Flow Induced Vibrations in Pipes: a Finite Element Approach

Ivan Grant
Cleveland State University

Follow this and additional works at: https://engagedscholarship.csuohio.edu/etdarchive

Part of the Mechanical Engineering Commons

How does access to this work benefit you? Let us know!

Recommended Citation

This Thesis is brought to you for free and open access by EngagedScholarship@CSU. It has been accepted for inclusion in ETD Archive by an authorized administrator of EngagedScholarship@CSU. For more information, please contact library.es@csuohio.edu.
FLOW INDUCED VIBRATIONS IN PIPES, A FINITE ELEMENT APPROACH

IVAN GRANT

Bachelor of Science in Mechanical Engineering
Nagpur University
Nagpur, India June, 2006

submitted in partial fulfillment of requirements for the degree
MASTERS OF SCIENCE IN MECHANICAL ENGINEERING
at the
CLEVELAND STATE UNIVERSITY
May, 2010
This thesis has been approved
for the department of MECHANICAL ENGINEERING
and the College of Graduate Studies by:

Thesis Chairperson, Majid Rashidi, Ph.D.

Department & Date

Asuquo B.E比亚纳, Ph.D.

Department & Date

Rama S.Gorca, Ph.D.

Department & Date
ACKNOWLEDGMENTS

I would like to thank my advisor Dr. Majid Rashidi and Dr. Paul Bellini, who provided essential support and assistance throughout my graduate career, and also for their guidance which immensely contributed towards the completion of this thesis. This thesis would not have been realized without their support. I would also like to thank Dr. Asuquo B. Ebiana and Dr. Rama S. Gorla for being in my thesis committee. Thanks are also due to my parents, my brother and friends who have encouraged, supported and inspired me.
FLOW INDUCED VIBRATIONS IN PIPES, A FINITE ELEMENT APPROACH

IVAN GRANT

ABSTRACT

Flow induced vibrations of pipes with internal fluid flow is studied in this work. Finite Element Analysis methodology is used to determine the critical fluid velocity that induces the threshold of pipe instability. The partial differential equation of motion governing the lateral vibrations of the pipe is employed to develop the stiffness and inertia matrices corresponding to two of the terms of the equations of motion. The Equation of motion further includes a mixed-derivative term that was treated as a source for a dissipative function. The corresponding matrix with this dissipative function was developed and recognized as the potentially destabilizing factor for the lateral vibrations of the fluid carrying pipe. Two types of boundary conditions, namely simply-supported and cantilevered were considered for the pipe. The appropriate mass, stiffness, and dissipative matrices were developed at an elemental level for the fluid carrying pipe. These matrices were then assembled to form the overall mass, stiffness, and dissipative matrices of the entire system. Employing the finite element model developed in this work two series of parametric studies were conducted. First, a pipe with a constant wall thickness of 1 mm was analyzed. Then, the parametric studies were extended to a pipe with variable wall thickness. In this case, the wall thickness of the pipe was modeled to taper down from 2.54 mm to 0.01 mm. This study shows that the critical velocity of a pipe carrying fluid can be increased by a factor of six as the result of tapering the wall thickness.
# TABLE OF CONTENTS

ABSTRACT iv

LIST OF FIGURES vii

LIST OF TABLES ix

I INTRODUCTION 1

1.1 Overview of Internal Flow Induced Vibrations in Pipes . . . . . . 1
1.2 Literature Review . . . . . . . . . . . . . . . . . . . . . . . . . 2
1.3 Objective . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2
1.4 Composition of Thesis . . . . . . . . . . . . . . . . . . . . . . . 3

II FLOW INDUCED VIBRATIONS IN PIPES, A FINITE ELEMENT AP-
PROACH 4

2.1 Mathematical Modelling . . . . . . . . . . . . . . . . . . . . . . 4
2.1.1 Equations of Motion . . . . . . . . . . . . . . . . . . . 4
2.2 Finite Element Model . . . . . . . . . . . . . . . . . . . . . . . 12
2.2.1 Shape Functions . . . . . . . . . . . . . . . . . . . . . . 12
2.2.2 Formulating the Stiffness Matrix for a Pipe Carrying Fluid 14
2.2.3 Forming the Matrix for the Force that conforms the
Fluid to the Pipe . . . . . . . . . . . . . . . . . . . . . . . . . . 21
2.2.4 Dissipation Matrix Formulation for a Pipe carrying Fluid 26
2.2.5 Inertia Matrix Formulation for a Pipe carrying Fluid . 28

III FLOW INDUCED VIBRATIONS IN PIPES, A FINITE ELEMENT AP-
PROACH 31

v
3.1 Forming Global Stiffness Matrix from
Elemental Stiffness Matrices .............................................. 31

3.2 Applying Boundary Conditions to Global
Stiffness Matrix for simply supported pipe with fluid flow ........ 33

3.3 Applying Boundary Conditions to Global
Stiffness Matrix for a cantilever pipe with fluid flow ............... 34

3.4 MATLAB Programs for Assembling Global Matrices for Simply
Supported and Cantilever pipe carrying fluid .......................... 35

3.5 MATLAB program for a simply supported pipe carrying fluid . 35

3.6 MATLAB program for a cantilever pipe carrying fluid ............ 36

IV FLOW INDUCED VIBRATIONS IN PIPES, A FINITE ELEMENT AP-
PROACH...................................................................................... 37

4.1 Parametric Study ............................................................. 37

V FLOW INDUCED VIBRATIONS IN PIPES, A FINITE ELEMENT AP-
PROACH...................................................................................... 42

5.1 Tapered Pipe Carrying Fluid ............................................. 42

VI RESULTS AND DISCUSSIONS................................................. 47

6.1 Contribution of the Thesis ................................................. 50

6.2 Future Scope ...................................................................... 50

BIBLIOGRAPHY........................................................................... 51

Appendices................................................................................. 54

0.1 MATLAB program for Simply Supported Pipe Carrying Fluid 54

0.2 MATLAB Program for Cantilever Pipe Carrying Fluid .......... 61

0.3 MATLAB Program for Tapered Pipe Carrying Fluid ............ 68
LIST OF FIGURES

2.1 Pinned-Pinned Pipe Carrying Fluid * .......................... 5
2.2 Pipe Carrying Fluid, Forces and Moments acting on Elements
   (a) Fluid (b) Pipe ** ........................................... 5
2.3 Force due to Bending ........................................... 7
2.4 Force that Conforms Fluid to the Curvature of Pipe ........ 9
2.5 Coriolis Force .................................................. 10
2.6 Inertia Force ................................................... 11
2.7 Pipe Carrying Fluid ............................................. 13
2.8 Beam Element Model ........................................... 14
2.9 Relationship between Stress and Strain, Hooks Law ........ 15
2.10 Plain sections remain plane ................................. 16
2.11 Moment of Inertia for an Element in the Beam ............. 17
2.12 Pipe Carrying Fluid Model ................................. 21

3.1 Representation of Simply Supported Pipe Carrying Fluid ... 33
3.2 Representation of Cantilever Pipe Carrying Fluid ........... 34
3.3 Pinned-Free Pipe Carrying Fluid* ............................ 36

4.1 Reduction of Fundamental Frequency for a Pinned-Pinned
   Pipe with increasing Flow Velocity .......................... 39
4.2 Shape Function Plot for a Cantilever Pipe with increasing
   Flow Velocity ................................................. 40
4.3 Reduction of Fundamental Frequency for a Cantilever Pipe
   with increasing Flow Velocity ............................... 41

5.1 Representation of Tapered Pipe Carrying Fluid ............... 42
5.2 Introducing a Taper in the Pipe Carrying Fluid ............... 43

6.1 Representation of Pipe Carrying Fluid and Tapered Pipe Carrying Fluid ......................................................... 47
LIST OF TABLES

4.1 Reduction of Fundamental Frequency for a Pinned-Pinned Pipe with increasing Flow Velocity ................. 38
4.2 Reduction of Fundamental Frequency for a Pinned-Free Pipe with increasing Flow Velocity ......................... 40
5.1 Reduction of Fundamental Frequency for a Tapered pipe with increasing Flow Velocity .......................... 46
6.1 Reduction of Fundamental Frequency for a Tapered Pipe with increasing Flow Velocity .......................... 48
6.2 Reduction of Fundamental Frequency for a Pinned-Pinned Pipe with increasing Flow Velocity ................. 49
CHAPTER I

INTRODUCTION

1.1 Overview of Internal Flow Induced Vibrations in Pipes

The flow of a fluid through a pipe can impose pressures on the walls of the pipe causing it to deflect under certain flow conditions. This deflection of the pipe may lead to structural instability of the pipe. The fundamental natural frequency of a pipe generally decreases with increasing velocity of fluid flow. There are certain cases where decrease in this natural frequency can be very important, such as very high velocity flows through flexible thin-walled pipes such as those used in feed lines to rocket motors and water turbines. The pipe becomes susceptible to resonance or fatigue failure if its natural frequency falls below certain limits. With large fluid velocities the pipe may become unstable. The most familiar form of this instability is the whipping of an unrestricted garden hose. The study of dynamic response of a fluid conveying pipe in conjunction with the transient vibration of ruptured pipes reveals that if a pipe ruptures through its cross section, then a flexible length of unsupported pipe is left spewing out fluid and is free to whip about and impact other structures. In power plant plumbing pipe whip is a possible mode of failure. A
study of the influence of the resulting high velocity fluid on the static and dynamic characteristics of the pipes is therefore necessary.

1.2 Literature Review

Initial investigations on the bending vibrations of a simply supported pipe containing fluid were carried out by Ashley and Haviland[2]. Subsequently, Housner[3] derived the equations of motion of a fluid conveying pipe more completely and developed an equation relating the fundamental bending frequency of a simply supported pipe to the velocity of the internal flow of the fluid. He also stated that at certain critical velocity, a statically unstable condition could exist. Long[4] presented an alternate solution to Housner’s[3] equation of motion for the simply supported end conditions and also treated the fixed-free end conditions. He compared the analysis with experimental results to confirm the mathematical model. His experimental results were rather inconclusive since the maximum fluid velocity available for the test was low and change in bending frequency was very small. Other efforts to treat this subject were made by Benjamin, Niordson[6] and Ta Li. Other solutions to the equations of motion show that type of instability depends on the end conditions of the pipe carrying fluid. If the flow velocity exceeds the critical velocity pipes supported at both ends bow out and buckle[1]. Straight Cantilever pipes fall into flow induced vibrations and vibrate at a large amplitude when flow velocity exceeds critical velocity[8-11].

1.3 Objective

The objective of this thesis is to implement numerical solutions method, more specifically the Finite Element Analysis (FEA) to obtain solutions for different pipe configurations and fluid flow characteristics. The governing dynamic equation describing the induced structural vibrations due to internal fluid flow has been formed and dis-
cussed. The governing equation of motion is a partial differential equation that is fourth order in spatial variable and second order in time. Parametric studies have been performed to examine the influence of mass distribution along the length of the pipe carrying fluid.

1.4 Composition of Thesis

This thesis is organized according to the following sequences. The equations of motions are derived in chapter (II) for pinned-pinned and fixed-pinned pipe carrying fluid. A finite element model is created to solve the equation of motion. Elemental matrices are formed for pinned-pinned and fixed-pinned pipe carrying fluid. Chapter (III) consists of MATLAB programs that are used to assemble global matrices for the above cases. Boundary conditions are applied and based on the user defined parameters fundamental natural frequency for free vibration is calculated for various pipe configurations. Parametric studies are carried out in the following chapter and results are obtained and discussed.
CHAPTER II

FLOW INDUCED VIBRATIONS IN PIPES, A FINITE ELEMENT APPROACH

In this chapter, a mathematical model is formed by developing equations of a straight fluid conveying pipe and these equations are later solved for the natural frequency and onset of instability of a cantilever and pinned-pinned pipe.

2.1 Mathematical Modelling

2.1.1 Equations of Motion

Consider a pipe of length L, modulus of elasticity E, and its transverse area moment I. A fluid flows through the pipe at pressure p and density \( \rho \) at a constant velocity \( v \) through the internal pipe cross-section of area A. As the fluid flows through the deflecting pipe it is accelerated, because of the changing curvature of the pipe and the lateral vibration of the pipeline. The vertical component of fluid pressure applied to the fluid element and the pressure force F per unit length applied on the fluid element by the tube walls oppose these accelerations. Referring to figures (2.1) and
Figure 2.1: **Pinned-Pinned Pipe Carrying Fluid** *

\[ F - \rho A \frac{\partial^2 Y}{\partial x^2} = \rho A (\frac{\partial}{\partial t} + v \frac{\partial}{\partial x})^2 Y \]  

(2.1)

The pressure gradient in the fluid along the length of the pipe is opposed by the shear stress of the fluid friction against the tube walls. The sum of the forces parallel to the pipe axis for a constant flow velocity gives

Figure 2.2: **Pipe Carrying Fluid, Forces and Moments acting on Elements**  
(a) Fluid (b) Pipe **

---

* Flow Induced Vibrations, Robert D. Blevins, Krieger, 1977, P 289
** Flow Induced Vibrations, Robert D. Blevins, Krieger, 1977, P 289
A\frac{\partial p}{\partial x} + \varphi S = 0 \quad (2.2)

Where S is the inner perimeter of the pipe, and \varphi is the shear stress on the internal surface of the pipe. The equations of motions of the pipe element are derived as follows.

\frac{\partial T}{\partial x} + \varphi S - Q \frac{\partial^2 Y}{\partial x^2} = 0 \quad (2.3)

Where Q is the transverse shear force in the pipe and T is the longitudinal tension in the pipe. The forces on the element of the pipe normal to the pipe axis accelerate the pipe element in the Y direction. For small deformations,

\frac{\partial Q}{\partial x} + T \frac{\partial^2 Y}{\partial x^2} - F = m \frac{\partial^2 Y}{\partial t^2} \quad (2.4)

Where m is the mass per unit length of the empty pipe. The bending moment M in the pipe, the transverse shear force Q and the pipe deformation are related by

Q = - \frac{\partial M}{\partial x} = EI \frac{\partial^3 Y}{\partial x^3} \quad (2.5)

Combining all the above equations and eliminating Q and F yields:

\begin{align*}
EI \frac{\partial^4 Y}{\partial x^4} + (\rho A - T) \frac{\partial^2 Y}{\partial x^2} + \rho A (\frac{\partial}{\partial t} + v \frac{\partial}{\partial x})^2 Y + m \frac{\partial Y}{\partial t^2} = 0
\end{align*} \quad (2.6)

The shear stress may be eliminated from equation 2.2 and 2.3 to give

\frac{\partial (\rho A - T)}{\partial x} = 0 \quad (2.7)

At the pipe end where x=L, the tension in the pipe is zero and the fluid pressure is equal to ambient pressure. Thus p=T=0 at x=L,

\rho A - T = 0 \quad (2.8)
The equation of motion for a free vibration of a fluid conveying pipe is found out by substituting $\rho A - T = 0$ from equation 2.8 in equation 2.6 and is given by the equation 2.9

$$EI\frac{\partial^4 Y}{\partial x^4} + \rho Av^2\frac{\partial^2 Y}{\partial x^2} + 2\rho Av\frac{\partial^2 Y}{\partial x\partial t} + M\frac{\partial^2 Y}{\partial t^2} = 0$$

where the mass per unit length of the pipe and the fluid in the pipe is given by $M = m + \rho A$. The next section describes the forces acting on the pipe carrying fluid for each of the components of eq(2.9)

![Figure 2.3: Force due to Bending](image)

Representation of the First Term in the Equation of Motion for a Pipe Carrying Fluid
The term $EI \frac{\partial^4 Y}{\partial x^4}$ is a force component acting on the pipe as a result of bending of the pipe. Fig(2.3) shows a schematic view of this force F1.
Figure 2.4: Force that Conforms Fluid to the Curvature of Pipe

Representation of the Second Term in the Equation of Motion for a Pipe Carrying Fluid

The term $\rho A v^2 \frac{\partial^2 Y}{\partial x^2}$ is a force component acting on the pipe as a result of flow around a curved pipe. In other words the momentum of the fluid is changed leading to a force component $F_2$ shown schematically in Fig(2.4) as a result of the curvature in the pipe.
The term $2\rho Av \frac{\partial^2 Y}{\partial x \partial t}$ is the force required to rotate the fluid element as each point in the span rotates with angular velocity. This force is a result of Coriolis Effect. Fig(2.5) shows a schematic view of this force F3.
The term $M \frac{\partial^2 Y}{\partial t^2}$ is a force component acting on the pipe as a result of Inertia of the pipe and the fluid flowing through it. Fig(2.6) shows a schematic view of this force F4.
2.2 Finite Element Model

Consider a pipeline span that has a transverse deflection $Y(x,t)$ from its equilibrium position. The length of the pipe is $L$, modulus of elasticity of the pipe is $E$, and the area moment of inertia is $I$. The density of the fluid flowing through the pipe is $\rho$ at pressure $p$ and constant velocity $v$, through the internal pipe cross section having area $A$. Flow of the fluid through the deflecting pipe is accelerated due to the changing curvature of the pipe and the lateral vibration of the pipeline. From the previous section we have the equation of motion for free vibration of a fluid conveying pipe:

$$EI \frac{\partial^4 Y}{\partial x^4} + \rho A v^2 \frac{\partial^2 Y}{\partial x^2} + 2\rho A v \frac{\partial^2 Y}{\partial x \partial t} + M \frac{\partial^2 Y}{\partial t^2} = 0 \quad (2.10)$$

2.2.1 Shape Functions

The essence of the finite element method is to approximate the unknown by an expression given as

$$w = \sum_{i=1}^{n} N_i a_i$$

where $N_i$ are the interpolating shape functions prescribed in terms of linear independent functions and $a_i$ are a set of unknown parameters.

We shall now derive the shape functions for a pipe element.
Consider an pipe of length \( L \) and let at point \( R \) be at distance \( x \) from the left end. \( L_2=x/L \) and \( L_1=1-x/L \).

Forming Shape Functions

\[
N_1 = L_1^2(3 - 2L_1) \\
N_2 = L_1^2L_2L \\
N_3 = L_2^2(3 - 2L_2) \\
N_4 = -L_1L_2^2L
\]

Substituting the values of \( L_1 \) and \( L_2 \) we get

\[
N_1 = (1 - x/l)^2(1 + 2x/l) \\
N_2 = (1 - x/l)^2x/l \\
N_3 = (x/l)^2(3 - 2x/l) \\
N_4 = -(1 - x/l)(x/l)^2
\]
2.2.2 Formulating the Stiffness Matrix for a Pipe Carrying Fluid

![Beam Element Model](image)

For a two dimensional beam element, the displacement matrix in terms of shape functions can be expressed as

\[
[W(x)] = \begin{bmatrix}
N1 & N2 & N3 & N4 \\
w1 & \theta1 & w2 & \theta2 \\
\end{bmatrix}
\]

(2.19)

where \(N1, N2, N3\) and \(N4\) are the displacement shape functions for the two dimensional beam element as stated in equations (2.15) to (2.18). The displacements and rotations at end 1 is given by \(w1, \theta1\) and at end 2 is given by \(w2, \theta2\).

Consider the point R inside the beam element of length L as shown in figure(2.7) Let the internal strain energy at point R is given by \(U_R\).

The internal strain energy at point R can be expressed as:

\[
U_R = \frac{1}{2} \sigma \epsilon
\]

(2.20)

where \(\sigma\) is the stress and \(\epsilon\) is the strain at the point R.
Also;

\[ \sigma = E \epsilon \]  \hspace{1cm} (2.21)

Relation between stress and strain for elastic material, Hooks Law

Substituting the value of \( \sigma \) from equation(2.21) into equation(2.20) yields

\[ U_R = \frac{1}{2} E \epsilon^2 \]  \hspace{1cm} (2.22)
Assuming plane sections remain same,

\[ \epsilon = \frac{du}{dx} \]  \hspace{1cm} (2.23)

\[ u = z \frac{dw}{dx} \]  \hspace{1cm} (2.24)

\[ \epsilon = z \frac{d^2 w}{dx^2} \]  \hspace{1cm} (2.25)

To obtain the internal energy for the whole beam we integrate the internal strain energy at point R over the volume.

The internal strain energy for the entire beam is given as:

\[ \int_{vol} U_R dv = U \]  \hspace{1cm} (2.26)

Substituting the value of \( \epsilon \) from equation (2.25) into (2.26) yields

\[ U = \int_{vol} \frac{1}{2} E \epsilon^2 dv \]  \hspace{1cm} (2.27)

Volume can be expressed as a product of area and length.

\[ dv = dA \, dx \]  \hspace{1cm} (2.28)
based on the above equation we now integrate equation (2.27) over the area and over
the length.

\[ U = \int_0^L \int_A \frac{1}{2} E c^2 dA dx \]  \hspace{1cm} (2.29)

Substituting the value of \( \epsilon \) from equation (2.25) into equation (2.28) yields

\[ U = \int_0^L \int_A \frac{1}{2} E (\frac{d^2 w}{d x^2})^2 dA dx \]  \hspace{1cm} (2.30)

Moment of Inertia \( I \) for the beam element is given as

\[ I = \int z^2 dA \]  \hspace{1cm} (2.31)

Substituting the value of \( I \) from equation (2.31) into equation (2.30) yields

\[ U = EI \int_0^L \frac{1}{2} (\frac{d^2 w}{d x^2})^2 dx \]  \hspace{1cm} (2.32)

The above equation for total internal strain energy can be rewritten as

\[ U = EI \int_0^L \frac{1}{2} (\frac{d^2 w}{d x^2}) (\frac{d^2 w}{d x^2}) dx \]  \hspace{1cm} (2.33)
The potential energy of the beam is nothing but the total internal strain energy. Therefore,

\[ \Pi = EI \int_0^L \frac{1}{2} \left( \frac{d^2 w}{dx^2} \right)^2 dx \]  
(2.34)

If \( A \) and \( B \) are two matrices then applying matrix property of the transpose, yields

\[(AB)^T = B^T A^T \]  
(2.35)

We can express the Potential Energy expressed in equation (2.34) in terms of displacement matrix \( W(x) \) equation (2.19) as,

\[ \Pi = \frac{1}{2} EI \int_0^L (W'')^T (W'') dx \]  
(2.36)

From equation (2.19) we have

\[
[W] = \begin{bmatrix} N1 & N2 & N3 & N4 \end{bmatrix} \begin{bmatrix} w1 \\ \theta1 \\ w2 \\ \theta2 \end{bmatrix}
\]  
(2.37)

\[
[W]^T = \begin{bmatrix} N1 \\ N2 \\ N3 \\ N4 \end{bmatrix} \begin{bmatrix} w1 & \theta1 & w2 & \theta2 \end{bmatrix}
\]  
(2.38)

Substituting the values of \( W \) and \( W^T \) from equation (2.37) and equation (2.38) in equation (2.36) yields

\[
\Pi = \frac{1}{2} EI \int_0^L \begin{bmatrix} w1 \\ \theta1 \\ w2 \\ \theta2 \end{bmatrix} \begin{bmatrix} N1'' \\ N2'' \\ N3'' \\ N4'' \end{bmatrix} \begin{bmatrix} w1 \\ \theta1 \\ w2 \\ \theta2 \end{bmatrix} dx
\]  
(2.39)
where $N_1$, $N_2$, $N_3$ and $N_4$ are the displacement shape functions for the two dimensional beam element as stated in equations (2.15) to (2.18). The displacements and rotations at end 1 is given by $w_1$, $\theta_1$ and at end 2 is given by $w_2$, $\theta_2$.

$$\Pi = \frac{1}{2} EI \int_0^L \begin{bmatrix} w_1 & \theta_1 & w_2 & \theta_2 \end{bmatrix} \begin{bmatrix} (N_1'') & N_1''N_2'' & N_1''N_3'' & N_1''N_4'' \\ N_2''N_1'' & (N_2'')^2 & N_2''N_3'' & N_2''N_4'' \\ N_3''N_1'' & N_3''N_2'' & (N_3'')^2 & N_3''N_4'' \\ N_4''N_1'' & N_4''N_2'' & N_4''N_3'' & (N_4'')^2 \end{bmatrix} \begin{bmatrix} w_1 \\ \theta_1 \\ w_2 \\ \theta_2 \end{bmatrix} \, dx$$

(2.40)

where

$$[K] = \int_0^L \begin{bmatrix} (N_1'') & N_1''N_2'' & N_1''N_3'' & N_1''N_4'' \\ N_2''N_1'' & (N_2'')^2 & N_2''N_3'' & N_2''N_4'' \\ N_3''N_1'' & N_3''N_2'' & (N_3'')^2 & N_3''N_4'' \\ N_4''N_1'' & N_4''N_2'' & N_4''N_3'' & (N_4'')^2 \end{bmatrix} \, dx$$

(2.41)

$$N_1 = (1 - x/l)^2(1 + 2x/l)$$

(2.42)

$$N_2 = (1 - x/l)^2x/l$$

(2.43)

$$N_3 = (x/l)^2(3 - 2x/l)$$

(2.44)

$$N_4 = -(1 - x/l)(x/l)^2$$

(2.45)

The element stiffness matrix for the beam is obtained by substituting the values of shape functions from equations (2.42) to (2.45) into equation(2.41) and integrating every element in the matrix in equation(2.40) over the length L.
The Element stiffness matrix for a beam element;

\[
[K^e] = \frac{EI}{l^3} \begin{bmatrix}
12 & 6l & -12 & 6l \\
6l & 4l^2 & -6l & 2l^2 \\
-12 & -6l & 12 & -6l \\
6l & 2l^2 & -6l & 4l^2
\end{bmatrix}
\] (2.46)
2.2.3 Forming the Matrix for the Force that conforms the Fluid to the Pipe

Consider a pipe carrying fluid and let R be a point at a distance x from a reference plane AB as shown in figure(2.12).

Due to the flow of the fluid through the pipe a force is introduced into the pipe causing the pipe to curve. This force conforms the fluid to the pipe at all times.

Let W be the transverse deflection of the pipe and \( \theta \) be angle made by the pipe due to the fluid flow with the neutral axis. \( \hat{i} \) and \( \hat{j} \) represent the unit vectors along the X and Y axis and \( \hat{r} \) and \( \hat{\theta} \) represent the two unit vectors at point R along the r and \( \theta \) axis. At point R, the vectors \( \hat{r} \) and \( \hat{\theta} \) can be expressed as

\[
\hat{r} = cos\theta \hat{i} + sin\theta \hat{j} \tag{2.47}
\]

\[
\hat{\theta} = -sin\theta \hat{i} + cos\theta \hat{j} \tag{2.48}
\]

Expression for slope at point R is given by;

\[
tan\theta = \frac{dW}{dx} \tag{2.49}
\]
Since the pipe undergoes a small deflection, hence $\theta$ is very small.

Therefore;

$$\tan \theta = \theta \quad (2.50)$$

ie

$$\theta = \frac{dW}{dx} \quad (2.51)$$

The displacement of a point $R$ at a distance $x$ from the reference plane can be expressed as;

$$\hat{R} = W \hat{j} + r \hat{r} \quad (2.52)$$

We differentiate the above equation to get velocity of the fluid at point $R$

$$\hat{\dot{R}} = \hat{W} \hat{j} + \hat{r} \hat{r} \quad (2.53)$$

where $v_f$ is the velocity of the fluid flow. Also at time $t$;

$$\dot{r} = \frac{d\hat{r}}{dt} \quad (2.54)$$

ie

$$\dot{r} = \frac{d\hat{r}}{d\theta} \frac{d\theta}{dt} = \dot{\theta} \hat{\theta} \quad (2.55)$$

Substituting the value of $\dot{r}$ in equation(2.53) yields

$$\hat{\dot{R}} = \hat{W} \hat{j} + \dot{r} \hat{r} + r \dot{\theta} \hat{\theta} \quad (2.56)$$

Substituting the value of $\dot{r}$ and $\dot{\theta}$ from equations(2.47) and (2.48) into equation(2.56) yields;

$$\hat{\dot{R}} = \hat{W} \hat{j} + \dot{r} [\cos \theta \hat{i} + \sin \theta \hat{j}] + r \dot{\theta} [-\sin \theta \hat{i} + \cos \theta \hat{j}] \quad (2.57)$$

Since $\theta$ is small

The velocity at point $R$ is expressed as;

$$\hat{R} = \hat{R}_x \hat{i} + \hat{R}_y \hat{j} \quad (2.58)$$
\[
\hat{R} = (\dot{r} - r\dot{\theta})\hat{i} + (\dot{W} + \dot{r}\theta + r\dot{\theta})\hat{j}
\]  
(2.60)

The Y component of velocity R causes the pipe carrying fluid to curve. Therefore,

\[
T = \frac{1}{2} \rho_f A \dot{R}_y \dot{R}_y
\]  
(2.61)

where T is the kinetic energy at the point R and \( \dot{R}_y \) is the Y component of velocity, \( \rho_f \) is the density of the fluid, A is the area of cross-section of the pipe.

Substituting the value of \( \dot{R}_y \) from equation (2.60) yields;

\[
T = \frac{1}{2} \rho_f A [\dot{W}^2 + \dot{r}^2\theta^2 + r^2\dot{\theta}^2 + 2\dot{W}\dot{r}\theta + 2\dot{W}\dot{\theta}r + 2\dot{r}\dot{\theta}\theta]
\]  
(2.62)

Substituting the value of \( \dot{r} \) from equation (2.54) and selecting the first, second and the fourth terms yields;

\[
T = \frac{1}{2} \rho_f A [\dot{W}^2 + v_f^2\theta^2 + 2\dot{W}v_f\theta]
\]  
(2.63)

Now substituting the value of \( \dot{\theta} \) from equation (2.51) into equation (2.63) yields;

\[
T = \frac{1}{2} \rho_f A \left[ \left( \frac{dW}{dt} \right)^2 + v_f^2 \left( \frac{dW}{dx} \right)^2 + 2v_f \left( \frac{dW}{dt} \right) \left( \frac{dW}{dx} \right) \right]
\]  
(2.64)

From the above equation we have these two terms;

\[
\frac{1}{2} \rho_f A v_f^2 \left( \frac{dW}{dx} \right)^2
\]  
(2.65)

\[
2\rho_f A v_f \left( \frac{dW}{dt} \right) \left( \frac{dW}{dx} \right)
\]  
(2.66)

The force acting on the pipe due to the fluid flow can be calculated by integrating the expressions in equations (2.65) and (2.66) over the length L.

\[
\int_L \frac{1}{2} \rho_f A v_f^2 \left( \frac{dW}{dx} \right)^2
\]  
(2.67)

The expression in equation (2.67) represents the force that causes the fluid to conform to the curvature of the pipe.

\[
\int_L 2\rho_f A v_f \left( \frac{dW}{dt} \right) \left( \frac{dW}{dx} \right)
\]  
(2.68)
The expression in equation (2.68) represents the coriolis force which causes the fluid in the pipe to whip.

The equation (2.67) can be expressed in terms of displacement shape functions derived for the pipe

\[ \Pi = T - V \]

\[ \Pi = \int_L \frac{1}{2} \rho_f A v_f^2 \left( \frac{dW}{dx} \right)^2 \]  \hspace{1cm} (2.69)

Rearranging the equation;

\[ \Pi = \rho_f A v_f^2 \int_L \frac{1}{2} \left( \frac{dW}{dx} \right) \left( \frac{dW}{dx} \right) \]  \hspace{1cm} (2.70)

For a pipe element, the displacement matrix in terms of shape functions can be expressed as

\[ [W(x)] = \begin{bmatrix} N_1 & N_2 & N_3 & N_4 \end{bmatrix} \begin{bmatrix} w_1 \\ \theta_1 \\ w_2 \\ \theta_2 \end{bmatrix} \]  \hspace{1cm} (2.71)

where N1, N2, N3 and N4 are the displacement shape functions pipe element as stated in equations (2.15) to (2.18). The displacements and rotations at end 1 is given by w1, \( \theta_1 \) and at end 2 is given by w2, \( \theta_2 \). Refer to figure (2.8).

Substituting the shape functions determined in equations (2.15) to (2.18)

\[ \Pi = \rho_f A v_f^2 \int_0^L \begin{bmatrix} w_1 & \theta_1 & w_2 & \theta_2 \end{bmatrix} \begin{bmatrix} N_1' \\ N_2' \\ N_3' \\ N_4' \end{bmatrix} \begin{bmatrix} w_1 \\ \theta_1 \\ w_2 \\ \theta_2 \end{bmatrix} dx \]  \hspace{1cm} (2.72)
\[ \Pi = \rho_f A v_f^2 \int_0^L \begin{bmatrix} w_1 & \theta_1 & w_2 & \theta_2 \end{bmatrix} \begin{bmatrix} (N1')^2 & N1'N2' & N1'N3' & N1'N4' \\ N2'N1' & (N2')^2 & N2'N3' & N2'N4' \\ N3'N1' & N3'N2' & (N3')^2 & N3'N4'' \\ N4'N1' & N4'N2' & N4'N3' & (N4')^2 \end{bmatrix} \begin{bmatrix} w_1 \\ \theta_1 \\ w_2 \\ \theta_2 \end{bmatrix} \ dx \] (2.73)

where

\[ [K_2] = \rho_f A v_f^2 \int_0^L \begin{bmatrix} (N1')^2 & N1'N2' & N1'N3' & N1'N4' \\ N2'N1' & (N2')^2 & N2'N3' & N2'N4' \\ N3'N1' & N3'N2' & (N3')^2 & N3'N4'' \\ N4'N1' & N4'N2' & N4'N3' & (N4')^2 \end{bmatrix} \ dx \] (2.74)

The matrix \( K_2 \) represents the force that conforms the fluid to the pipe.

Substituting the values of shape functions equations (2.15) to (2.18) and integrating it over the length gives us the elemental matrix for the above force.

\[ [K_2]^e = \frac{\rho A v^2}{30l} \begin{bmatrix} 36 & 3 & -36 & 3 \\ 3 & 4 & -3 & -1 \\ -36 & -3 & 36 & -3 \\ 3 & -1 & -3 & 4 \end{bmatrix} \] (2.75)
2.2.4 Dissipation Matrix Formulation for a Pipe carrying Fluid

The dissipation matrix represents the force that causes the fluid in the pipe to whip creating instability in the system. To formulate this matrix we recall equation (2.64) and (2.68)

The dissipation function is given by:

\[ D = \int_{L} 2\rho_f A v_f \left( \frac{dW}{dt} \right) \left( \frac{dW}{dx} \right) \]  \hspace{1cm} (2.76)

Where \( L \) is the length of the pipe element, \( \rho_f \) is the density of the fluid, \( A \) area of cross-section of the pipe, and \( v_f \) velocity of the fluid flow.

Recalling the displacement shape functions mentioned in equations (2.15) to (2.18);

\[ N_1 = (1 - x/l)^2(1 + 2x/l) \]  \hspace{1cm} (2.77)
\[ N_2 = (1 - x/l)^2 x/l \]  \hspace{1cm} (2.78)
\[ N_3 = (x/l)^2(3 - 2x/l) \]  \hspace{1cm} (2.79)
\[ N_4 = -(1 - x/l)(x/l)^2 \]  \hspace{1cm} (2.80)

The Dissipation Matrix can be expressed in terms of its displacement shape functions as shown in equations (2.77) to (2.80).

\[ D = 2\rho_f A v_f \int_{0}^{L} \begin{bmatrix} w_1 & \theta_1 & w_2 & \theta_2 \end{bmatrix} \begin{bmatrix} N_1' \\ N_2' \\ N_3' \\ N_4' \end{bmatrix} \begin{bmatrix} N_1 & N_2 & N_3 & N_4 \end{bmatrix} \begin{bmatrix} w_1 \\ \theta_1 \\ w_2 \\ \theta_2 \end{bmatrix} dx \]  \hspace{1cm} (2.81)

\[ 2\rho_f A v_f \int_{0}^{L} \begin{bmatrix} w_1 & \theta_1 & w_2 & \theta_2 \end{bmatrix} \begin{bmatrix} (N_1')^2 & N_1'N_2' & N_1'N_3' & N_1'N_4' \\ N_2'N_1' & (N_2')^2 & N_2'N_3' & N_2'N_4' \\ N_3'N_1' & N_3'N_2' & (N_3')^2 & N_3'N_4' \\ N_4'N_1' & N_4'N_2' & N_4'N_3' & (N_4')^2 \end{bmatrix} \begin{bmatrix} w_1 \\ \theta_1 \\ w_2 \\ \theta_2 \end{bmatrix} dx \]  \hspace{1cm} (2.82)
Substituting the values of shape functions from equations (2.77) to (2.80) and integrating over the length L yields:

\[
[D]^e = \frac{\rho A v}{30} \begin{bmatrix}
-30 & 6 & 30 & -6 \\
6 & 0 & 6 & -1 \\
-30 & -6 & 30 & 6 \\
6 & 1 & -6 & 0 \\
\end{bmatrix}
\]  

(2.83)

\([D]^e\) represents the elemental dissipation matrix.
2.2.5 Inertia Matrix Formulation for a Pipe carrying Fluid

Consider an element in the pipe having an area $dA$, length $x$, volume $dv$ and mass $dm$. The density of the pipe is $\rho$ and let $W$ represent the transverse displacement of the pipe. The displacement model for the

Assuming the displacement model of the element to be

$$\vec{W}(x,t) = [N]\vec{\omega}(t) \quad (2.84)$$

where $\vec{W}$ is the vector of displacements, $[N]$ is the matrix of shape functions and $\vec{\omega}$ is the vector of nodal displacements which is assumed to be a function of time.

Let the nodal displacement be expressed as;

$$\vec{W} = we^{iwt} \quad (2.85)$$

Nodal Velocity can be found by differentiating the equation() with time.

$$\dot{\vec{W}} = (iw)we^{iwt} \quad (2.86)$$

Kinetic Energy of a particle can be expressed as a product of mass and the square of velocity

$$T = \frac{1}{2}mv^2 \quad (2.87)$$

Kinetic energy of the element can be found out by integrating equation(2.87) over the volume. Also, mass can be expressed as the product of density and volume ie $dm = \rho dv$

$$T = \int_v \frac{1}{2} \rho \dot{\vec{W}}^2 dv \quad (2.88)$$

The volume of the element can be expressed as the product of area and the length.

$$dv = dA*dx \quad (2.89)$$

Substituting the value of volume $dv$ from equation(2.89) into equation(2.88) and integrating over the area and the length yields;

$$T = \frac{-u^2}{2} \int_A \int_L \rho \dot{\vec{W}}^2 dA*dx \quad (2.90)$$
\[
\int_A \rho dA = \rho A
\]  
(2.91)

Substituting the value of \( \int_A \rho dA \) in equation (2.90) yields;

\[
T = -\frac{\rho A w^2}{2} \int_L \dot{\vec{W}}^2 dx
\]  
(2.92)

Equation (2.92) can be written as;

\[
T = -\frac{\rho A w^2}{2} \int_L \dot{\vec{W}} \dot{\vec{W}} dx
\]  
(2.93)

The Lagrange equations are given by

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{w}} \right) - \left( \frac{\partial L}{\partial w} \right) = (0)
\]  
(2.94)

where

\[
L = T - V
\]  
(2.95)

is called the Lagrangian function, \( T \) is the kinetic energy, \( V \) is the potential energy, \( W \) is the nodal displacement and \( \dot{W} \) is the nodal velocity. The kinetic energy of the element "e" can be expressed as

\[
T^e = -\frac{\rho A w^2}{2} \int_L \dot{\vec{W}}^T \dot{\vec{W}} dx
\]  
(2.96)

and where \( \rho \) is the density and \( \dot{\vec{W}} \) is the vector of velocities of element e. The expression for \( T \) using the eq (2.19) to (2.21) can be written as;

\[
T^e = -\frac{\rho A w^2}{2} \int_L \begin{bmatrix} w1 & \theta1 & w2 & \theta2 \\
N1 \\
N2 \\
N3 \\
N4 \\
\end{bmatrix} \begin{bmatrix} N1 \\
N2 \\
N3 \\
N4 \\
\end{bmatrix} \begin{bmatrix} w1 \\
\theta1 \\
w2 \\
\theta2 \\
\end{bmatrix} dx
\]  
(2.97)
Rewriting the above expression we get;

\[
T^e = -\frac{\rho A w^2}{2} \int_L \begin{bmatrix} w1 & \theta1 & w2 & \theta2 \end{bmatrix} \begin{bmatrix} (N1)^2 & N1N2 & N1N3 & N1N4 \\ N2N1 & (N2)^2 & N2N3 & N2N4 \\ N3N1 & N3N2 & (N3)^2 & N3N4 \\ N4N1 & N4N2 & N4N3 & (N4)^2 \end{bmatrix} \begin{bmatrix} w1 \\ \theta1 \\ w2 \\ \theta2 \end{bmatrix} dx
\]

(2.98)

Recalling the shape functions derived in equations (2.15) to (2.18)

\[
N1 = (1 - x/l)^2(1 + 2x/l)
\]

(2.99)

\[
N2 = (1 - x/l)^2x/l
\]

(2.100)

\[
N3 = (x/l)^2(3 - 2x/l)
\]

(2.101)

\[
N4 = -(1 - x/l)(x/l)^2
\]

(2.102)

Substituting the shape functions from eqs (2.99) to (2.102) into eqs (2.98) yields the elemental mass matrix for a pipe.

\[
[M]^e = \frac{Ml}{420} \begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix}
\]

(2.103)
CHAPTER III

FLOW INDUCED VIBRATIONS IN PIPES, A FINITE ELEMENT APPROACH

3.1 Forming Global Stiffness Matrix from Elemental Stiffness Matrices

In order to form a Global Matrix, we start with a 6x6 null matrix, with its six degrees of freedom being translation and rotation of each of the nodes. So our Global Stiffness matrix looks like this:

\[ K_{\text{Global}} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix} \]  

(3.1)
The two 4x4 element stiffness matrices are:

\[
[k_1^e] = \frac{EI}{l^3} \begin{bmatrix}
12 & 6l & -12 & 6l \\
6l & 4l^2 & -6l & 2l^2 \\
-12 & -6l & 12 & -6l \\
6l & 2l^2 & -6l & 4l^2
\end{bmatrix}
\] (3.2)

\[
[k_2^e] = \frac{EI}{l^3} \begin{bmatrix}
12 & 6l & -12 & 6l \\
6l & 4l^2 & -6l & 2l^2 \\
-12 & -6l & 12 & -6l \\
6l & 2l^2 & -6l & 4l^2
\end{bmatrix}
\] (3.3)

We shall now build the global stiffness matrix by inserting element 1 first into the global stiffness matrix.

\[
K_{Global} = \frac{EI}{l^3} \begin{bmatrix}
12 & 6l & -12 & 6l & 0 & 0 \\
6l & 4l^2 & -6l & 2l^2 & 0 & 0 \\
-12 & -6l & 12 & -6l & 0 & 0 \\
6l & 2l^2 & -6l & 4l^2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\] (3.4)

Inserting element 2 into the global stiffness matrix

\[
K_{Global} = \frac{EI}{l^3} \begin{bmatrix}
12 & 6l & -12 & 6l & 0 & 0 \\
6l & 4l^2 & -6l & 2l^2 & 0 & 0 \\
-12 & -6l & (12 + 12) & (-6l + 6l) & -12 & 6l \\
6l & 2l^2 & (-6l + 6l) & (4l^2 + 4l^2) & -6l & 2l^2 \\
0 & 0 & -12 & -6l & 12 & -6l \\
0 & 0 & 6l & 2l^2 & -6l & 4l^2
\end{bmatrix}
\] (3.5)
3.2 Applying Boundary Conditions to Global Stiffness Matrix for simply supported pipe with fluid flow

When the boundary conditions are applied to a simply supported pipe carrying fluid, the 6x6 Global Stiffness Matrix formulated in eq(3.5) is modified to a 4x4 Global Stiffness Matrix. It is as follows;

\[
K_{Global} = \frac{EI}{l^3} \begin{bmatrix}
4l^2 & -6l & 2l^2 & 0 \\
-6l & (12 + 12) & (-6l + 6l) & 6l \\
2l^2 & (-6l + 6l) & (4l^2 + 4l^2) & 2l^2 \\
0 & 6l & 2l^2 & 4l^2
\end{bmatrix} \tag{3.6}
\]

Since the pipe is supported at the two ends the pipe does not deflect causing its two translational degrees of freedom to go to zero. Hence we end up with the Stiffness Matrix shown in eq(3.6)
3.3 Applying Boundary Conditions to Global Stiffness Matrix for a cantilever pipe with fluid flow

When the boundary conditions are applied to a Cantilever pipe carrying fluid, the 6x6 Global Stiffness Matrix formulated in eq(3.5) is modified to a 4x4 Global Stiffness Matrix. It is as follows:

\[
K_{\text{GlobalS}} = \frac{EI}{l^3} \begin{bmatrix}
(12 + 12) & (-6l + 6l) & -12 & 6l \\
(-6l + 6l) & (4l^2 + 4l^2) & -6l & 2l^2 \\
-12 & -6l & 12 & -6l \\
6l & 2l^2 & -6l & 4l^2
\end{bmatrix}
\] (3.7)

Since the pipe is supported at one end, the pipe does not deflect or rotate at that end causing translational and rotational degrees of freedom at that end to go to zero. Hence we end up with the Stiffness Matrix shown in eq(3.8).
3.4 MATLAB Programs for Assembling Global Matrices for Simply Supported and Cantilever pipe carrying fluid

In this section, we implement the method discussed in section (3.1) to (3.3) to form global matrices from the developed elemental matrices of a straight fluid conveying pipe and these assembled matrices are later solved for the natural frequency and onset of instability of a cantilever and simply supported pipe carrying fluid utilizing MATLAB Programs. Consider a pipe of length \( L \), modulus of elasticity \( E \) has fluid flowing with a velocity \( v \) through its inner cross-section having an outside diameter \( od \), and thickness \( t1 \). The expression for critical velocity and natural frequency of the simply supported pipe carrying fluid is given by;

\[
wn = \frac{(3.14)^2}{L^2} \sqrt{\frac{E \times I}{M}} \quad (3.8)
\]

\[
vc = \frac{3.14}{L} \sqrt{\frac{E \times I}{\rho A}} \quad (3.9)
\]

3.5 MATLAB program for a simply supported pipe carrying fluid

The number of elements, density, length, modulus of elasticity of the pipe, density and velocity of fluid flowing through the pipe and the thickness of the pipe can be defined by the user.

Refer to Appendix 1 for the complete MATLAB Program.
3.6 MATLAB program for a cantilever pipe carrying fluid

![Figure 3.3: Pinned-Free Pipe Carrying Fluid*](image)

The number of elements, density, length, modulus of elasticity of the pipe, density and velocity of fluid flowing through the pipe and the thickness of the pipe can be defined by the user. The expression for critical velocity and natural frequency of the cantilever pipe carrying fluid is given by;

\[
wn = \left(\frac{(1.875)^2}{L^2}\right)\sqrt{\frac{E \cdot I}{M}}
\] (3.10)

Where,

\[
wn = \left(\frac{(an^2)}{L^2}\right)\sqrt{\frac{EI}{M}}an = 1.875, 4.694, 7.855
\]

\[
v_c = (1.875/L)\sqrt{\frac{E \cdot I}{\rho A}}
\] (3.11)

Refer to Appendix 2 for the complete MATLAB Program.

---

* Flow Induced Vibrations, Robert D. Blevins, Krieger, 1977, P 297
CHAPTER IV

FLOW INDUCED VIBRATIONS IN PIPES, A FINITE ELEMENT APPROACH

4.1 Parametric Study

Parametric study has been carried out in this chapter.

The study is carried out on a single span steel pipe with a 0.01 m (0.4 in.) diameter and a .0001 m (0.004 in.) thick wall.

The other parameters are:

<table>
<thead>
<tr>
<th>Density of the pipe $\rho_p (Kg/m^3)$</th>
<th>Density of the fluid $\rho_f (Kg/m^3)$</th>
<th>Length of the pipe L (m)</th>
<th>Number of elements n</th>
<th>Modulus of Elasticity $E$ (Gpa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8000</td>
<td>1000</td>
<td>2</td>
<td>10</td>
<td>207</td>
</tr>
</tbody>
</table>

MATLAB program for the simply supported pipe with fluid flow is utilized for these set of parameters with varying fluid velocity.

Results from this study are shown in the form of graphs and tables. The fundamental frequency of vibration and the critical velocity of fluid for a simply supported pipe...
carrying fluid are:

\( \omega_n \) 21.8582 rad/sec

\( v_c \) 16.0553 m/sec

Table 4.1: Reduction of Fundamental Frequency for a Pinned-Pinned Pipe with increasing Flow Velocity

<table>
<thead>
<tr>
<th>Velocity of Fluid (v)</th>
<th>Velocity Ratio (v/vc)</th>
<th>Frequency (w)</th>
<th>Frequency Ratio (w/( \omega_n ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>21.8806</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0.1246</td>
<td>21.5619</td>
<td>0.9864</td>
</tr>
<tr>
<td>4</td>
<td>0.2491</td>
<td>20.5830</td>
<td>0.9417</td>
</tr>
<tr>
<td>6</td>
<td>0.3737</td>
<td>18.8644</td>
<td>0.8630</td>
</tr>
<tr>
<td>8</td>
<td>0.4983</td>
<td>16.2206</td>
<td>0.7421</td>
</tr>
<tr>
<td>10</td>
<td>0.6228</td>
<td>12.1602</td>
<td>0.5563</td>
</tr>
<tr>
<td>12</td>
<td>0.7474</td>
<td>3.7349</td>
<td>0.1709</td>
</tr>
<tr>
<td>14</td>
<td>0.8720</td>
<td>0.3935</td>
<td>0.0180</td>
</tr>
<tr>
<td>16.0553</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
The fundamental frequency of vibration and the critical velocity of fluid for a Cantilever pipe carrying fluid are:

\[ \omega_n = 7.7940 \text{ rad/sec} \]

\[ v_c = 9.5872 \text{ m/sec} \]
Figure 4.2: **Shape Function Plot for a Cantilever Pipe with increasing Flow Velocity**

Table 4.2: **Reduction of Fundamental Frequency for a Pinned-Free Pipe with increasing Flow Velocity**

<table>
<thead>
<tr>
<th>Velocity of Fluid(v)</th>
<th>Velocity Ratio(v/vc)</th>
<th>Frequency(w)</th>
<th>Frequency Ratio(w/wn)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>7.7940</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0.2086</td>
<td>7.5968</td>
<td>0.9747</td>
</tr>
<tr>
<td>4</td>
<td>0.4172</td>
<td>6.9807</td>
<td>0.8957</td>
</tr>
<tr>
<td>6</td>
<td>0.6258</td>
<td>5.8549</td>
<td>0.7512</td>
</tr>
<tr>
<td>8</td>
<td>0.8344</td>
<td>3.8825</td>
<td>0.4981</td>
</tr>
<tr>
<td>9</td>
<td>0.9388</td>
<td>1.9897</td>
<td>0.2553</td>
</tr>
<tr>
<td>9.5872</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Figure 4.3: Reduction of Fundamental Frequency for a Cantilever Pipe with increasing Flow Velocity
CHAPTER V

FLOW INDUCED VIBRATIONS IN PIPES, A FINITE ELEMENT APPROACH

5.1 Tapered Pipe Carrying Fluid

Consider a pipe of length L, modulus of elasticity E. A fluid flows through the pipe at a velocity v and density $\rho$ through the internal pipe cross-section. As the fluid flows through the deflecting pipe it is accelerated, because of the changing curvature.
of the pipe and the lateral vibration of the pipeline. The vertical component of fluid pressure applied to the fluid element and the pressure force F per unit length applied on the fluid element by the tube walls oppose these accelerations.

The input parameters are given by the user.

<table>
<thead>
<tr>
<th>Density of the pipe $\rho_p (Kg/m^3)$</th>
<th>Density of the fluid $\rho_f (Kg/m^3)$</th>
<th>Length of the pipe L (m)</th>
<th>Number of elements n</th>
<th>Modulus of Elasticity E (Gpa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8000</td>
<td>1000</td>
<td>2</td>
<td>10</td>
<td>207</td>
</tr>
</tbody>
</table>

For these user defined values we introduce a taper in the pipe so that the material property and the length of the pipe with the taper or without the taper remain the same. This is done by keeping the inner diameter of the pipe constant and varying the outer diameter. Refer to figure (5.2)

The pipe tapers from one end having a thickness $x$ to the other end having a thickness $t = 0.01 mm$ such that the volume of material is equal to the volume of material

![Pipe Carrying Fluid](image_url)

![Tapered Pipe Carrying Fluid](image_url)

Figure 5.2: Introducing a Taper in the Pipe Carrying Fluid
for a pipe with no taper.

The thickness $x$ of the tapered pipe is now calculated:

From figure(5.2) we have

- Outer Diameter of the pipe with no taper (OD) 10 mm
- Inner Diameter of the pipe (ID) 9.8 mm
- Outer Diameter of thick end of the Tapered pipe ($OD_1$)
- Length of the pipe (L) 2000 mm
- Thickness of thin end of the taper (t) 0.01 mm
- Thickness of thick end of the taper $x$ mm

Volume of the pipe without the taper:

$$V_1 = \frac{\pi}{4}(OD^2 - ID^2)L$$ (5.1)

Volume of the pipe with the taper:

$$V_2 = \left[\frac{\pi}{4}(OD_1^2) + \frac{\pi}{4}(ID + 2t)^2\right] \frac{L}{3} - \left[\frac{\pi}{4}(ID^2)\right]$$ (5.2)

Since the volume of material distributed over the length of the two pipes is equal

We have,

$$V_1 = V_2$$ (5.3)

Substituting the value for $V_1$ and $V_2$ from equations (5.1) and (5.2) into equation (5.3) yields

$$\frac{\pi}{4}(OD^2 - ID^2)L = \left[\frac{\pi}{4}(OD_1^2) + \frac{\pi}{4}(ID + 2t)^2\right] \frac{L}{3} - \left[\frac{\pi}{4}(ID^2)\right]$$ (5.4)

The outer diameter for the thick end of the tapered pipe can be expressed as

$$OD_1 = ID + 2x$$ (5.5)
Substituting values of outer diameter(OD), inner diameter(ID), length(L) and thickness(t) into equation (5.6) yields

$$\frac{\pi}{4}(10^2 - 9.8^2)2000 = \left[\frac{\pi}{4}(9.8 + 2x)^2 + \frac{\pi}{4}(9.8 + 0.02)^2\right] \frac{2000}{3} - \left[\frac{\pi}{4}(9.8^2)\right]$$

(5.6)

Solving equation (5.6) yields

$$x = 2.24mm$$

(5.7)

Substituting the value of thickness x into equation (5.5) we get the outer diameter $OD_1$ as

$$OD_1 = 14.268mm$$

(5.8)

Thus, the taper in the pipe varies from an outer diameter of 14.268 mm to 9.82 mm.
The following MATLAB program is utilized to calculate the fundamental natural frequency of vibration for a tapered pipe carrying fluid. Refer to Appendix 3 for the complete MATLAB program. Results obtained from the program are given in table (5.1)

Table 5.1: Reduction of Fundamental Frequency for a Tapered pipe with increasing Flow Velocity

<table>
<thead>
<tr>
<th>Velocity of Fluid(v)</th>
<th>Velocity Ratio(v/vc)</th>
<th>Frequency(w)</th>
<th>Frequency Ratio(w/wn)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>40.8228</td>
<td>.8100</td>
</tr>
<tr>
<td>20</td>
<td>0.1935</td>
<td>40.083</td>
<td>0.7784</td>
</tr>
<tr>
<td>40</td>
<td>0.3870</td>
<td>37.7783</td>
<td>0.7337</td>
</tr>
<tr>
<td>60</td>
<td>0.5806</td>
<td>33.5980</td>
<td>0.6525</td>
</tr>
<tr>
<td>80</td>
<td>0.7741</td>
<td>26.5798</td>
<td>0.5162</td>
</tr>
<tr>
<td>100</td>
<td>0.9676</td>
<td>10.7122</td>
<td>0.2080</td>
</tr>
<tr>
<td>103.3487</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The fundamental frequency of vibration and the critical velocity of fluid for a tapered pipe carrying fluid obtained from the MATLAB program are:

\[ \omega_n = 51.4917 \text{ rad/sec} \]

\[ v_c = 103.3487 \text{ m/sec} \]
In the present work, we have utilized numerical method techniques to form the basic elemental matrices for the pinned-pinned and pinned-free pipe carrying fluid. Matlab programs have been developed and utilized to form global matrices from these elemental matrices and fundamental frequency for free vibration has been calculated for various pipe configurations and varying fluid flow velocities.

Consider a pipe carrying fluid having the following user defined parameters.

![Diagram of pipe carrying fluid and tapered pipe carrying fluid](image)

Figure 6.1: Representation of Pipe Carrying Fluid and Tapered Pipe Carrying Fluid
<table>
<thead>
<tr>
<th>Density of the pipe $\rho_p (Kg/m^3)$</th>
<th>Density of the fluid $\rho_f (Kg/m^3)$</th>
<th>Length of the pipe $L (m)$</th>
<th>Number of elements $n$</th>
<th>Modulus of Elasticity $E$ (Gpa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8000</td>
<td>1000</td>
<td>2</td>
<td>10</td>
<td>207</td>
</tr>
</tbody>
</table>

Refer to *Appendix 1* and *Appendix 3* for the complete MATLAB program.

Parametric study carried out on a pinned-pinned and tapered pipe for the same material of the pipe and subjected to the same conditions reveal that the tapered pipe is more stable than a pinned-pinned pipe.

Comparing the following set of tables justifies the above statement.

The fundamental frequency of vibration and the critical velocity of fluid for a tapered and a pinned-pinned pipe carrying fluid are:

$\omega_{n_t}$ 51.4917 rad/sec

$\omega_{n_p}$ 21.8582 rad/sec

$v_{c_t}$ 103.3487 m/sec

$v_{c_p}$ 16.0553 m/sec

**Table 6.1: Reduction of Fundamental Frequency for a Tapered Pipe with increasing Flow Velocity**

<table>
<thead>
<tr>
<th>Velocity of Fluid ($v$)</th>
<th>Velocity Ratio ($v/v_c$)</th>
<th>Frequency ($\omega$)</th>
<th>Frequency Ratio ($\omega/\omega_n$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>40.8228</td>
<td>0.8100</td>
</tr>
<tr>
<td>20</td>
<td>0.1935</td>
<td>40.083</td>
<td>0.7784</td>
</tr>
<tr>
<td>40</td>
<td>0.3870</td>
<td>37.7783</td>
<td>0.7337</td>
</tr>
<tr>
<td>60</td>
<td>0.5806</td>
<td>33.5980</td>
<td>0.6525</td>
</tr>
<tr>
<td>80</td>
<td>0.7741</td>
<td>26.5798</td>
<td>0.5162</td>
</tr>
<tr>
<td>100</td>
<td>0.9676</td>
<td>10.7122</td>
<td>0.2080</td>
</tr>
<tr>
<td>103.3487</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 6.2: **Reduction of Fundamental Frequency for a Pinned-Pinned Pipe with increasing Flow Velocity**

<table>
<thead>
<tr>
<th>Velocity of Fluid (v)</th>
<th>Velocity Ratio (v/vc)</th>
<th>Frequency (w)</th>
<th>Frequency Ratio (w/wn)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>21.8806</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0.1246</td>
<td>21.5619</td>
<td>0.9864</td>
</tr>
<tr>
<td>4</td>
<td>0.2491</td>
<td>20.5830</td>
<td>0.9417</td>
</tr>
<tr>
<td>6</td>
<td>0.3737</td>
<td>18.8644</td>
<td>0.8630</td>
</tr>
<tr>
<td>8</td>
<td>0.4983</td>
<td>16.2206</td>
<td>0.7421</td>
</tr>
<tr>
<td>10</td>
<td>0.6228</td>
<td>12.1602</td>
<td>0.5563</td>
</tr>
<tr>
<td>12</td>
<td>0.7474</td>
<td>3.7349</td>
<td>0.1709</td>
</tr>
<tr>
<td>14</td>
<td>0.8720</td>
<td>0.3935</td>
<td>0.0180</td>
</tr>
<tr>
<td>16.0553</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The fundamental frequency for vibration and critical velocity for the onset of instability in tapered pipe is approximately three times larger than the pinned-pinned pipe, thus making it more stable.
6.1 Contribution of the Thesis

- Developed Finite Element Model for vibration analysis of a Pipe Carrying Fluid.
- Implemented the above developed model to two different pipe configurations: Simply Supported and Cantilever Pipe Carrying Fluid.
- Developed MATLAB Programs to solve the Finite Element Models.
- Determined the effect of fluid velocities and density on the vibrations of a thin walled Simply Supported and Cantilever pipe carrying fluid.
- The critical velocity and natural frequency of vibrations were determined for the above configurations.
- Study was carried out on a variable wall thickness pipe and the results obtained show that the critical fluid velocity can be increased when the wall thickness is tapered.

6.2 Future Scope

- Turbulence in Two-Phase Fluids
  In single-phase flow, fluctuations are a direct consequence of turbulence developed in fluid, whereas the situation is clearly more complex in two-phase flow since the fluctuation of the mixture itself is added to the inherent turbulence of each phase.
- Extend the study to a time dependent fluid velocity flowing through the pipe.
BIBLIOGRAPHY


Appendices
0.1 MATLAB program for Simply Supported Pipe Carrying Fluid

MATLAB program for Simply Supported Pipe Carrying Fluid.

% The following MATLAB Program calculates the Fundamental
% Natural frequency of vibration, frequency ratio (w/wn)
% and velocity ratio (v/vc), for a
% simply supported pipe carrying fluid.
% In order to perform the above task the program assembles
% Elemental Stiffness, Dissipation, and Inertia matrices
% to form Global Matrices which are used to calculate
% Fundamental Natural
% Frequency w.

clc;

num_elements = input( 'Input number of elements for beam:' );
% num_elements = The user enters the number of elements
% in which the pipe
% has to be divided.

n=1:num_elements+1; % Number of nodes (n) is equal to number of
% elements plus one

node1=1:num_elements;
node2=2:num_elements+1;
max_node1=max(node1);
max_node2=max(node2);
max_node_used=max([max_node1 max_node2]);
mnu=max_node_used;
k=zeros(2*mnu); % Creating a Global Stiffness Matrix of zeros
m = zeros(2*mnu); % Creating Global Mass Matrix of zeros
x = zeros(2*mnu); % Creating Global Matrix of zeros
% for the force that conforms fluid
% to the curvature of the
% pipe
d = zeros(2*mnu); % Creating Global Dissipation Matrix of zeros
%(Coriolis Component)
t = num_elements*2;
L = 2; % Total length of the pipe in meters
l = L/num_elements; % Length of an element
t1 = .0001; % thickness of the pipe in meter
od = .01; % outer diameter of the pipe
id = od - 2*t1 % inner diameter of the pipe
I = pi*(od^4-id^4)/64 % moment of inertia of the pipe
E = 207*10^9; % Modulus of elasticity of the pipe
roh = 8000; % Density of the pipe
rohw = 1000; % density of water (Fluid)
M = roh*pi*(od^2-id^2)/4 + rohw*pi*.25*id^2; % mass per unit length of
% the pipe + fluid
rohA = rohw*pi*.25*id^2;
l = L/num_elements;
v = 0 % velocity of the fluid flowing through the pipe
% v = 16.0553
z = rohA/M
i = sqrt(-1);
wn = ((3.14)^2/L^2)*sqrt(E*I/M) % Natural Frequency
vc = (3.14/L)*sqrt(E*I/rohA) % Critical Velocity
% Assembling Global Stiffness, Dissipation and Inertia Matrices

for j = 1:num_elements
    dof1 = 2 * node1(j) - 1;
    dof2 = 2 * node1(j);
    dof3 = 2 * node2(j) - 1;
    dof4 = 2 * node2(j);

    % Stiffness Matrix Assembly
    k(dof1, dof1) = k(dof1, dof1) + (12 * E * I / l^3);
    k(dof2, dof1) = k(dof2, dof1) + (6 * E * I / l^2);
    k(dof3, dof1) = k(dof3, dof1) - (12 * E * I / l^3);
    k(dof4, dof1) = k(dof4, dof1) + (6 * E * I / l^2);

    k(dof1, dof2) = k(dof1, dof2) + (6 * E * I / l^2);
    k(dof2, dof2) = k(dof2, dof2) + (4 * E * I / l);
    k(dof3, dof2) = k(dof3, dof2) - (6 * E * I / l^2);
    k(dof4, dof2) = k(dof4, dof2) + (2 * E * I / l);

    k(dof1, dof3) = k(dof1, dof3) - (12 * E * I / l^3);
    k(dof2, dof3) = k(dof2, dof3) - (6 * E * I / l^2);
    k(dof3, dof3) = k(dof3, dof3) + (12 * E * I / l^3);
    k(dof4, dof3) = k(dof4, dof3) - (6 * E * I / l^2);

    k(dof1, dof4) = k(dof1, dof4) + (6 * E * I / l^2);
    k(dof2, dof4) = k(dof2, dof4) + (2 * E * I / l);
    k(dof3, dof4) = k(dof3, dof4) - (6 * E * I / l^2);
    k(dof4, dof4) = k(dof4, dof4) + (4 * E * I / l);

    %
% Matrix assembly for the second term i.e
% for the force that conforms
% fluid to the curvature of the pipe
x(dof1, dof1)=x(dof1, dof1)+ ((36*rohA*v^2)/30*1);
x(dof2, dof1)=x(dof2, dof1)+ ((3*rohA*v^2)/30*1);
x(dof3, dof1)=x(dof3, dof1)+ ((−36*rohA*v^2)/30*1);
x(dof4, dof1)=x(dof4, dof1)+ ((3*rohA*v^2)/30*1);

x(dof1, dof2)=x(dof1, dof2)+ ((3*rohA*v^2)/30*1);
x(dof2, dof2)=x(dof2, dof2)+ ((4*rohA*v^2)/30*1);
x(dof3, dof2)=x(dof3, dof2)+ ((−3*rohA*v^2)/30*1);
x(dof4, dof2)=x(dof4, dof2)+ ((−1*rohA*v^2)/30*1);

x(dof1, dof3)=x(dof1, dof3)+ ((−36*rohA*v^2)/30*1);
x(dof2, dof3)=x(dof2, dof3)+ ((−3*rohA*v^2)/30*1);
x(dof3, dof3)=x(dof3, dof3)+ ((36*rohA*v^2)/30*1);
x(dof4, dof3)=x(dof4, dof3)+ ((−3*rohA*v^2)/30*1);

x(dof1, dof4)=x(dof1, dof4)+ ((3*rohA*v^2)/30*1);
x(dof2, dof4)=x(dof2, dof4)+ ((−1*rohA*v^2)/30*1);
x(dof3, dof4)=x(dof3, dof4)+ ((−3*rohA*v^2)/30*1);
x(dof4, dof4)=x(dof4, dof4)+ ((4*rohA*v^2)/30*1);

% Dissipation Matrix Assembly
% dof1
d(dof1, dof1)=d(dof1, dof1)+ (2*(-30*rohA*v)/60);
d(dof2, dof1)=d(dof2, dof1)+ (2*(6*rohA*v)/60);
d(dof3, dof1)=d(dof3, dof1)+ (2*(30*rohA*v)/60);
\[ d(\text{dof}_4, \text{dof}_1) = d(\text{dof}_4, \text{dof}_1) + \left( 2 \times \frac{-6 \times \text{rohA} \times \text{v}}{60} \right); \]
\[ d(\text{dof}_1, \text{dof}_2) = d(\text{dof}_1, \text{dof}_2) + \left( 2 \times \frac{-6 \times \text{rohA} \times \text{v}}{60} \right); \]
\[ d(\text{dof}_2, \text{dof}_2) = d(\text{dof}_2, \text{dof}_2) + \left( 2 \times \frac{0 \times \text{rohA} \times \text{v}}{60} \right); \]
\[ d(\text{dof}_3, \text{dof}_2) = d(\text{dof}_3, \text{dof}_2) + \left( 2 \times \frac{6 \times \text{rohA} \times \text{v}}{60} \right); \]
\[ d(\text{dof}_4, \text{dof}_2) = d(\text{dof}_4, \text{dof}_2) + \left( 2 \times \frac{-1 \times \text{rohA} \times \text{v}}{60} \right); \]
\[ d(\text{dof}_1, \text{dof}_3) = d(\text{dof}_1, \text{dof}_3) + \left( 2 \times \frac{-30 \times \text{rohA} \times \text{v}}{60} \right); \]
\[ d(\text{dof}_2, \text{dof}_3) = d(\text{dof}_2, \text{dof}_3) + \left( 2 \times \frac{-6 \times \text{rohA} \times \text{v}}{60} \right); \]
\[ d(\text{dof}_3, \text{dof}_3) = d(\text{dof}_3, \text{dof}_3) + \left( 2 \times \frac{30 \times \text{rohA} \times \text{v}}{60} \right); \]
\[ d(\text{dof}_4, \text{dof}_3) = d(\text{dof}_4, \text{dof}_3) + \left( 2 \times \frac{6 \times \text{rohA} \times \text{v}}{60} \right); \]
\[ d(\text{dof}_1, \text{dof}_4) = d(\text{dof}_1, \text{dof}_4) + \left( 2 \times \frac{6 \times \text{rohA} \times \text{v}}{60} \right); \]
\[ d(\text{dof}_2, \text{dof}_4) = d(\text{dof}_2, \text{dof}_4) + \left( 2 \times \frac{1 \times \text{rohA} \times \text{v}}{60} \right); \]
\[ d(\text{dof}_3, \text{dof}_4) = d(\text{dof}_3, \text{dof}_4) + \left( 2 \times \frac{-6 \times \text{rohA} \times \text{v}}{60} \right); \]
\[ d(\text{dof}_4, \text{dof}_4) = d(\text{dof}_4, \text{dof}_4) + \left( 2 \times \frac{0 \times \text{rohA} \times \text{v}}{60} \right); \]

\% Inertia Matrix Assembly

\[ m(\text{dof}_1, \text{dof}_1) = m(\text{dof}_1, \text{dof}_1) + \left( 156 \times \text{M} \times \text{l} / 420 \right); \]
\[ m(\text{dof}_2, \text{dof}_1) = m(\text{dof}_2, \text{dof}_1) + \left( 22 \times \text{l}^2 \times \text{M} / 420 \right); \]
\[ m(\text{dof}_3, \text{dof}_1) = m(\text{dof}_3, \text{dof}_1) + \left( 54 \times \text{l} \times \text{M} / 420 \right); \]
\[ m(\text{dof}_4, \text{dof}_1) = m(\text{dof}_4, \text{dof}_1) + \left( -13 \times \text{l}^2 \times \text{M} / 420 \right); \]
\[ m(\text{dof}_1, \text{dof}_2) = m(\text{dof}_1, \text{dof}_2) + \left( 22 \times \text{l}^2 \times \text{M} / 420 \right); \]
\[ m(\text{dof}_2, \text{dof}_2) = m(\text{dof}_2, \text{dof}_2) + \left( 4 \times \text{M} \times \text{l}^3 / 420 \right); \]
\[ m(\text{dof}_3, \text{dof}_2) = m(\text{dof}_3, \text{dof}_2) + \left( 13 \times \text{l}^2 \times \text{M} / 420 \right); \]
\[ m(\text{dof}_4, \text{dof}_2) = m(\text{dof}_4, \text{dof}_2) + \left( -3 \times \text{M} \times \text{l}^3 / 420 \right); \]
\[ m(\text{dof1}, \text{dof3}) = m(\text{dof1}, \text{dof3}) + \left(54 \times \frac{M \times l}{420}\right); \]
\[ m(\text{dof2}, \text{dof3}) = m(\text{dof2}, \text{dof3}) + \left(13 \times \frac{l^2}{2} \times \frac{M}{420}\right); \]
\[ m(\text{dof3}, \text{dof3}) = m(\text{dof3}, \text{dof3}) + \left(156 \times \frac{l}{2} \times \frac{M}{420}\right); \]
\[ m(\text{dof4}, \text{dof3}) = m(\text{dof4}, \text{dof3}) + \left(-22 \times \frac{l^2}{2} \times \frac{M}{420}\right); \]
\[ m(\text{dof1}, \text{dof4}) = m(\text{dof1}, \text{dof4}) + \left(-13 \times \frac{l^2}{2} \times \frac{M}{420}\right); \]
\[ m(\text{dof2}, \text{dof4}) = m(\text{dof2}, \text{dof4}) + \left(-3 \times \frac{M \times l^3}{420}\right); \]
\[ m(\text{dof3}, \text{dof4}) = m(\text{dof3}, \text{dof4}) + \left(-22 \times \frac{l^2}{2} \times \frac{M}{420}\right); \]
\[ m(\text{dof4}, \text{dof4}) = m(\text{dof4}, \text{dof4}) + \left(4 \times \frac{M \times l^3}{420}\right); \]

end

\[ k(1:1,:)=[]; \text{\% Applying Boundary conditions} \]
\[ k(:,1:1) = []; \]
\[ k((2 \times \text{mnu} - 2):(2 \times \text{mnu} - 2), :) = []; \]
\[ k(:,(2 \times \text{mnu} - 2):(2 \times \text{mnu} - 2)) = []; \]
\[ k \text{\% Global Stiffness Matrix} \]
\[ x(1:1,:) = []; \]
\[ x(:,1:1) = []; \]
\[ x((2 \times \text{mnu} - 2):(2 \times \text{mnu} - 2), :) = []; \]
\[ x(:,(2 \times \text{mnu} - 2):(2 \times \text{mnu} - 2)) = []; \]
\[ x; \text{\% Global Matrix for the} \]
\[ \% Force that conforms fluid to pipe \]
\[ x1 = -x \]
\[ d(1:1,:) = []; \]
\[ d(:,1:1) = []; \]
\[ d((2 \times \text{mnu} - 2):(2 \times \text{mnu} - 2), :) = []; \]
d (: , (2*mnu-2):(2*mnu-2)) = []; d  \quad \% \ Global \ Dissipation \ Matrix 

d1 = (-d) 
Kglobal = k + 10*x1; 
m(1:1,:) = []; 
m( :, 1:1) = []; 
m((2*mnu-2):(2*mnu-2), :) = []; 
m(:, (2*mnu-2):(2*mnu-2)) = []; 
m;  \quad \% \ Global \ Mass \ Matrix 

eye(t); 
zeros(t); 
H = [-inv(m)*(d1) - inv(m)*Kglobal; eye(t) zeros(t)]; 
Evalue = eig(H) \% Eigenvalues 
vratio = v/vc \% Velocity Ratio 
iv2 = imag(Evalue); 
iv21 = min(abs(iv2)); 
w1 = (iv21) \% Fundamental Natural frequency 
wn 
wratio = w1/wn \% Frequency Ratio 
vc
0.2 MATLAB Program for Cantilever Pipe Carrying Fluid

MATLAB Program for Cantilever Pipe Carrying Fluid.

% The following MATLAB Program calculates the Fundamental Natural frequency of vibration, frequency ratio (w/wn) % and velocity ratio (v/vc), for a cantilever pipe % carrying fluid.
% In order to perform the above task the program assembles % Elemental Stiffness, Dissipation, and Inertia matrices % to form Global Matrices which are used % to calculate Fundamental Natural % Frequency w.

clc;

num_elements = input(’Input number of elements for Pipe:’);

% num_elements = The user enters the number of elements % in which the pipe has to be divided.

n=1:num_elements+1; % Number of nodes (n) is % equal to number of elements plus one

node1=1:num_elements; % Parameters used in the loops

node2=2:num_elements+1;

max_node1=max(node1);

max_node2=max(node2);

max_node_used=max([max_node1 max_node2]);

mnu=max_node_used;

k=zeros(2*mnu); % Creating a Global Stiffness Matrix of zeros
m=zeros(2*mnu); % Creating Global Mass Matrix of zeros
x=zeros(2*mnu); % Creating Global Matrix of zeros
    % for the force that conforms fluid
    % to the curvature of the pipe
d=zeros(2*mnu); % Creating Global Dissipation Matrix
    % of zeros (Coriolis Component)
t=num_elements*2;
L=2;        % Total length of the pipe in meters
l=L/num_elements; % Length of an element
t1=.0001;   % thickness of the pipe in meter
od=.01;    % outer diameter of the pipe
id=od−2*t1  % inner diameter of the pipe
I=pi*(od^4−id^4)/64 % moment of inertia of the pipe
E=207*10^9; % Modulus of elasticity of the pipe
roh=8000;  % Density of the pipe
rohw=1000; % density of water (Fluid)
M=roh*pi*(od^2−id^2)/4 + rohw*pi*2.5*id^2 % mass per unit length
    % of the pipe + fluid
rohA=rohw*pi*(.25*id^2);
v=9 % velocity of the fluid flowing through the pipe
s=v*L*sqrt((rohA)/(E*I));
l=L/num_elements; % Elemental Length
z=rohA/M;  % beta in the expression
i=sqrt(-1);
wn=((1.875)^2/L^2)*sqrt(E*I/M) % wn=((an^2)/L^2)*sqrt(EI/M)
    % here an=1.875, 4.694, 7.855
vc=(1.875/L)*sqrt(E*I/rohA) % Critical Velocity
% Assembling Global Stiffness, Dissipation and Inertia Matrices

for j=1:num_elements
    dof1=2*nodel(j)-1;
    dof2=2*nodel(j);
    dof3=2*node2(j)-1;
    dof4=2*node2(j);

    % Stiffness Matrix Assembly
    k(dof1,dof1)=k(dof1,dof1)+ (12*E*I/l^3);
    k(dof2,dof1)=k(dof2,dof1)+ (6*E*I/l^2);
    k(dof3,dof1)=k(dof3,dof1)+ (-12*E*I/l^3);
    k(dof4,dof1)=k(dof4,dof1)+ (6*E*I/l^2);
    k(dof1,dof2)=k(dof1,dof2)+ (6*E*I/l^2);
    k(dof2,dof2)=k(dof2,dof2)+ (4*E*I/l);
    k(dof3,dof2)=k(dof3,dof2)+ (-6*E*I/l^2);
    k(dof4,dof2)=k(dof4,dof2)+ (2*E*I/l);
    k(dof1,dof3)=k(dof1,dof3)+ (-12*E*I/l^3);
    k(dof2,dof3)=k(dof2,dof3)+ (-6*E*I/l^2);
    k(dof3,dof3)=k(dof3,dof3)+ (12*E*I/l^3);
    k(dof4,dof3)=k(dof4,dof3)+ (-6*E*I/l^2);
    k(dof1,dof4)=k(dof1,dof4)+ (6*E*I/l^2);
    k(dof2,dof4)=k(dof2,dof4)+ (2*E*I/l);
    k(dof3,dof4)=k(dof3,dof4)+ (-6*E*I/l^2);
    k(dof4,dof4)=k(dof4,dof4)+ (4*E*I/l);

    % Dissipation Matrix Assembly
    % Inertia Matrix Assembly

end
% Matrix assembly for the second term i.e
% for the force that conforms fluid to the
% curvature of the pipe

\[
x(\text{dof1}, \text{dof1}) = x(\text{dof1}, \text{dof1}) + ((36 \ast \rho A \ast \nu^2) / 30 \ast 1);
\]
\[
x(\text{dof2}, \text{dof1}) = x(\text{dof2}, \text{dof1}) + ((3 \ast \rho A \ast \nu^2) / 30 \ast 1);
\]
\[
x(\text{dof3}, \text{dof1}) = x(\text{dof3}, \text{dof1}) + ((-36 \ast \rho A \ast \nu^2) / 30 \ast 1);
\]
\[
x(\text{dof4}, \text{dof1}) = x(\text{dof4}, \text{dof1}) + ((3 \ast \rho A \ast \nu^2) / 30 \ast 1);
\]
\[
x(\text{dof1}, \text{dof2}) = x(\text{dof1}, \text{dof2}) + ((3 \ast \rho A \ast \nu^2) / 30 \ast 1);
\]
\[
x(\text{dof2}, \text{dof2}) = x(\text{dof2}, \text{dof2}) + ((4 \ast \rho A \ast \nu^2) / 30 \ast 1);
\]
\[
x(\text{dof3}, \text{dof2}) = x(\text{dof3}, \text{dof2}) + ((-3 \ast \rho A \ast \nu^2) / 30 \ast 1);
\]
\[
x(\text{dof4}, \text{dof2}) = x(\text{dof4}, \text{dof2}) + ((-1 \ast \rho A \ast \nu^2) / 30 \ast 1);
\]
\[
x(\text{dof1}, \text{dof3}) = x(\text{dof1}, \text{dof3}) + ((-36 \ast \rho A \ast \nu^2) / 30 \ast 1);
\]
\[
x(\text{dof2}, \text{dof3}) = x(\text{dof2}, \text{dof3}) + ((-3 \ast \rho A \ast \nu^2) / 30 \ast 1);
\]
\[
x(\text{dof3}, \text{dof3}) = x(\text{dof3}, \text{dof3}) + ((36 \ast \rho A \ast \nu^2) / 30 \ast 1);
\]
\[
x(\text{dof4}, \text{dof3}) = x(\text{dof4}, \text{dof3}) + ((-3 \ast \rho A \ast \nu^2) / 30 \ast 1);
\]
\[
x(\text{dof1}, \text{dof4}) = x(\text{dof1}, \text{dof4}) + ((3 \ast \rho A \ast \nu^2) / 30 \ast 1);
\]
\[
x(\text{dof2}, \text{dof4}) = x(\text{dof2}, \text{dof4}) + ((-1 \ast \rho A \ast \nu^2) / 30 \ast 1);
\]
\[
x(\text{dof3}, \text{dof4}) = x(\text{dof3}, \text{dof4}) + ((-3 \ast \rho A \ast \nu^2) / 30 \ast 1);
\]
\[
x(\text{dof4}, \text{dof4}) = x(\text{dof4}, \text{dof4}) + ((4 \ast \rho A \ast \nu^2) / 30 \ast 1);
\]

% Dissipation Matrix Assembly

\[
d(\text{dof1}, \text{dof1}) = d(\text{dof1}, \text{dof1}) + (2 \ast (-30 \ast \rho A \ast \nu) / 60); 
\]
\[
d(\text{dof2}, \text{dof1}) = d(\text{dof2}, \text{dof1}) + (2 \ast (6 \ast \rho A \ast \nu) / 60); 
\]
\[
d(\text{dof3}, \text{dof1}) = d(\text{dof3}, \text{dof1}) + (2 \ast (30 \ast \rho A \ast \nu) / 60); 
\]
\[ d(\text{dof}4, \text{dof}1) = d(\text{dof}4, \text{dof}1) + \frac{2(6 \ast \rho A \ast v)}{60}; \]

\[ d(\text{dof}1, \text{dof}2) = d(\text{dof}1, \text{dof}2) + \frac{2(6 \ast \rho A \ast v)}{60}; \]

\[ d(\text{dof}2, \text{dof}2) = d(\text{dof}2, \text{dof}2) + \frac{2(0 \ast \rho A \ast v)}{60}; \]

\[ d(\text{dof}3, \text{dof}2) = d(\text{dof}3, \text{dof}2) + \frac{2(6 \ast \rho A \ast v)}{60}; \]

\[ d(\text{dof}4, \text{dof}2) = d(\text{dof}4, \text{dof}2) + \frac{2(-1 \ast \rho A \ast v)}{60}; \]

\[ d(\text{dof}1, \text{dof}3) = d(\text{dof}1, \text{dof}3) + \frac{2(-30 \ast \rho A \ast v)}{60}; \]

\[ d(\text{dof}2, \text{dof}3) = d(\text{dof}2, \text{dof}3) + \frac{2(-6 \ast \rho A \ast v)}{60}; \]

\[ d(\text{dof}3, \text{dof}3) = d(\text{dof}3, \text{dof}3) + \frac{2(30 \ast \rho A \ast v)}{60}; \]

\[ d(\text{dof}4, \text{dof}3) = d(\text{dof}4, \text{dof}3) + \frac{2(6 \ast \rho A \ast v)}{60}; \]

\[ d(\text{dof}1, \text{dof}4) = d(\text{dof}1, \text{dof}4) + \frac{2(6 \ast \rho A \ast v)}{60}; \]

\[ d(\text{dof}2, \text{dof}4) = d(\text{dof}2, \text{dof}4) + \frac{2(1 \ast \rho A \ast v)}{60}; \]

\[ d(\text{dof}3, \text{dof}4) = d(\text{dof}3, \text{dof}4) + \frac{2(-6 \ast \rho A \ast v)}{60}; \]

\[ d(\text{dof}4, \text{dof}4) = d(\text{dof}4, \text{dof}4) + \frac{2(0 \ast \rho A \ast v)}{60}; \]

\[\% Inertia Matrix Assembly\]

\[ m(\text{dof}1, \text{dof}1) = m(\text{dof}1, \text{dof}1) + \frac{156 \ast M \ast l}{420}; \]

\[ m(\text{dof}2, \text{dof}1) = m(\text{dof}2, \text{dof}1) + \frac{22 \ast l^2 \ast M}{420}; \]

\[ m(\text{dof}3, \text{dof}1) = m(\text{dof}3, \text{dof}1) + \frac{54 \ast l \ast M}{420}; \]

\[ m(\text{dof}4, \text{dof}1) = m(\text{dof}4, \text{dof}1) + \frac{-13 \ast l^2 \ast M}{420}; \]

\[ m(\text{dof}1, \text{dof}2) = m(\text{dof}1, \text{dof}2) + \frac{22 \ast l^2 \ast M}{420}; \]

\[ m(\text{dof}2, \text{dof}2) = m(\text{dof}2, \text{dof}2) + \frac{4 \ast M \ast l^3}{420}; \]

\[ m(\text{dof}3, \text{dof}2) = m(\text{dof}3, \text{dof}2) + \frac{13 \ast l^2 \ast M}{420}; \]

\[ m(\text{dof}4, \text{dof}2) = m(\text{dof}4, \text{dof}2) + \frac{-3 \ast M \ast l^3}{420}; \]
\[m(\text{dof}_1, \text{dof}_3) = m(\text{dof}_1, \text{dof}_3) + \left(\frac{54 \times M \times l}{420}\right);
\]
\[m(\text{dof}_2, \text{dof}_3) = m(\text{dof}_2, \text{dof}_3) + \left(\frac{13 \times l^2 \times M}{420}\right);
\]
\[m(\text{dof}_3, \text{dof}_3) = m(\text{dof}_3, \text{dof}_3) + \left(\frac{156 \times l \times M}{420}\right);
\]
\[m(\text{dof}_4, \text{dof}_3) = m(\text{dof}_4, \text{dof}_3) + \left(-\frac{22 \times l^2 \times M}{420}\right);
\]
\[m(\text{dof}_1, \text{dof}_4) = m(\text{dof}_1, \text{dof}_4) + \left(-\frac{13 \times l^2 \times M}{420}\right);
\]
\[m(\text{dof}_2, \text{dof}_4) = m(\text{dof}_2, \text{dof}_4) + \left(-\frac{3 \times M \times l^3}{420}\right);
\]
\[m(\text{dof}_3, \text{dof}_4) = m(\text{dof}_3, \text{dof}_4) + \left(-\frac{22 \times l^2 \times M}{420}\right);
\]
\[m(\text{dof}_4, \text{dof}_4) = m(\text{dof}_4, \text{dof}_4) + \left(\frac{4 \times M \times l^3}{420}\right);
\]

\textbf{end}

\texttt{k(1:2,:) = [];} \textit{%; Applying Boundary conditions}
\texttt{k(:,1:2) = [];}  
\texttt{k;} \hspace{1cm} \textit{%; Global Stiffness Matrix}
\texttt{x(1:2,:) = [];} 
\texttt{x(:,1:2) = [];} 
\texttt{x;} \hspace{1cm} \textit{%; Global Matrix for the Force}
\texttt{x1=-x;} \hspace{1cm} \textit{%; that conforms fluid to pipe}
\texttt{d1=-d;} \hspace{1cm} \textit{%; Global Dissipation Matrix}
\texttt{Kglobal=k+x1;} \hspace{1cm} \textit{%; Global Matrix formed by combining}
\hspace{1cm} \textit{%; the Stiffness matrix and the Matrix}
\hspace{1cm} \textit{%; for the Force that conforms fluid to pipe}
\( m(1:2,:) = []; \)
\( m(:,1:2) = []; \)
\( m; \quad \text{% Global Inertia Matrix} \)
\( \text{eye}(t); \)
\( \text{zeros}(t); \)
\( H = [-\text{inv}(m) \ast (d1) - \text{inv}(m) \ast K\text{global}; \text{eye}(t) \text{ zeros}(t)]; \)
\( \text{Evalue} = \text{eig}(H); \quad \text{% Finding Eigenvalues} \)
\( \text{Evecs} = \text{eigs}(H); \)
\( w = \text{imag}(\text{Evalue}); \)
\( w1 = \text{min}(|w|) \quad \text{% Fundamental Natural Frequency} \)
\( \text{frequency} = w1/(2*\pi); \)
\( \text{ wn} \)
\( \text{ vc} \)
\( \text{ v} \)
\( v \text{ ratio} = v/\text{vc} \quad \text{% Velocity Ratio} \)
\( \text{wratio} = w1/wn \quad \text{% Frequency Ratio} \)
\( y = (((w1^2)\ast M)/(\text{E}\ast \text{I}))^{1/4}; \)
\( \text{for} \quad x = 0:.2:L \)
\( Z = (\cos(y\ast x) - \cosh(y\ast x)); \)
\( R = (-\cos(y\ast L) - \cosh(y\ast L))/(\sin(y\ast L) - \sinh(y\ast L)); \)
\( X = .5*Z + .5*R*(\sin(y\ast x) - \sinh(y\ast x)); \)
\( \text{end} \)
0.3 MATLAB Program for Tapered Pipe Carrying Fluid

MATLAB Program for Tapered Pipe Carrying Fluid.

% The following MATLAB Program calculates the Fundamental Natural frequency of vibration, frequency ratio (w/wn) % and velocity ratio (v/vc), for a Tapered pipe carrying fluid. % In order to perform the above task the program assembles % Elemental Stiffness, Dissipation, and Inertia matrices % to form Global Matrices which are used to % calculate Fundamental Natural % Frequency w.
clc;
num_elements = input( 'Input number of elements for beam:' );
n=1 : num_elements + 1;
nodel = 1 : num_elements;
node2 = 2 : num_elements + 1;
max_nodel = max(nodel);
max_node2 = max(node2);
max_node_used = max([max_nodel max_node2]);
mmu = max_node_used;
k=zeros(2*mmu); % Creating a Global Stiffness Matrix of zeros
m=zeros(2*mmu); % Creating Global Mass Matrix of zeros
x=zeros(2*mmu); % Creating Global Matrix of zeros
% for the force that conforms fluid to the % curvature of the % pipe
% Creating Global Dissipation Matrix of zeros

% (Coriolis Component)

t=num_elements*2;
L=2; % Total length of the pipe in meters
l=L/num_elements; % Length of an element
%t1=.0001; % thickness of the pipe in meter
%od=.01; % outer diameter of the pipe
%id=od-2*t1; % inner diameter of the pipe
%I=pi*(od^4-id^4)/64; % moment of inertia of the pipe
E=207*10^9; % Modulus of elasticity of the pipe
roh=8000; % Density of the pipe
rohw=1000; % density of water(FLuid)
rohA=rohw*pi*(.25*.0098^2);
l=L/num_elements;
%v=103.5
v=0
od1=.01427 % Outer diameter(1) for the tapered pipe
ode=.00982 % Outer diameter(2) for the tapered pipe
i=sqrt(-1);
I=pi*(od1^4-.00982^4)/64 % Varying Moment of Inertia
 % for the tapered pipe
M=roh*pi*(od1^2-.00982^2)/4 + rohw*pi*.25*.0098^2;
wn=((3.14)^2/L^2)*sqrt(E*I/M);
vc=(3.14/L)*sqrt(E*I/rohA)
od1=.01427
% Assembling Global Stiffness, Dissipation and Inertia Matrices

for j=1:num_elements
\[ I(j) = \pi (od(j)^4 - 0.0098^4)/64; \]
\[ M(j) = roh*\pi*(od(j)^2 - 0.0098^2)/4 + rohw*\pi*25*0.0098^2; \]
\[ od(j+1) = \sqrt{((3/num\_elements)*(od(j)^2 - 0.0098^2)) + (3*0.0098^2) - od(j)^2}; \]

\% Variable Outer diameter over the tapered pipe
\[
dof1 = 2*node1(j) - 1; \\
dof2 = 2*node1(j); \\
dof3 = 2*node2(j) - 1; \\
dof4 = 2*node2(j); \\
\%
\% Stiffness Matrix Assembly
\[
k(dof1, dof1) = k(dof1, dof1) + (12*E*I(j)/l^3); \\
k(dof2, dof1) = k(dof2, dof1) + (6*E*I(j)/l^2); \\
k(dof3, dof1) = k(dof3, dof1) + (-12*E*I(j)/l^3); \\
k(dof4, dof1) = k(dof4, dof1) + (6*E*I(j)/l^2); \\
k(dof1, dof2) = k(dof1, dof2) + (6*E*I(j)/l^2); \\
k(dof2, dof2) = k(dof2, dof2) + (4*E*I(j)/l); \\
k(dof3, dof2) = k(dof3, dof2) + (-6*E*I(j)/l^2); \\
k(dof4, dof2) = k(dof4, dof2) + (2*E*I(j)/l); \\
k(dof1, dof3) = k(dof1, dof3) + (-12*E*I(j)/l^3); \\
k(dof2, dof3) = k(dof2, dof3) + (-6*E*I(j)/l^2); \\
k(dof3, dof3) = k(dof3, dof3) + (12*E*I(j)/l^3); \\
k(dof4, dof3) = k(dof4, dof3) + (-6*E*I(j)/l^2); \\
k(dof1, dof4) = k(dof1, dof4) + (6*E*I(j)/l^2); \\
k(dof2, dof4) = k(dof2, dof4) + (2*E*I(j)/l); \]
\[ k(\text{dof}3, \text{dof}4) = k(\text{dof}3, \text{dof}4) + (-6E*\text{I}(j)/l^2); \]
\[ k(\text{dof}4, \text{dof}4) = k(\text{dof}4, \text{dof}4) + (4E*\text{I}(j)/l); \]

\% Matrix assembly for the second term i.e
\% for the force that conforms fluid to the
\% curvature of the pipe
\[ x(\text{dof}1, \text{dof}1) = x(\text{dof}1, \text{dof}1) + ((36*\text{rohA}*v^2)/30*1); \]
\[ x(\text{dof}2, \text{dof}1) = x(\text{dof}2, \text{dof}1) + ((3*\text{rohA}*v^2)/30*1); \]
\[ x(\text{dof}3, \text{dof}1) = x(\text{dof}3, \text{dof}1) + ((-36*\text{rohA}*v^2)/30*1); \]
\[ x(\text{dof}4, \text{dof}1) = x(\text{dof}4, \text{dof}1) + ((3*\text{rohA}*v^2)/30*1); \]
\[ x(\text{dof}1, \text{dof}2) = x(\text{dof}1, \text{dof}2) + ((3*\text{rohA}*v^2)/30*1); \]
\[ x(\text{dof}2, \text{dof}2) = x(\text{dof}2, \text{dof}2) + ((4*\text{rohA}*v^2)/30*1); \]
\[ x(\text{dof}3, \text{dof}2) = x(\text{dof}3, \text{dof}2) + ((-3*\text{rohA}*v^2)/30*1); \]
\[ x(\text{dof}4, \text{dof}2) = x(\text{dof}4, \text{dof}2) + ((-1*\text{rohA}*v^2)/30*1); \]
\[ x(\text{dof}1, \text{dof}3) = x(\text{dof}1, \text{dof}3) + ((-36*\text{rohA}*v^2)/30*1); \]
\[ x(\text{dof}2, \text{dof}3) = x(\text{dof}2, \text{dof}3) + ((-3*\text{rohA}*v^2)/30*1); \]
\[ x(\text{dof}3, \text{dof}3) = x(\text{dof}3, \text{dof}3) + ((36*\text{rohA}*v^2)/30*1); \]
\[ x(\text{dof}4, \text{dof}3) = x(\text{dof}4, \text{dof}3) + ((-3*\text{rohA}*v^2)/30*1); \]
\[ x(\text{dof}1, \text{dof}4) = x(\text{dof}1, \text{dof}4) + ((3*\text{rohA}*v^2)/30*1); \]
\[ x(\text{dof}2, \text{dof}4) = x(\text{dof}2, \text{dof}4) + ((-1*\text{rohA}*v^2)/30*1); \]
\[ x(\text{dof}3, \text{dof}4) = x(\text{dof}3, \text{dof}4) + ((-3*\text{rohA}*v^2)/30*1); \]
\[ x(\text{dof}4, \text{dof}4) = x(\text{dof}4, \text{dof}4) + ((4*\text{rohA}*v^2)/30*1); \]

\% Dissipation Matrix Assembly
\[ d(\text{dof}_1, \text{dof}_1) = d(\text{dof}_1, \text{dof}_1) + \left(2 \ast \left(-30 \ast \text{roh}_A \ast v\right) / 60\right) ; \]
\[ d(\text{dof}_2, \text{dof}_1) = d(\text{dof}_2, \text{dof}_1) + \left(2 \ast \left(6 \ast \text{roh}_A \ast v\right) / 60\right) ; \]
\[ d(\text{dof}_3, \text{dof}_1) = d(\text{dof}_3, \text{dof}_1) + \left(2 \ast \left(30 \ast \text{roh}_A \ast v\right) / 60\right) ; \]
\[ d(\text{dof}_4, \text{dof}_1) = d(\text{dof}_4, \text{dof}_1) + \left(2 \ast \left(-6 \ast \text{roh}_A \ast v\right) / 60\right) ; \]

\[ d(\text{dof}_1, \text{dof}_2) = d(\text{dof}_1, \text{dof}_2) + \left(2 \ast \left(-6 \ast \text{roh}_A \ast v\right) / 60\right) ; \]
\[ d(\text{dof}_2, \text{dof}_2) = d(\text{dof}_2, \text{dof}_2) + \left(2 \ast \left(0 \ast \text{roh}_A \ast v\right) / 60\right) ; \]
\[ d(\text{dof}_3, \text{dof}_2) = d(\text{dof}_3, \text{dof}_2) + \left(2 \ast \left(6 \ast \text{roh}_A \ast v\right) / 60\right) ; \]
\[ d(\text{dof}_4, \text{dof}_2) = d(\text{dof}_4, \text{dof}_2) + \left(2 \ast \left(-1 \ast \text{roh}_A \ast v\right) / 60\right) ; \]

\[ d(\text{dof}_1, \text{dof}_3) = d(\text{dof}_1, \text{dof}_3) + \left(2 \ast \left(-30 \ast \text{roh}_A \ast v\right) / 60\right) ; \]
\[ d(\text{dof}_2, \text{dof}_3) = d(\text{dof}_2, \text{dof}_3) + \left(2 \ast \left(-6 \ast \text{roh}_A \ast v\right) / 60\right) ; \]
\[ d(\text{dof}_3, \text{dof}_3) = d(\text{dof}_3, \text{dof}_3) + \left(2 \ast \left(30 \ast \text{roh}_A \ast v\right) / 60\right) ; \]
\[ d(\text{dof}_4, \text{dof}_3) = d(\text{dof}_4, \text{dof}_3) + \left(2 \ast \left(6 \ast \text{roh}_A \ast v\right) / 60\right) ; \]

\[ d(\text{dof}_1, \text{dof}_4) = d(\text{dof}_1, \text{dof}_4) + \left(2 \ast \left(6 \ast \text{roh}_A \ast v\right) / 60\right) ; \]
\[ d(\text{dof}_2, \text{dof}_4) = d(\text{dof}_2, \text{dof}_4) + \left(2 \ast \left(1 \ast \text{roh}_A \ast v\right) / 60\right) ; \]
\[ d(\text{dof}_3, \text{dof}_4) = d(\text{dof}_3, \text{dof}_4) + \left(2 \ast \left(-6 \ast \text{roh}_A \ast v\right) / 60\right) ; \]
\[ d(\text{dof}_4, \text{dof}_4) = d(\text{dof}_4, \text{dof}_4) + \left(2 \ast \left(0 \ast \text{roh}_A \ast v\right) / 60\right) ; \]

\[
\% \text{ Inertia Matrix Assembly}
\]
\[ m(\text{dof}_1, \text{dof}_1) = m(\text{dof}_1, \text{dof}_1) + \left(156 \ast M(j) \ast 1 / 420\right) ; \]
\[ m(\text{dof}_2, \text{dof}_1) = m(\text{dof}_2, \text{dof}_1) + \left(22 \ast 1 \ast 2 \ast M(j) / 420\right) ; \]
\[ m(\text{dof}_3, \text{dof}_1) = m(\text{dof}_3, \text{dof}_1) + \left(54 \ast 1 \ast M(j) / 420\right) ; \]
\[ m(\text{dof}_4, \text{dof}_1) = m(\text{dof}_4, \text{dof}_1) + \left(-13 \ast 1 \ast 2 \ast M(j) / 420\right) ; \]
\[ m(\text{dof}_1, \text{dof}_2) = m(\text{dof}_1, \text{dof}_2) + \left(22 \ast 1 \ast 2 \ast M(j) / 420\right) ; \]
\[ m(\text{dof}_2, \text{dof}_2) = m(\text{dof}_2, \text{dof}_2) + (4 \times M(j) \times l^{3/4} / 420); \]
\[ m(\text{dof}_3, \text{dof}_2) = m(\text{dof}_3, \text{dof}_2) + (13 \times l^2 \times M(j) / 420); \]
\[ m(\text{dof}_4, \text{dof}_2) = m(\text{dof}_4, \text{dof}_2) + (-3 \times M(j) \times l^{3/4} / 420); \]
\[ m(\text{dof}_1, \text{dof}_3) = m(\text{dof}_1, \text{dof}_3) + (54 \times M(j) \times l / 420); \]
\[ m(\text{dof}_2, \text{dof}_3) = m(\text{dof}_2, \text{dof}_3) + (13 \times l^2 \times M(j) / 420); \]
\[ m(\text{dof}_3, \text{dof}_3) = m(\text{dof}_3, \text{dof}_3) + (156 \times l \times M(j) / 420); \]
\[ m(\text{dof}_4, \text{dof}_3) = m(\text{dof}_4, \text{dof}_3) + (-22 \times l^2 \times M(j) / 420); \]
\[ m(\text{dof}_1, \text{dof}_4) = m(\text{dof}_1, \text{dof}_4) + (-13 \times l^2 \times M(j) / 420); \]
\[ m(\text{dof}_2, \text{dof}_4) = m(\text{dof}_2, \text{dof}_4) + (-3 \times M(j) \times l^{3/4} / 420); \]
\[ m(\text{dof}_3, \text{dof}_4) = m(\text{dof}_3, \text{dof}_4) + (-22 \times l^2 \times M(j) / 420); \]
\[ m(\text{dof}_4, \text{dof}_4) = m(\text{dof}_4, \text{dof}_4) + (4 \times M(j) \times l^{3/4} / 420); \]

end

\[ k(1:1,:) = []; \]
\[ k(:,1:1) = []; \]
\[ k((2\times mnu - 2):(2\times mnu - 2),:) = []; \]
\[ k(:,(2\times mnu - 2):(2\times mnu - 2)) = []; \]

\[ k \quad \% \ Global \ Stiffness \ Matrix \]

\[ x(1:1,:) = []; \]
\[ x(:,1:1) = []; \]
\[ x((2\times mnu - 2):(2\times mnu - 2),:) = []; \]
\[ x(:,(2\times mnu - 2):(2\times mnu - 2)) = []; \]

\[ x; \quad \% \ Global \ Matrix \ for \ the \ Force \]
\[ \% \ that \ conforms \ fluid \ to \ pipe \]

\[ x1 = -x \]
\begin{verbatim}
d(1:1,:)=[];
d(:,1:1)=[ ];
d((2*mnu-2):(2*mnu-2), :)=[];
d(:,(2*mnu-2):(2*mnu-2))=[ ];
d;

% Global Dissipation Matrix
d1=-d;
Kglobal=k+1.7*x1;
m(1:1,:)=[ ];
m(:,1:1)=[ ];
m((2*mnu-2):(2*mnu-2), :)=[ ];
m(:,(2*mnu-2):(2*mnu-2))=[ ];
m
% Global Mass Matrix

eye(t);
zeros(t);
H=[-inv(m)*(d1) -inv(m)*(Kglobal); eye(t) zeros(t)];
Evalue=eig(H) % Eigenvalues
w=imag(Evalue);
w1=min(abs(w));
w1; % Fundamental Natural frequency
wn
vc
v
vratio=v/vc % Velocity Ratio
w1
wratio=w1/wn % Frequency Ratio
\end{verbatim}