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Comparison of Methods for Developing Estimated Parameter $\bar{X}$ Control Charts Propsed by Nedumaran & Pignatiello, Albers & Kallenberg and Tsai Et Al.

Ozlem Temiz
Cleveland State University

COMPARISION OF METHODS FOR DEVELOPING ESTIMATED PARAMETER

$\overline{X}$ CONTROL CHARTS

PROPOSED BY

NEDUMARAN & PIGNATIELLO, ALBERS & KALLENBERG and TSAI ET AL.

OZLEM TEMIZ

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University Of Gaziantep, Turkey

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COMPARISION OF THREE METHODS FOR ESTIMATED PARAMETER $\overline{X}$

CONTROL CHARTS

COMPARISION OF METHODS FOR DEVELOPING ESTIMATED PARAMETER $\overline{X}$ CONTROL CHARTS PROPOSED BY NEDUMARAN & PIGNATIELLO, ALBERS & KALLENBERG and TSAI ET AL.

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ABSTRACT

The subject of this thesis is the comparison of the development method used to determine the value of $\overline{X}$ control chart limits when the underlying process parameters are unknown and must be estimated from data obtained from a Phase I “training” sample.

Historically it was accepted that estimates of process parameters using a training sample of 20-30 subgroups produced chart limits that were essentially as good as those that would be obtained using the actual distribution parameters themselves.

More recently Quesenberry has shown that control limits obtained from samples of this size produce SPC procedures with Run Length (RL) distributions significantly worse than would be expected. A number of articles (Nedumaran & Pignatiello(2001), Tsai Et al (August 2005) and Albers & Kallenberg (December 2000,December 2003)) have since appeared, each proposing a different method of calculating chart control limits for Shewart $\overline{X}$ charts that will produce desirable in-control RL characteristics while minimizing training sample size. The out-of-control performance for the above plans, however, was only addressed in one of the articles. In addition, the different authors employed differing performance measures. Among these are percentiles of the conditional RL distribution, percentiles of marginal RL distribution, and exceedance

probabilities. Because of these differences, Jensen et al. (2006) has suggested the comparison of these methods as an area of research.

I propose in the research for this thesis to compare these three proposed methods in detail comparing their performance by developing empirical probability of signal distributions for both in-control and out-of-control situations. Generation of these distributions will be accomplished through discrete simulation. The final result will consist of recommendations concerning the best of the methods to use in individual environments.

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CHAPTER I

INTRODUCTION

The $\overline{X}$ control chart, also referred to as Shewhart control chart, is a graphical tool to monitor the activity of an ongoing process.

When the underlying process parameters are known, it is easy to set up the control limits of an $\overline{X}$ control chart, the center line (CL) is set at μ, and the upper control limit (UCL) and the lower control limit (LCL) are set at;

$$UCL = \mu + k\,\sigma_{\bar{x}}$$

$$LCL = \mu - k\,\sigma_{\bar{x}}$$

$\sigma_{\bar{x}} = \sigma/\sqrt{n}$,
Generally k=3-10

However, in practice, generally the parameters μ and σ of the underlying process are unknown. In this situation, a control chart is often developed in a two phase procedure, in which the phases are known as Phase I and Phase II. In phase I, the parameters μ and σ are estimated from in-control historical reference samples and the results are used to estimate the control limits in phase II.

In Phase I, data from m initial subgroups of size n are collected and the mean of each subgroup is calculated then the grand average of those is used to estimate the process mean ($\hat{\mu} = \bar{\bar{X}}$) and the average of the subgroup sample standard deviations is used to estimate sigma ($\hat{s}$ = S)

Then,

$UCL = \bar{\bar{X}} + 3\hat{s}_{\bar{x}}$

$LCL = \bar{\bar{X}} - 3\hat{s}_{\bar{x}}$

$\hat{s}_{\bar{x}}$ = Sbar/ (c4√n), Where c4 is a constant that depends only on subgroup size n. Values of c4 can be

In Phase II, subgroups from the new data are collected periodically and the resulting $\bar{X}$ is plotted on a $\bar{X}$ control chart constructed in Phase I. As long as the points plot within the control limits, the process is assumed to be in-control, and no action is necessary. If points fall outside of the control limits, the process is assumed to be out-of-control requiring corrective action.

The effects of the number and sizes of subgroups in determining the $\bar{X}$ control chart limits have been investigated. Early studies presented by Hillier-1964, Yang/Hillier-1970, and Montgomery-1996. proposed a classical formula for Phase I calculations that requires 20-30 data subgroups of size 5 or greater. Quesenberry later showed that if the parameters μ and σ are estimated from such a small number of subgroups, there might be unexpected and undesirable effects in phase II. Control chart performance in phase II relies on the assumptions that are made in Phase I.

Quesenberry used simulation to study the performance of the $\bar{X}$ control charts developed using estimated parameters for several values of m number of subgroups of sample size n=5. His goal was find the minimum m for which such charts would perform as well as one developed with “true limits’’ (known parameters case). Quesenberry showed that m should be at least 100 when n=5 to accomplish that goal. He suggested that for other values of n, m should be at least 400/ (n-1) based on the speculation that the minimum degrees of freedom of the variance estimator should always be the same.

The problem with using the classical two phase approach is that the process is not being monitored during phase I. To minimize this problem, many approaches have been proposed including those of Nedumaran & Pignatiello (2001), Tsai et al. (2004, 2005) and Albers/Kallenberg (2004a, 2004b, 2004c). Descriptions of these authors' methods can be found in the following sections of the Appendix:

Method proposed by Nedumaran & Pignatiello - See appendix A-1

Method proposed by Albers and Kallenberg - See appendix A-2

Methods proposed by Tsai et al. - See appendix A-3

By employing modified calculation schemes for Phase I, all three methods attempt to shorten this phase allowing earlier monitoring of the process in question. Each author used Monte Carlo simulation to study his proposed method employing performance measure or measures to allow him to compare his results to those that would be expected from "known parameter" developed charts.

Jensen et al. (2006) pointed out that there has not been a detailed comparison of the three methods to determine which one is better under different circumstances. We have used Monte Carlo simulation to compare these methods under different conditions based on comparison of the resulting probability of signal distributions.

1.1 Construction of Xbar Charts Via Estimators

In order to construct an $\overline{X}$ control chart when the parameters μ and σ are unknown, common practice is to estimate them using data from Phase I reference samples once this done and the process is determined to be in-control, control limits are calculated for use in phase II.

When the process parameters are unknown, we have used the following equations to calculate control limits.

$$\widehat{UCL} = \bar{\bar{X}} + 3\frac{\hat{\sigma}}{\sqrt{n}}, \quad \widehat{CL} = \bar{\bar{X}}, \quad \widehat{LCL} = \bar{\bar{X}} - 3\frac{\hat{\sigma}}{\sqrt{n}},$$

$$\text{where } \overline{\overline{X}} = \frac{1}{m}\sum_{i=1}^{m}\overline{X}_i = \frac{1}{m}\sum_{i=1}^{m}\sum_{j=1}^{n}X_{ij},$$

$\bar{\bar{X}}$, the average of the subgroup means is an approximately normally distributed unbiased estimator of the parameter μ. (Central Limit Theorem). The estimate of sigma is calculated as $s' = \bar{S}/c4$

where c4 is a function of the sample size n and

$$\bar{S} = 1/m(S_1+S_2+\ldots.+S_m)$$

1.2. Evaluation Criteria Used in Literature

What to use as the best methods of evaluation of control chart performance is a matter of frequent discussion in literature. Run length distributions are often offered as a candidate. The run length (RL) of a control chart is a random variable that represents the number of plotted statistics until a signal occurs.

In the literature, proponents of the various methods generally fall into three groups. These are known as conditional RL distribution, marginal RL distributions and both conditional and marginal RL distributions.

The conditional distribution of RL is defined as a distribution which depends on the specific control limits developed in phase I. It is the probability mass function F (RLI $U\hat{C}L$ =ucl, $L\hat{C}L$ =lcl) where ucl and lcl are respectively the realized values of the random variables ($U\hat{C}L$ and $L\hat{C}L$) obtained during the phase I procedure. This distribution defines the RL probabilities of an individual chart, once $U\hat{C}L$ and $L\hat{C}L$ has been calculated. One would need the actual values of μ and σ to calculate the conditional distribution of a single chart. Jensen et al. (2006), however, pointed out that Jones et al (2001) give a method by which standardized values of $U\hat{C}L$ and $L\hat{C}L$ can be hypothesized and percentile points (e.g. 25 % and 75 %) of the conditional RL distribution can be found by calculation or estimation by simulation. The benefit of this method is that the control chart practitioner can look into both best and worst case performance for charts with a given methods. However, our main question is how we can use a RL distribution to compare $\overline{X}$ control chart development methods. A particular method may produce a chart having superior characteristics at a given RL percentile but an inferior one at another percentile making the comparison only partially useful.

The marginal RL distribution is that probability mass function obtained by averaging the conditional distribution over all possible values of $U\hat{C}L$ and $L\hat{C}L$. The main advantage of the marginal RL distribution from the conditional RL dist. is that the knowledge average of control chart performance does not require knowledge of the actual values of parameters σ and μ. Furthermore, Marginal analysis allows calculating or estimating the performance measures for an average control chart developed using a particular method. Although the practitioner will never have an average chart, the marginal distribution approach allows a common basis to compare the result obtained by the various methods of developing control charts.

When parameters are known RL is a geometrically distributed random variable. This is also the case for the RL of any single chart developed using estimated parameters i. e., when the RL distribution being considered is the conditional one. In both of these cases, there is a known fixed relationship between the average RL (ARL) and the standard deviation of the RL (SDRL) which can be expressed as

$$SDRL= (ARL\ (ARL-1))^{1/2}$$

If the parameters are estimated, the marginal RL distribution, however, is not geometric and thus the probability of a signal (1/ARL) does not have a meaningful interpretation. In this situation, RL and its measures must be interpreted carefully. The main measure performance for an RL distribution is the Average Run length (ARL) which is defined as an expected value of the random variable that indicates the sample number on which the first (false) out-of-control point appears for a process that is operating-in-control. ARL is the average over a large number of charts of single false alarm per chart, the first one that the chart produces. This indicates that a practitioner can expect to obtain a signal, on average, once in every 370 (in the known parameter case with 3 sigma limits) plotted statistics in-control situations when known parameters case. For an efficient control chart, one would like to have the in-control ARL to be large and the out-of-control A RL to be small.

In the literature, ARL is used as the most important performance measure of control chart. Since ARL is not geometric with estimated parameters in order to measure performance of control charts, it is recommended to use ARL with standard deviation of run length (SDRL) if we want to use marginal RL distribution with estimated parameters.

CHAPTER II

TASKS METHODS AND ASSUMPTIONS USED

2.1. Summary of Tasks

The thesis research described herein consists of the following:

1) Determination of an appropriate measure for comparing the relative merits of three literature proposed methods for developing a $\overline{X}$ control chart.

2) Running of series of Monte Carlo simulation studies to estimate the values of the selected measures and presentation of the comparative results;

3) Recommendations concerning the best of the three methods to employ under various situations.

2.2. Criteria Used for Comparing Methods

One of the major concerns in literature is the selection of appropriate measures to evaluate the performance of an $\overline{X}$ control chart. For the work in this thesis the measure of merit described below has been used. If the mean and variance of the in-control distribution for the quality characteristic of interest are known, they are used to calculate LCL and UCL. In that case, assuming normality, the probability of a signal, i.e., an alarm, may be found by

$$p = 1 - [\Phi((UCL - m)/s) - \Phi((LCL - m)/s)]$$

Where Φ is the cumulative standardized normal distribution. In the case of an in-control process, p represents the type I error α. For an out-of-control, process p represents the power of the test.

In this thesis, we have run simulations creating charts from sample data estimating σ and μ. Each time we create a new chart in Phase I using different data from the same process we produce different values for LCL and UCL. They become random variables producing a different probability of alarm p when inserted in the above equation, i.e., p is a random variable with its own distribution. Attempts to fit different distribution forms have shown that lognormal provides an excellent fit to our empirical data. In what follows, we used the parameters and plotted cdf's of these distributions to compare the merits of alternatives.

2.3. Control Chart Development and Use Procedures

When the parameters μ and σ of the underlying process are unknown, some assumptions are made to construct an Xbar control chart whose performance is close to one developed with known parameters. As mentioned previously, the development usually is done in two phases designated Phase I and Phase II. Control limits are calculated using parameter estimates from an in-control Phase I historical sample. In Phase II statistics based on new samples are compared with these limits monitoring to detect out-of-control situations.

When a process engineer wants to apply the classical two-phase procedure to the development of the estimated parameter $\overline{X}$ control charts, proposed by Nedumaran & Pignatiello, Albers & Kallenberg and Tsai et al., he typically uses a similar approach.

Let n_q ($n_q = 400 / (ns - 1)$) be the Quesenberry recommended number of rational samples of size n_s needed to establish $\overline{X}$ control chart limits that will be (according to Quesenberry) similar in performance to limits calculated using known values of μ and σ. Our three authors have each recommended their own method for establishing control chart limits that could allow use of a control chart before $m < n_q$ samples and recalculation of those limits at Integer((nq-m)/k) intervals, k samples in length, thereafter. When n_q samples have been obtained one final calculation is made using all of the data gathered to that point establishing the Quesenberry limits which are used thereafter.

In our work we have divided Phase I into the Phase Ia, Phase Ib1, Phase Ib2, Phase Ib3 etc. From sample data, the sample average and sample standard deviation are calculated as estimates of the mean and standard deviation of the process respectively. $\overline{X}$ chart control limits are said herein to be developed using the "standard" method when they are calculated as $\overline{\overline{X}}$ +/- 3*S/(c4*n_s^.5) where S is the "pooled" value of the sample standard deviations and where $\overline{\overline{X}}$ is the grand average of m samples of size n_s. At the end of Phase Ia, the control limits of the $\overline{X}$ control chart are calculated. The Phase I data used to generate the limits is then retrospectively checked against them. When a chart showing the process to be in-control is found, process monitoring begins and continues until either an out-of-control alarm is generated or k new subgroups have been checked. We designate this portion of the procedure as Phase Ib-1. If no alarm has occurred by the end of Phase Ib-1 we enter Phase 1b-2 which continues until either an alarm is generated, or k additional samples have been processed, or the total number of samples including those in Phase I-a has exceeded the

Quesenberry requirement of $m > 400/(n_s\text{-}1)$. In general, if the end of Phase I-bx is reached and the Quesenberry criterion has not, Phase-1b(x+1) is started. At the end of each Phase I sub phase new calculations of the chart control limits are made using the total number of Phase I-a and b) subgroups processed up to that point. When the total number of subgroups processed within Phase I reaches the Quesenberry criterion, Phase II begins. For example, if we start with m = 70, n = 5 and k = 10, seventy subgroups of size 5 are processed in Phase Ia before parameter estimates are made and control limits are first calculated. At that point there remains 400/(5-1)-70 = 30 subgroups to be processed before the Quesenberry criterion is satisfied. This means, barring an alarm, three Phase 1b sub phases each using 10 subgroups will be employed.

CHAPTER III
RESEARCH QUESTIONS

A control chart designer's goal is to economically and effectively monitor an ongoing process identifying unusual process performance. To do this a control chart must be able to distinguish between situations in which the process is operating as expected and when it is not operating as expected. To be effective a control chart needs to be usable as early as possible. Charts requiring fewer Phase Ia subgroups (small values of m) are more effective in this regard. It needs also to be able to detect relatively small significant deviations in the process be monitored. Charts with higher power are more effective in this regard. These requirements give rise to the following three research questions.

The first question is: "For each author's method and possible values of n_s what are the minimum values of m used to develop the initial control limit calculations that will produce results similar to charts created with the standard method and $m = n_q$?" For this work similarity means that the probability distributions of the generated charts' Type I errors are similar to that of charts generated with the standard method and $m = n_q$.

Given the answer to question 1, a second question concerns the power of charts developed using the different authors' methods. To enable monitoring earlier than $m = n_q$

the width of control limits employed by the different authors may be larger (larger values of UCL – LCL) than those of Quesenberry. For this reason the probability of detecting an out-of-control situation on a particular check (i.e. the power of the test) may be reduced. A second research question then is “which of the methods produces the highest out-of-control probabilities of alarm (power} when developed using its particular value of minimum m found as an answer to the first question?”

The third research question is “How many control limit recalculations should be made before reaching n_q or, in other words, what is the optimum value of k? The parameter k at maximum equals n_q – m. Minimum k equals 1.” One consideration is that recalculating the limits should allow tighter limits with (higher power) with each recalculation while still maintaining the same type I error. This would be true since there is less uncertainty as the total number of data used for the control limit calculations increases. These tighter limits may have the effect of increasing the power of the test. There is also the possibility, however, that a process that goes out of control shortly after the first m subgroups might go undetected at first. In such a case recalculating the limits before n_s might cause the control limits to “adapt” to the out-of-control process increasing the time it takes to detect the problem. Another issue is that the smaller the value of k the more effort recalculating control limits required to establish the final control chart. . The third question, therefore, is “What net effect does the value of k have on the power of the control scheme?”

CHAPTER IV

SIMULATION METHOD AND DETAILS

We used Monte Carlo simulation to compare our three authors' methods under different conditions.

An Arena simulation model has been constructed. This model has four control variables, i.e., n_s, m, k, delta_mu. Simulation begins by generating m rational samples of size n_s from an in-control distribution N(0,1) and calculating from them an initial set of control limits for each of the three methods using the algorithms supplied by each author. (This completed Arena model is showed in appendix B)

The supplying distribution is then changed to one that is out-of-control, i.e., N(delta_mu,1) (called hereafter the "second distribution"). delta_mu represents a shift in mean expressed in multiples of sigma. If a simulation is being run to examine type I error rate delta_mu is set to zero. If the run is to evaluate the power of the test, delta_mu is chosen to be non-zero.

The initially calculated limits are then compared to the second distribution and the probability of an alarm is calculated for each of the three methods. k additional rational subgroups are then generated after which the control limits are recalculated using all of the data generated up to the last point. These new limits are then compared to the second distribution and the probability of an alarm again calculated. The process of generating k new subgroups, recalculating the limits every Integer($(n_q$-m)/k) intervals, k subgroups in length, thereafter, and then calculating the probability of an alarm against the new limits is repeated until the generation of another k samples would cause the total number generated to exceed n_q. When this happens enough subgroups have been generated to bring the total number to n_q at which point the final limits are calculated which become the Quesenberry limits. The calculated values of the alarm probabilities for each author are then averaged over the run to calculate the average probability of an alarm for that run for each of the three plans' operations.

For a fixed set of simulation control variable values, the above program simulates generating many charts with different control limits. These different control limits produce in turn different probabilities of an alarm when the same samples from a particular distribution are tested against them.

Because of the random nature of the control limits, the probability of an alarm is itself a random variable with its own distribution. If the actual σ and μ for a process were known exactly and used to calculate control limits, the probability of an alarm for a given delta_mu would be a single value. For the estimated parameter case as m becomes smaller, the variance of the probability of alarm distribution becomes larger. For an in-control situation, the increase in the upper tail represents an increase in type I error. For an out-of-control situation the larger lower tail represents a reduction in chart power. For this reason comparing the results of the various simulations requires comparing both the average and variance of the probability of alarm distributions

A log-normal distribution (showed below) has been found to provide a good fit for the probability of alarm data generated by these simulations. Observations of the fitted distribution behavior under changes in the controls have been used to generate answers and conclusions in regards to the above questions.

CHAPTER V

GENERATION AND EVALUATION OF RESULTS

5.1 Overview

In order to provide answers to the first question concerning minimum m, simulations were run for three common values of n_s, i.e., n_s=5, n_s=7 and n_s=10. delta_mu for these simulations was held at zero. That choice results in an in-control simulation with the resulting probability of alarm representing the type I error rate α. Runs were made using various values for. The probability of an alarm was recorded for each method and each run. The methods simulated were that of Tsai, Nedumaran &Pignatiellio and Albers &Kallenberg. For reference purposes, simulations were also run with m=n_q representing the distribution of probability of alarm for the standard method with Quesenberry's recommendation for m (labeled Q standard). For each set of control values1000 replications were made.

5.2. Results from Question 1 Simulations

The simulation results for n=5, n=7 and n=10 and various values of m are shown below in Figure1, 2, 3,4,5- Table I; Figure 6, 7,8,9,10- Table II; Figure 11, 12,13, 14, 15- table III respectively. For these runs The simulations were run with no interim recalculation of limits using n_q-m as the value for k in all cases.

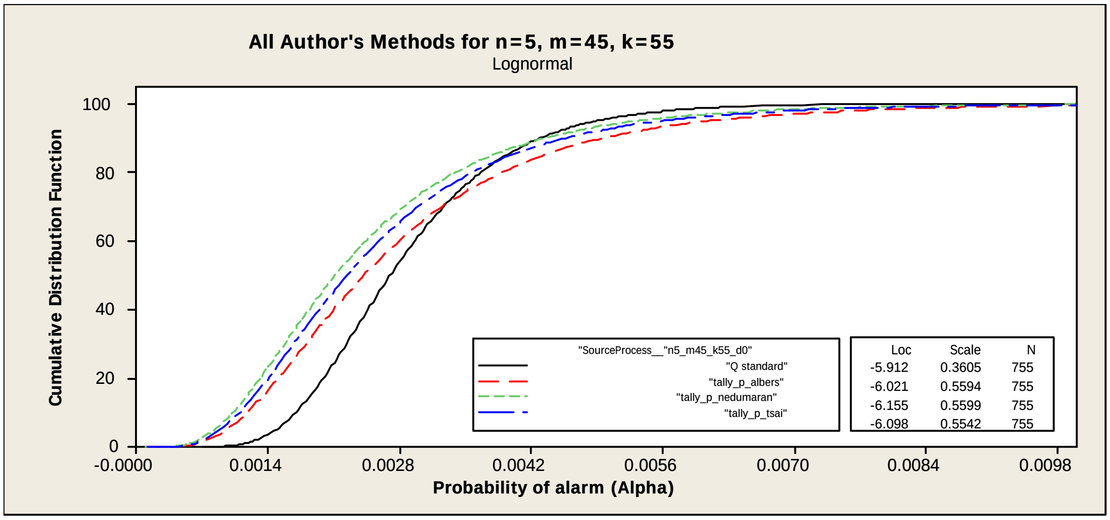


Figure 1

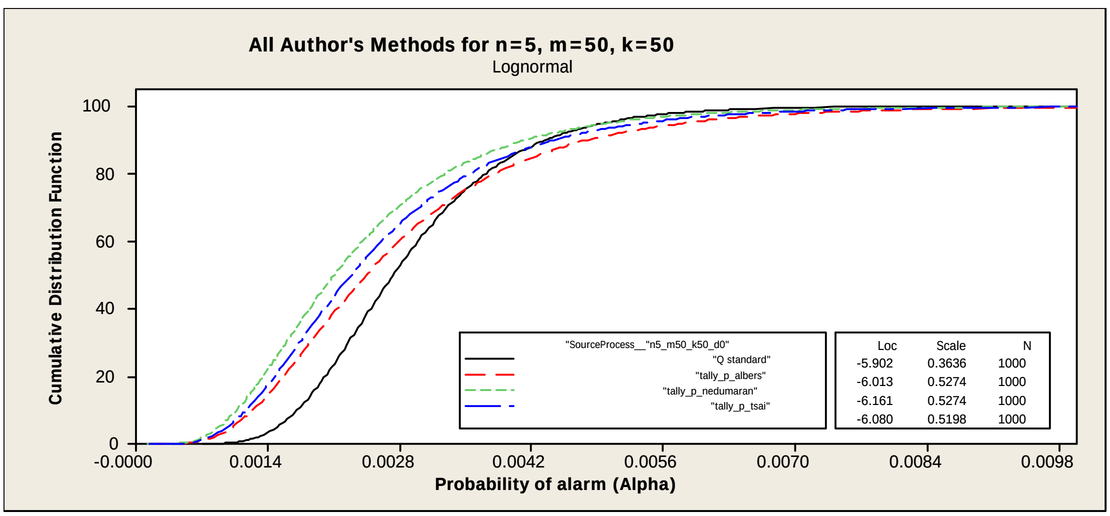


Figure 2

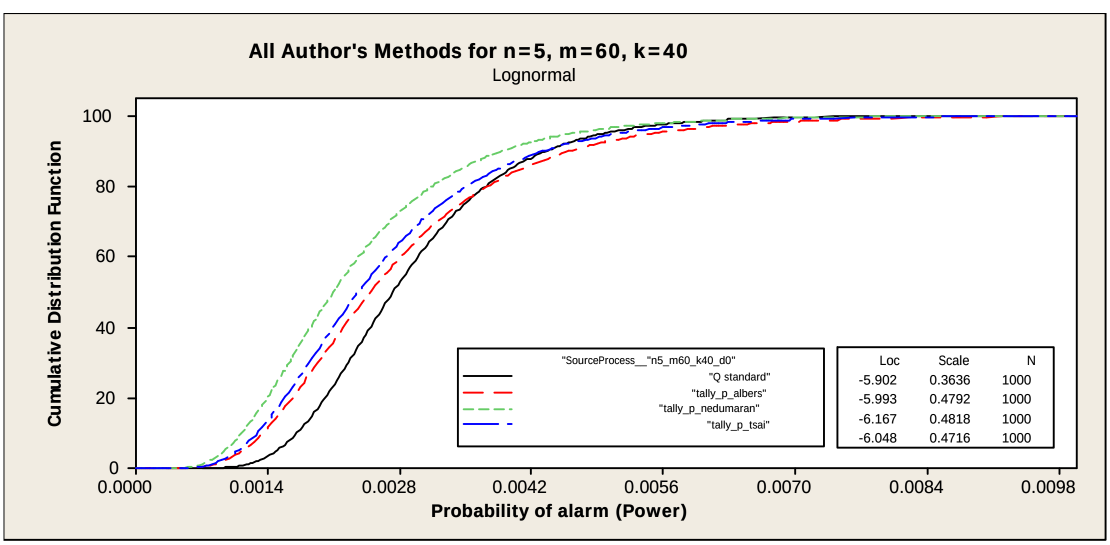


Figure 3

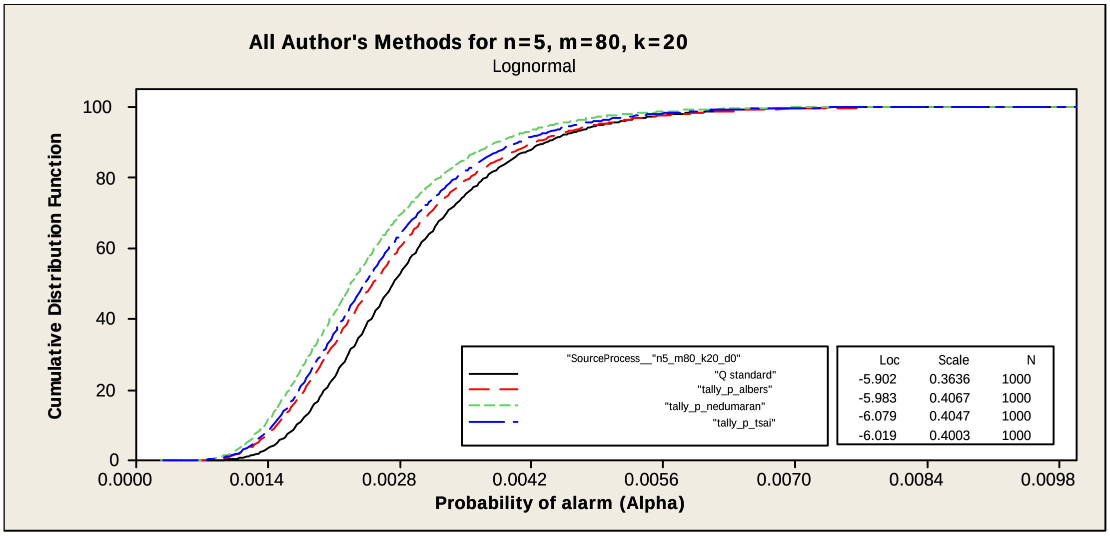


Figure 4

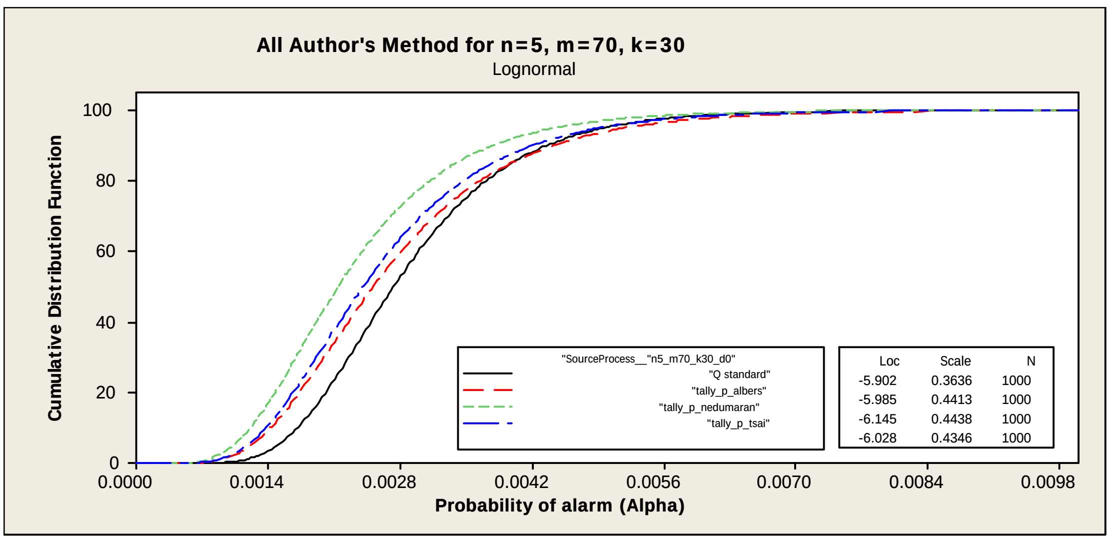


Figure 5

	Authors	m=45	m=50	m=60	m=70	m=80
Avg. Probability of alarm n=5	Q standard	0.0029	0.0029	0.0029	0.0029	0.0029
	Albers & Kallenberg	0.0028	0.0028	0.0028	0.0028	0.0027
	Nedumaran & Pignatiello	0.0025	0.0024	0.0024	0.0024	0.0025
	Tsai Et al.	0.0026	0.0026	0.0026	0.0026	0.0026
Standard deviation n=5	Q standard	0.0011	0.0011	0.0011	0.0011	0.0011
	Albers & Kallenberg	0.0017	0.0016	0.0014	0.0013	0.0012
	Nedumaran & Pignatiello	0.0015	0.0014	0.0012	0.0011	0.0010
	Tsai Et al.	0.0016	0.0015	0.0013	0.0012	0.0011

Table I

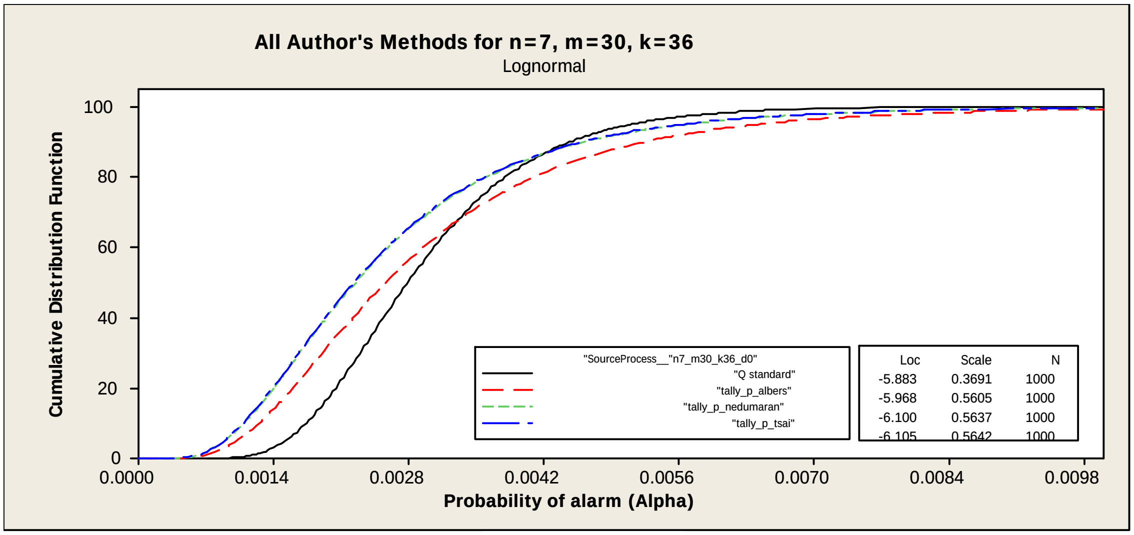


Figure 6

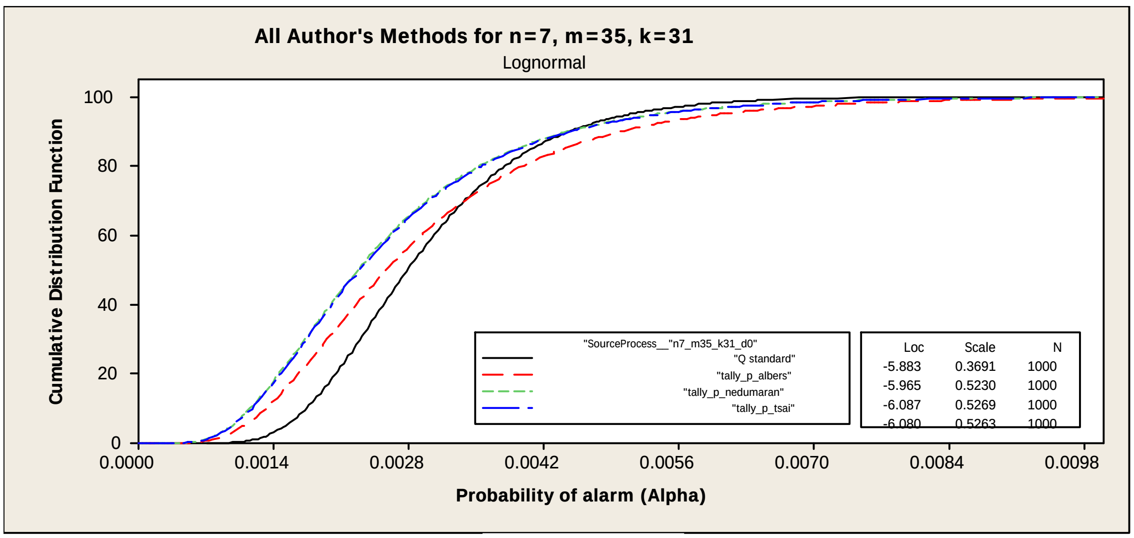


Figure 7

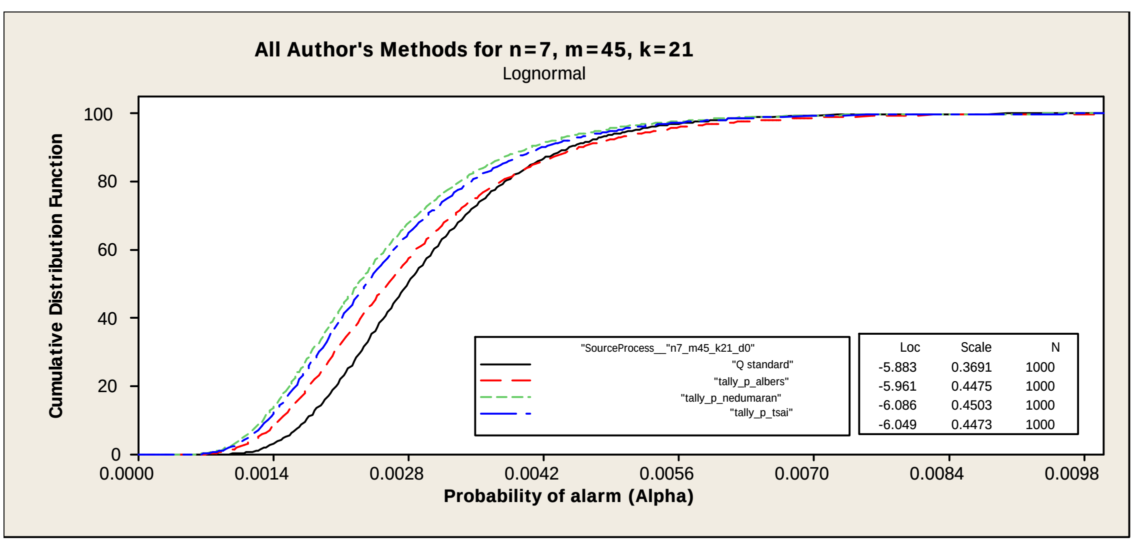


Figure 8

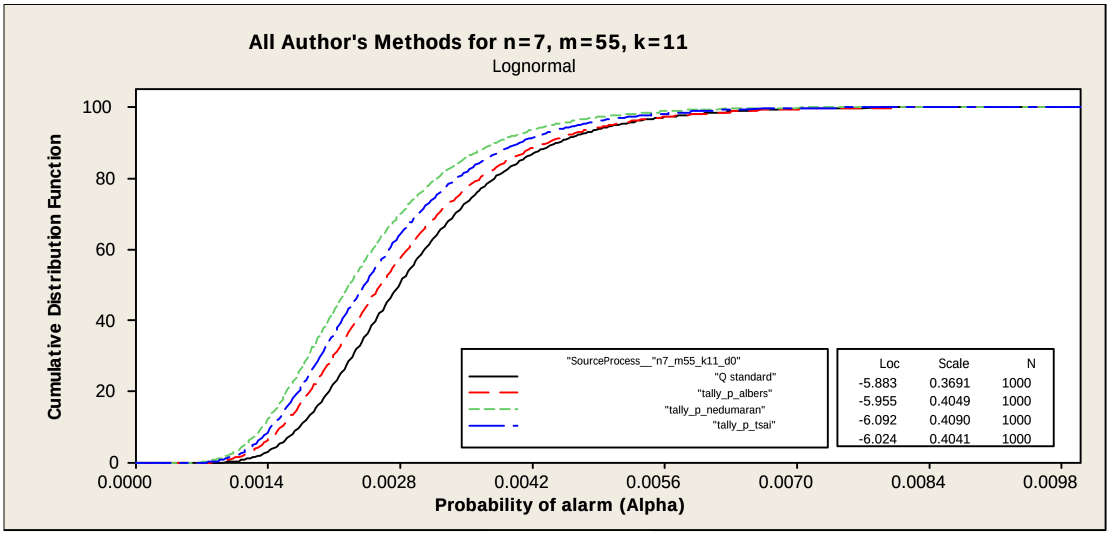


Figure 9

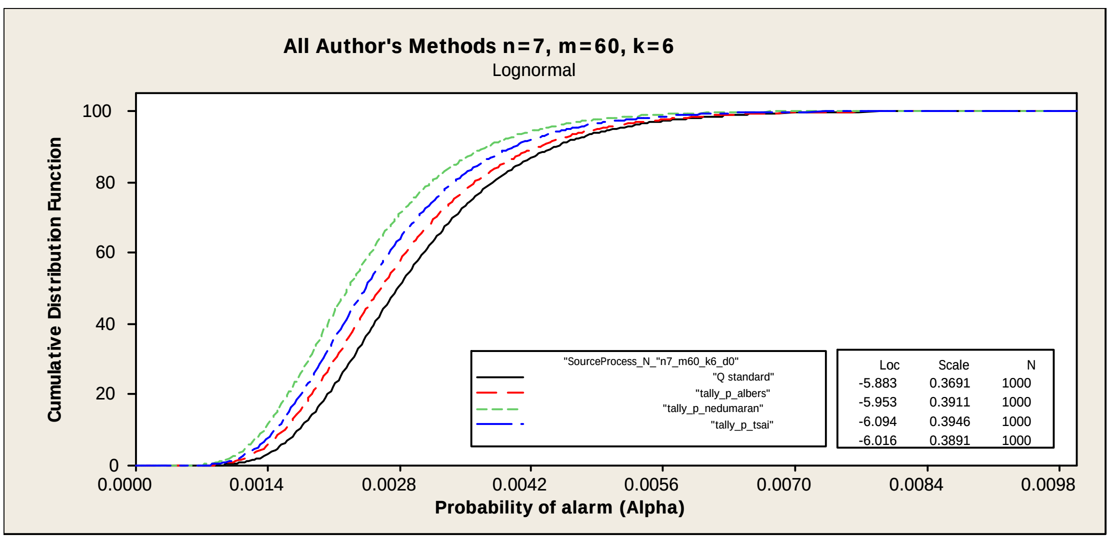


Figure 10

	Authors	m=30	m=35	m=45	m=55	m=60
Avg. Probability of signal n=7	Q standard	0.0030	0.0030	0.0030	0.0030	0.0030
	Albers & Kallenberg	0.0030	0.0029	0.0028	0.0028	0.0028
	Nedumaran & Pignatiellio	0.0026	0.0026	0.0025	0.0025	0.0024
	Tsai Et al.	0.0026	0.0026	0.0026	0.0026	0.0026
Standard deviation n=7	Q standard	0.0011	0.0011	0.0011	0.0011	0.0011
	Albers & Kallenberg	0.0018	0.0017	0.0013	0.0012	0.0011
	Nedumaran & Pignatiellio	0.0016	0.0015	0.0012	0.0010	0.0010
	Tsai Et al.	0.0016	0.0015	0.0012	0.0011	0.0011

Table II

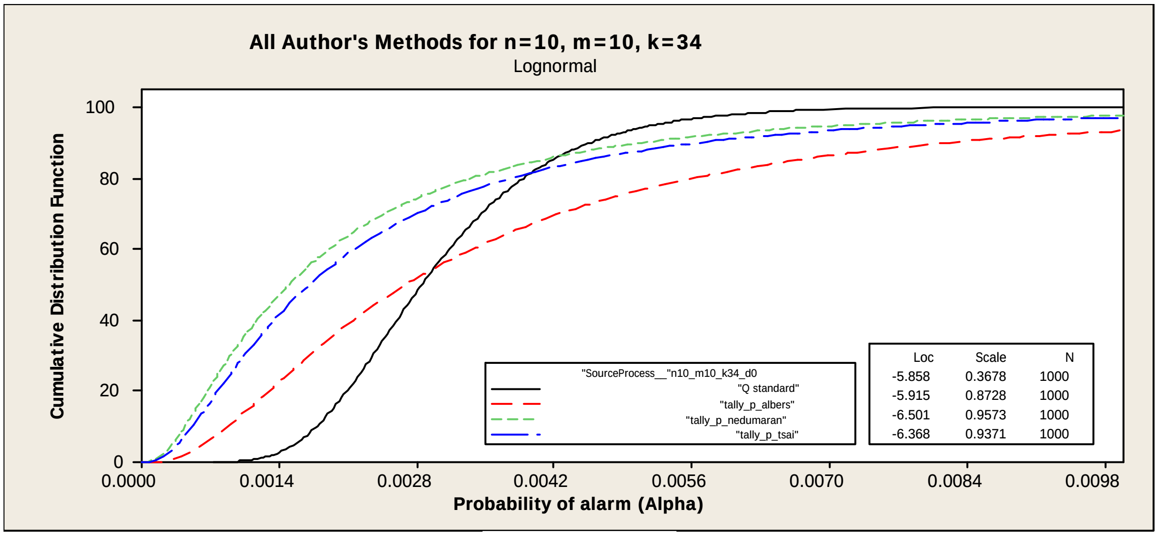


Figure 11

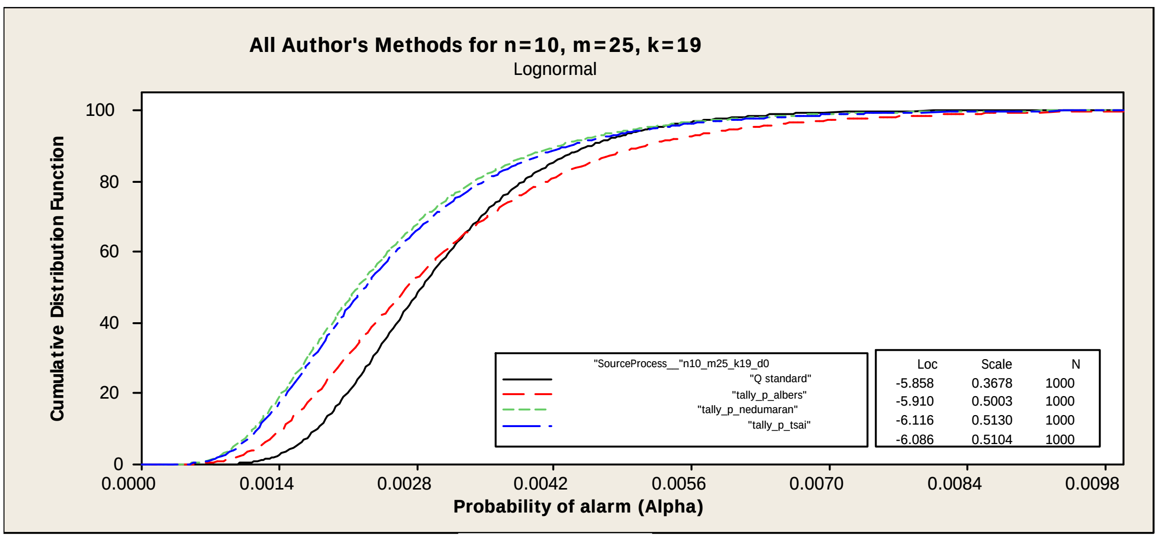


Figure 12

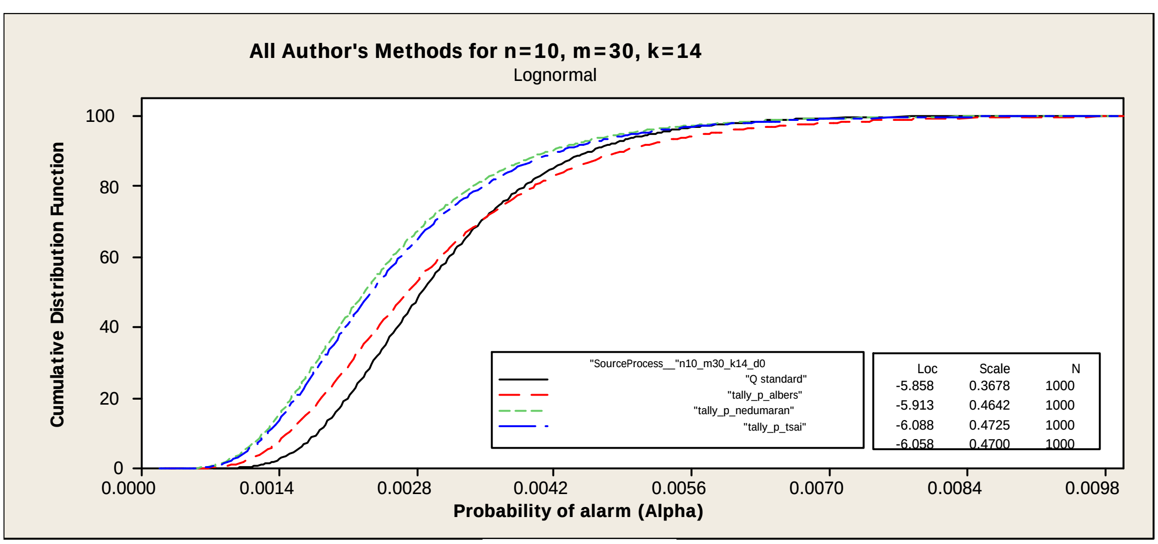


Figure 13

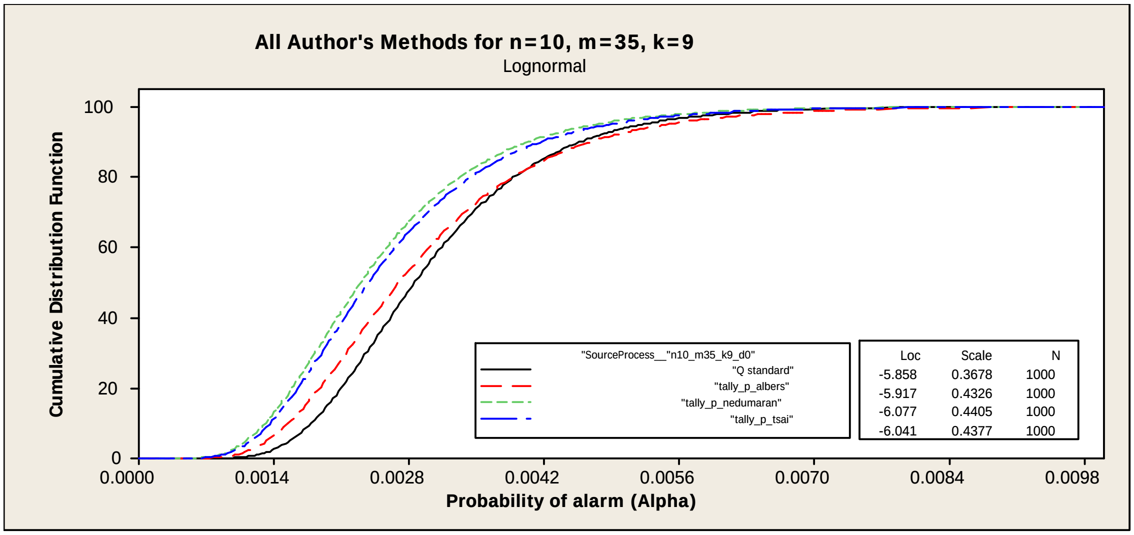


Figure 14

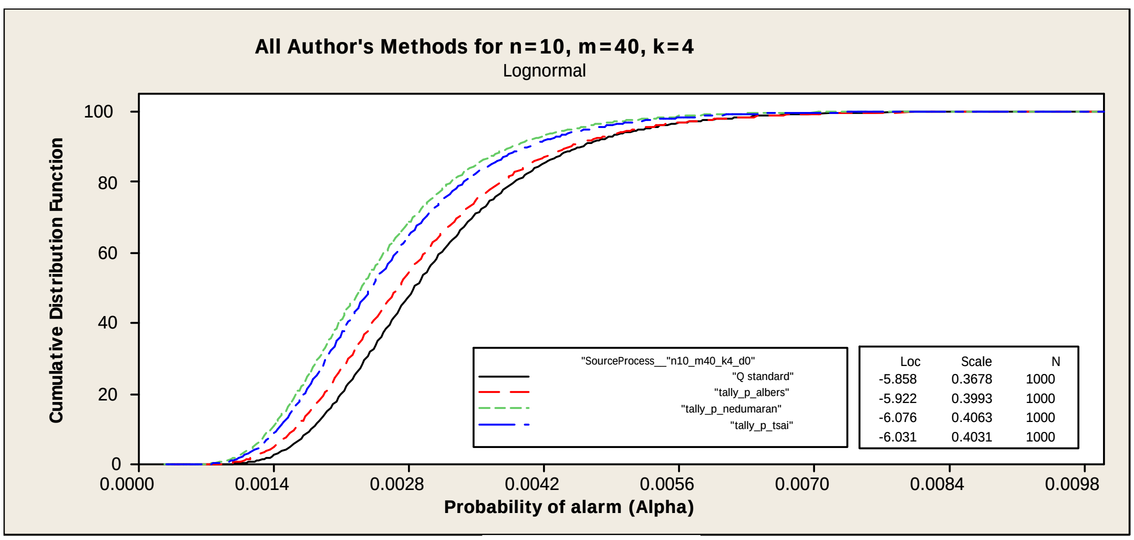


Figure 15

	Authors	m=10	m=25	m=30	m=35	m=40
Avg. probability of alarm n=10	Q standard	0.0031	0.0031	0.0031	0.0031	0.0031
	Albers & Kallenberg	0.0039	0.0031	0.0029	0.0030	0.0029
	Nedumaran & Pignatiellio	0.0024	0.0025	0.0025	0.0025	0.0025
	Tsai Et al.	0.0027	0.0026	0.0026	0.0026	0.0026
Standard deviation n=10	Q standard	0.0012	0.0012	0.0012	0.0012	0.0012
	Albers & Kallenberg	0.0042	0.0016	0.0012	0.0013	0.0012
	Nedumaran & Pignatiellio	0.0029	0.0014	0.0011	0.0012	0.0011
	Tsai Et al.	0.0032	0.0014	0.0011	0.0012	0.0011

Table III

Figures 1, 6, and 11 show a plots of the fitted cumulative distribution functions made with a relatively small value of m for methods except the Q standard method which is always run with m = $n_q = 400/(n_s-1)$. At this value of m it can be seen that all of the other methods are inferior to the Q standard method. This is true because they all exhibit larger variance which results a larger number of charts with higher in-control probability of alarm and therefore a higher Type I error rate . This can be seen visually by inspection of the cumulative distribution curves. To be equal or superior to the performance of the Q standard method the plot for another method would have to have all of its points on or above and to the left of those of the Q standard plot. This is not the case in Figures 1, 6, and 11. After the simulation runs that generated these charts, additional runs were made using increasing values of m and still holding k = n_q-m while observing the performance of the various methods. This process was continued for each value of n_s until the plots for all methods equaled or exceed the performance of the Q standard method. All of the simulation results for question 1 can be found in Figures 1-15. The minimum value of m at which a given method's performance matches the Quesenberry method performance was then recorded in Table V as the minimum m for that method.

n	Minimum m		
	Albers & Kallenberg	Nedumaran & Pignatiello	Tsai et al
5	80	50	70
7	55	45	45
10	40	25	30

Table IV

These results were than ranked 1 -3 with the highest rank for a particular value of n_s given to the author whose method has the lowest minimum m. The ranks were than averaged over n_s to estimate the overall average rank for each author's method which is shown table V.

Evaluation Table for Question 1

n	Albers &Kallenberg	Nedumaran &Pignatiellio	Tsai Et al.
5	1	3	2
7	1	2.5	2.5
10	1	3	2
score	(1+1+1)/3=1	(3+2.5+3)/3=2.83	2+2.5+2=2.166

Table V

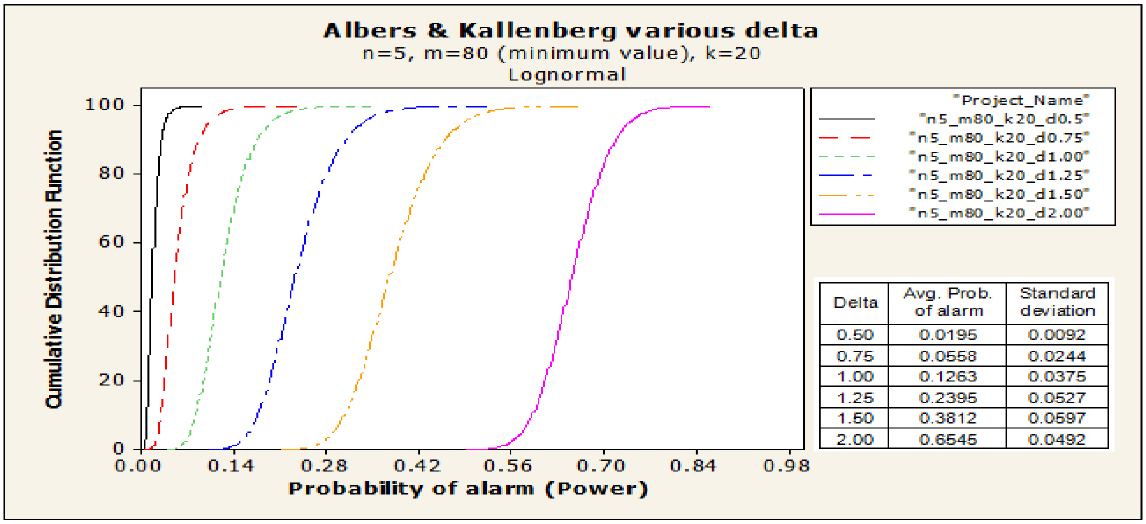
5.3 Results from Question 2 Simulations

To evaluate the power performance for each plan the following experimental design matrix was established. Table VI shows the simulation parameter (n_s, min m, k, delta_mu) values for each simulation study. As in question one simulations, no recalculation of limits was performed resulting in k = n_q – m. Again 1000 replications were run for each parameter combination. In addition, lognormal distributions were fitted to the data and cumulative distribution plots formed. The results of these runs are shown in Figures 16-24.

Simulation Parameters for Question 2

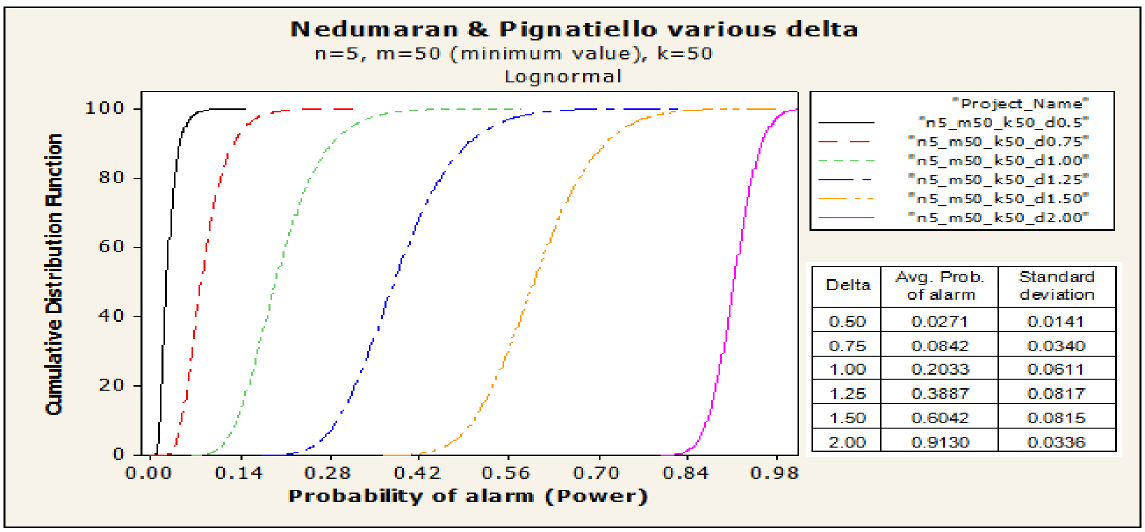
n	**ALBERS**		**NEDUMARAN**		**TSAI**		**DELTA**
	m	**k**	**m**	**k**	**m**	**k**	
5	80	20	50	50	70	30	**0.5, 0.75, 1.00, 1.25, 1.5, 2.00**
7	55	11	45	21	45	21	**0.5, 0.75, 1.00, 1.25, 1.5, 2.00**
10	40	4	25	19	30	14	**0.5, 0.75, 1.00, 1.25, 1.5, 2.00**

Table VI



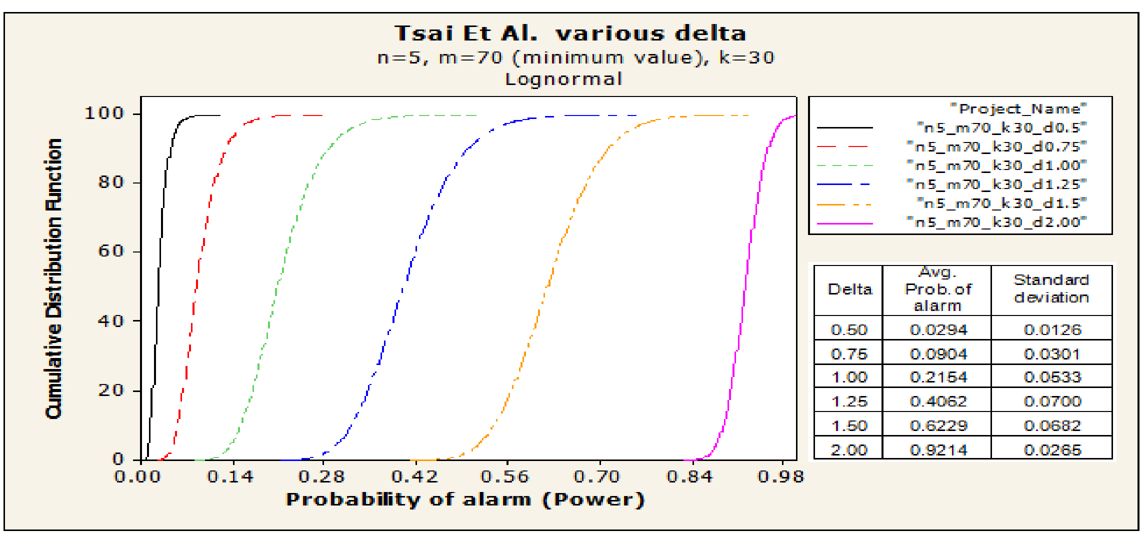
Delta	Avg. Prob. of alarm	Standard deviation
0.50	0.0195	0.0092
0.75	0.0558	0.0244
1.00	0.1263	0.0375
1.25	0.2395	0.0527
1.50	0.3812	0.0597
2.00	0.6545	0.0492

Figure 16



Delta	Avg. Prob. of alarm	Standard deviation
0.50	0.0271	0.0141
0.75	0.0842	0.0340
1.00	0.2033	0.0611
1.25	0.3887	0.0817
1.50	0.6042	0.0815
2.00	0.9130	0.0336

Figure 17



Delta	Avg. Prob. of alarm	Standard deviation
0.50	0.0294	0.0126
0.75	0.0904	0.0301
1.00	0.2154	0.0533
1.25	0.4062	0.0700
1.50	0.6229	0.0682
2.00	0.9214	0.0265

Figure 18

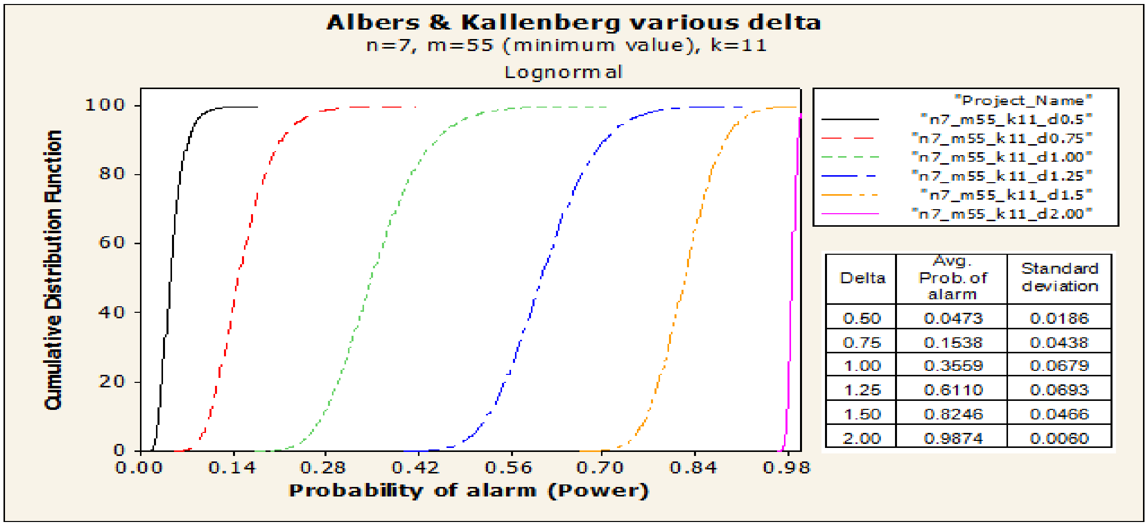


Figure 19

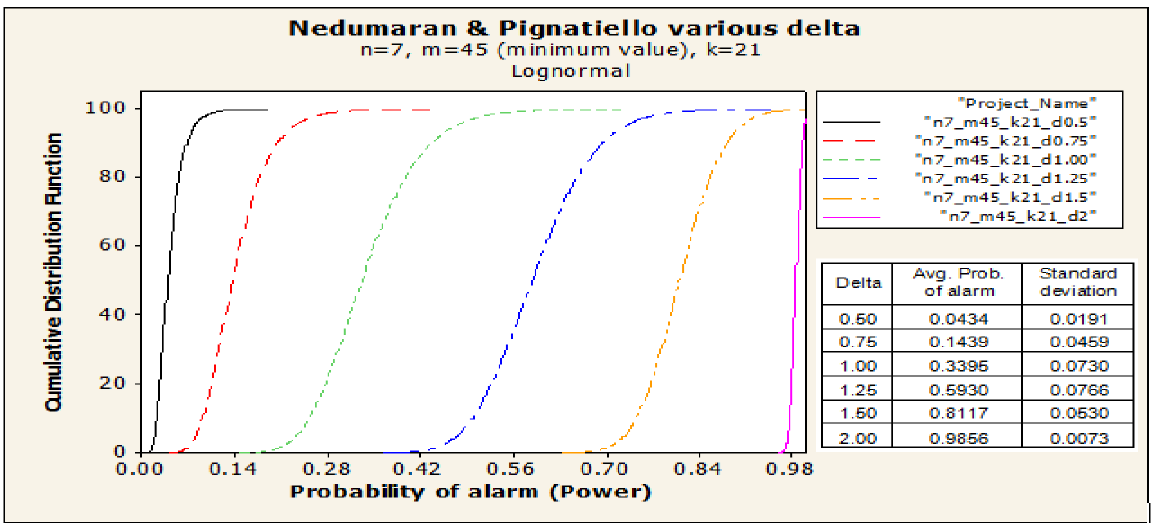


Figure 20

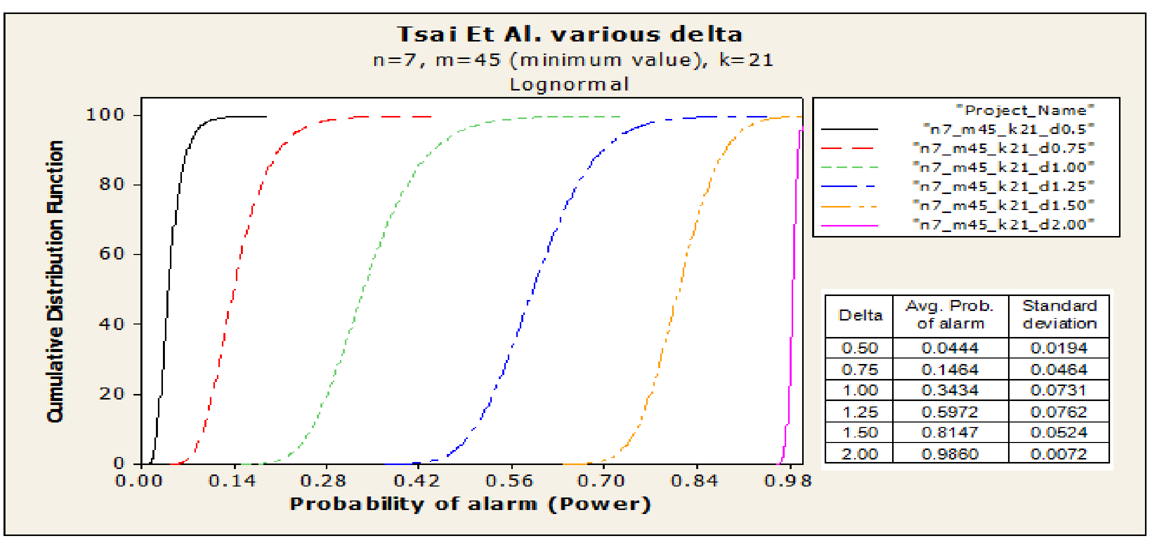


Figure 21

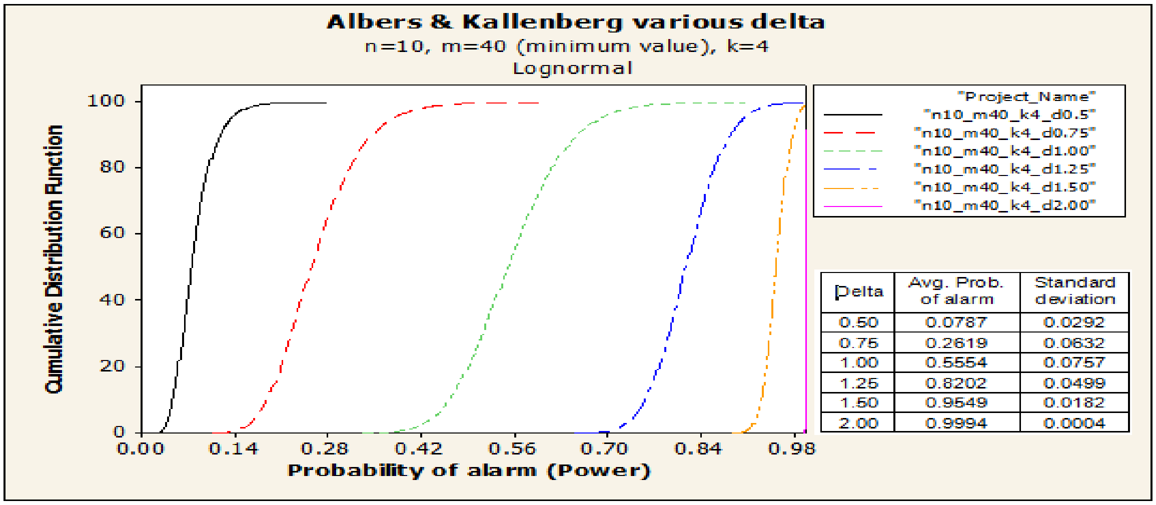


Figure 22

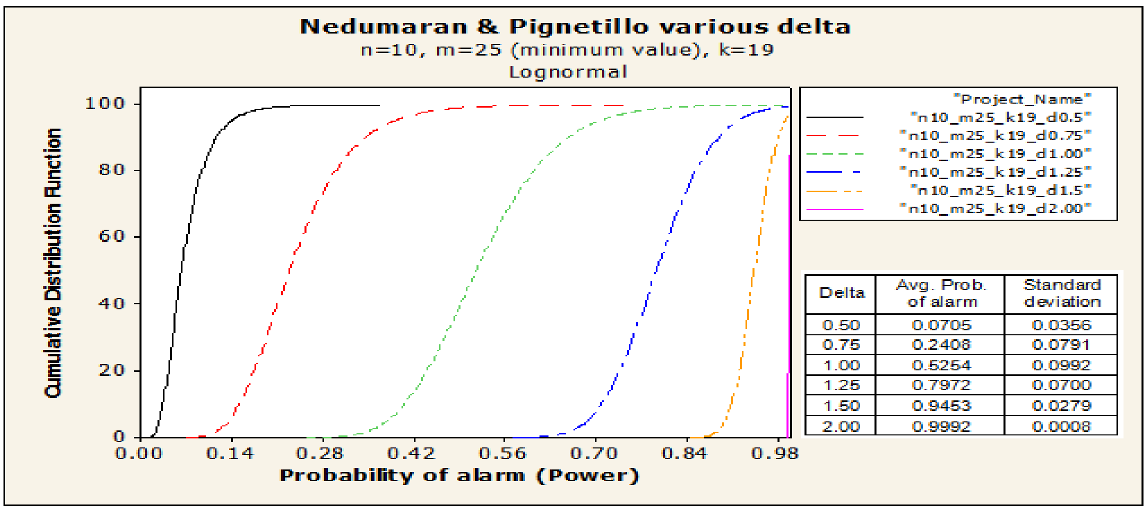


Figure 23

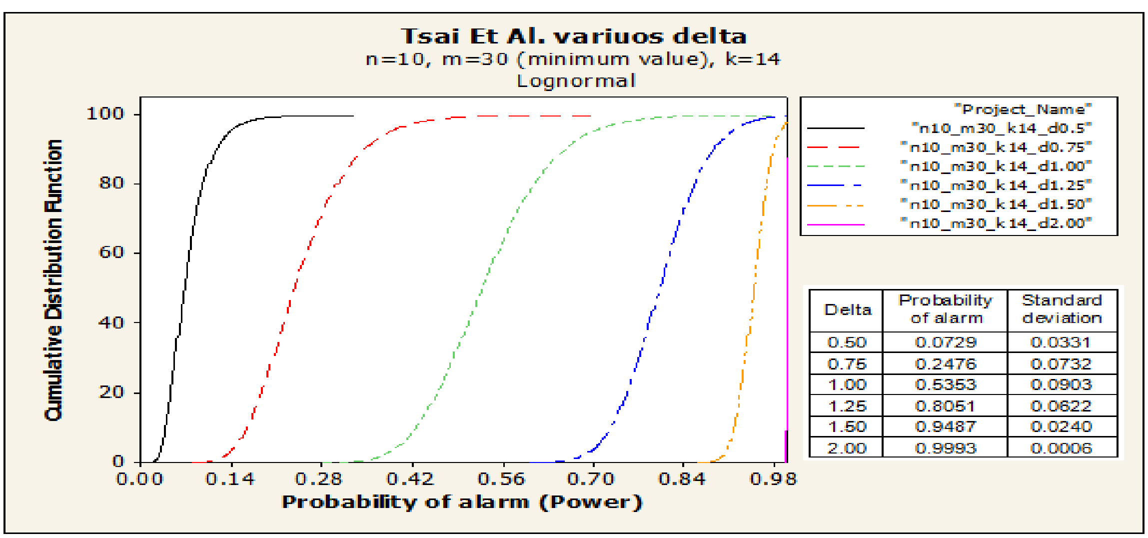


Figure 24

For a given value of ns, each of the above graphs shows a cumulative distribution functions for the probability of alarm for the indicated author's method. Each plot on a graph shows the cdf for a certain value of delta_mu. Of course, as would be expected, the larger the delta_mu (indicating the magnitude of the process out-of-control condition), the larger is the power. Comparing the plots for the three authors for a given value of ns one can see which is superior in regards to power. In an out-of-control process condition a high probability of signal/alarm is desirable. If an author's curve set, therefore, is to the right and under another author's, his method is producing superior results. Table VII displays the relative merits of the different methods in regards to question 2.

Evaluation Table for Question 2

n	Albers &Kallenberg	Nedumaran &Pignatiellio	Tsai Et al.
5	2	1	3
7	3	1	2
10	3	1	2
Average Score	(2+3+3)/3=2.666	(1+1+1)/3=1	3+2+2=2.33

Table VII

5.4 Results from Question 3 Simulations

All of the previous simulations were run with no recalculation of limits after the initial calculation after minimum m subgroups. This infers that there was only one portion of PhaseIb, i.e., Phase Ib-1 consisting of k = n_q-m subgroups. To address research question 3 a number of simulations have been run during which Phase Ib was divided into various numbers of subgroups with recalculation of control limits being done at the end of each one of the sub phases of Phase1b. Because a different value for minimum m was determined in the work on the previous questions, the value of n_q-m, the length of Phase 1b, is also different. Because of this the length of Phase 1b and number of sub phases run differ by author's method also. Table VIII shows the simulation parameter design matrix. Figures 25-33 and Tables IX-XVII show the results of the simulations.

Parameters for Question 3 Simulation Runs
(Table Entries are Values of k)

Number of sub phases	**TSAI**			**NEDUMARAN**			**ALBERS**			**delta_mu**
	n=5 m=70 n_q-m =30	n=7 m=35 n_q-m =31	n=10 m=30 n_q-m =14	n=5 m=50 n_q-m =50	n=7 m=45 n_q-m =21	n=10 m=25 n_q-m =19	n=5 m=80 n_q-m =20	n=7 m=55 n_q-m =11	n=10 m=40 n_q-m =4	
1	30	31	14	50	21	19	20	11	4	**0.75** **1.00** **1.25**
2	15	15	7	25	10	12	10	5	2	**0.75** **1.00** **1.25**
3	10	10	4	16	7	6	6	3	1	**0.75** **1.00** **1.25**
4	7	7	3	12	5	6	5	2		**0.75** **1.00** **1.25**
5		6		10	4		4	1		**0.75** **1.00** **1.25**

Table VIII

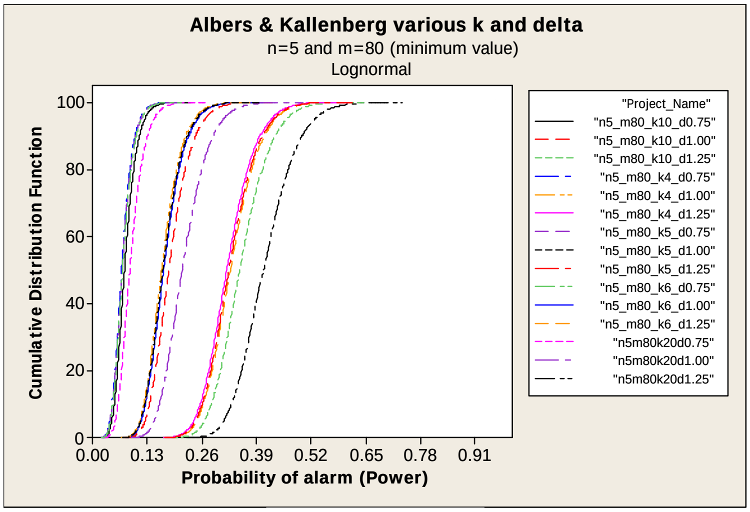


Figure 25

Albers & Kallenberg =5		
Parameters	Avg. Prob. of alarm	Standard deviation
k10_d0.75	0.0792	0.0246
k10_d1.00	0.1871	0.0441
k10_d1.25	0.3556	0.0602
k4_d0.75	0.0719	0.0223
k4_d1.00	0.1694	0.0404
k4_d1.25	0.3247	0.0568
k5_d0.75	0.0731	0.0226
k5_d1.25	0.1723	0.0410
k5_d1.25	0.3298	0.0573
k6_d0.75	0.0740	0.0229
k6_d1.00	0.1745	0.0414
k6_d1.25	0.3334	0.0576
k20_d0.75	0.0927	0.0290
k20_d1.00	0.2199	0.0511
k20_d1.25	0.4123	0.0666

Table IX

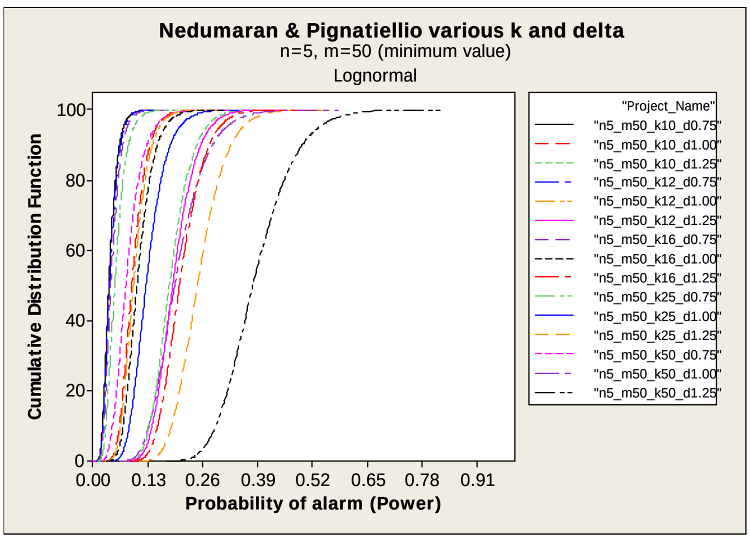


Figure 26

Nedumaran & Pignatiellio n=5		
Parameters	Avg. Prob. of alarm	Standard deviation
k10_d0.75	0.0418	0.0163
k10_d1.00	0.0967	0.0299
k10_d1.25	0.1874	0.0446
k12_d0.75	0.0435	0.0169
k12_d1.25	0.1009	0.0310
k12_d1.25	0.1953	0.0459
k16_d0.75	0.0470	0.0183
k16_d1.00	0.1097	0.0335
k16_d1.25	0.2118	0.0487
k25_d0.75	0.0560	0.0220
k25_d1.00	0.1321	0.0399
k25_d1.25	0.2538	0.0562
k50_d0.75	0.0842	0.0340
k50_d1.00	0.2033	0.0611
k50_d1.25	0.3887	0.0817

Table X

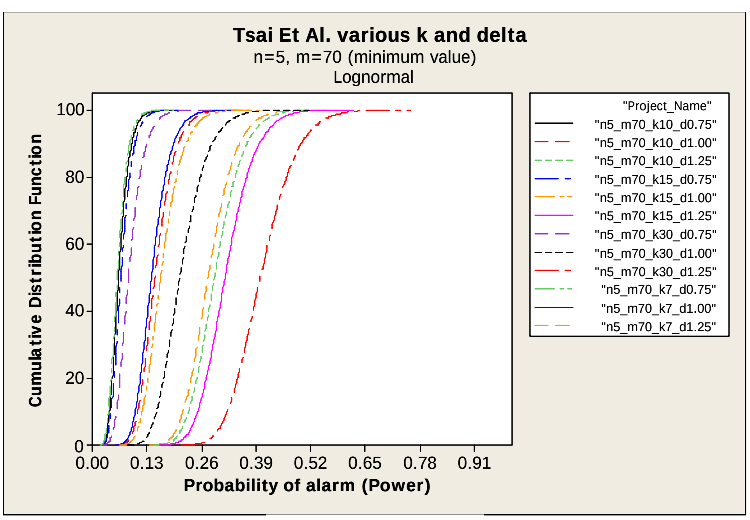


Figure 27

Tsai et al.n=5		
Parameters	Avg. Prob. of alarm	Standard Deviation
k10_d0.75	0.0654	0.0215
k10_d1.00	0.1539	0.0391
k10_d1.25	0.2959	0.0557
k15_d0.75	0.0711	0.0233
k15_d1.00	0.1680	0.0421
k15_d1.25	0.3216	0.0587
k30_d0.75	0.0904	0.0301
k30_d1.00	0.2154	0.0533
k30_d1.25	0.4062	0.0700
k7_d0.75	0.0619	0.0204
k7_d1.00	0.1454	0.0373
k7_d1.25	0.2805	0.0537

Table XI

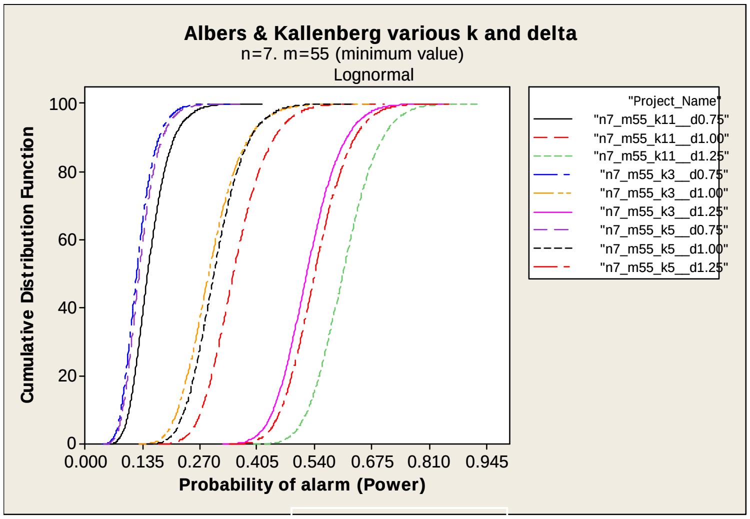
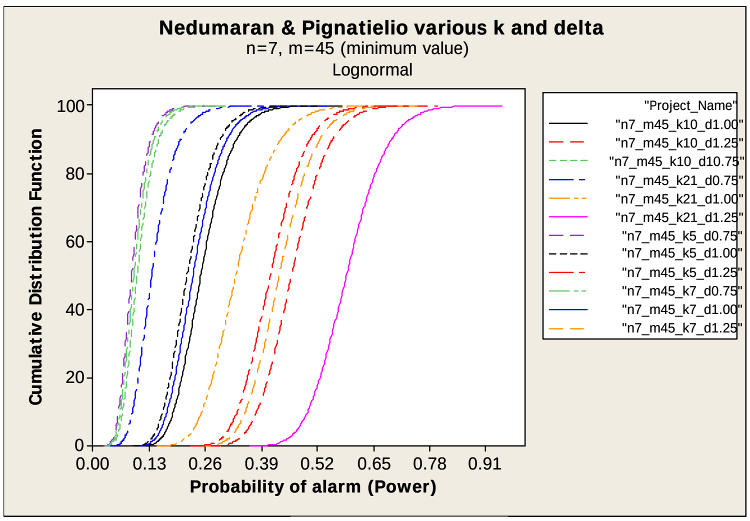


Figure 28

Albers & Kallenberg n=7		
Parameters	Avg. Prob. of alarm	Standard Deviation
k11_d0.75	0.1538	0.0438
k11_d1.00	0.3559	0.0679
k11_d1.25	0.6110	0.0693
k3_d0.75	0.1265	0.0366
k3_d1.00	0.2988	0.0675
k3_d1.25	0.5268	0.0680
k5_d0.75	0.1325	0.0382
k5_d1.00	0.3093	0.0617
k5_d1.25	0.5454	0.0684

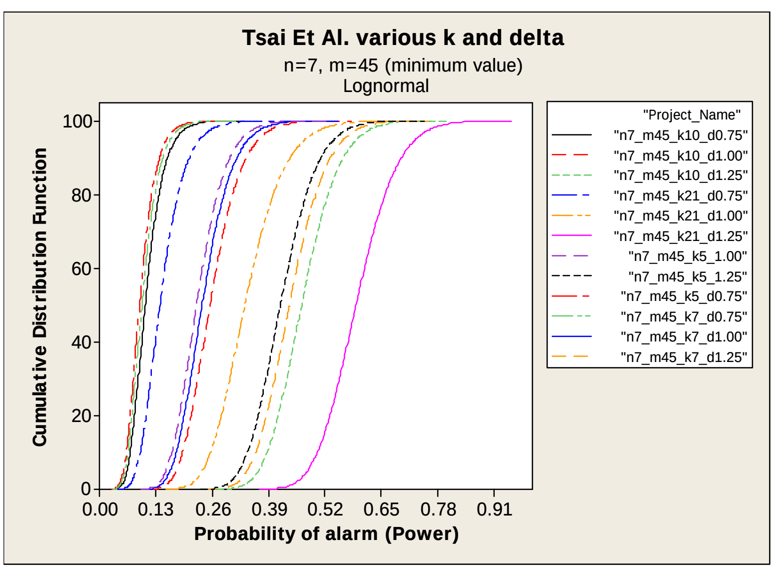
Table XII



Nedumaran & Pignatiello n=7		
Parameters	Avg. Prob. of alarm	Standard Deviation
k10_d1.00	0.2570	0.0584
k10_d1.25	0.4659	0.0694
k10_d10.75	0.1081	0.0348
k21_d0.75	0.1439	0.0459
k21_d1.00	0.3395	0.0730
k21_d1.25	0.5930	0.0766
k5_d0.75	0.0945	0.0308
k5_d1.00	0.2254	0.0532
k5_d1.25	0.4168	0.0669
k7_d0.75	0.1001	0.0325
k7_d1.00	0.2384	0.0554
k7_d1.25	0.4376	0.0682

Table XIII

Figure 29



Tsai et al. n=7		
Parameters	Avg. Prob. of alarm	Standard Deviation
k10_d0.75	0.1108	0.0353
k10_d1.00	0.2615	0.0588
k10_d1.25	0.4718	0.0694
k21_d0.75	0.1464	0.0464
k21_d1.00	0.3434	0.0731
k21_d1.25	0.5972	0.0762
k5_1.00	0.2303	0.0538
k5_1.25	0.4233	0.0670
k5_d0.75	0.0973	0.0314
k7_d0.75	0.1029	0.0331
k7_d1.00	0.2434	0.0560
k7_d1.25	0.4439	0.0682

Table XIV

Figure 30

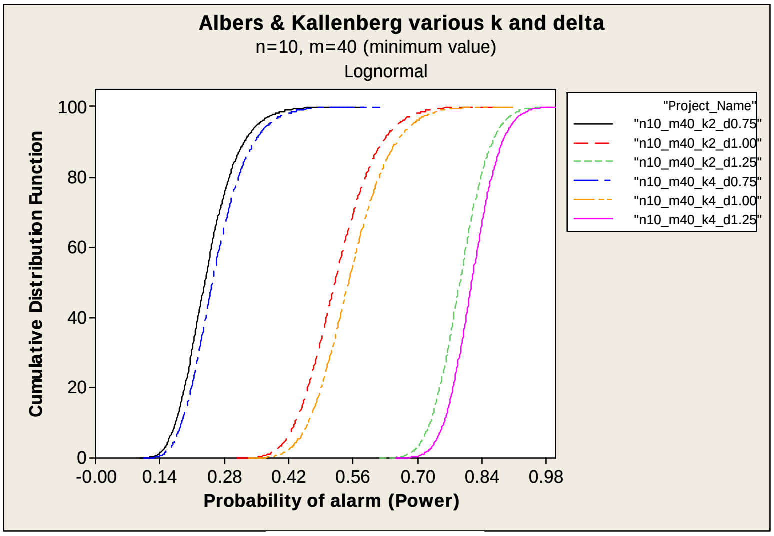


Figure 31

Albers & Kallenberg n=10		
Parameters	Avg. Prob. of alarm	Standard deviation
n10_m40_	0.2447	0.0599
k2_d1.00	0.5261	0.0747
nk2_d1.25	0.7941	0.0530
k4_d0.75	0.2619	0.0632
k4_d1.00	0.5554	0.0757
k4_d1.25	0.8202	0.0499

Table XV

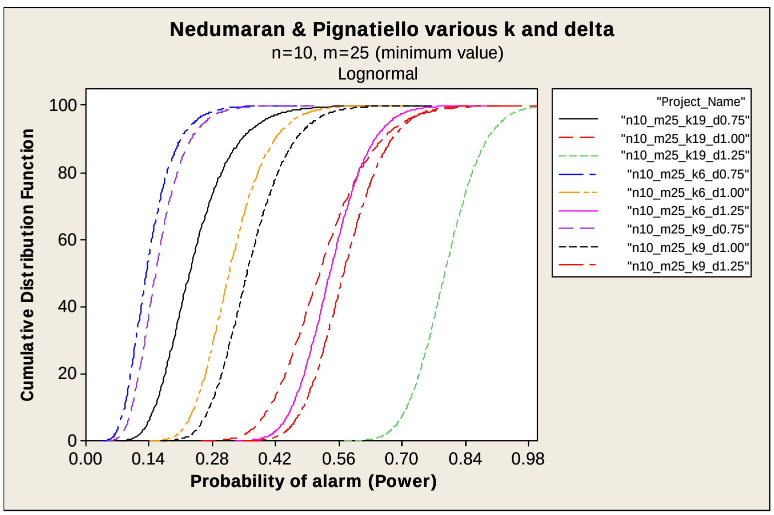


Figure 32

Nedumaran & Pignatiellio n=10		
Parameters	Avg. Prob. of alarm	Standard Deviation
k19_d0.75	0.2408	0.0791
k19_d1.00	0.5254	0.0992
k19_d1.25	0.7972	0.0700
k6_d0.75	0.1419	0.0521
k6_d1.00	0.3242	0.0713
k6_d1.25	0.5444	0.0725
k9_d0.75	0.1621	0.0538
k9_d1.00	0.3643	0.0761
k9_d1.25	0.5753	0.0774

Table XVI

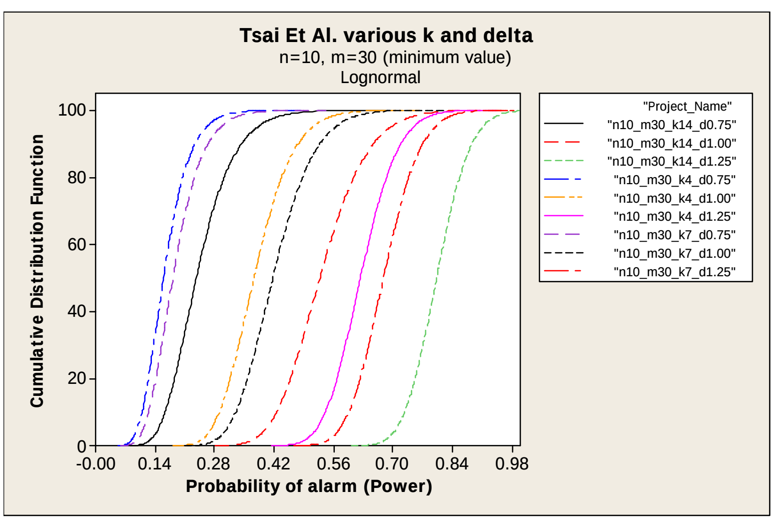


Figure 33

Tsai et al. n=10		
Parameters	Avg. Prob. of alarm	Standard Deviation
k14_d0.75	0.2476	0.0732
k14_d1.00	0.5353	0.0903
k14_d1.25	0.8051	0.0622
k4_d0.75	0.1675	0.0509
k4_d1.00	0.3798	0.0738
k4_d1.25	0.6274	0.0698
k7_d0.75	0.1902	0.0571
k7_d1.00	0.4246	0.0787
k7_d1.25	0.6810	0.0685

Table XVII

Inspection of the above plots shows that for all methods the power of the test for out-of-control conditions deteriorates as the number of sub phases in Phase Ib increases from one to some larger number. Real time plots of the UCL and LCL made during simulations of an out-of-control showed in many cases that recalculation produced limits that adjusted to the out-of-control process without producing an alarm. This effect evidently out weighs any positive benefit of recalculation. These results would seem to lead to the conclusion that recalculation of limits between the original calculation and calculation of the Quesenberry limits at nq is detrimental and should not be done. While this is true for both Albers' and Nedumaran's methods there is another consideration for Tsai's. While the first two methods allow the user of the chart to start checks of every new data point against calculated limits as soon as the limits are calculated at m minimum, Tsai's method does not. It requires waiting until the end of the current Phase1b sub phase and then transforming all of the data gathered in that sub phase simultaneously before checking any individual datum against control limits. If only one Phase 1b-1 is used, this check would be made at the time Quesenberry limits were calculated which essentially prohibits any possibility of process monitoring before nq. For the reasons mentioned above we recommend as an answer to question 3 that only one Phase Ib sub phase be used for the methods of Albers and Nedumaran and that two be used for Tsai's method. We combine this recommendation with the results of Table VI to create the following Table XVIII Evaluation Table for "How soon can we start monitoring the process?"

Evaluation Table for “How soon can we start monitoring the process?”

n	Albers &Kallenberg	Nedumaran &Pignatiellio	Tsai Et al.
5	2	3	1
7	2	3	1
10	2	3	1
score	(2+2+2)/3=2	(3+3+3)/3=3	(1+1+1)/3=1

Table XVIII

Because the power of a test is lower when two sub phases are used Tsai’s method being used with two sub phases will always have lower power than the other two methods. Combining that fact with Table XIX we have developed the following Evaluation Table to answer the question “Which method has the greater power?”

Evaluation Table for “Which Method has the Greater Power

n	**Albers &Kallenberg**	**Nedumaran &Pignatiellio**	**Tsai Et al.**
5	3	2	1
7	3	2	1
10	3	2	1
Average Score	3	2	1

Table XIX

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

6.1 General Considerations Concerning Application Environments

The answer to the question “Which one of these methods is best in practice?” we must consider another factor in addition to the data obtained in the above simulation studies. That factor is the environments in which the chart will be developed and used. We have identified three such possible environments.

First they are characterized by the level of technical knowledge of the control chart developer. In the case of the Tsai et al. method the level of knowledge of the user of the chart is also important. Second these environments are characterized by whether the chart development and/or use is automated by application of a computer. These factors are important to varying degrees in evaluating the three author’s methods.

We see these three possible environments.:

1, Neither the developer of the chart or its user have a knowledge of statistics or computing resources available.

2. The developer of the chart has the knowledge of statistics and computing resources available but the user does not.

3. Both the developer and user have statistical knowledge and computing resources.

In what follows we evaluate and make a separate recommendation in regards to which of the author's methods is best for each of these three environments. We do this using Pugh Matricies of the form shown in Table XX

Pugh Matrix Format

CRITERIA	**WEIGHT**	**METHODS**		
		Albers & Kallenberg	**Nedumaran & Pignatiellio**	**Tsai et al.**
How soon we can monitor the process		2	3	1
Power		3	2	1
How easy is it to build a chart				
How easy is it to use a chart?"				
Score				

Table XX

The first column lists four factors upon which our comparison judgments are to be made. The second column contains a weighting factor indicating the importance of the first column factor in the environment being evaluated. The weights are estimated on a scale of 0-10 with 10 being the most important. The next columns contain ranks for the different methods for the row's criteria. The ranks range from 1-3 with three being the highest. The score row contains the score calculated as the sum of the row values multiplied by the row weights. The values in the first and second table rows are taken from Tables XIX and XX. The environment determines the values in the next two rows. The values in the weight column vary by environment.

6.2 Recommendation for Environment 1

Table XXI shows the Pugh matrix for environment 1. In this environment the statistical knowledge of both chart developer and user is limited. Tsai's method requires knowledge and computer resources in both Phases Ia and Ib. Nedumaran's method requires them in Phase Ia but not Ib. The Pugh matrix for this environment is shown in Table XXI below.

Pugh Matrix Environment 1
(Fractional ranks represent ties)

CRITERIA	WEIGHT	METHODS		
		Albers & Kallenberg	**Nedumaran & Pignatiellio**	**Tsai et al.**
How soon we can monitor the process	5	2	3	1
Power	5	3	2	1
How easy is it to build a chart	10	3	1.5	1.5
How easy is it to use a chart?"	10	2.5	2.5	1
Score		80	65	35

Table XXI

Based on the contents of Table XXI Albers & Kallenberg's method is recommended for environment 1.

6.3 Recommendation for Environment 2

In the second environment statistical knowledge and computing power is available during Phase Ia but not in Phase Ib. Because of this "How easy is it to build a chart" is weighted lower. The Pugh matrix for this environment is shown in Table XXII below

Pugh Matrix Environment 2

(Fractional ranks represent ties)

CRITERIA	WEIGHT	METHODS		
		Albers & Kallenberg	Nedumaran & Pignatiellio	Tsai et al.
How soon we can monitor the process	5	2	3	1
Power	5	3	2	1
How easy is it to build a chart	0	3	1.5	1.5
How easy is it to use a chart?”	10	2.5	2.5	1
Score		50	50	20

Table XXII

The above analysis gives a tie between Albers and Nedumaran. Albers has greater power and therefore higher probability of detecting smaller out-of-control process. Nedumaran allows earlier monitoring of the process. These differences are small so either could be chosen in this environment.

6.4 Recommendation for Environment 3

In this environment statistical knowledge and computing resource is available for all of Phase I. This might occur when development of a chart and its use are completely automated. In environment 3 the weights for the last two rows of the matrix are set at their lowest possible value indicating these factors are not important. The Pugh Matrix for environment 3 is shown below in Table XXIII

Pugh Matrix Environment 3
(Fractional ranks represent ties)

CRITERIA	WEIGHT	METHODS		
		Albers & Kallenberg	Nedumaran & Pignatiellio	Tsai et al.
How soon we can monitor the process?	5	2	3	1
Power	5	3	2	1
How easy is it to build a chart ?	0	3	1.5	1.5
How easy is it to use a chart?	0	2.5	2.5	1
Score		25	25	10

Table XXIII

Again, in this environment Albers and Nedumaran is tie with Albers being superior in minimum m and Nedumaran in power. Again, either method is appropriate for this environment.

6.5 Final Conclusions

The method of Tsai et al. is only able to actually check for the possibility of an out-of-control process after a number of subgroups (k) have been accumulated. Its competitors can check each new subgroup as data is received. This fact delays decision making considerably unless small values of k are used producing short Phase Ib sub phases. Small values of k, however, cause frequent recalculation of limits which in turn increases the probability that the recalculated limits will track the out-of-control process rather that create and alarm which means the power will be lower. The above brings the conclusion that Tsai’s method is not as good as the others.

Albers' method requires the smallest amount of knowledge to create and use charts. It is, therefore, the best for environment 1 where knowledge and computer resources are limited.

Nedumaran & Pignatiello's method requires more knowledge in the chart development stage but otherwise has only small differences with Albers'. If that knowledge is available it is an good alternative to Albers' in environment 2. In any case it is a good alternative to Albers' in environment 3.

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APPENDIX

APPENDIX A
AUTHOR'S METHOD

Appendix A-1

NEDUMARAN&PIGNATIELLO (2001) (1)

- **Objectives:** Construct control limits of the $\overline{X}$ control charts that match any specific percentile point of run length distribution of the true limits even when the limits are estimated using data from only a few subgroups.
- **Method:** This approach is constructed based on multivariate t-distribution. Prospective control limits are constructed from m initial subgroups number for a future subgroups number (k). Authors assumed there is equicorrelatedmultivariate normal distribution between all future subgroups. Control limitsare

$$\widehat{UCL} = \overline{\overline{X}} + h'_{\gamma,m,k,v}\sqrt{\frac{m+1}{mn}}\sqrt{\overline{V}}$$
$$\widehat{LCL} = \overline{\overline{X}} - h'_{\gamma,m,k,v}\sqrt{\frac{m+1}{mn}}\sqrt{\overline{V}}$$

- **Simulation:** Samples are considered as identically and normally distributed. m=initial subgroups=variable, n=sample size=3,5,7, k=future subgroups=variable, α=0.0027(for each k), γ= probability of signal within k subgroups,
- Constructed future control limits from m initial subgroups (μ=0, σ^2=1) for a future the number of subgroups.
- Repeat 10000 times to compare this result with the standard two phase approach based on the probability of signal within k subgroups.

NEDUMARAN&PIGNATIELLO (2001) (2)

- **Analysis:** The authors showed that their proposed control limits perform similar to the true limits even for small m whereas the standard approachesissue relatively large number of false alarms after short runs based on simulation
- **Comment:** Their results are close true to those of limits of control chart. But nevertheless they use of the multivariate t distribution which is difficult and perhaps not possible for typical practice.

Appendix A-2

ALBERS&KALLENBERG (2004-5) (1)

- **Objective:** Reduce the number of Phase I samples required while maintaining the in-control ARL to that of a known parameter chart. This is to be accomplished by increasing the width of UCL-LCL (e.g. +/-(u_p+c) σ_{hat}) beyond that of a similar known-parameter chart (e.g. +/- u_p σ_{hat}).

- **Definitions:** Random variable Error=(Pn-p)/P where:
  - P is the probability of a false alarm using a certain chart constructed with known parameters (e.g., .=00135 if u_p=3.0).
  - Pn is the probability of a false alarm using the wider control limits calculated to match the above false alarm rate using an estimated parameter control chart.

- **Chart Development –Bias Method**
- Define Bias as (Expectation{Pn}-P)/P
  - Choose c to force Bias to zero
  - Calculate Control Chart Limits

ALBERS &KALLENBERG (2004-5) (2)

- **Chart Development –Exceedance Approach**
  - Specify and "unpleasant"value of Pn e.g., Pn >(1+e)P or error = ε
  - Specify the maximum proportion of future charts that should exceed this error = α
  - Choose c to satisfy Probability(|(Pn-p)|/p>= ε)<α. This probability is called by the authors the "exceedance"probability.
  - Calculate Control Limits

- **Bias Approach:**

$$ControlLimit = \bar{x} \pm u_{p/2}\left(1+\left(\tfrac{1}{2}+\tfrac{u_{p/2}^{2}}{8}\right)\Big/n\right)\times s$$

- **Exceedance Probability Approach**

$$ControlLimit = \bar{x} \pm u_{p/2}\left(1+\frac{u_a}{(2n)^{1/2}}-\frac{e}{u_{p/2}^{2}}\right)\times s$$

ALBERS & KALLENBERG (2004 -5) (3)

- **Analysis:** While the bias approach allows for n=40 during phase I it does not limit the worst case error that can be incurred for a single control chartdeveloped in that manner. The diffidence probability approach is much more stringent in that it places constraints on a remote percentile of the error distribution rather than some parameter averaged over all possible charts. As the authors point out, this tight control of the in-control error rate is at the expense of the chart power for detecting out-of-control situations. To what degree this power limitation reduces the usefulness of the method is to be determined.

Appendix A-3

TSAI ET AL.(2005)

(1)

- **Objectives:** Start monitoring process at the early stage before the numbersamples which is recommended by Quesenberry ($m=400/(n-1)$)
- **Method:** This approach is constructed based on Student's t-distribution to build control limits for any number of subgroups.
- Control limits are calculated from m initial subgroups number with n size and k future subgroups. First $\bar{\bar{X}}$ is calculated from m subgroups and $U_i=X_i-\bar{\bar{X}}$
- They did matrix transform from correlated sequence to uncorrelated sequence ($\mathbf{W}=\Sigma^{-1/2}\mathbf{U}$)
- Center line is considered as 0.Control limits are;

$$\text{Control limits}=0 \pm t_{\alpha/2}\,(\bar{\nabla}\ ((m+1)/mn))^{1/2}$$

- **Simulation:** Samples are considered as identically and normally distributed from m subgroups=10-75; samples sizes, n=3,5,7 and k future subgroups=5-25 from in-control process ($\mu=0,\sigma^2=1$).
- $\alpha=0.0027$, γ= probability of signal within k subgroups; $g=1-(1-\alpha)^k$
- Plot the W_i at the control chart
- Repeat this procedure 10.000
- Tsai et al. compare those results of their second approach with standard two phase approach and Nedumaran &Pignatiello's approach based on the probability of signal within k subgroups.

TSAI ET AL.(2005)

(2)

- **Analysis:** The authors showed that their proposed control limits perform similar to the true limits even for small m without dropping any a number of subgroups. In this approach we need a few initial subgroups, student's t distribution and we have do matrix transform to plot individual points.

- **Comment**: The results of this method are close to the true limits (known parameters case) of control chart without dropping any a number of subgroups. However, in order to transform the correlated sequence to an uncorrelated sequence The quality practitioner has to know and use matrix algebra which takes long time or He/She has to use an advance math program. This is not useful for a typical quality practitioner.

APPENDIX B
ARENA MODEL

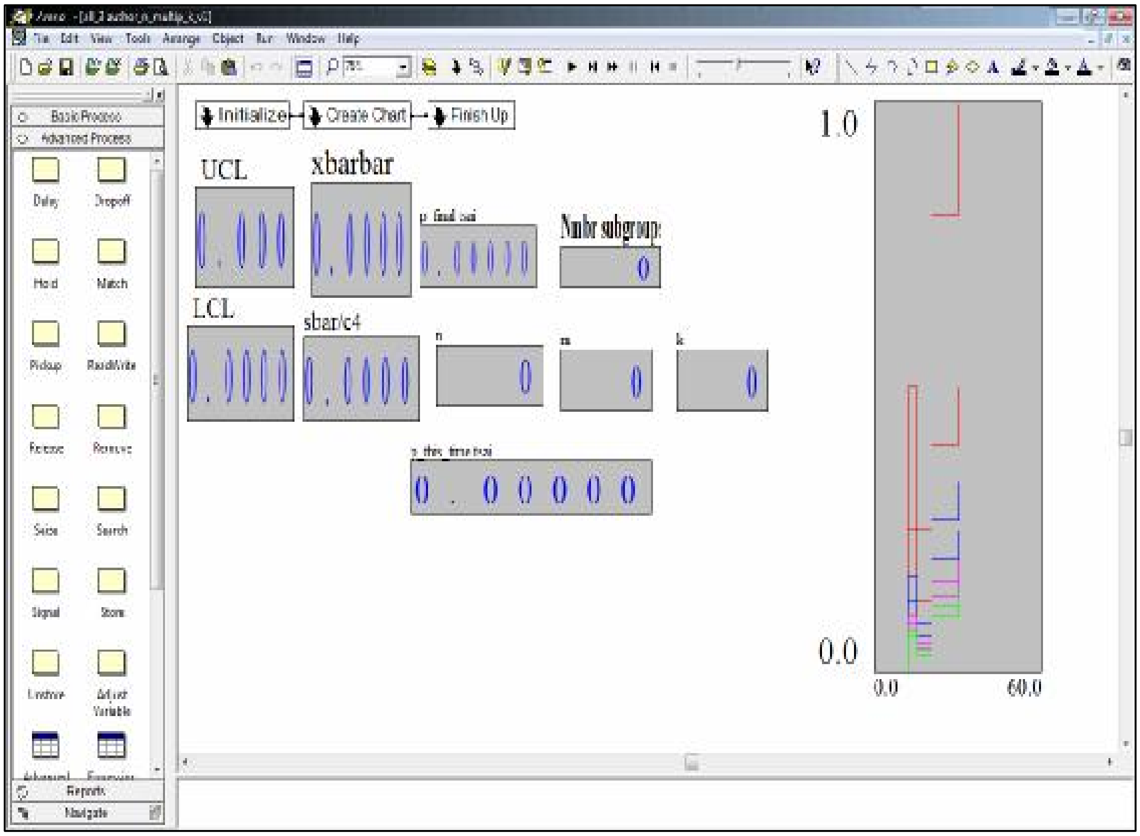

Figure 1 Whole model

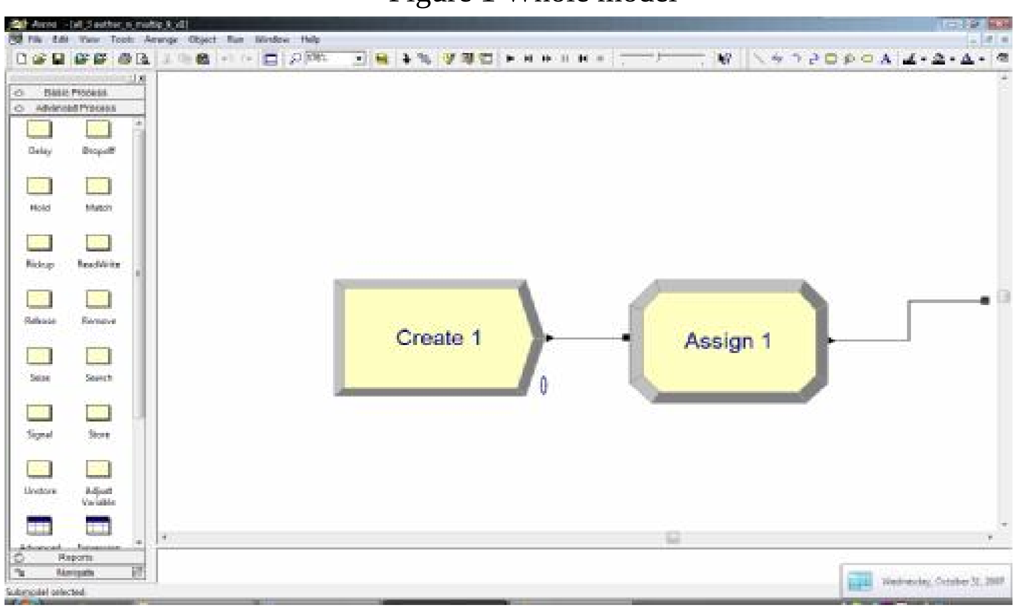

Figure 2- Initialize subgroup

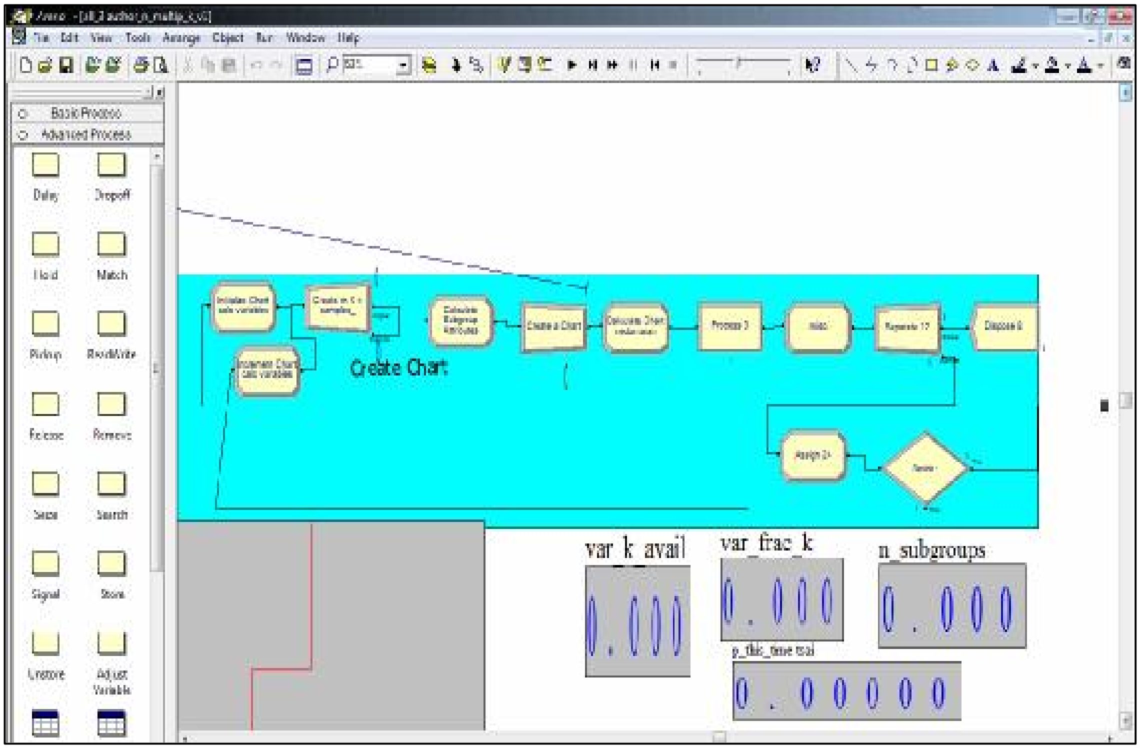

Figure 3-Create subgroup

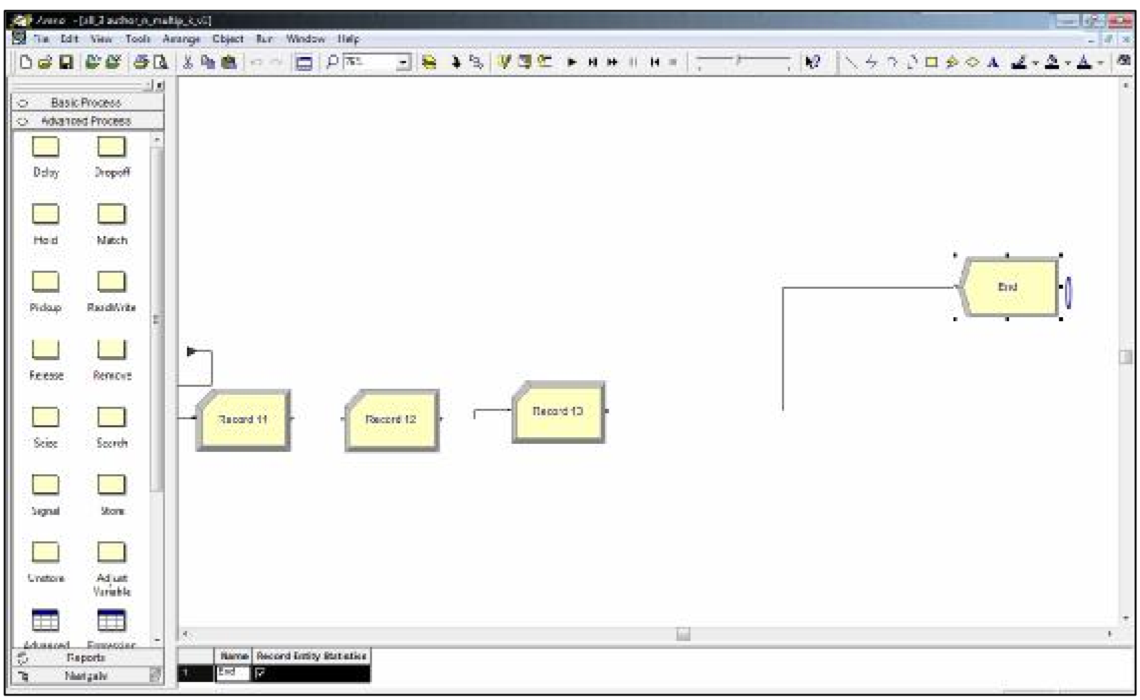

Figure III-Create subgroup