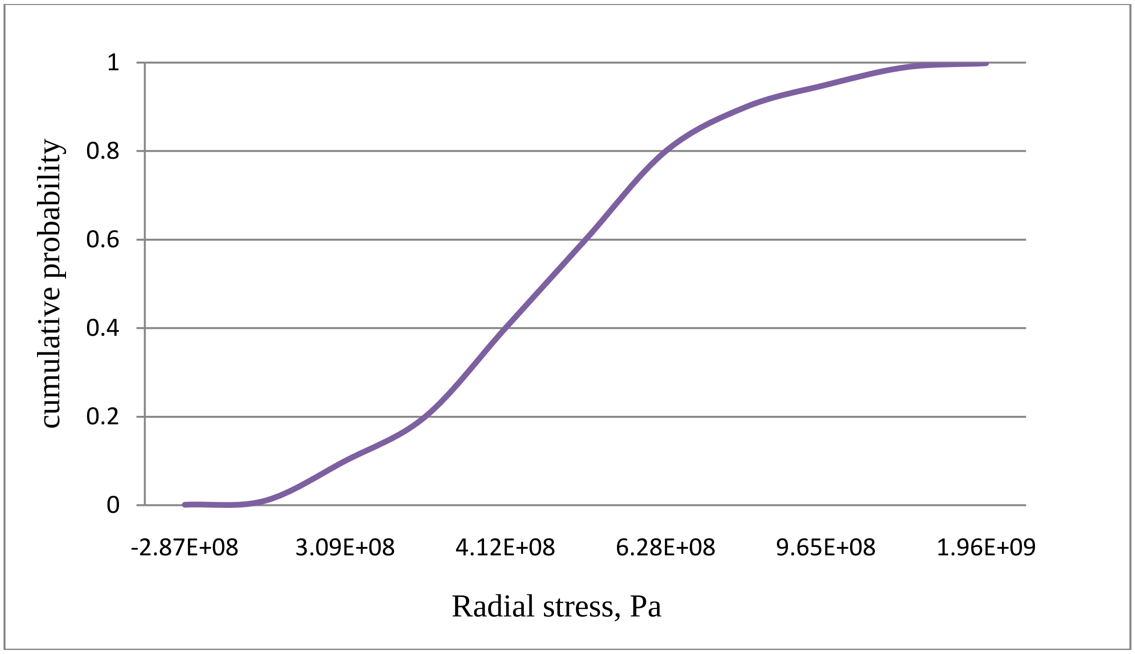

Fig.4.1.0 Cumulative probability Vs Maximum Radial stress of Triangular Fin

Sensitivity factors Vs Random variables for radial stress of Triangular Fin profile.

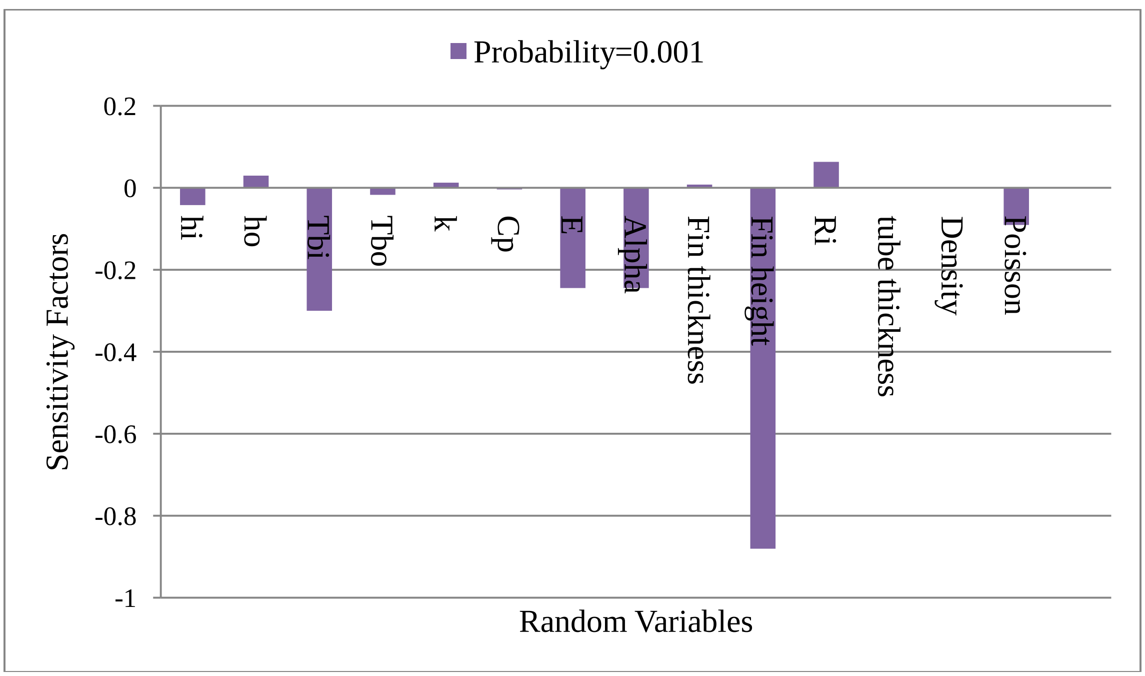

FIG.4.1.1 Sensitivity Factor for 0.001 Probability

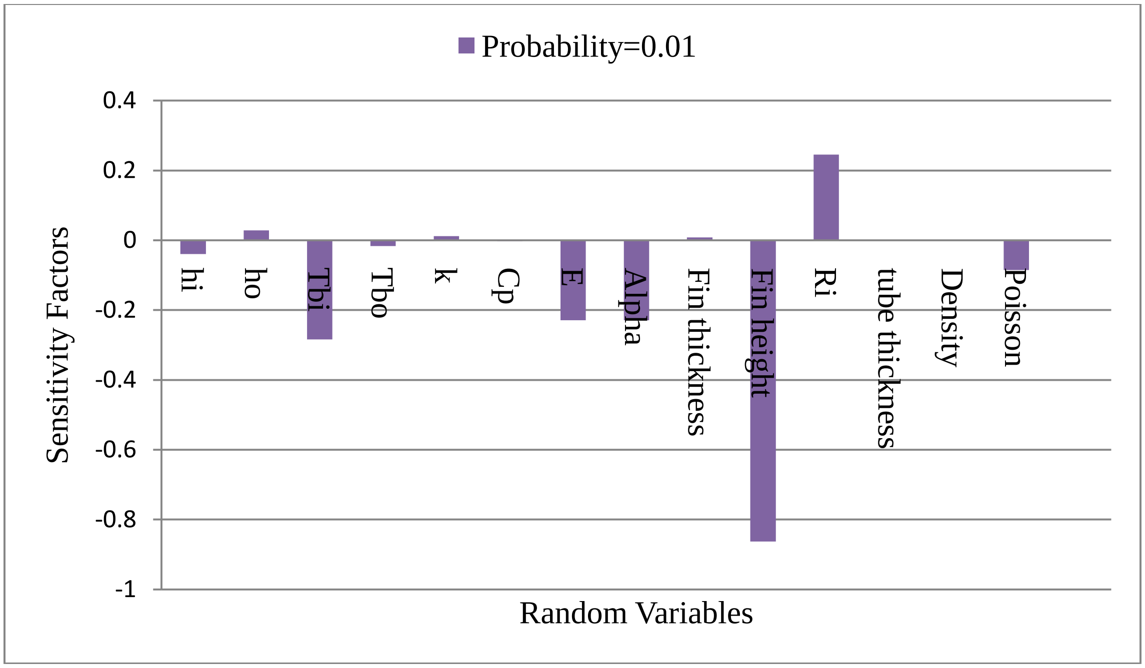

FIG.4.1.2 Sensitivity Factor for 0.01 Probability

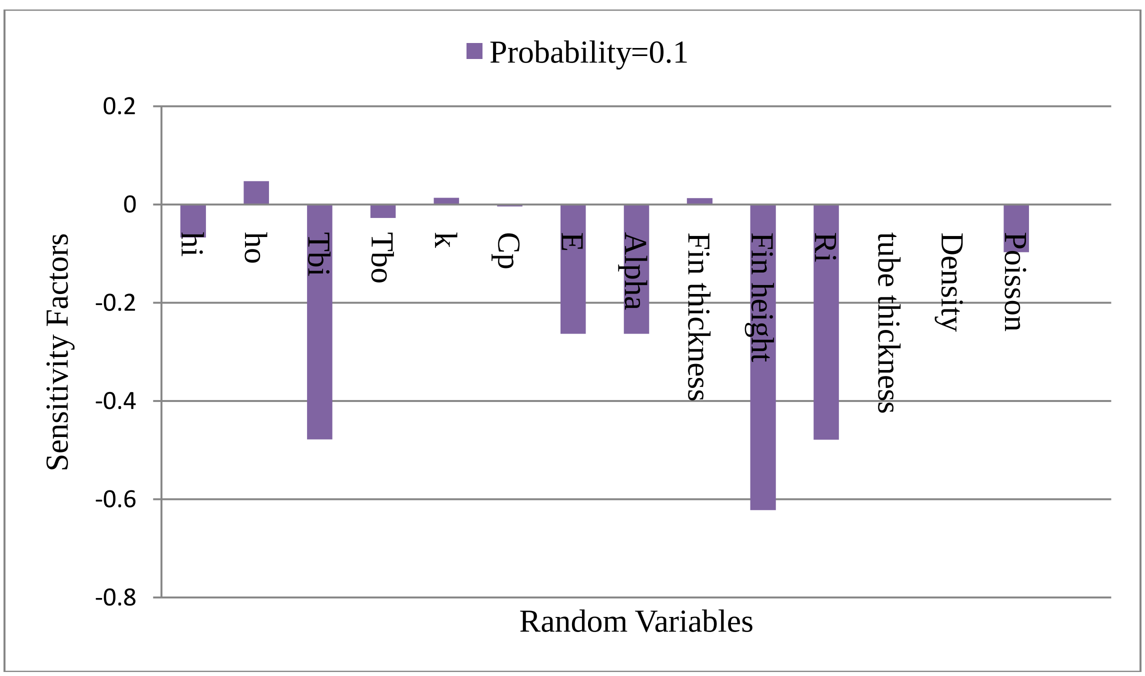

FIG.4.1.3 Sensitivity Factor for 0.1 Probability

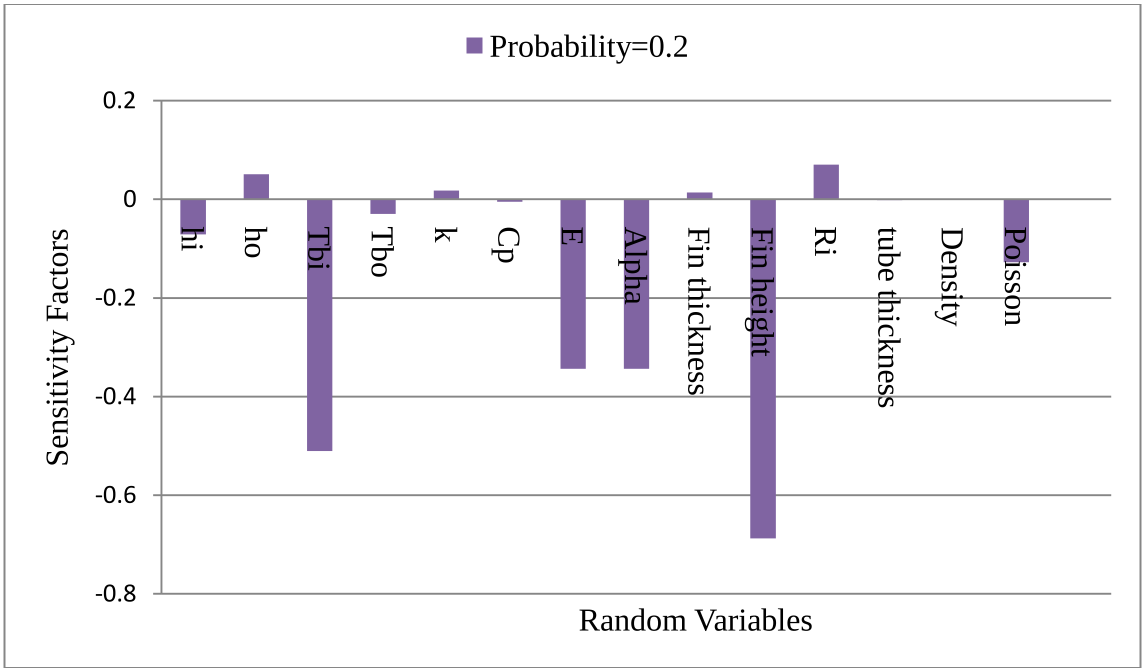

FIG.4.1.4 Sensitivity Factor for 0.2 Probability

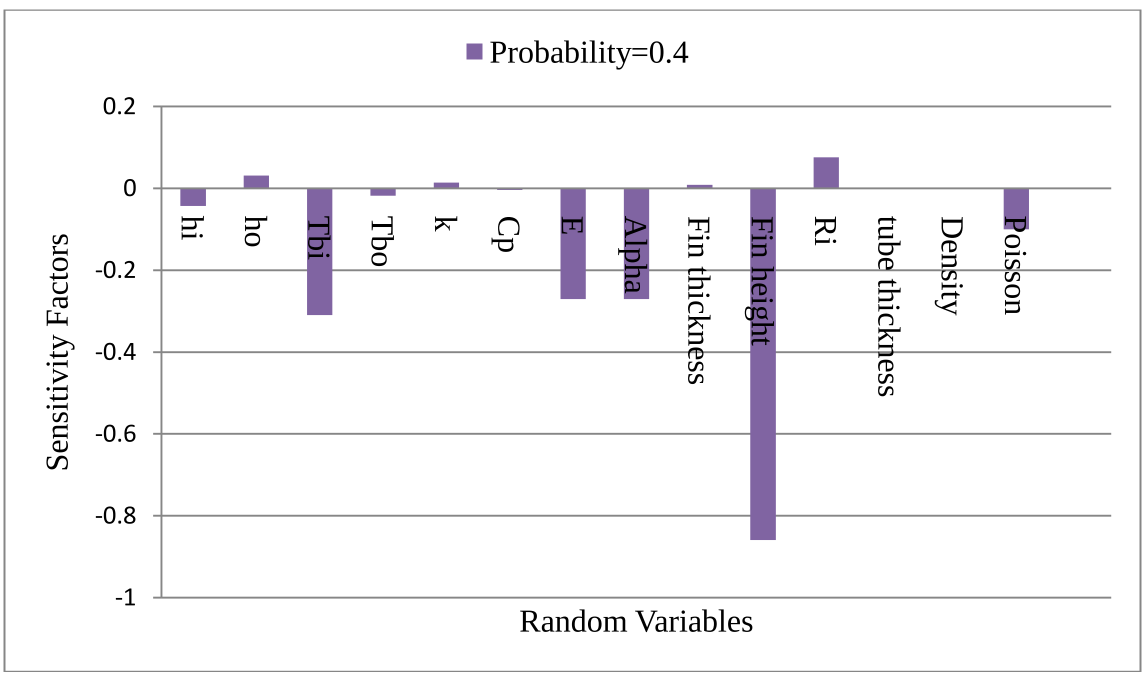

FIG.4.1.5 Sensitivity Factor for 0.4 Probability

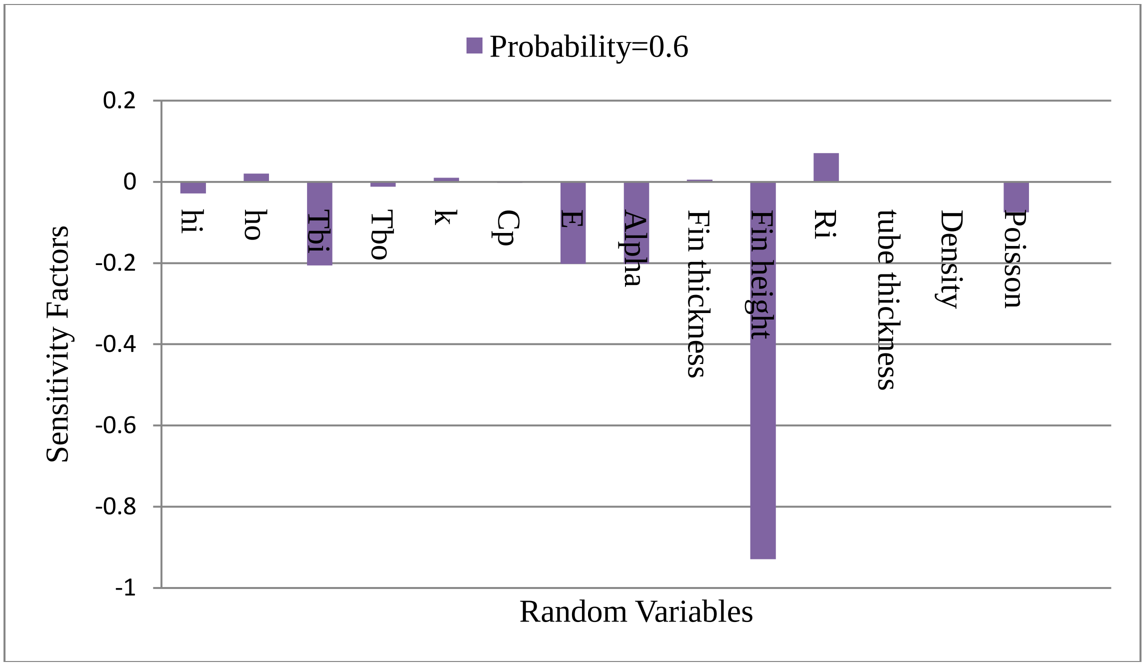

FIG.4.1.6 Sensitivity Factor for 0.6 Probability

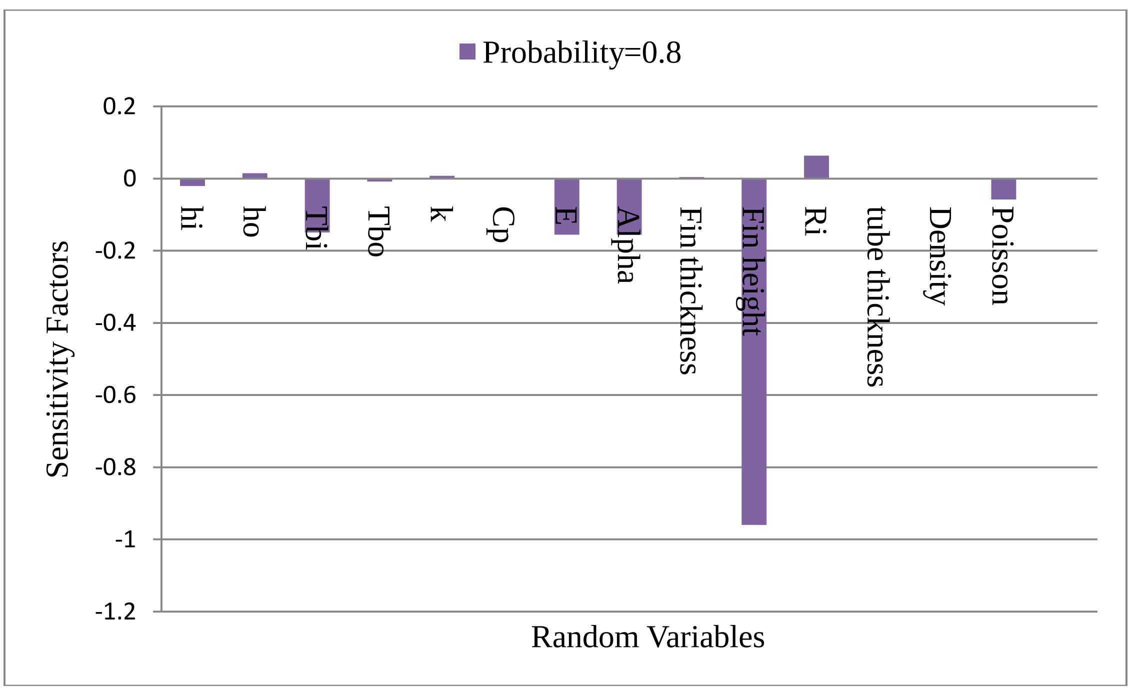

FIG.4.1.7 Sensitivity Factor for 0.8 Probability

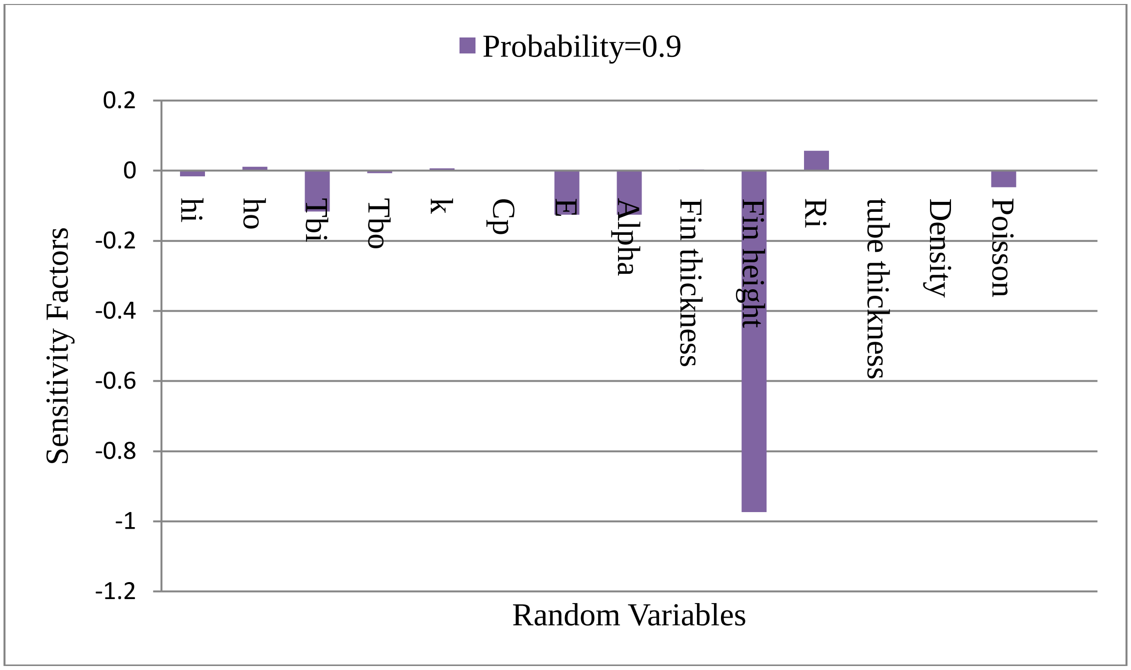

FIG.4.1.8 Sensitivity Factor for 0.9 Probability

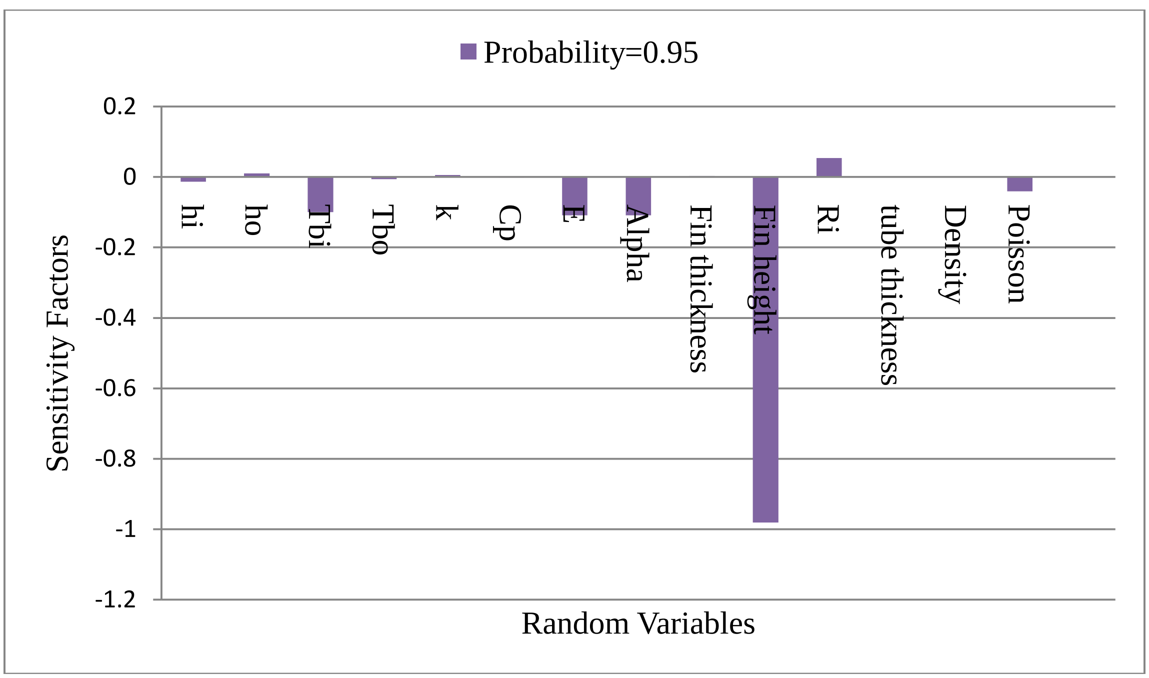

FIG.4.1.9 Sensitivity Factor for 0.95 Probability

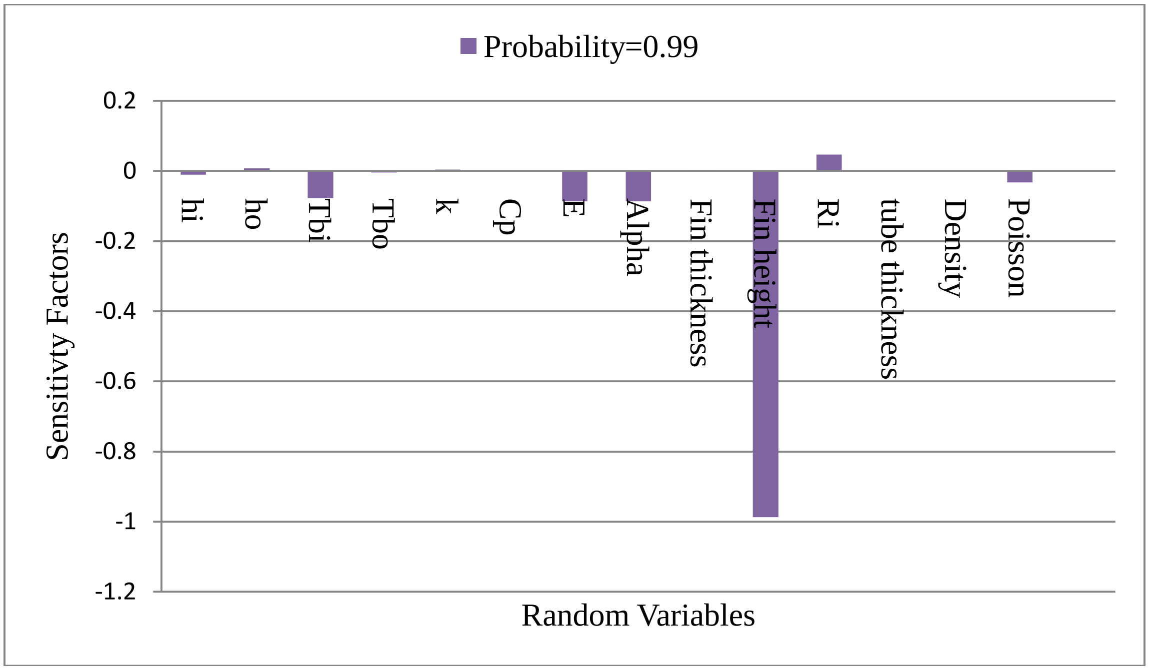

FIG.4.1.10 Sensitivity Factor for 0.99 Probability

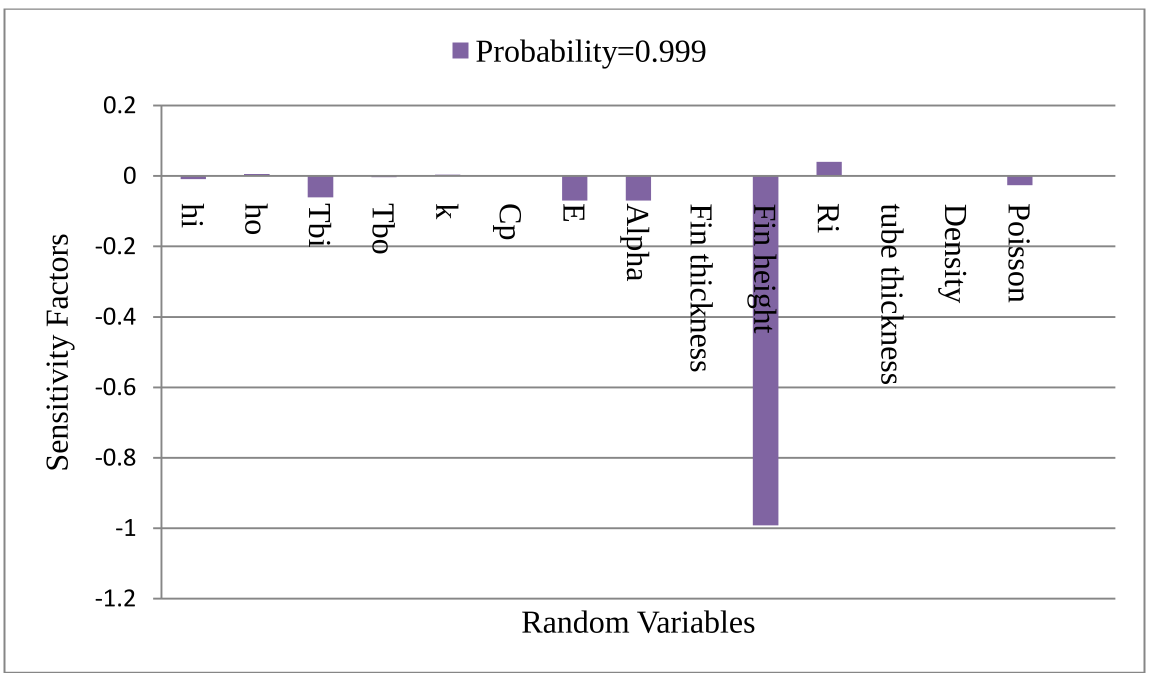

FIG.4.1.11 Sensitivity Factor for 0.999 Probability

CHAPTER V

CONCLUSIONS

The aim of this thesis is the probabilistic evaluation of the finite element solution for a thermally and mechanically loaded heat exchanger tube with circular fins. Cumulative distribution functions and sensitivity factors were computed for stresses generated due to heat transfer analysis by considering 15 random variables in the areas of thermal, material, and structural variables that govern the circular fins.

The study was done to predict the uncertainty in the stresses of the circular fins of different profiles under non-ideal conditions due to variation in the random variables. The first step was to perform a finite element analysis using COSMOS Works to determine the maximum temperatures, Hoop and Radial stresses for each run. The NESSUS probabilistic engineering analysis software was then used to simulate uncertainties in the random variables. Probabilistic design is a way to quantify the effect of uncertainties. Probabilistic design is necessary to study the effect of the variables on maximum temperatures and stresses. In sum, a design can be cost effectively accomplished if the effects of uncertainties are known.

The sensitivity factors versus random variables for the probabilities from 0.001 to 0.999 were found. Following conclusions have been drawn from the results.

Hoop stress of Rectangular profile:

Inside bulk temperature, elastic modulus, thermal expansion coefficient and outer bulk temperature has a lot of influence on thermal stresses.

Radial stress of Rectangular profile:

Fin height, tube thickness, inner radius of tube and inside bulk temperature has a lot of influence on thermal stresses.

Hoop stress of Triangular profile:

Inside bulk temperature, elastic modulus, thermal expansion coefficient, outer bulk temperatures have a lot of influence on thermal stresses.

Radial stress of Triangular profile:

Fin height, inside bulk temperature, elastic modulus, and thermal expansion coefficient have a lot of influence on thermal stresses

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APPENDIX

Output of NESSUS

1. Response and Sensitivity factors for Hoop stress of Rectangular fin profile:

Response (Z) median, mean, and std. dev. based on mean-value method
0.485220E+09 0.475136E+09 0.989536E+08

Response/Probability level: 11

Level	Z-value	u (std. normal)	Probability
1	0.172330E+09	-3.09025	0.001000000
2	0.247807E+09	-2.32635	0.010000000
3	0.351571E+09	-1.28155	0.100000000
4	0.397888E+09	-0.84162	0.200000000
5	0.459765E+09	-0.25335	0.400000000
6	0.509956E+09	0.25335	0.600000000
7	0.565136E+09	0.84162	0.800000000
8	0.608435E+09	1.28155	0.900000000
9	0.644307E+09	1.64485	0.950000000
10	0.710213E+09	2.32635	0.990000000
11	0.785706E+09	3.09025	0.999000000

Most probable point (MPP) or design point

Level 1 :(Z-value = 0.17233E+09, u = -3.0903, Probability = 0.001)

R.V. name	X-value	Std.Dev.from Mean	Sensitivity factor
HI	0.9684746E+03	-0.315254	-0.099934
HO	0.1007119E+03	0.071189	0.022567
TBI	0.9536083E+03	-2.053264	-0.650878
TBO	0.3210116E+03	0.700387	0.222020
K	0.4296409E+02	0.229545	0.072765
CP	0.2540000E+04	0.000000	0.000000
E	0.3612996E+03	-1.538650	-0.487747
ALPHA	0.4061448E+01	-1.538650	-0.487747
FIN THIC	0.1035659E+01	0.356588	0.113037
FIN HEIG	0.4575639E+02	0.399180	0.126539
RI	0.3905663E+02	-0.235841	-0.074761
TUBE THI	0.4035329E+01	0.088322	0.027998
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1709561E+00	0.056241	0.017828
TENSILE	0.9500000E+03	0.000000	0.000000

Level 2 :(Z-value = 0.24781E+09, u =-2.3263, Probability=0.010)

R.V. name	X-value	Std. Dev. from Mean	Sensitivity factor
HI	0.9763109E+03	-0.236891	-0.098899
HO	0.1005358E+03	0.053577	0.022368

TBI	0.1012606E+04	-1.561616	-0.651959
TBO	0.3159806E+03	0.532685	0.222391
K	0.4273058E+02	0.173947	0.072621
CP	0.2540000E+04	0.000000	0.000000
E	0.3770310E+03	-1.170234	-0.488561
ALPHA	0.4238288E+01	-1.170234	-0.488561
FIN THIC	0.1026952E+01	0.269524	0.112524
FIN HEIG	0.4523195E+02	0.279988	0.116892
RI	0.3931158E+02	-0.172105	-0.071852
TUBE THI	0.4026159E+01	0.065396	0.027302
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1707271E+00	0.042768	0.017855
TENSILE	0.9500000E+03	0.000000	0.000000

Level 3 :(Z-value = 0.35157E+09, u =-1.2816, Probability = 0.100)

R.V.name	X-value	Std.Dev.from Mean	Sensitivity factor
HI	0.9868472E+03	-0.131528	-0.097464
HO	0.1002981E+03	0.029810	0.022089
TBI	0.1094227E+04	-0.881445	-0.653164
TBO	0.3090202E+03	0.300674	0.222804
K	0.4241031E+02	0.097692	0.072392
CP	0.2540000E+04	0.000000	0.000000
E	0.3987950E+03	-0.660539	-0.489469
ALPHA	0.4482941E+01	-0.660539	-0.489469
FIN THIC	0.1015084E+01	0.150839	0.111774
FIN HEIG	0.4462814E+02	0.142759	0.105786
RI	0.3963204E+02	-0.091991	-0.068166
TUBE THI	0.4014243E+01	0.035608	0.026386
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1704103E+00	0.024136	0.017885
TENSILE	0.9500000E+03	0.000000	0.000000

Level 4 :(Z-value = 0.39789E+09, u = -0.8416, Probability = 0.2)

R.V. name	X-value	Std.Dev.from Mean	Sensitivity factor
HI	0.9914592E+03	-0.085408	-0.096823
HO	0.1001937E+03	0.019375	0.021964
TBI	0.1130813E+04	-0.576559	-0.653616
TBO	0.3059002E+03	0.196674	0.222959
K	0.4226778E+02	0.063758	0.072280
CP	0.2540000E+04	0.000000	0.000000
E	0.4085508E+03	-0.432065	-0.489810
ALPHA	0.4592609E+01	-0.432065	-0.489810
FIN THIC	0.1009829E+01	0.098290	0.111426
FIN HEIG	0.4439417E+02	0.089583	0.101556
RI	0.3976485E+02	-0.058787	-0.066644
TUBE THI	0.4009172E+01	0.022930	0.025995
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1702684E+00	0.015786	0.017896
TENSILE	0.9500000E+03	0.000000	0.000000

Level 5: (Z-value = 0.45977E+09, u = -0.2533, Probability = 0.4)

R.V. name	X-value	Std.Dev.from Mean	Sensitivity factor
HI	0.9975315E+03	-0.024685	-0.095971
HO	0.1000561E+03	0.005607	0.021797
TBI	0.1179809E+04	-0.168262	-0.654165
TBO	0.3017219E+03	0.057397	0.223148
K	0.4207792E+02	0.018552	0.072124
CP	0.2540000E+04	0.000000	0.000000
E	0.4216158E+03	-0.126094	-0.490225
ALPHA	0.4739475E+01	-0.126094	-0.490225
FIN THIC	0.1002854E+01	0.028540	0.110956
FIN HEIG	0.4410901E+02	0.024774	0.096316
RI	0.3993344E+02	-0.016641	-0.064696
TUBE THI	0.4002622E+01	0.006555	0.025486
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1700783E+00	0.004606	0.017909
TENSILE	0.9500000E+03	0.000000	0.000000

Level 6: (Z-value = 0.50996E+09, u = 0.2533, Probability = 0.6)

R.V.name	X-value	Std.Dev.from Mean	Sensitivity factor
HI	0.1002382E+04	0.023823	-0.095284
HO	0.9994584E+02	-0.005416	0.021662
TBI	0.1219639E+04	0.163659	-0.654570
TBO	0.2983252E+03	-0.055828	0.223287
K	0.4192440E+02	-0.018000	0.071994
CP	0.2540000E+04	0.000000	0.000000
E	0.4322370E+03	0.122645	-0.490531
ALPHA	0.4858870E+01	0.122645	-0.490530
FIN THIC	0.9972354E+00	-0.027646	0.110571
FIN HEIG	0.4389833E+02	-0.023106	0.092415
RI	0.4006320E+02	0.015799	-0.063189
TUBE THI	0.3997491E+01	-0.006272	0.025085
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1699238E+00	-0.004480	0.017918
TENSILE	0.9500000E+03	0.000000	0.000000

Level 7: (Z-value = 0.56514E+09, u = 0.8416, Probability = 0.8)

R.V. name	X-value	Std.Dev.from Mean	Sensitivity factor
HI	0.1007639E+04	0.076387	-0.094533
HO	0.9982615E+02	-0.017385	0.021514
TBI	0.1263510E+04	0.529249	-0.654974
TBO	0.2945838E+03	-0.180539	0.223427
K	0.4175617E+02	-0.058056	0.071847
CP	0.2540000E+04	0.000000	0.000000
E	0.4439356E+03	0.396618	-0.490836
ALPHA	0.4990377E+01	0.396618	-0.490836
FIN THIC	0.9911000E+00	-0.089000	0.110142
FIN HEIG	0.4368538E+02	-0.071504	0.088491
RI	0.4019913E+02	0.049783	-0.061610
TUBE THI	0.3992030E+01	-0.019924	0.024657
DENSITY	0.3210000E+04	0.000000	0.000000

POISSON	0.1697537E+00	-0.014486	0.017927
TENSILE	0.9500000E+03	0.000000	0.000000

Level 8: (Z-value = 0.60844E+09, u = 1.2816, Probability = 0.9)

R.V.name	X-value	Std.Dev.from Mean	Sensitivity factor
HI	0.1011707E+04	0.117074	-0.093949
HO	0.9973334E+02	-0.026666	0.021399
TBI	0.1297987E+04	0.816558	-0.655264
TBO	0.2916436E+03	-0.278548	0.223526
K	0.4162458E+02	-0.089385	0.071729
CP	0.2540000E+04	0.000000	0.000000
E	0.4531294E+03	0.611929	-0.491056
ALPHA	0.5093726E+01	0.611929	-0.491056
FIN THIC	0.9863168E+00	-0.136832	0.109804
FIN HEIG	0.4353020E+02	-0.106772	0.085681
RI	0.4030121E+02	0.075302	-0.060428
TUBE THI	0.3987872E+01	-0.030321	0.024331
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1696201E+00	-0.022348	0.017934
TENSILE	0.9500000E+03	0.000000	0.000000

Level 9: (Z-value = 0.64431E+09, u = 1.6449, Probability = 0.95)

R.V.name	X-value	Std.Dev.from Mean	Sensitivity factor
HI	0.1015041E+04	0.150414	-0.093469
HO	0.9965717E+02	-0.034283	0.021304
TBI	0.1326584E+04	1.054863	-0.655500
TBO	0.2892048E+03	-0.359841	0.223608
K	0.4151586E+02	-0.115272	0.071631
CP	0.2540000E+04	0.000000	0.000000
E	0.4607552E+03	0.790519	-0.491235
ALPHA	0.5179449E+01	0.790519	-0.491235
FIN THIC	0.9823748E+00	-0.176252	0.109524
FIN HEIG	0.4341004E+02	-0.134082	0.083319
RI	0.4038275E+02	0.095688	-0.059461
TUBE THI	0.3984509E+01	-0.038727	0.024065
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1695092E+00	-0.028869	0.017939
TENSILE	0.9500000E+03	0.000000	0.000000

Level 10: (Z-value = 0.71021E+09, u = 2.3263, Probability = 0.99)

R.V.name	X-value	Std.Dev.from Mean	Sensitivity factor
HI	0.1021081E+04	0.210807	-0.092594
HO	0.9951894E+02	-0.048106	0.021130
TBI	0.1379192E+04	1.493263	-0.655895
TBO	0.2847182E+03	-0.509394	0.223744
K	0.4131682E+02	-0.162661	0.071447
CP	0.2540000E+04	0.000000	0.000000
E	0.4747841E+03	1.119065	-0.491534
ALPHA	0.5337151E+01	1.119065	-0.491534
FIN THIC	0.9751825E+00	-0.248175	0.109007

FIN HEIG	0.4320452E+02	-0.180790	0.079410
RI	0.4052617E+02	0.131542	-0.057778
TUBE THI	0.3978516E+01	-0.053711	0.023592
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1693054E+00	-0.040861	0.017948
TENSILE	0.9500000E+03	0.000000	0.000000

Level 11: (Z-value = 0.78571E+09, u = 3.0903, Probability = 0.999)

R.V.name	X-value	Std.Dev.from Mean	Sensitivity factor
HI	0.1027863E+04	0.278627	-0.091603
HO	0.9936331E+02	-0.063669	0.020932
TBI	0.1439547E+04	1.996223	-0.656291
TBO	0.2795708E+03	-0.680972	0.223881
K	0.4109002E+02	-0.216661	0.071231
CP	0.2540000E+04	0.000000	0.000000
E	0.4908792E+03	1.496000	-0.491834
ALPHA	0.5518080E+01	1.496000	-0.491834
FIN THIC	0.9670247E+00	-0.329753	0.108412
FIN HEIG	0.4298836E+02	-0.229919	0.075590
RI	0.4068101E+02	0.170254	-0.055974
TUBE THI	0.3971931E+01	-0.070174	0.023071
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1690715E+00	-0.054617	0.017956
TENSILE	0.9500000E+03	0.000000	0.000000

2. Response and Sensitivity factors for Radial stress of Rectangular fin profile

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Response (Z) median, mean, and std. dev. based on mean-value method
   0.743257E+09   0.460129E+09   0.176549E+09
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Response/Probability level: 11

Level	Z-value	u (std. normal)	Probability
1	-0.331215E+09	-3.09025	0.001000000
2	-0.138573E+09	-2.32635	0.010000000
3	0.295039E+09	-1.28155	0.100000000
4	0.417632E+09	-0.84162	0.200000000
5	0.640579E+09	-0.25335	0.400000000
6	0.743408E+09	0.25335	0.600000000
7	0.274980E+09	0.84162	0.800000000
8	0.274943E+09	1.28155	0.900000000
9	0.274897E+09	1.64485	0.950000000
10	0.274864E+09	2.32635	0.990000000
11	0.274815E+09	3.09025	0.999000000

Most probable point (MPP) or design point

Level 1: (Z-value = -.33122E+09, u = -3.0903, Probability = 0.001)

R.V.name	X-value	Std.Dev.from Mean	Sensitivity factor
HI	0.9935033E+03	-0.064967	-0.021962
HO	0.1004348E+03	0.043479	0.014698
TBI	0.1146723E+04	-0.443973	-0.150086
TBO	0.2991970E+03	-0.026765	-0.009048
K	0.4208554E+02	0.020366	0.006885
CP	0.2540000E+04	0.000000	0.000000
E	0.4127426E+03	-0.333897	-0.112875
ALPHA	0.4639730E+01	-0.333897	-0.112875
FIN THIC	0.9790758E+00	-0.209242	-0.070735
FIN HEIG	0.3256197E+02	-2.599553	-0.878785
RI	0.4190228E+02	0.475569	0.160767
TUBE THI	0.3546027E+01	-1.134931	-0.383666
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1685315E+00	-0.086380	-0.029201
TENSILE	0.9500000E+03	0.000000	0.000000

Level 2: (Z-value = -.13857E+09, u = -2.3263, Probability = 0.01)

R.V. name	X-value	Std.Dev.from Mean	Sensitivity factor
HI	0.9935055E+03	-0.064945	-0.024510
HO	0.1004346E+03	0.043465	0.016403
TBI	0.1146741E+04	-0.443825	-0.167495
TBO	0.2991973E+03	-0.026756	-0.010098
K	0.4208551E+02	0.020359	0.007683
CP	0.2540000E+04	0.000000	0.000000
E	0.4127473E+03	-0.333786	-0.125967
ALPHA	0.4639783E+01	-0.333786	-0.125967

FIN THIC	0.9790896E+00	-0.209104	-0.078914
FIN HEIG	0.3412241E+02	-2.244906	-0.847204
RI	0.4190146E+02	0.475365	0.179398
TUBE THI	0.3547885E+01	-1.130288	-0.426559
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1685320E+00	-0.086351	-0.032588
TENSILE	0.9500000E+03	0.000000	0.000000

Level 3: (Z-value = 0.29504E+09, u = -1.2816, Probability = 0.1)

R.V.name	X-value	Std.Dev.from Mean	Sensitivity factor
HI	0.9935175E+03	-0.064825	-0.036614
HO	0.1004339E+03	0.043385	0.024505
TBI	0.1146839E+04	-0.443011	-0.250219
TBO	0.2991988E+03	-0.026707	-0.015085
K	0.4208535E+02	0.020322	0.011478
CP	0.2540000E+04	0.000000	0.000000
E	0.4127735E+03	-0.333174	-0.188181
ALPHA	0.4640077E+01	-0.333174	-0.188181
FIN THIC	0.9791646E+00	-0.208354	-0.117681
FIN HEIG	0.3919684E+02	-1.091626	-0.616565
RI	0.4189701E+02	0.474252	0.267864
TUBE THI	0.3554260E+01	-1.114350	-0.629400
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1685347E+00	-0.086193	-0.048683
TENSILE	0.9500000E+03	0.000000	0.000000

Level 4: (Z-value = 0.41763E+09, u = -0.8416, Probability = 0.2)

R.V.name	X-value	Std.Dev.from Mean	Sensitivity factor
HI	0.9935508E+03	-0.064492	-0.045139
HO	0.1004317E+03	0.043165	0.030212
TBI	0.1147109E+04	-0.440760	-0.308494
TBO	0.2992029E+03	-0.026572	-0.018598
K	0.4208492E+02	0.020219	0.014151
CP	0.2540000E+04	0.000000	0.000000
E	0.4128458E+03	-0.331480	-0.232008
ALPHA	0.4640890E+01	-0.331480	-0.232008
FIN THIC	0.9793850E+00	-0.206150	-0.144287
FIN HEIG	0.4214028E+02	-0.422663	-0.295827
RI	0.4188447E+02	0.471117	0.329741
TUBE THI	0.3567388E+01	-1.081530	-0.756978
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1685421E+00	-0.085757	-0.060022
TENSILE	0.9500000E+03	0.000000	0.000000

Level 5: (Z-value= 0.64058E+09, u = -0.2533, Probability = 0.4)

R.V.name	X-value	Std.Dev.from Mean	Sensitivity factor
HI	0.9960851E+03	-0.039149	-0.071331
HO	0.1002633E+03	0.026333	0.047979
TBI	0.1167761E+04	-0.268655	-0.489492
TBO	0.2995141E+03	-0.016196	-0.029510
K	0.4205178E+02	0.012329	0.022463

CP	0.2540000E+04	0.000000	0.000000
E	0.4183726E+03	-0.202046	-0.368130
ALPHA	0.4703018E+01	-0.202046	-0.368130
FIN THIC	0.9910619E+00	-0.089381	-0.162853
FIN HEIG	0.4395179E+02	-0.010957	-0.019964
RI	0.4103019E+02	0.257549	0.469256
TUBE THI	0.3896010E+01	-0.259976	-0.473679
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1691100E+00	-0.052355	-0.095391
TENSILE	0.9500000E+03	0.000000	0.000000

Level 6: (Z-value = 0.74341E+09, u = 0.2533, Probability = 0.6)

R.V.name	X-value	Std.Dev.from Mean	Sensitivity factor
HI	0.1000007E+04	0.000069	-0.081073
HO	0.9999953E+02	-0.000047	0.054954
TBI	0.1200057E+04	0.000478	-0.559897
TBO	0.3000009E+03	0.000029	-0.033754
K	0.4199991E+02	-0.000022	0.025709
CP	0.2540000E+04	0.000000	0.000000
E	0.4270154E+03	0.000360	-0.421079
ALPHA	0.4800173E+01	0.000360	-0.421079
FIN THIC	0.1000011E+01	0.000109	-0.128119
FIN HEIG	0.4400004E+02	0.000008	-0.009343
RI	0.3999842E+02	-0.000394	0.461934
TUBE THI	0.4000096E+01	0.000240	-0.280456
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1700016E+00	0.000093	-0.109387
TENSILE	0.9500000E+03	0.000000	0.000000

Level 7: (Z-value = 0.27498E+09, u = 0.8416, Probability =0.8)

R.V.name	X-value	Std.Dev.from Mean	Sensitivity factor
HI	0.9935152E+03	-0.064848	-0.035622
HO	0.1004340E+03	0.043401	0.023840
TBI	0.1146820E+04	-0.443170	-0.243437
TBO	0.2991985E+03	-0.026717	-0.014676
K	0.4208538E+02	0.020329	0.011167
CP	0.2540000E+04	0.000000	0.000000
E	0.4127684E+03	-0.333293	-0.183081
ALPHA	0.4640020E+01	-0.333293	-0.183081
FIN THIC	0.9791485E+00	-0.208515	-0.114539
FIN HEIG	0.3886193E+02	-1.167743	-0.641452
RI	0.4189790E+02	0.474474	0.260633
TUBE THI	0.3553026E+01	-1.117435	-0.613817
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1685342E+00	-0.086224	-0.047364
TENSILE	0.9500000E+03	0.000000	0.000000

Level 8: (Z-value =0.27494E+09, u = 1.2816, Probability =0.9)

R.V.name	X-value	Std.Dev.from Mean	Sensitivity factor
HI	0.9935152E+03	-0.064848	-0.035620
HO	0.1004340E+03	0.043401	0.023839
TBI	0.1146820E+04	-0.443170	-0.243425
TBO	0.2991985E+03	-0.026717	-0.014675
K	0.4208538E+02	0.020329	0.011167
CP	0.2540000E+04	0.000000	0.000000
E	0.4127684E+03	-0.333293	-0.183072
ALPHA	0.4640020E+01	-0.333293	-0.183072
FIN THIC	0.9791485E+00	-0.208515	-0.114533
FIN HEIG	0.3886129E+02	-1.167888	-0.641500
RI	0.4189789E+02	0.474474	0.260620
TUBE THI	0.3553028E+01	-1.117431	-0.613784
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1685342E+00	-0.086224	-0.047361
TENSILE	0.9500000E+03	0.000000	0.000000

Level 9: (Z-value = 0.27490E+09, u = 1.6449, Probability = 0.95)

R.V.name	X-value	Std.Dev.from Mean	Sensitivity factor
HI	0.9935152E+03	-0.064848	-0.035618
HO	0.1004340E+03	0.043401	0.023838
TBI	0.1146820E+04	-0.443169	-0.243410
TBO	0.2991985E+03	-0.026717	-0.014674
K	0.4208538E+02	0.020329	0.011166
CP	0.2540000E+04	0.000000	0.000000
E	0.4127684E+03	-0.333292	-0.183060
ALPHA	0.4640020E+01	-0.333292	-0.183060
FIN THIC	0.9791486E+00	-0.208514	-0.114526
FIN HEIG	0.3886051E+02	-1.168066	-0.641558
RI	0.4189789E+02	0.474473	0.260604
TUBE THI	0.3553030E+01	-1.117426	-0.613744
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1685342E+00	-0.086224	-0.047358
TENSILE	0.9500000E+03	0.000000	0.000000

Level 10: (Z-value = 0.27486E+09, u = 2.3263, Probability = 0.99)

R.V.name	X-value	Std.Dev.from Mean	Sensitivity factor
HI	0.9935152E+03	-0.064848	-0.035616
HO	0.1004340E+03	0.043401	0.023837
TBI	0.1146820E+04	-0.443169	-0.243399
TBO	0.2991985E+03	-0.026717	-0.014674
K	0.4208538E+02	0.020329	0.011165
CP	0.2540000E+04	0.000000	0.000000
E	0.4127684E+03	-0.333292	-0.183052
ALPHA	0.4640020E+01	-0.333292	-0.183052
FIN THIC	0.9791486E+00	-0.208514	-0.114521
FIN HEIG	0.3885994E+02	-1.168196	-0.641601
RI	0.4189789E+02	0.474473	0.260592
TUBE THI	0.3553031E+01	-1.117423	-0.613715
DENSITY	0.3210000E+04	0.000000	0.000000

POISSON	0.1685342E+00	-0.086224	-0.047356
TENSILE	0.9500000E+03	0.000000	0.000000

Level 11: (Z-value = 0.27481E+09, u = 3.0903, Probability = 0.999)

R.V.name	**X-value**	**Std.Dev.from Mean**	**Sensitivity factor**
HI	0.9935152E+03	-0.064848	-0.035614
HO	0.1004340E+03	0.043401	0.023835
TBI	0.1146820E+04	-0.443169	-0.243383
TBO	0.2991985E+03	-0.026717	-0.014673
K	0.4208538E+02	0.020329	0.011165
CP	0.2540000E+04	0.000000	0.000000
E	0.4127684E+03	-0.333292	-0.183040
ALPHA	0.4640020E+01	-0.333292	-0.183040
FIN THIC	0.9791486E+00	-0.208514	-0.114513
FIN HEIG	0.3885908E+02	-1.168391	-0.641665
RI	0.4189789E+02	0.474473	0.260574
TUBE THI	0.3553033E+01	-1.117418	-0.613671
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1685342E+00	-0.086224	-0.047353
TENSILE	0.9500000E+03	0.000000	0.000000

3. Response and Sensitivity factors for Hoop stress of Triangular fin profile

Response (Z) median, mean, and std. dev. based on mean-value method
0.606117E+09   0.607497E+09   0.123860E+09

Response/Probability level: 11

Level	Z-value	u (std. normal)	Probability
1	0.225591E+09	-3.09025	0.001000000
2	0.319318E+09	-2.32635	0.010000000
3	0.448041E+09	-1.28155	0.100000000
4	0.501924E+09	-0.84162	0.200000000
5	0.575495E+09	-0.25335	0.400000000
6	0.638282E+09	0.25335	0.600000000
7	0.710526E+09	0.84162	0.800000000
8	0.765404E+09	1.28155	0.900000000
9	0.810914E+09	1.64485	0.950000000
10	0.895805E+09	2.32635	0.990000000
11	0.991288E+09	3.09025	0.999000000

Most probable point (MPP) or design point

Level 1: (Z-value = 0.22559E+09, u = -3.0903, Probability = 0.001

R.V.name	X-value	Std.Dev.from Mean	Sensitivity factor
HI	0.9701678E+03	-0.298322	-0.096695
HO	0.1018367E+03	0.183673	0.059534
TBI	0.9562029E+03	-2.031642	-0.658512
TBO	0.3208947E+03	0.696488	0.225751
K	0.4234535E+02	0.082227	0.026652
CP	0.2539965E+04	-0.000137	-0.000044
E	0.3630657E+03	-1.497291	-0.485314
ALPHA	0.4081300E+01	-1.497291	-0.485314
FIN THIC	0.1016678E+01	0.166780	0.054058
FIN HEIG	0.4598390E+02	0.450885	0.146145
RI	0.3963311E+02	-0.091722	-0.029730
TUBE THI	0.4088403E+01	0.221007	0.071635
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1710977E+00	0.064569	0.020929
TENSILE	0.9500000E+03	0.000000	0.000000

Level 2: (Z-value = 0.31932E+09, u =-2.3263, Probability =0.01)

R.V.name	X-value	Std.Dev.from Mean	Sensitivity factor
HI	0.9778502E+03	-0.221498	-0.095355
HO	0.1013863E+03	0.138628	0.059679
TBI	0.1016818E+04	-1.526519	-0.657165
TBO	0.3156997E+03	0.523322	0.225290
K	0.4226005E+02	0.061916	0.026655
CP	0.2539974E+04	-0.000103	-0.000044
E	0.3787563E+03	-1.129829	-0.486390
ALPHA	0.4257682E+01	-1.129829	-0.486390
FIN THIC	0.1012500E+01	0.125001	0.053813

FIN HEIG	0.4549888E+02	0.340654	0.146651
RI	0.3972780E+02	-0.068049	-0.029295
TUBE THI	0.4066765E+01	0.166913	0.071856
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1708281E+00	0.048714	0.020971
TENSILE	0.9500000E+03	0.000000	0.000000

Level 3: (Z-value = 0.44804E+09, u = -1.2816, Probability = 0.10)

R.V.name	X-value	Std.Dev.from Mean	Sensitivity factor
HI	0.9880371E+03	-0.119629	-0.093570
HO	0.1007655E+03	0.076554	0.059878
TBI	0.1099463E+04	-0.837806	-0.655307
TBO	0.3086165E+03	0.287217	0.224653
K	0.4214314E+02	0.034081	0.026657
CP	0.2539986E+04	-0.000057	-0.000045
E	0.4003671E+03	-0.623722	-0.487857
ALPHA	0.4500614E+01	-0.623722	-0.487857
FIN THIC	0.1006837E+01	0.068372	0.053478
FIN HEIG	0.4482888E+02	0.188382	0.147347
RI	0.3985314E+02	-0.036715	-0.028717
TUBE THI	0.4036902E+01	0.092255	0.072159
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1704571E+00	0.026886	0.021030
TENSILE	0.9500000E+03	0.000000	0.000000

Level 4: (Z-value =0.50192E+09, u =-0.8416, Probability =0.2)

R.V.name	X-value	Std.Dev.from Mean	Sensitivity factor
HI	0.9921809E+03	-0.078191	-0.092842
HO	0.1005050E+03	0.050499	0.059961
TBI	0.1133852E+04	-0.551237	-0.654527
TBO	0.3056693E+03	0.188975	0.224385
K	0.4209430E+02	0.022451	0.026658
CP	0.2539990E+04	-0.000038	-0.000045
E	0.4094340E+03	-0.411382	-0.488466
ALPHA	0.4602537E+01	-0.411382	-0.488466
FIN THIC	0.1004492E+01	0.044922	0.053339
FIN HEIG	0.4454709E+02	0.124339	0.147638
RI	0.3990405E+02	-0.023987	-0.028482
TUBE THI	0.4024351E+01	0.060878	0.072285
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1703014E+00	0.017731	0.021054
TENSILE	0.9500000E+03	0.000000	0.000000

Level 5 :(Z-value = 0.57549E+09, u = -0.2533, Probability = 0.4)

R.V.name	X-value	Std.Dev.from Mean	Sensitivity factor
HI	0.9977280E+03	-0.022720	-0.091867
HO	0.1001486E+03	0.014857	0.060074
TBI	0.1180606E+04	-0.161613	-0.653459
TBO	0.3016621E+03	0.055404	0.224019
K	0.4202769E+02	0.006593	0.026659

CP	0.2539997E+04	-0.000011	-0.000045
E	0.4218328E+03	-0.121012	-0.489296
ALPHA	0.4741914E+01	-0.121012	-0.489296
FIN THIC	0.1001315E+01	0.013145	0.053151
FIN HEIG	0.4416109E+02	0.036612	0.148035
RI	0.3997214E+02	-0.006966	-0.028167
TUBE THI	0.4007168E+01	0.017920	0.072458
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1700887E+00	0.005215	0.021087
TENSILE	0.9500000E+03	0.000000	0.000000

Level 6: (Z-value = 0.63828E+09, u = 0.2533, Probability = 0.6)

R.V.name	**X-value**	**Std.Dev.from Mean**	**Sensitivity factor**
HI	0.1002364E+04	0.023637	-0.091050
HO	0.9984380E+02	-0.015620	0.060170
TBI	0.1220328E+04	0.169401	-0.652546
TBO	0.2982578E+03	-0.058074	0.223706
K	0.4197093E+02	-0.006921	0.026659
CP	0.2540003E+04	0.000012	-0.000045
E	0.4324316E+03	0.127204	-0.490000
ALPHA	0.4861058E+01	0.127204	-0.490000
FIN THIC	0.9986244E+00	-0.013756	0.052990
FIN HEIG	0.4383052E+02	-0.038518	0.148374
RI	0.4002897E+02	0.007244	-0.027903
TUBE THI	0.3992461E+01	-0.018848	0.072605
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1699068E+00	-0.005481	0.021115
TENSILE	0.9500000E+03	0.000000	0.000000

Level 7: (Z-value = 0.71053E+09, u = 0.8416, Probability =0.8)

R.V.name	**X-value**	**Std.Dev.from Mean**	**Sensitivity factor**
HI	0.1007589E+04	0.075887	-0.090129
HO	0.9949246E+02	-0.050754	0.060279
TBI	0.1265825E+04	0.548543	-0.651494
TBO	0.2943584E+03	-0.188052	0.223345
K	0.4190572E+02	-0.022447	0.026660
CP	0.2540010E+04	0.000038	-0.000045
E	0.4446457E+03	0.413248	-0.490806
ALPHA	0.4998359E+01	0.413248	-0.490806
FIN THIC	0.9955538E+00	-0.044462	0.052806
FIN HEIG	0.4344888E+02	-0.125255	0.148763
RI	0.4009297E+02	0.023244	-0.027606
TUBE THI	0.3975490E+01	-0.061274	0.072774
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1696973E+00	-0.017805	0.021147
TENSILE	0.9500000E+03	0.000000	0.000000

Level 8: (Z-value = 0.76540E+09, u = 1.2816, Probability = 0.9)

R.V.name	X-value	Std.Dev.from Mean	Sensitivity factor
HI	0.1011482E+04	0.114821	-0.089442
HO	0.9922510E+02	-0.077490	0.060363
TBI	0.1300239E+04	0.835321	-0.650693
TBO	0.2914090E+03	-0.286365	0.223071
K	0.4185626E+02	-0.034224	0.026660
CP	0.2540015E+04	0.000058	-0.000045
E	0.4539373E+03	0.630851	-0.491416
ALPHA	0.5102809E+01	0.630851	-0.491416
FIN THIC	0.9932388E+00	-0.067612	0.052668
FIN HEIG	0.4315805E+02	-0.191353	0.149059
RI	0.4014062E+02	0.035155	-0.027385
TUBE THI	0.3962565E+01	-0.093587	0.072902
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1695380E+00	-0.027178	0.021171
TENSILE	0.9500000E+03	0.000000	0.000000

Level 9: (Z-value = 0.81091E+09, u =1.6449, Probability =0.95)

R.V.name	X-value	Std.Dev.from Mean	Sensitivity factor
HI	0.1014663E+04	0.146626	-0.088882
HO	0.9900308E+02	-0.099692	0.060431
TBI	0.1328681E+04	1.072338	-0.650027
TBO	0.2889714E+03	-0.367619	0.222843
K	0.4181528E+02	-0.043980	0.026660
CP	0.2540019E+04	0.000074	-0.000045
E	0.4616515E+03	0.811510	-0.491920
ALPHA	0.5189525E+01	0.811510	-0.491920
FIN THIC	0.9913304E+00	-0.086696	0.052553
FIN HEIG	0.4291626E+02	-0.246304	0.149304
RI	0.4017952E+02	0.044879	-0.027205
TUBE THI	0.3951824E+01	-0.120440	0.073008
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1694057E+00	-0.034958	0.021191
TENSILE	0.9500000E+03	0.000000	0.000000

Level 10 :(Z-value = 0.89581E+09, u = 2.3263, Probability = 0.99)

R.V.name	X-value	Std.Dev.from Mean	Sensitivity factor
HI	0.1020482E+04	0.204817	-0.087856
HO	0.9858820E+02	-0.141180	0.060559
TBI	0.1381500E+04	1.512499	-0.648785
TBO	0.2844445E+03	-0.518516	0.222417
K	0.4173897E+02	-0.062150	0.026659
CP	0.2540027E+04	0.000105	-0.000045
E	0.4760616E+03	1.148984	-0.492855
ALPHA	0.5351512E+01	1.148984	-0.492855
FIN THIC	0.9877981E+00	-0.122019	0.052340
FIN HEIG	0.4246381E+02	-0.349134	0.149761
RI	0.4025061E+02	0.062653	-0.026875
TUBE THI	0.3931735E+01	-0.170663	0.073206
DENSITY	0.3210000E+04	0.000000	0.000000

POISSON	0.1691587E+00	-0.049488	0.021228
TENSILE	0.9500000E+03	0.000000	0.000000

Level 11: (Z-value = 0.99129E+09, u = 3.0903, Probability = 0.9990)

R.V.name	**X-value**	**Std.Dev.from Mean**	**Sensitivity factor**
HI	0.1026856E+04	0.268558	-0.086734
HO	0.9812045E+02	-0.187955	0.060702
TBI	0.1440544E+04	2.004532	-0.647384
TBO	0.2793842E+03	-0.687194	0.221937
K	0.4165332E+02	-0.082543	0.026658
CP	0.2540035E+04	0.000140	-0.000045
E	0.4923008E+03	1.529292	-0.493901
ALPHA	0.5534060E+01	1.529292	-0.493901
FIN THIC	0.9838673E+00	-0.161327	0.052102
FIN HEIG	0.4195267E+02	-0.465301	0.150274
RI	0.4032840E+02	0.082099	-0.026515
TUBE THI	0.3909057E+01	-0.227358	0.073427
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1688804E+00	-0.065857	0.021269
TENSILE	0.9500000E+03	0.000000	0.000000

4. Response and Sensitivity factors for Radial stress of Triangular fin profile

Response (Z) median, mean, and std. dev. based on mean-value method
0.390937E+09 0.920821E+09 0.143753E+09

Response/Probability level: 11

Level	Z-value	u (std. normal)	Probability
1	-0.287496E+09	-3.09025	0.001000000
2	-0.496507E+08	-2.32635	0.010000000
3	0.309019E+09	-1.28155	0.100000000
4	0.354221E+09	-0.84162	0.200000000
5	0.412001E+09	-0.25335	0.400000000
6	0.498317E+09	0.25335	0.600000000
7	0.628163E+09	0.84162	0.800000000
8	0.799506E+09	1.28155	0.900000000
9	0.965234E+09	1.64485	0.950000000
10	0.135767E+10	2.32635	0.990000000
11	0.195861E+10	3.09025	0.999000000

Most probable point (MPP) or design point

Level 1: (Z-value = -.28750E+09, u =-3.0903, Probability =0.0010)

R.V.name	X-value	Std.Dev.from Mean	Sensitivity factor
HI	0.9837141E+03	-0.162859	-0.041729
HO	0.1011735E+03	0.117349	0.030068
TBI	0.1059405E+04	-1.171628	-0.300204
TBO	0.2979766E+03	-0.067448	-0.017282
K	0.4220533E+02	0.048888	0.012526
CP	0.2536656E+04	-0.013167	-0.003374
E	0.3862223E+03	-0.954982	-0.244693
ALPHA	0.4341609E+01	-0.954982	-0.244693
FIN THIC	0.1003212E+01	0.032118	0.008230
FIN HEIG	0.2888554E+02	-3.435105	-0.880170
RI	0.4098926E+02	0.247315	0.063369
TUBE THI	0.3998179E+01	-0.004552	-0.001166
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1639799E+00	-0.354122	-0.090736
TENSILE	0.9500000E+03	0.000000	0.000000

Level 2: (Z-value = -.49651E+08, u = -2.3263, Probability =0.0100)

R.V.name	X-value	Std.Dev.from Mean	Sensitivity factor
HI	0.9906593E+03	-0.093407	-0.039426
HO	0.1006739E+03	0.067390	0.028445
TBI	0.1119277E+04	-0.672689	-0.283932
TBO	0.2988383E+03	-0.038725	-0.016345
K	0.4211661E+02	0.027764	0.011719
CP	0.2538102E+04	-0.007471	-0.003154
E	0.4038403E+03	-0.542382	-0.228932
ALPHA	0.4539657E+01	-0.542382	-0.228932

FIN THIC	0.1001846E+01	0.018455	0.007790
FIN HEIG	0.3500684E+02	-2.043901	-0.862701
RI	0.4232656E+02	0.581640	0.245502
TUBE THI	0.3999003E+01	-0.002493	-0.001052
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1665828E+00	-0.201015	-0.084845
TENSILE	0.9500000E+03	0.000000	0.000000

Level 3: (Z-value = 0.30902E+09, u = -1.2816, Probability = 0.100)

R.V.name	X-value	Std.Dev.from Mean	Sensitivity factor
HI	0.9961722E+03	-0.038278	-0.066877
HO	0.1002736E+03	0.027357	0.047796
TBI	0.1167177E+04	-0.273523	-0.477880
TBO	0.2995276E+03	-0.015746	-0.027510
K	0.4203235E+02	0.007703	0.013459
CP	0.2539476E+04	-0.002063	-0.003604
E	0.4205708E+03	-0.150568	-0.263061
ALPHA	0.4727728E+01	-0.150568	-0.263061
FIN THIC	0.1000746E+01	0.007460	0.013033
FIN HEIG	0.4243396E+02	-0.355918	-0.621835
RI	0.3890385E+02	-0.274037	-0.478778
TUBE THI	0.3999818E+01	-0.000455	-0.000795
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1690545E+00	-0.055617	-0.097169
TENSILE	0.9500000E+03	0.000000	0.000000

Level 4: (Z-value = 0.35422E+09, u = -0.8416, Probability = 0.200)

R.V.name	X-value	Std.Dev.from Mean	Sensitivity factor
HI	0.9978434E+03	-0.021566	-0.071152
HO	0.1001548E+03	0.015482	0.051077
TBI	0.1181439E+04	-0.154671	-0.510294
TBO	0.2997329E+03	-0.008904	-0.029376
K	0.4202241E+02	0.005336	0.017603
CP	0.2539636E+04	-0.001433	-0.004728
E	0.4225483E+03	-0.104255	-0.343959
ALPHA	0.4749958E+01	-0.104255	-0.343959
FIN THIC	0.1000423E+01	0.004230	0.013956
FIN HEIG	0.4308299E+02	-0.208411	-0.687594
RI	0.4008511E+02	0.021276	0.070196
TUBE THI	0.3999829E+01	-0.000427	-0.001408
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1693440E+00	-0.038588	-0.127310
TENSILE	0.9500000E+03	0.000000	0.000000

Level 5: (Z-value = 0.41200E+09, u = -0.2533, Probability = 0.400)

R.V.name	X-value	Std.Dev.from Mean	Sensitivity factor
HI	0.1000581E+04	0.005814	-0.042959
HO	0.9995804E+02	-0.004196	0.031007
TBI	0.1205026E+04	0.041884	-0.309483
TBO	0.3000723E+03	0.002411	-0.017816
K	0.4199212E+02	-0.001876	0.013863

CP	0.2540128E+04	0.000506	-0.003737
E	0.4285648E+03	0.036647	-0.270784
ALPHA	0.4817591E+01	0.036647	-0.270784
FIN THIC	0.9998851E+00	-0.001149	0.008493
FIN HEIG	0.4451166E+02	0.116286	-0.859236
RI	0.3995882E+02	-0.010296	0.076076
TUBE THI	0.4000073E+01	0.000183	-0.001354
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1702311E+00	0.013597	-0.100467
TENSILE	0.9500000E+03	0.000000	0.000000

Level 6: (Z-value = 0.49832E+09, u = 0.2533, Probability = 0.600)

R.V.name	X-value	Std.Dev.from Mean	Sensitivity factor
HI	0.1001564E+04	0.015640	-0.028548
HO	0.9988689E+02	-0.011311	0.020645
TBI	0.1213543E+04	0.112856	-0.205996
TBO	0.3001949E+03	0.006497	-0.011859
K	0.4197622E+02	-0.005663	0.010336
CP	0.2540388E+04	0.001529	-0.002791
E	0.4317221E+03	0.110587	-0.201854
ALPHA	0.4853082E+01	0.110587	-0.201854
FIN THIC	0.9996899E+00	-0.003101	0.005660
FIN HEIG	0.4624060E+02	0.509227	-0.929486
RI	0.3984537E+02	-0.038657	0.070560
TUBE THI	0.4000250E+01	0.000625	-0.001142
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1706984E+00	0.041082	-0.074987
TENSILE	0.9500000E+03	0.000000	0.000000

Level 7: (Z-value = 0.62816E+09, u = 0.8416, Probability = 0.800)

R.V.name	X-value	Std.Dev.from Mean	Sensitivity factor
HI	0.1002030E+04	0.020303	-0.020696
HO	0.9985304E+02	-0.014696	0.014981
TBI	0.1217593E+04	0.146611	-0.149454
TBO	0.3002532E+03	0.008440	-0.008604
K	0.4196723E+02	-0.007801	0.007953
CP	0.2540536E+04	0.002109	-0.002150
E	0.4335047E+03	0.152334	-0.155288
ALPHA	0.4873120E+01	0.152334	-0.155288
FIN THIC	0.9995969E+00	-0.004031	0.004109
FIN HEIG	0.4814278E+02	0.941540	-0.959798
RI	0.3974947E+02	-0.062633	0.063847
TUBE THI	0.4000375E+01	0.000937	-0.000955
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1709628E+00	0.056632	-0.057731
TENSILE	0.9500000E+03	0.000000	0.000000

Level 8 :(Z-value = 0.79951E+09, u = 1.2816, Probability = 0.900)

R.V.name	X-value	Std.Dev.from Mean	Sensitivity factor
HI	0.1002284E+04	0.022837	-0.016160
HO	0.9983461E+02	-0.016539	0.011703
TBI	0.1219798E+04	0.164981	-0.116744
TBO	0.3002849E+03	0.009498	-0.006721
K	0.4196188E+02	-0.009075	0.006422
CP	0.2540624E+04	0.002455	-0.001737
E	0.4345666E+03	0.177203	-0.125393
ALPHA	0.4885058E+01	0.177203	-0.125393
FIN THIC	0.9995463E+00	-0.004537	0.003211
FIN HEIG	0.5005728E+02	1.376655	-0.974153
RI	0.3967639E+02	-0.080904	0.057249
TUBE THI	0.4000457E+01	0.001144	-0.000809
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1711204E+00	0.065904	-0.046635
TENSILE	0.9500000E+03	0.000000	0.000000

Level 9: (Z-value = 0.96523E+09, u = 1.6449, Probability = 0.95)

R.V.name	X-value	Std.Dev.from Mean	Sensitivity factor
HI	0.1002414E+04	0.024143	-0.013753
HO	0.9982510E+02	-0.017490	0.009963
TBI	0.1220935E+04	0.174456	-0.099377
TBO	0.3003013E+03	0.010043	-0.005721
K	0.4195899E+02	-0.009764	0.005562
CP	0.2540671E+04	0.002642	-0.001505
E	0.4351405E+03	0.190645	-0.108599
ALPHA	0.4891510E+01	0.190645	-0.108599
FIN THIC	0.9995201E+00	-0.004799	0.002734
FIN HEIG	0.5157473E+02	1.721531	-0.980654
RI	0.3962449E+02	-0.093879	0.053477
TUBE THI	0.4000509E+01	0.001273	-0.000725
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1712057E+00	0.070921	-0.040399
TENSILE	0.9500000E+03	0.000000	0.000000

Level 10 :(Z-value = 0.13577E+10, u = 2.3263, Probability = 0.99)

R.V.name	X-value	Std.Dev.from Mean	Sensitivity factor
HI	0.1002578E+04	0.025776	-0.010688
HO	0.9981321E+02	-0.018679	0.007745
TBI	0.1222357E+04	0.186306	-0.077248
TBO	0.3003218E+03	0.010725	-0.004447
K	0.4195520E+02	-0.010667	0.004423
CP	0.2540733E+04	0.002888	-0.001197
E	0.4358928E+03	0.208262	-0.086352
ALPHA	0.4899966E+01	0.208263	-0.086352
FIN THIC	0.9994874E+00	-0.005126	0.002125
FIN HEIG	0.5448213E+02	2.382302	-0.987777
RI	0.3955121E+02	-0.112198	0.046521
TUBE THI	0.4000576E+01	0.001440	-0.000597
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1713174E+00	0.077496	-0.032132
TENSILE	0.9500000E+03	0.000000	0.000000

Level 11: (Z-value = 0.19586E+10, u = 3.0903, Probability = 0.9990)

R.V.name	X-value	Std.Dev.from Mean	Sensitivity factor
HI	0.1002696E+04	0.026961	-0.008427
HO	0.9980458E+02	-0.019542	0.006108
TBI	0.1223389E+04	0.194909	-0.060922
TBO	0.3003366E+03	0.011220	-0.003507
K	0.4195233E+02	-0.011350	0.003548
CP	0.2540781E+04	0.003074	-0.000961
E	0.4364620E+03	0.221593	-0.069263
ALPHA	0.4906365E+01	0.221593	-0.069263
FIN THIC	0.9994637E+00	-0.005363	0.001676
FIN HEIG	0.5796586E+02	3.174059	-0.992108
RI	0.3948598E+02	-0.128504	0.040166
TUBE THI	0.4000629E+01	0.001573	-0.000492
DENSITY	0.3210000E+04	0.000000	0.000000
POISSON	0.1714021E+00	0.082474	-0.025779
TENSILE	0.9500000E+03	0.000000	0.000000