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Effect of Axial Oscillation on Performance of Hydrodynamic Journal Bearings

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EFFECT OF AXIAL OSCILLATION ON PERFORMANCE OF HYDRODYNAMIC JOURNAL BEARINGS

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ABSTRACT

The hydrodynamic coefficients of a journal bearing set with the bearing subjected to axial oscillation are determined in this work. The Navier-Stokes equations were used to account for the axial component of the relative motion between the journal and the bearing. Numerical solution of the Navier-Stokes equations yielded the pressure distribution within the cavity of the bearing. This pressure distribution was then integrated over the surface of the bearing to obtain the two orthogonal force components. The derivatives of these force components were then taken with respect to the displacement and velocity of the journal that are co-linear with these forces. This yielded the hydrodynamic coefficients.

To conduct a parametric study, the bearing design parameters were chosen such that the Sommerfeld number was equal to one.   The axial oscillation of the bearing was modeled as a purely sinusoidal motion, with an amplitude of one millimeter and a frequency range of 0-304 rads/s.

This study shows that for a bearing axial frequency up to 304 rads/s the bearing dynamic coefficients exhibit an increase in their deviation from the values of a stationary bearing.

This study further shows that at the frequency of 300 rads/s the stiffness and damping coefficients vary dramatically over the course of cycle for the bearing set described in this work. This study shows K_{xy} changes from 1.711073 to 5.235242 back to 1.711073 again to 5.235242 and then back to 1.711073 as the bearing completes one-half cycle of travel.

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NOMENCLATURE

A	=Maximum bearing oscillation velocity
B_{xx} , B_{yy}	=Direct damping coefficients
B_{xy} , B_{yx}	=Cross-coupled damping coefficients
C	=Radial clearance
D = 2 R	=Bearing diameter
e	=Eccentricity of the journal center
F_x , F_y	=Fluid film force components
$f_i = F_i/(\sigma W)$	=Dimensionless fluid film force component
γ	=Bearing oscillation frequency
G_z	=Turbulence coefficient
$\bar{h}$	=Film thickness
$h = \bar{h}$ /C	=Dimensionless film thickness
K_{xx} , K_{yy}	=Direct stiffness coefficients
K_{xy} , K_{yx}	=Cross-coupled stiffness coefficients
L	=Bearing length
$\bar{p}$	=Oil film pressure
$p = \bar{p}/\mu\omega(\frac{R}{C})^2(\frac{L}{D})^2$	=Dimensionless oil film pressure
R	=Bearing radius
Re = $\rho\omega RC/\mu$	=Reynolds number
t	=time
$\overline{w}_{rot}$	=Axial mean velocity due to journal rotation
$w_{rot} = \overline{w}_{rot}/(R\omega/2)$	=Dimensionless axial mean velocity due to journal rotation
$\overline{w}_{osc}$	=Axial mean velocity due to journal rotation
$w_{osc} = \overline{w}_{osc}/(R\omega/2)$	=Dimensionless axial mean velocity due to journal rotation

$\bar{x}, \bar{y}, \bar{z}$	=Coordinates
x, y, z = $\bar{x}/C, \bar{y}/C, \bar{z}/L$	=Dimensionless coordinates
$\varepsilon = e/C$	=Eccentricity ratio
θ	=Angular coordinate
μ	=Dynamic viscosity
ρ	=Oil density
σ	=Modified Sommerfeld number
$\tau = \omega t$	=Dimensionless time
ω	=Journal angular velocity

CHAPTER I

INTRODUCTION

Journal bearings are used in a wide variety of applications to support rotating shafts. When appropriately designed and maintained, journal bearings offer high load carrying capacity across a wide range of radial speeds with virtually unlimited life. These benefits can be realized with a simple sleeve supporting the journal and an adequate supply of lubricant, forgoing the expense and limited life of ball bearings or roller bearings.

Bearing failure, in general, is caused by the bearing becoming unstable. Bearing instability can be caused by a variety of factors. The factor that is relevant to this work is increased lubricant temperature. Increased lubricant temperature leads to a decrease in viscosity. The decrease in lubricant viscosity alters the stiffness and damping coefficients that govern the dynamics of a given journal bearing and can lead to catastrophic failures.

Journal bearing stability is influenced by the stiffness and damping properties of the oil film developed under a hydrodynamic lubrication regime. The stiffness and damping are characterized by eight coefficients, four for stiffness and four for damping. A means of generating these coefficients for a typical journal bearing is readily available. These coefficients are used in the design of journal bearings.

The approach for determining stiffness and damping coefficients of typical bearing proceeds as follows:

- Using the bearing design parameters, the pressure distribution of the oil film is determined using the standard Reynolds equation.
- The pressure distribution is integrated along the circumferential area of the bearing to come up with two orthogonal force components.
- Derivation of the force components with respect to orbital displacement of the journal to determine the stiffness coefficients.
- Derivation of the force components with respect to orbital velocity of the journal to determine the damping coefficients.

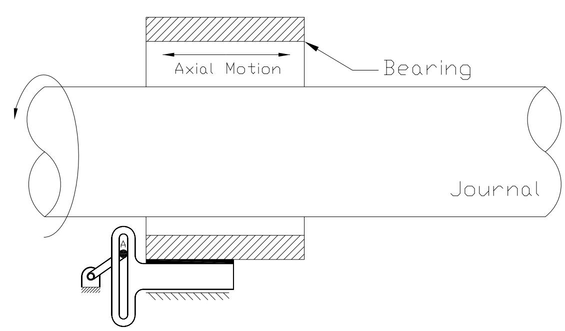

Figure 1. Schematic view of journal bearing subjected to axial oscillation

In previous research in this area the bearing has always been considered stationary with the journal undergoing spinning and orbital motion within the radial clearance of the bearing. This is seen in previous works, such as those of Jang [4] involving stability analysis when herringbone grooves are introduced. Sinnadurai [10] involved molten zinc as the lubricant. Lund [5], Morton [6] involving measurements made during operation, Najji [7] involving non-Newtonian fluids, and Parkins [8] involving both theoretical and experimental work.

One of the most important performance parameters of a journal bearing is its side flow. The side flow helps to maintain the temperature rise of a bearing at an acceptable level. The higher the side flow, the lower the temperature rise. One way of ensuring a high rate of side flow is to make the bearing short. However, shortening of a bearing will result in reduction of its load carrying capacity. An innovative approach for maintaining a high level of load carrying capacity while ensuring a low level of temperature rise could be moving the bearing in an axial direction of the journal in a reciprocating manner. Although this approach may be useful for temperature management of the bearing, its influence on the stiffness and damping coefficients is yet to be determined. The axial oscillation of the bearing could be achieved in several ways, using a magnetic actuator or something else along those lines. It is the purpose of this work to investigate the influence of the bearing's axial motion on the dynamic coefficients of the bearing. This work, however, does not involve the design aspects of the bearing osciallator mechanism. The mathematical approach employed in this thesis relies on the use of the Navier-Stokes equation rather than the typical Reynolds equation used in previous research in this field. This is because in Reynolds equation there is no parameter accounting for a moving boundary condition, namely the axial oscillation of the bearing. The impact of this oscillation upon the stiffness and damping coefficients has not been investigated before.

The work shown here will begin investigating the use of axial oscillation of journal bearings to reduce failures caused by the phenomenon of decreased lubricant viscosity due to temperature increase. This phenomenon precludes the use of journal bearings in applications where, otherwise, they would be desirable. This work may also lead to an expanded range of applications for which a journal bearing is a suitable solution.

CHAPTER II

MATHEMATICAL MODELING

2.1 Setup of journal bearing set with axial oscillation

The system analyzed in this work consists of journal bearing consisting of a journal rotating at a prescribed angular velocity and a bearing that is axially oscillating in a sinusoidal fashion at various maximum velocities over a constant range of travel. Under the rigid supports assumption, in a reference fixed to bearings, a right handed rectangular Cartesian coordinates system is assumed, as shown in Figure 2. In a standard journal bearing, the oil separates the rotating journal from the stationary bearing and the pressure developed in the radial clearance of the bearing supports the static load, as well as any dynamic load due to external disturbances. The journal bearing to be modeled here will feature a bearing that oscillates along the journal. This oscillation may or may not have an impact upon the bearing's ability to carry static and dynamic loads.

The approach taken here will be to use the Navier-Stokes equation to derive the pressure distribution due to journal rotation and bearing oscillation. Navier –Stokes will allow the moving boundary condition resulting from the axial oscillation to be accounted for. This pressure distribution will be developed as two separate components, one due to journal rotation and one due to bearing oscillation, that can be added together to yield the time variable pressure. This time variable pressure will be integrated along the circumferential area of the bearing to develop two orthogonal force components. The derivative of these two force components with respect to orbital journal displacement will yield the stiffness coefficients. The derivative of these two force components with respect to orbital journal velocity will yield the damping coefficients. Maple 11 was the software package used to perform the calculations.

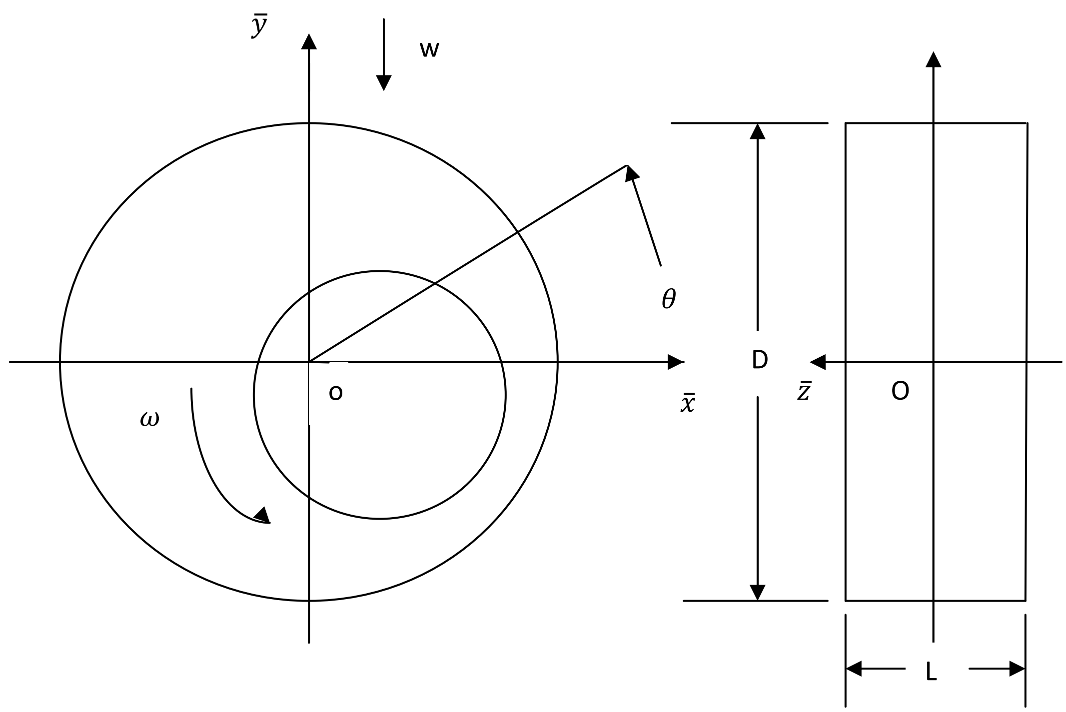

Figure 2. Notation for the journal bearing

Fluid viscosity and type of fluid (Newtonian or non Newtonian) are the two major lubricant characteristics that influence the hydrodynamic properties of a journal bearing set. The viscosity of the oil is dependent upon its grade and temperature, it is independent of pressure.

The mechanical properties of the oil film attributable to journal rotation are modeled by the standard stiffness and damper sets as shown in Figure 3. These stiffness and damper sets determine the dynamic behavior of the journal. This can be modeled as a two degrees of freedom system. Sinnadurai's method Sinnadurai[10] will be used here. Sinadurai's work used molten zinc as the lubricant in a galvanizing operation. As an effort to validate the construction of the model, when Sinnadurai's method had been reproduced, a check was made using his parameters. The stiffness and damping coefficients that were obtained matched those reported by Sinnadurai. After adding the allowance for the oscillation of the bearing, Sinnadurai's parameters were used again, with zero oscillation. The stiffness and damping coefficients that that were obtained again matched those reported by Sinnadurai.

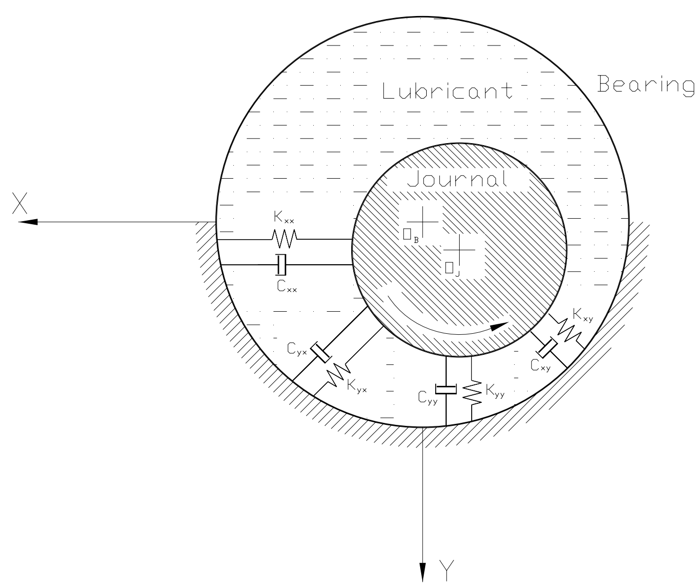

Figure 3. Stiffness and damping properties of journal bearing oil film (linearized version)

2.2 Formulation of a mathematical model

For a standard journal bearing set, under the short bearing assumption, the momentum and continuity equations in dimensionless form respectively are (Constantinescu[3];Szeri[11]):

$$Re\frac{C}{R}\left[\frac{\partial(hw)}{\partial\tau}+\frac{1}{2}\frac{\partial(hw)}{\partial\theta}+\frac{1}{4}\frac{D}{L}\frac{\partial(hw^2)}{\partial z}\right]=-\frac{L}{D}h\frac{\partial p}{\partial z}-12G_z\frac{w}{h} \quad (1)$$

$$\frac{\partial h}{\partial\theta}+2\frac{\partial h}{\partial\tau}+\frac{1}{2}\frac{\partial}{\partial z}(h*w*w)=0 \quad (2)$$

Where the dimensionless film thickness, h, is given by,

$$h=1-x\cos\theta-y\sin\theta \quad (3)$$

For the turbulent coefficient G, Capone et al.[1,2], have proposed an expression which takes into consideration the effects of the oil film non-laminar flow, even when the turbulence is not developed in the whole gap.

$$G=1+\gamma shRe^{0.96} \quad (4)$$

In the above equations, gamma is constant with the value of $3.47*10^{-4}$ and 's' is defined as follows:

$$s = \frac{1}{\pi} tan^{-1}\left[\frac{2}{\pi}\frac{Re_t - Re_l}{Re_t + Re_l}\frac{\frac{Re}{Re^*}}{1 - \frac{Re}{Re^*}}\right] \quad (5)$$

Where,

$$Re_t = 41.2[\frac{R/_C}{(1-\varepsilon)^3 + 4/_3\left(\frac{L}{D}\right)^2 \dot{\varepsilon}^2(1-\varepsilon)}]^{1/2}$$

$$Re_l = 41.2[\frac{R/_C}{(1+\varepsilon)^3 + 4/_3\left(\frac{L}{D}\right)^2 \dot{\varepsilon}^2(1+\varepsilon)}]^{1/2}$$

$$Re^* = \frac{Re_t + Re_l}{2}$$

$$\varepsilon = (x^2 + y^2)^{1/2}$$

$$\dot{\varepsilon} = \frac{x\dot{x} + y\dot{y}}{(x^2+y^2)^{1/2}}$$

Re_t represents the Reynolds number for the part of the flow which is turbulent, and Re_l represents the Reynolds number for the part of the flow which is laminar.

Integrating the continuity equation (Eq. 2) with respect to z and applying boundary conditions $w_{z=0}$=0, we obtain the dimensionless axial mean velocity resulting from rotation of the journal as,

$$w_{rot} = -2\frac{L}{D}\frac{1}{h}\left(\frac{\partial h}{\partial \theta} + 2\frac{\partial h}{\partial \tau}\right)z \quad (6)$$

The following equation is obtained by substituting the above equation into the momentum equation (Eq. 1).

$$\frac{\partial p_{rot}}{\partial z} = Re\frac{C}{R}\frac{-D}{Lh}\left[\frac{-2Lz}{D}\left(\frac{\partial^2 h}{\partial\tau\partial\theta} + 2\frac{\partial^2 h}{\partial\tau^2}\right) + \frac{-Lz}{D}\left(\frac{\partial^2 h}{\partial\tau\partial\theta} + 2\frac{\partial^2 h}{\partial\theta^2}\right) + 2\frac{Lz}{Dh}\left(\left(\frac{\partial h}{\partial\theta}\right)^2 + 4\frac{\partial h}{\partial\theta}\frac{\partial h}{\partial\tau} + 4\left(\frac{\partial h}{\partial\tau}\right)^2\right)\right] + 24\frac{G_z}{h^3}\left(\frac{\partial h}{\partial\theta} + 2\frac{\partial h}{\partial\tau}\right)z \qquad (7)$$

Integrating the above equation, with boundary conditions corresponding to zero values of pressure at the bearing edge, yields the following pressure distribution resulting from rotation of the journal which takes into account the effect of turbulence and inertia of the oil film.

$$p_{rot} = \frac{\vartheta}{2}z^2 - \frac{\vartheta L^2}{8} \qquad (8)$$

Where:

$$\vartheta = Re\frac{C}{Rh}[4\frac{\partial^2 h}{\partial\tau\partial\theta} + 4\frac{\partial^2 h}{\partial\tau^2} + \frac{\partial^2 h}{\partial\theta^2} - \frac{2}{h}(\left(\frac{\partial h}{\partial\theta}\right)^2 + 4\frac{\partial h}{\partial\theta}\frac{\partial h}{\partial\tau} + 4\left(\frac{\partial h}{\partial\tau}\right)^2)] + 24\frac{G_z}{h^3}\left(\frac{\partial h}{\partial\theta} + 2\frac{\partial h}{\partial\tau}\right)$$

The inclusion of (Eq. 4) for G_z in the above equation makes it valid for any value of the Reynolds number Re (Capone et al.[1,2]).

The pressure distribution expressed in (Eq. 8) is significant for a standard journal bearing in the interval ,

$$\alpha^* = \alpha^*(x, y, \dot{x}, \dot{y}, \ddot{x}, \ddot{y}) \leq \theta \ddot{\leq} \beta = \beta(x, y, \dot{x}, \dot{y}, \ddot{x}, \ddot{y})$$

where p is positive, as shown in the figure below. For the values of θ outside of this interval, p is taken to be zero.

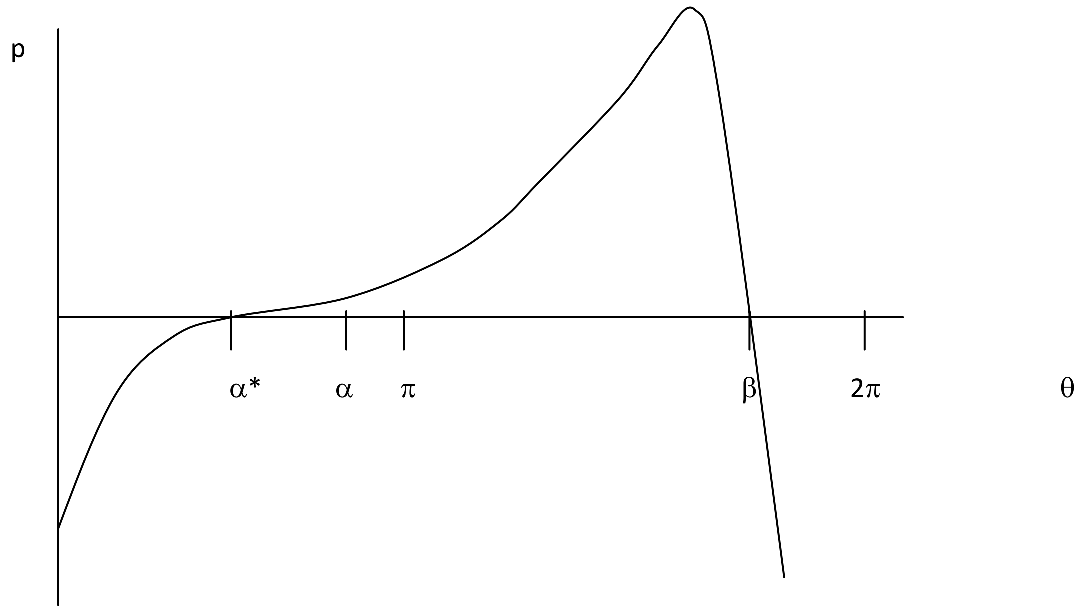

Figure 4. Qualitative pressure (due to rotation) trend accounting for turbulence

A similar process will be used to derive the pressure distribution due to bearing oscillation. In this case, the dimensionless axial mean velocity is defined.

$$w_{osc} = \frac{A}{R\omega} sin(\gamma\tau) \tag{9}$$

Where bearing velocity is defined as:

$$w_b = Asin(\gamma\tau)$$

No slip conditions are assumed. It is also assumed that oil momentum is negligible and the entire oil film reacts to changes in bearing velocity instantaneously. With these assumptions in place, average actual oil velocity due to bearing oscillation is:

$$\overline{w}_{osc} = \frac{A}{2} sin(\gamma\tau)$$

And dimensionless axial mean velocity is defined as:

$$w_{osc} = \frac{\overline{w}}{R\omega/_{2}}$$

The following equation is obtained by substituting (Eq. 9) into the momentum equation (Eq. 1)

$$\frac{\partial p_{osc}}{\partial z} =$$

$$\frac{-D}{Lh} Re \frac{C}{R} \frac{A}{R\omega} \left[h\gamma * \cos(\gamma\tau) + (\sin(\gamma\tau)) \frac{\partial h}{\partial \tau} + \frac{1}{2} (\sin(\gamma\tau)) \frac{\partial h}{\partial \theta} \right] - \frac{D}{Lh} 12 G_z \frac{A}{hR\omega} \sin(\gamma\tau) \qquad (10)$$

$\frac{\partial p_{osc}}{\partial z}$ is independent of z. Therefore, for a given bearing at a given time, the rate of change of pressure with respect to z is constant. When the bearing is at one end of its travel and its velocity is zero, the pressure due to oscillation will be zero. As it begins to accelerate, the pressure at one end will remain zero and the pressure at the other end will begin to increase. This increase will occur in a sinusoidal fashion, peaking at some maximum, reached when the bearing is in the middle of its travel and its velocity is at its maximum, and then decreasing to zero. This will occur in the opposite direction for the bearing's return trip.

Integrating (Eq. 10), with a boundary condition corresponding to zero value of pressure at one bearing edge, yields the following pressure distribution which takes into account the effect of turbulence and inertia of the oil film.

For $n\pi \leq \gamma\tau \leq (n+1)\pi$, where $n = 0, 2, 4, 6, 8 \ldots$ and $p_{osc} = 0 \; at \; z = \frac{-L}{2}$,

$$p_{osc} =$$

$$\frac{\partial p_{osc}}{\partial z} z + \frac{\partial p_{osc}}{\partial z} \frac{L}{2} \qquad (11)$$

For $n\pi \leq \gamma\tau \leq (n+1)\pi$, where $n = 1, 3, 5, 7, 9\ldots$ and $p_{osc} = 0\ at\ z = \frac{L}{2}$,

$$p_{osc} = \frac{\partial p_{osc}}{\partial z} z - \frac{\partial p_{osc}}{\partial z}\frac{L}{2} \quad (12)$$

The pressure due to bearing oscillation during a given half cycle of travel will vary in a linear fashion along the length of the bearing. During a given half cycle of travel the pressure at one end of the bearing, the down stream end for the given direction of bearing travel, will equal zero. The pressure at the opposite, or upstream end, will also equal zero for the instant when the bearing is at rest between changes of direction. As the bearing velocity increases, the pressure at the upstream end will increase until it reaches its maximum. This instant of maximum pressure will occur as the bearing velocity is at its maximum in the middle of its length of travel. As the bearing slows, the pressure at the upstream end will decrease, reaching zero as the bearing reaches the end of its travel. This rise and fall at the upstream end will occur in a sinusoidal fashion, matching the rise and fall of bearing velocity. All points along the bearing will experience a similar rise and fall, with the instantaneous pressure exhibiting a linear degradation from the maximum at the upstream end to zero at the down stream end. The subsequent half cycle will exhibit the same behavior, with the upstream and downstream ends reversing.

The pressure distributions due to journal rotation (Eq. 8) and bearing oscillation (Eq. 11 and 12) can then be added.

For $n\pi \leq \gamma\tau \leq (n+1)\pi$, where $n = 0, 2, 4, 6, 8 \ldots$ and $p_{osc} = 0\ at\ z = \frac{-L}{2}$,

$$p = \frac{\partial p_{osc}}{\partial z} z + \frac{\partial p_{osc}}{\partial z}\frac{L}{2} + \frac{\vartheta}{2} z^2 - \frac{\vartheta L^2}{8} \tag{13}$$

For $n\pi \leq \gamma\tau \leq (n+1)\pi$, where $n = 1, 3, 5, 7, 9 \ldots$ and $p_{osc} = 0\ at\ z = \frac{L}{2}$,

$$p = \frac{\partial p_{osc}}{\partial z} z - \frac{\partial p_{osc}}{\partial z}\frac{L}{2} + \frac{\vartheta}{2} z^2 - \frac{\vartheta L^2}{8} \tag{14}$$

The unsteady fluid film force components f_x and f_y can be obtained by integrating the pressure distribution as given by (Eq. 13 and 14), in the intervals of $-\frac{L}{2} \leq z \leq \frac{L}{2}$ and $\alpha^* \leq \theta \leq \beta$.

$$f_x = -\int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{\alpha^*}^{\beta} p(\cos\theta)\, d\theta\, dz \tag{15}$$

$$f_y = -\int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{\alpha^*}^{\beta} p(\sin\theta)\, d\theta\, dz \tag{16}$$

2.3 Evaluation of Stiffness and Damping coefficients

To evaluate the integrals in (Eq. 14 and 15), α^* is replaced with $\alpha = \beta - \pi$, where β is the value of θ for which p=0 and $\frac{\partial p}{\partial \theta} < 0$. By doing so, it is possible for the integrals in (Eq. 13 and 14) to be replaced with the derivatives of apposite functions (Capone et al.[1,2]).

Evaluating the integrals, with the above substitution, the fluid force components are obtained as follows:

$$f_x = \left\{(x - 2\dot{y})\frac{\partial^2 F_1}{\partial x \partial y} - (y + 2\dot{x})\frac{\partial^2 F_1}{\partial x^2}\right\} + 2\gamma s Re^{0.96}\left\{(x - 2\dot{y})\frac{\partial F_2}{\partial x} - (y + 2\dot{x})\frac{\partial F_2}{\partial x}\right\}$$
$$+ \frac{1}{6} Re \frac{C}{R}\left\{(x - 2\dot{y})(y + 2\dot{x})\frac{\partial F_4}{\partial x} - (x - 2\dot{y})\frac{\partial F_5}{\partial x} - (y + 2\dot{x})^2\frac{\partial F_6}{\partial x}\right\}$$
$$+ \frac{1}{6} Re \frac{C}{R}\left\{(\frac{x}{2} - 2\dot{y} - 2\ddot{x})F_6 + (\frac{y}{2} - 2\dot{x} - 2\ddot{y})F_4\right\} \qquad (17)$$

$$f_y = \left\{(x - 2\dot{y})\frac{\partial^2 F_1}{\partial y^2} - (y + 2\dot{x})\frac{\partial^2 F_1}{\partial x \partial y}\right\} + 2\gamma s Re^{0.96}\left\{(x - 2\dot{y})\frac{\partial F_2}{\partial y} - (y + 2\dot{x})\frac{\partial F_2}{\partial y}\right\}$$
$$+ \frac{1}{6} Re \frac{C}{R}\left\{(x - 2\dot{y})(y + 2\dot{x})\frac{\partial F_4}{\partial y} - (x - 2\dot{y})\frac{\partial F_5}{\partial y} - (y + 2\dot{x})^2\frac{\partial F_6}{\partial y}\right\}$$
$$+ \frac{1}{6} Re \frac{C}{R}\left\{(\frac{x}{2} - 2\dot{y} - 2\ddot{x})F_4 + (\frac{y}{2} - 2\dot{x} - 2\ddot{y})F_5\right\} \qquad (18)$$

Where:

$$F_1(x,y,\alpha) = \frac{2}{(1-x^2-y^2)^2}\left[\frac{\pi}{2} + tan^{-1}\frac{y\cos\alpha - x\sin\alpha}{(1-x^2-y^2)^{\frac{1}{2}}}\right] \qquad (19)$$

$$F_2(x,y,\alpha) = \frac{1}{(x^2+y^2)}\left[y(F_1-\pi) + x\, ln\frac{1+x\cos\alpha + y\sin\alpha}{1-x\cos\alpha - y\sin\alpha}\right] \qquad (20)$$

$$F_3(x,y,\alpha) = \frac{1}{(x^2+y^2)}\left[x(F_1-\pi) - y\, ln\frac{1+x\cos\alpha + y\sin\alpha}{1-x\cos\alpha - y\sin\alpha}\right] \qquad (21)$$

$$F_4(x,y,\alpha) = \frac{-xy}{(x^2+y^2)}F_1 + \frac{2xy}{(x^2+y^2)^2}[F_1 - \pi - 2(y\cos\alpha - x\sin\alpha)] + \frac{x^2-y^2}{(x^2+y^2)^2}[ln\frac{1+x\cos\alpha + y\sin\alpha}{1-x\cos\alpha - y\sin\alpha} - 2(x\cos\alpha + y\sin\alpha) \qquad (22)$$

$$F_5(x,y,\alpha) = \frac{x^2}{(x^2+y^2)}F_1 - \frac{x^2-y^2}{(x^2+y^2)^2}[F_1 - \pi - 2(y\cos\alpha - x\sin\alpha)] + \frac{2xy}{(x^2+y^2)^2}[ln\frac{1+x\cos\alpha + y\sin\alpha}{1-x\cos\alpha - y\sin\alpha} - 2(x\cos\alpha + y\sin\alpha)] \qquad (23)$$

$$F_6(x,y,\alpha) = \frac{y^2}{(x^2+y^2)}F_1 + \frac{x^2-y^2}{(x^2+y^2)^2}[F_1 - \pi - 2(y\cos\alpha - x\sin\alpha)]$$

$$-\frac{2xy}{(x^2+y^2)^2}[ln\frac{1+x\cos\alpha+y\sin\alpha}{1-x\cos\alpha-y\sin\alpha}$$

$$-2(x\cos\alpha+y\sin\alpha)] \qquad (24)$$

The bearing's stiffness and damping coefficients can now be obtained from the derivatives of the fluid force components. An infinitesimal variation in α results in an infinitesimal variation of a higher order in the components f_x and f_y of the fluid film force. It is also assumed that the flow regime does not vary around the steady state and coincides with the regime corresponding to the steady state itself. Hence, the dynamic coefficients can be evaluated taking α and s as constants.

$$K_{xx} = -\frac{\partial}{\partial x}f_x(x,y,\alpha,\dot{x},\dot{y},\ddot{x},\ddot{y},s)\Big|_0 \qquad (25)$$

$$K_{xy} = -\frac{\partial}{\partial y}f_x(x,y,\alpha,\dot{x},\dot{y},\ddot{x},\ddot{y},s)\Big|_0 \qquad (26)$$

$$K_{yx} = -\frac{\partial}{\partial x}f_y(x,y,\alpha,\dot{x},\dot{y},\ddot{x},\ddot{y},s)\Big|_0 \qquad (27)$$

$$K_{yy} = -\frac{\partial}{\partial y}f_y(x,y,\alpha,\dot{x},\dot{y},\ddot{x},\ddot{y},s)\Big|_0 \qquad (28)$$

$$B_{xx} = -\frac{\partial}{\partial \dot{x}} f_x(x, y, \alpha, \dot{x}, \dot{y}, \ddot{x}, \ddot{y}, s)\Big|_0 \quad (29)$$

$$B_{xy} = -\frac{\partial}{\partial \dot{y}} f_x(x, y, \alpha, \dot{x}, \dot{y}, \ddot{x}, \ddot{y}, s)\Big|_0 \quad (30)$$

$$B_{yx} = -\frac{\partial}{\partial \dot{x}} f_y(x, y, \alpha, \dot{x}, \dot{y}, \ddot{x}, \ddot{y}, s)\Big|_0 \quad (31)$$

$$B_{yy} = -\frac{\partial}{\partial \dot{y}} f_y(x, y, \alpha, \dot{x}, \dot{y}, \ddot{x}, \ddot{y}, s)\Big|_0 \quad (32)$$

Also, the dimensionless damping coefficients have simpler expressions as indicated below:

$$B_{xx} = \Big\{2\frac{\partial}{\partial x}\Big[\frac{\partial F_1}{\partial x} + 2s\gamma Re^{0.96}F_3 + \frac{Re}{3}\frac{C}{R}(yF_6 - xF_4)\Big] - \frac{Re}{3}\frac{C}{R}F_4\Big\}_0 \quad (33)$$

$$B_{xy} = \{2\frac{\partial}{\partial x}\Big[\frac{\partial F_1}{\partial y} + 2s\gamma Re^{0.96}F_2 + \frac{Re}{3}\frac{C}{R}(yF_4 - xF_5)\Big] + \frac{Re}{3}\frac{C}{R}F_6\}_0 \quad (34)$$

$$B_{yx} = \{2\frac{\partial}{\partial y}\Big[\frac{\partial F_1}{\partial x} + 2s\gamma Re^{0.96}F_3 + \frac{Re}{3}\frac{C}{R}(yF_6 - xF_4)\Big] - \frac{Re}{3}\frac{C}{R}F_5\}_0 \quad (35)$$

$$B_{yy} = \{2\frac{\partial}{\partial y}\left[\frac{\partial F_1}{\partial y} + 2s\gamma Re^{0.96} F_2 + \frac{Re}{3}\frac{C}{R}(yF_4 - xF_5)\right] + \frac{Re}{3}\frac{C}{R}F_4\}_0 \quad (36)$$

To determine the stiffness and damping coefficients for a given journal bearing set with axially oscillating bearing, the coordinates of the steady state equilibrium position (x_0, y_0, α_0) have to be found. For a given Reynold's number Re, R/C ratio and modified Sommerfeld number, the steady state equilibrium position can be obtained by solving the following system of non-linear equations.

$$f_{ox}(x_o, y_o, \alpha_o) = 0 \quad (37)$$

$$f_{oy}(x_o, y_o, \alpha_o) - 1/\sigma = 0 \quad (38)$$

$$p_o(x_o, y_o, \beta_o) = 0 \quad (39)$$

$$\alpha_o = \beta_o - \pi \quad (40)$$

CHAPTER III

SOLUTION AND STUDY OF MATHEMATICAL MODEL

3.1 Computer Simulation of the Mathematical Model

The fairly large expressions for the fluid force components with numerous substitutions and cumbersome partial differentiations, and the subsequent differentiations of these expressions to find the hydrodynamic coefficients, require quite lengthy mathematical computations. Fortunately, the availability of symbolic computational software has greatly reduced the time and effort required for mathematical calculations performed in this work. Maple 11 was the software package used in this work. This software package was capable of dealing with the numerous terms required, many of which are quite large and unwieldy. It allowed the parameters to be changed easily for each run, which was required as bearing oscillation frequency was changed. Maple is also capable of carrying out the various calculus operations that were called for and performing the linear algebra and matrix manipulation required. A sample computational session is provided in Appendix B.

3.2 Determination of Stiffness and Damping Coefficients

Finding the stiffness and damping coefficients requires evaluation of the coordinates of the steady state equilibrium position (x_o, y_o, α_o). These coordinates are to be evaluated from the system of equations, (Eq. 37-40), rewritten below.

$$f_{ox}(x_o, y_o, \alpha_o) = 0 \qquad (37)$$

$$f_{oy}(x_o, y_o, \alpha_o) - 1/\sigma = 0 \qquad (38)$$

$$p_o(x_o, y_o, \beta_o) = 0 \qquad (39)$$

$$\alpha_o = \beta_o - \pi \qquad (40)$$

The equations above form a system of non-linear equations and have no closed form solution. Therefore, a numerical method has to be employed to find the solution to these simultaneous equations.

The numerical method used in this work is the Newton-Raphson method, which requires an initial guess of the roots of the non-linear simultaneous equations (Scarborough). This is an iterative method and gives approximate values of the roots. When the solution converges to the actual roots, the accuracy of the solution can be improved by increasing the number of iterations. Since computer software is used in this work to solve the above non-linear equations, a high degree of accuracy is obtained in the solution with little additional effort. A sample evaluation of the steady state coordinates is included in Appendix B.

With accurate figures for the coordinates of the steady state equilibrium position (x_o, y_o, α_o), the stiffness and damping coefficients can be determined. This is accomplished by taking the derivative of the fluid force components with respect to changes in position of the journal for the stiffness coefficients. Next the derivative of the fluid force components with respect to changes in velocity of the journal to find the damping coefficients.

3.3 Journal bearing parameters

The theory described in the previous chapters is implemented here for some test cases. Baseline for these test cases will be a journal bearing with no axial oscillation. This bearing will then be analyzed when axial oscillation is introduced. This oscillation will have a fixed total travel. Frequency and velocity will be adjusted so the bearing can complete one cycle while exhibiting the prescribed travel.

Bearing Length, (L), meters	Bearing Diameter, (D), meters	Radial Clearance, (C), meters	Journal Angular Velocity (ω), rad/sec	Modified Sommerfeld Number, (σ)	Oil Density, (µ), kg/m^3	Oil Dynamic Viscosity, (µ), Pa*s
0.0254	0.0127	0.0000254	94.2	1	850	0.05

Table 1. Journal bearing constant parameters

The journal bearing described above is analyzed with the bearing oscillating at various frequencies and speeds. Journal travel is fixed at one millimeter for all cases.

Test Case Number	Bearing oscillation frequency, (γ), rad/sec	Bearing maximum velocity, (V) m/s
1	0	0
2	1	0.001
3	100	0.1
4	200	0.2
5	300	0.3
6	304	0.304

Table 2. Journal bearing oscillation parameters

The stiffness and damping coefficients resulting from the analyses performed using these parameters are presented in the next chapter.

Note: The bearing oscillation frequency of 304 rads/sec was selected after trial and error determined that any greater frequencies resulted in irrational stiffness and damping coefficients. Due to the methods used to compute the coefficients, these irrational coefficients are indicative of contact between the journal and the bearing. The other oscillation frequencies were selected to demonstrate the changes that occur to the coefficients as oscillation frequency changes.

CHAPTER IV

RESULTS

4.1 Hydrodynamic Coefficients of Stiffness and Damping

Using the parameters given in the previous chapter, the stiffness and damping coefficients are determined for each test case. The approach taken here was be to use the Navier-Stokes equation to derive the pressure distribution due to journal rotation and bearing oscillation. Navier –Stokes allows the moving boundary condition resulting from the axial oscillation to be accounted for. This pressure distribution was developed as two separate components, one due to journal rotation and one due to bearing oscillation, that were added together to yield the time variable pressure. This time variable pressure was integrated along the circumferential area of the bearing to develop two orthogonal force components. The derivative of these two force components with respect to orbital journal displacement yielded the stiffness coefficients. The derivative of these two force components with respect to orbital journal velocity yielded the damping coefficients.

The numerical results are shown in Tables 3 through 8, found in Appendix A. These tables present the stiffness and damping coefficients for each test case individually. A column with the stiffness and damping coefficient of a journal bearing with no axial oscillation is included in each table for ease of comparison. These coefficients are calculated at 5 points throughout the half cycle of oscillation under study.

4.2 Comparison of the Stiffness and Damping Coefficients

The main areas of interest for this study are the changes that occur to the stiffness and damping coefficients as the speed and frequency of the bearing oscillation is changed and the changes that occur as the bearing accelerates and decelerates over the course of the half cycle of oscillation under study. With that in mind, Figures 6 through 13 each show an individual stiffness or damping coefficient for each test case while changes throughout the half cycle are reflected by the change in the value of time multiplied by frequency of axial oscillation, as shown on the x-axis.

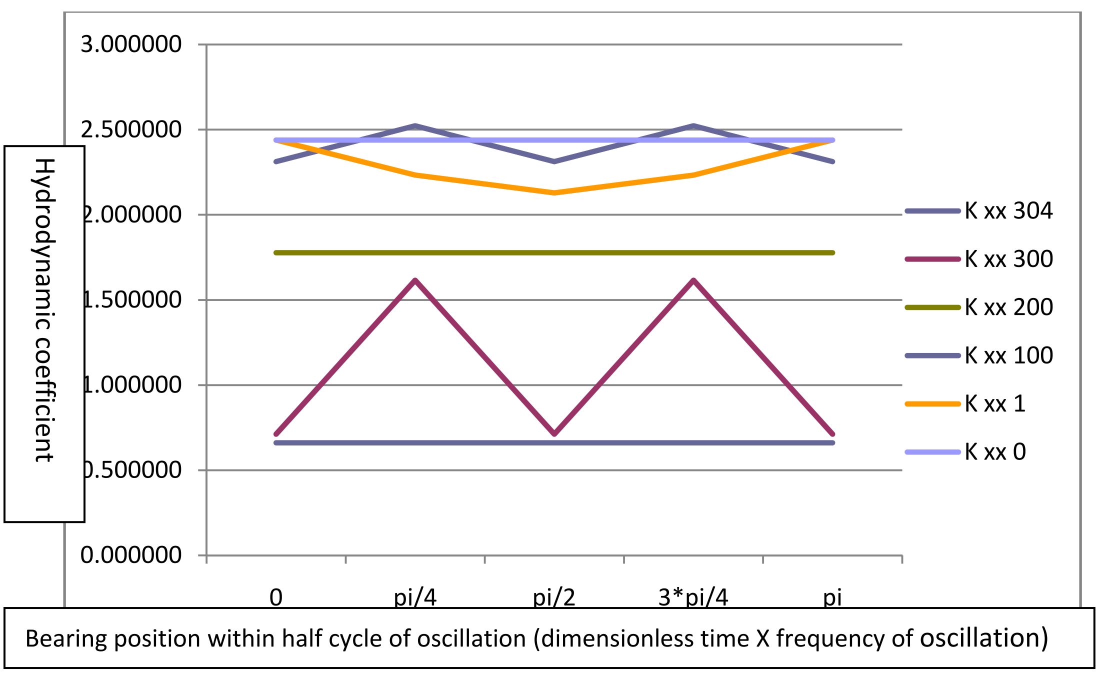

Figure 5. K_{xx}

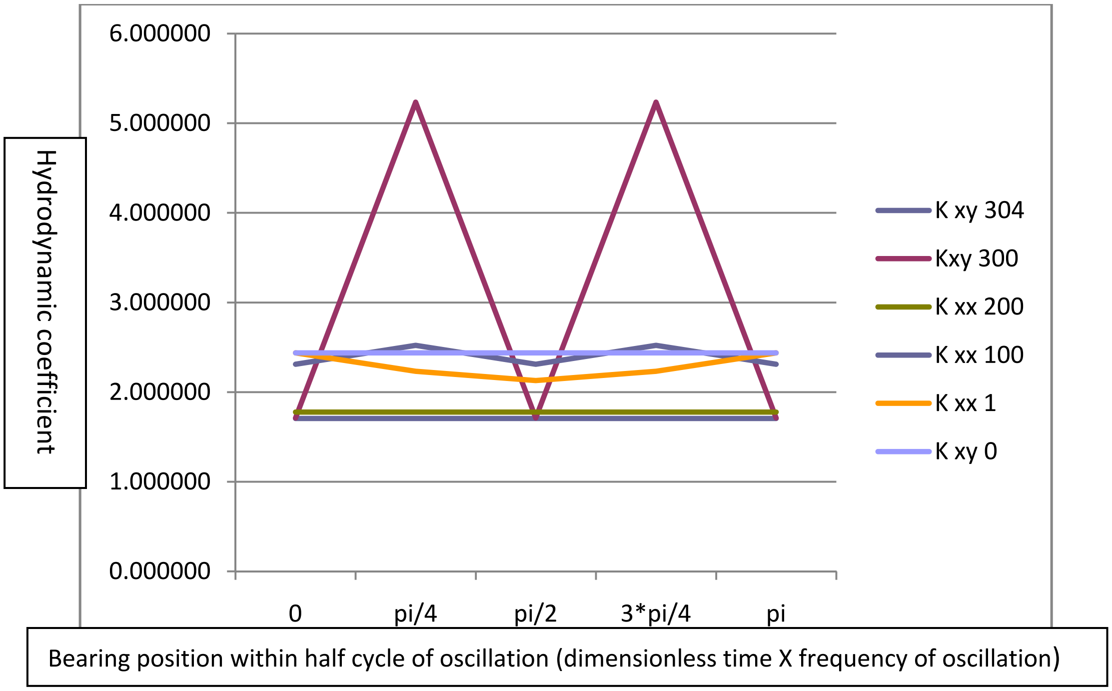

Figure 6. K_{xy}

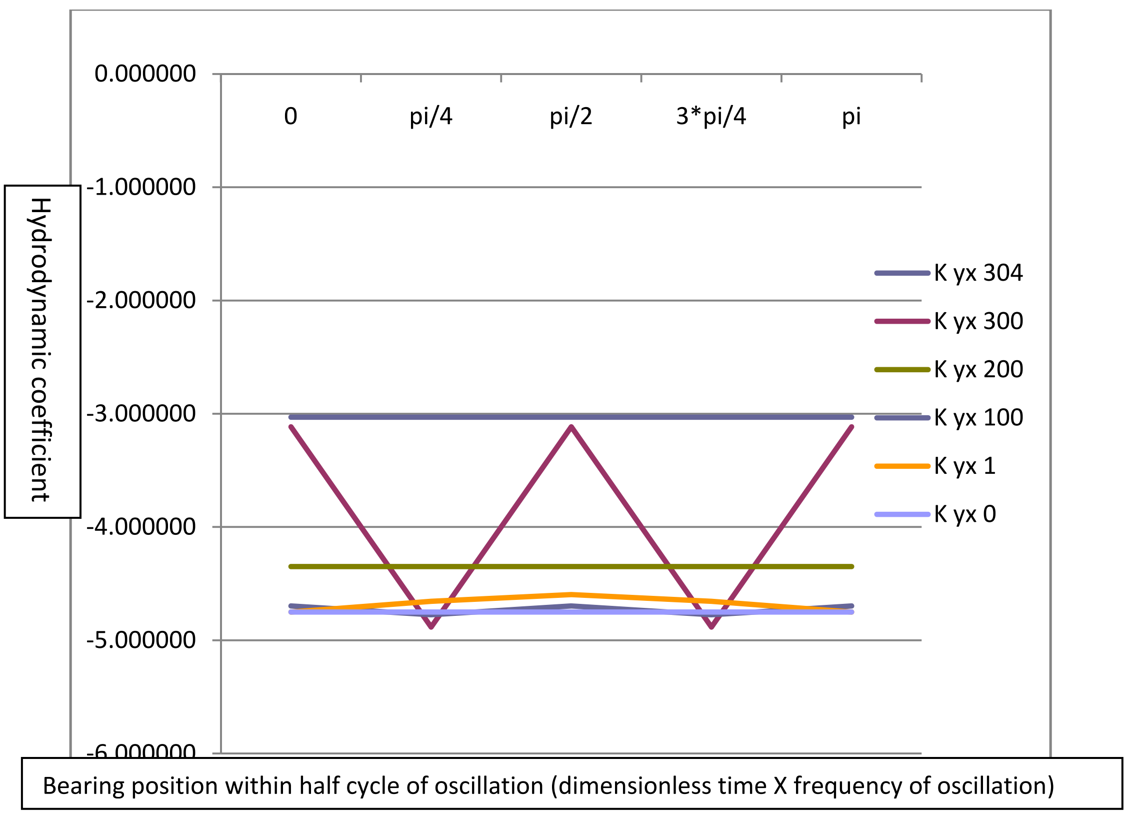

Figure 7. K_{yx}

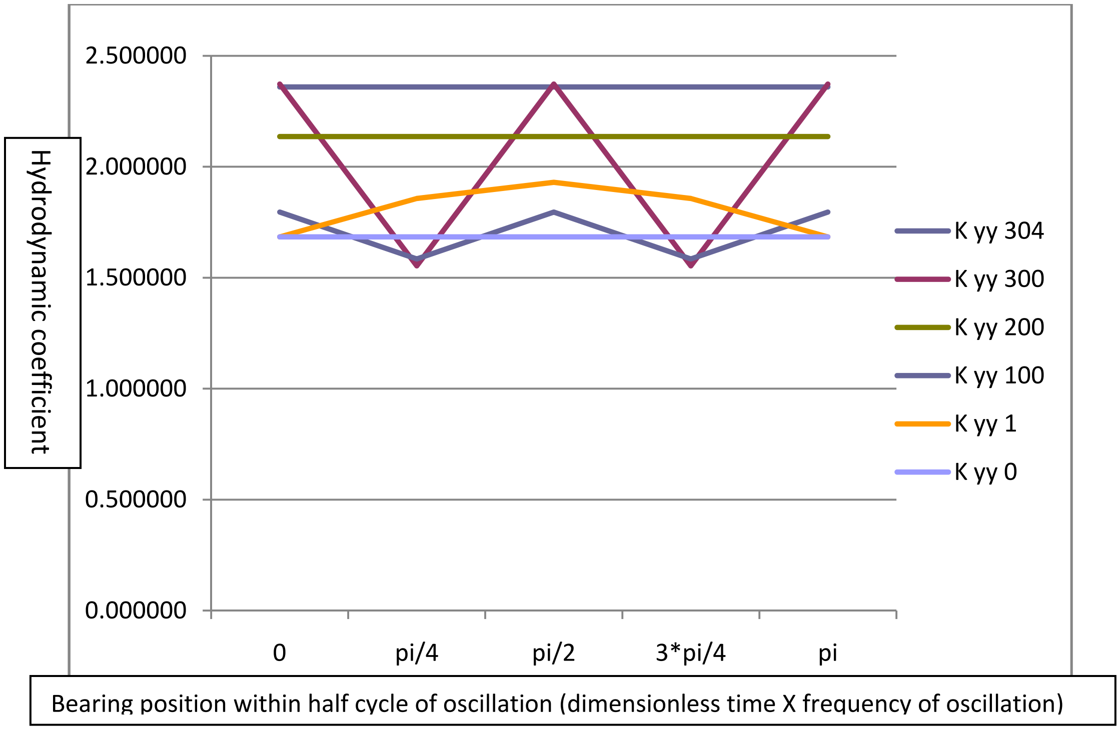

Figure 8. K_{yy}

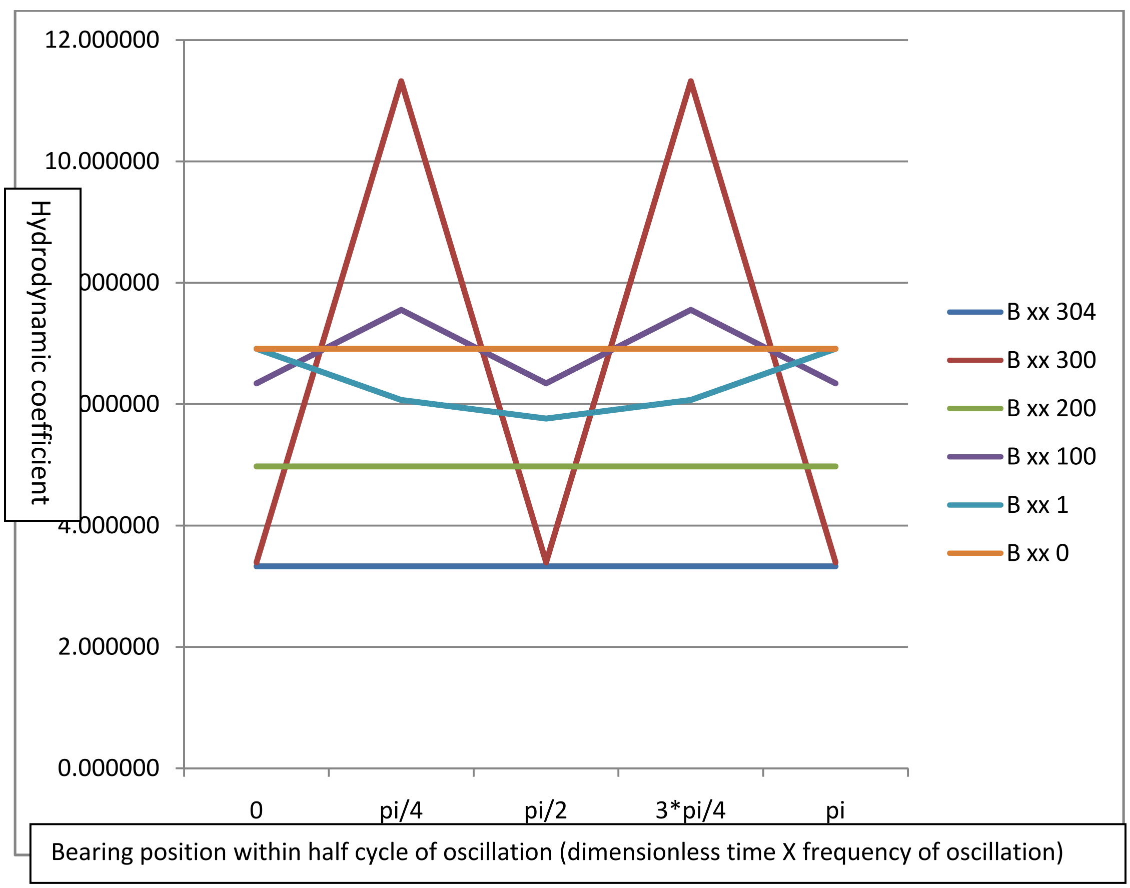

Figure 9. B_{xx}

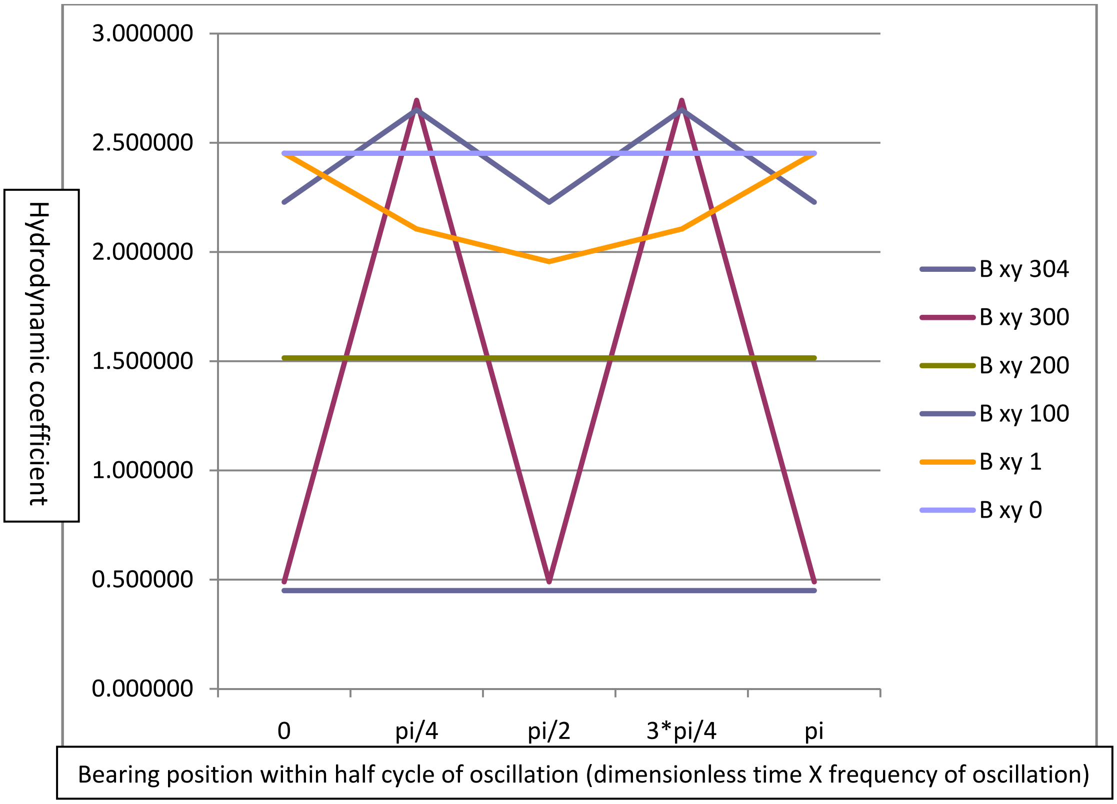

Figure 10. B_{xy}

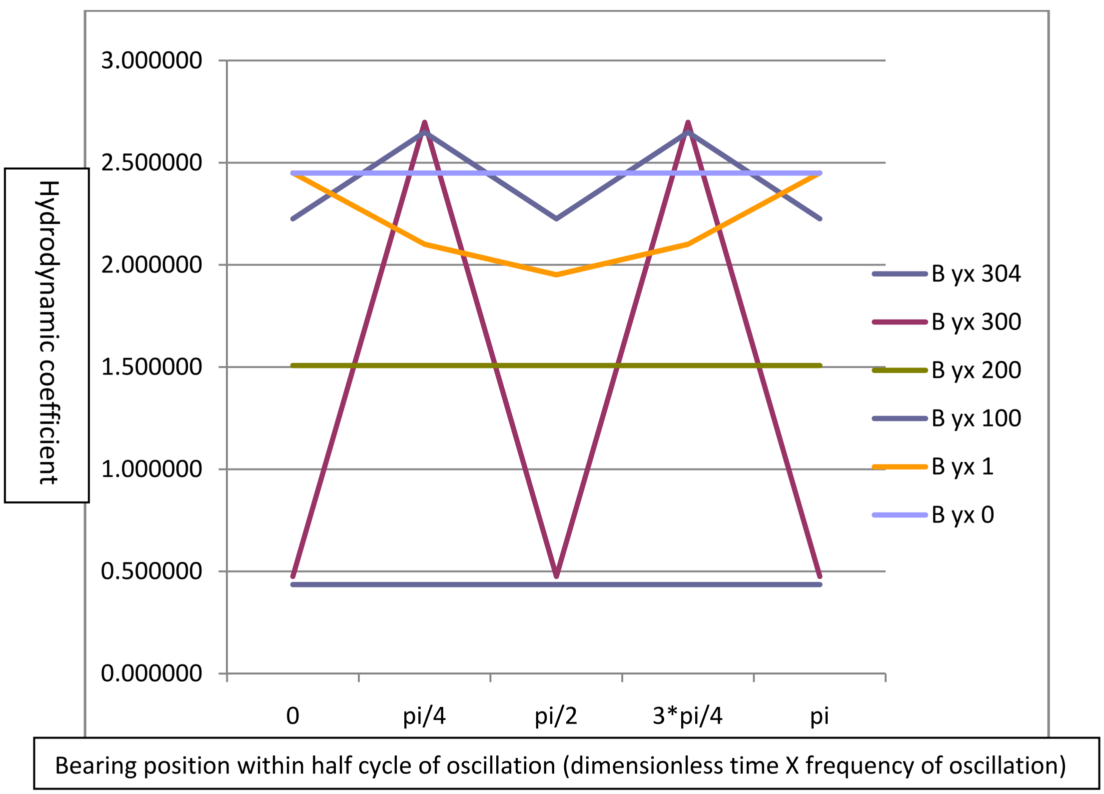

Figure 11. B_{yx}

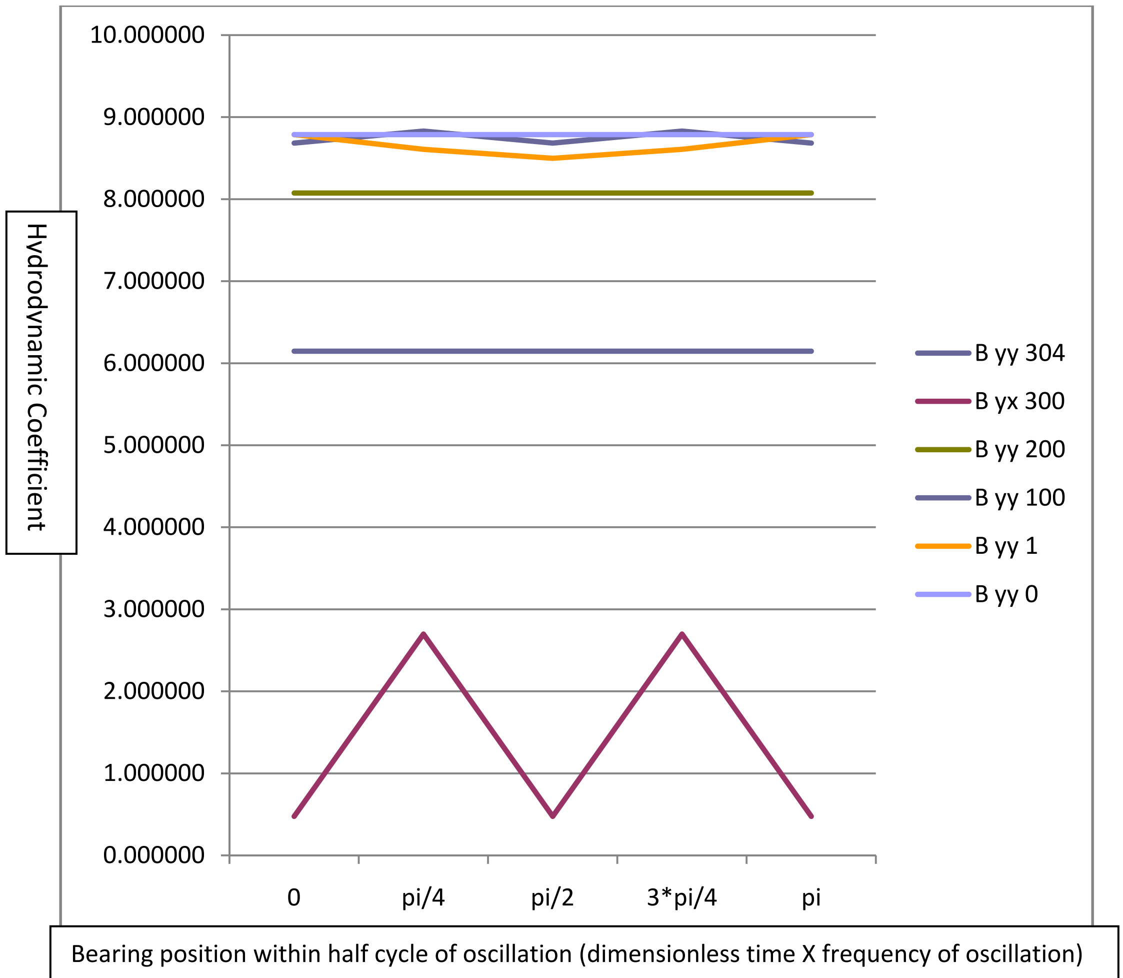

Figure 12. B_{yy}

4.3 Comparison of the Average Stiffness and Damping Coefficients

Finally, Figure 14 shows the average stiffness coefficient throughout the half cycle for a given test case. These average stiffness coefficients are shown for each bearing frequency. Bearing frequency is shown on the x-axis. Figure 15 shows the damping coefficients in a similar fashion. Numerical data is presented in Appendix A.

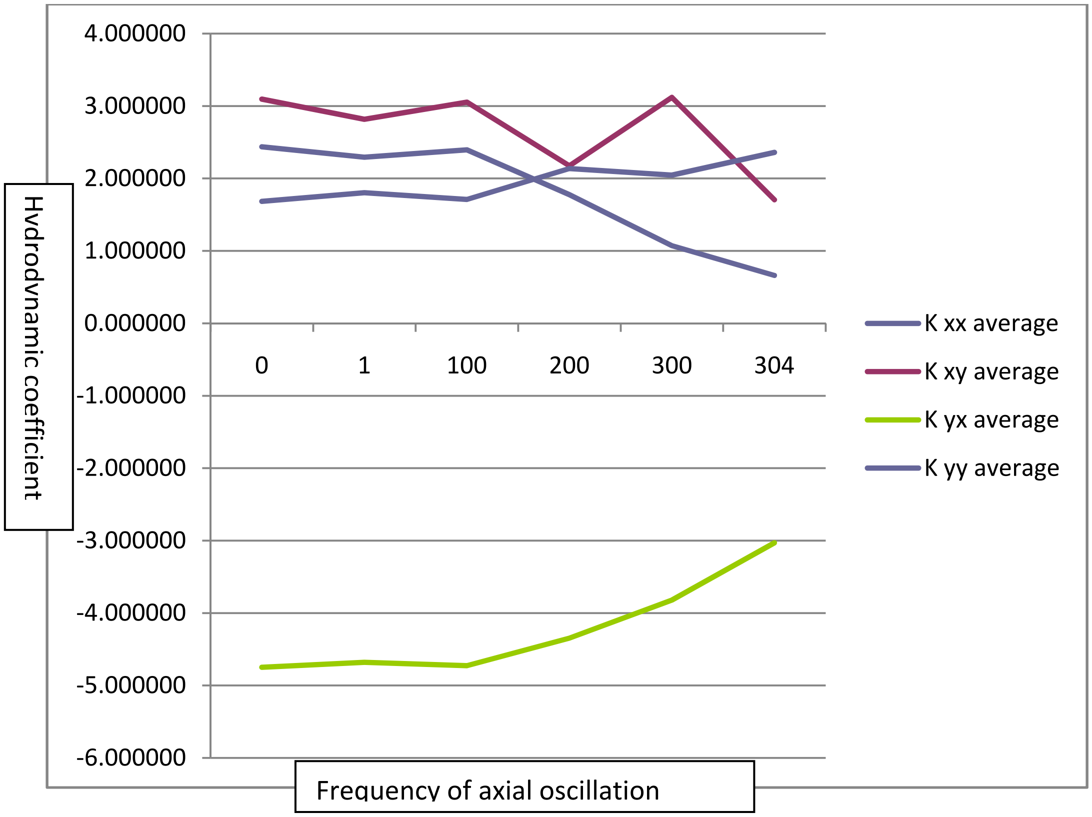

Figure 13. Average K at various frequencies of bearing oscillation.

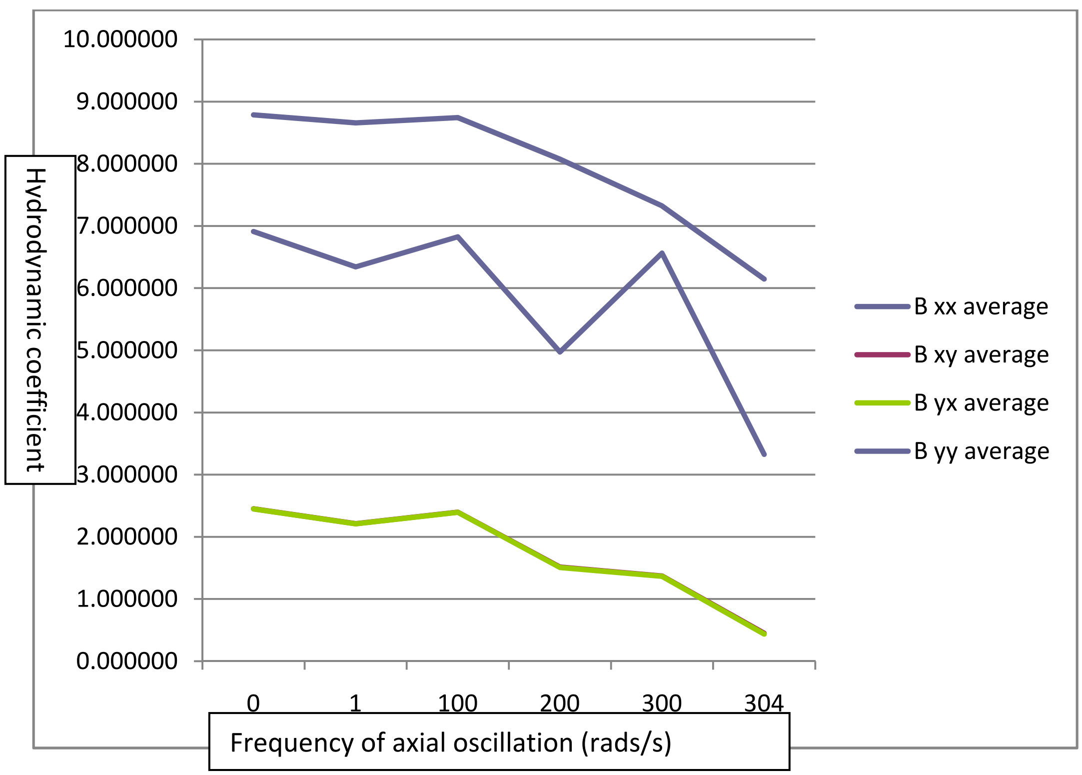

Figure 14. Average B at various frequencies of bearing oscillation.

CHAPTER V

DISCUSSION AND CONCLUSIONS

The dynamic behavior of journal bearings is analyzed under the short bearing assumption in this work. Turbulence within the oil film is accounted for. Oil film response to changes in the bearing velocity is assumed to be instantaneous. The approach taken here was be to use the Navier-Stokes equation to derive the pressure distribution due to journal rotation and bearing oscillation. Navier –Stokes allows the moving boundary condition resulting from the axial oscillation to be accounted for. This pressure distribution was developed as two separate components, one due to journal rotation and one due to bearing oscillation, that were added together to yield the time variable pressure. This time variable pressure was integrated along the circumferential area of the bearing to develop two orthogonal force components. The derivative of these two force components with respect to orbital journal displacement yielded the stiffness coefficients. The derivative of these two force components with respect to orbital journal velocity yielded the damping coefficients.

The main focus of this work was determining whether or not axially oscillating journal would have any effect on the stiffness and damping coefficients of this journal bearing and what these effects might be.

Analysis of the results clearly shows that oscillation of the bearing causes the stiffness and damping to vary from those seen during operation of a non-oscillating bearing. This variance is not always constant and changes throughout the course of the bearing's oscillation.

At certain frequencies the hydrodynamic coefficients remain relatively stable throughout the bearing's oscillation, at others there are dramatic changes. For the most part the deviation from a static bearing increases with the bearing frequency and velocity increases, for a given bearing travel. This deviation increases until the mathematical reaches its limit and the journal contacts the bearing.

The dramatic changes seen at some frequencies suggest that certain frequencies are unstable. More thorough investigation of this phenomenon may reveal something akin to natural frequencies for journal bearing oscillation. Oscillating the journal bearing at these frequencies will quickly lead to an unstable condition. Or it may be found that bearing velocity, or some combination of velocity and frequency (or even travel distance), is what determines the variance from a static bearing.

Further investigation, beyond the scope of this work, would be required to determine exactly what these frequencies might be for a given bearing and what hydrodynamic effects are causing them.

The work done in this thesis is an attempt to lay the groundwork for a theoretical analysis of an axially oscillating journal bearing. Axial oscillation of a journal bearing holds promise as an improved means of thermal management. It is important to determine the viability of these bearings by analyzing their stability. It is clear from this work that some magnitude of oscillation will be permissible. It remains to be seen what sort of heat transfer gains are to be had.

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APPENDICES

Appendix A

Numerical Results

Maximum bearing velocity	0 m/s	0 m/s	0 m/s	0 m/s	0 m/s	0 m/s
Bearing oscillation frequency	0 rads/s	0 rads/s	0 rads/s	0 rads/s	0 rads/s	0 rads/s
Time	0.000000	pi*0.25	pi*0.5	pi*0.75	pi	N/A
K xx 0	2.438032	2.438032	2.438032	2.438032	2.438032	2.438032
K xy 0	3.097234	3.097234	3.097234	3.097234	3.097234	3.097234
K yx 0	-4.750351	-4.75035	-4.75035	-4.75035	-4.75035	-4.75035
K yy 0	1.684296	1.684296	1.684296	1.684296	1.684296	1.684296
B xx 0	6.911347	6.911347	6.911347	6.911347	6.911347	6.911347
B xy 0	2.451952	2.451952	2.451952	2.451952	2.451952	2.451952
B yx 0	2.449843	2.449843	2.449843	2.449843	2.449843	2.449843
B yy 0	8.786275	8.786275	8.786275	8.786275	8.786275	8.786275

Table 3. Stiffness and damping coefficients with journal bearing oscillation frequency of 0 rads/s

Maximum bearing velocity	.001 m/s	.001 m/s	.001 m/s	.001 m/s	.001 m/s	0 m/s
Bearing oscillation frequency	1 rads/s	1 rads/s	1 rads/s	1 rads/s	1 rads/s	0 rads/s
Time	0.000000	pi*0.25	pi*0.5	pi*0.75	pi	N/A
K xx 1	2.438021	2.232088	2.128188	2.232110	2.438042	2.438032
K xy 1	3.097204	2.679584	2.532382	2.679619	3.097264	3.097234
K yx 1	-4.750347	-4.655874	-4.59647	-4.65588	-4.75035	-4.75035
K yy 1	1.684307	1.856851	1.929706	1.856835	1.684286	1.684296
B xx 1	6.911287	6.067150	5.762485	6.067221	6.911407	6.911347
B xy 1	2.451930	2.104735	1.955799	2.104768	2.451973	2.451952
B yx 1	2.449822	2.101184	1.951459	2.101217	2.449865	2.449843
B yy 1	8.786268	8.606985	8.498694	8.607007	8.786283	8.786275

Table 4. Stiffness and damping coefficients with journal bearing oscillation frequency of 1 rads/s and maximum bearing velocity of 0.001 m/s.

Maximum bearing velocity	.10 m/s	.10 m/s	.10 m/s	.10 m/s	.10 m/s	0 m/s
Bearing oscillation frequency	100 rads/s	100 rads/s	100 rads/s	100 rads/s	100 rads/s	0 rads/s
Time	0.000000	pi*0.25/100	pi*0.5/100	pi*0.75/100	pi/100	N/A
K xx 100	2.311833	2.521825	2.311833	2.521825	2.31183	2.438032
K xy 100	2.814427	3.415900	2.814427	3.415900	2.81442	3.097234
K yx 100	-4.69655	-4.774874	-4.69655	-4.774874	-4.69655	-4.75035
K yy 100	1.795763	1.584317	1.795763	1.584317	1.79576	1.684296
B xx 100	6.341767	7.553713	6.341767	7.553713	6.34176	6.911347
B xy 100	2.228318	2.650604	2.228318	2.650604	2.22831	2.451952
B yx 100	2.225335	2.649211	2.225335	2.649211	2.22533	2.449843
B yy 100	8.683179	8.829674	8.683179	8.829674	8.68317	8.786275

Table 5. Stiffness and damping coefficients with journal bearing oscillation frequency of 100 rads/s and maximum bearing velocity of 0.1 m/s.

Maximum bearing velocity	.20 m/s	.20 m/s	.20 m/s	.20 m/s	.20 m/s	0 m/s
Bearing oscillation frequency	200 rads/s	200 rads/s	200 rads/s	200 rads/s	200 rads/s	0 rads/s
Time	0.000000	pi*0.25/200	pi*0.5/200	pi*0.75/200	pi/200	N/A
K xx 200	1.776998	1.776998	1.776998	1.776998	1.776998	2.438032
K xy 200	2.173293	2.173293	2.173293	2.173293	2.173293	3.097234
K yx 200	-4.348657	-4.348657	-4.348657	-4.348657	-4.34865	-4.75035
K yy 200	2.135879	2.135879	2.135879	2.135879	2.135879	1.684296
B xx 200	4.973489	4.973489	4.973489	4.973489	4.973489	6.911347
B xy 200	1.514739	1.514739	1.514739	1.514739	1.514739	2.451952
B yx 200	1.507332	1.507332	1.507332	1.507332	1.507332	2.449843
B yy 200	8.074289	8.074289	8.074289	8.074289	8.074289	8.786275

Table 6. Stiffness and damping coefficients with journal bearing oscillation frequency of 200 rads/s and maximum bearing velocity of 0.2 m/s.

Maximum bearing velocity	.30 m/s	.30 m/s	.30 m/s	.30 m/s	.30 m/s	0 m/s
Bearing oscillation frequency	300 rads/s	300 rads/s	300 rads/s	300 rads/s	300 rads/s	0 rads/s
Time	0.000000	pi*0.25/300	pi*0.5/300	pi*0.75/300	pi/300	N/A
K xx 300	0.711549	1.615201	0.711549	1.615107	0.711549	2.438032
Kxy 300	1.711073	5.235242	1.711073	5.235338	1.711073	3.097234
K yx 300	-3.114670	-4.883148	-3.114670	-4.883208	-3.11467	-4.75035
K yy 300	2.372425	1.553882	2.372425	1.553935	2.372425	1.684296
B xx 300	3.393065	11.320337	3.393065	11.320811	3.393065	6.911347
B xy 300	0.490081	2.694219	0.490081	2.694118	0.490081	2.451952
B yx 300	0.475440	2.697989	0.475440	2.697890	0.475440	2.449843
B yy 300	6.260492	8.914602	6.260492	8.914438	6.260492	8.786275

Table 7. Stiffness and damping coefficients with journal bearing oscillation frequency of 304 rads/s and maximum bearing velocity of 0.304 m/s.

Maximum bearing velocity	.304 m/s	.304 m/s	.304 m/s	.304 m/s	.304 m/s	0 m/s
Bearing oscillation frequency	304 rads/s	304 rads/s	304 rads/s	304 rads/s	304 rads/s	0 rads/s
Time	0.000000	pi*0.25/304	pi*0.5/304	pi*0.75/304	pi/304	N/A
K xx 304	0.660973	0.660973	0.660973	0.660973	0.660973	2.438032
K xy 304	1.705502	1.705502	1.705502	1.705502	1.705502	3.097234
K yx 304	-3.030431	-3.030431	-3.030431	-3.030431	-3.03043	-4.75035
K yy 304	2.359401	2.359401	2.359401	2.359401	2.359401	1.684296
B xx 304	3.326645	3.326645	3.326645	3.326645	3.326645	6.911347
B xy 304	0.449999	0.449999	0.449999	0.449999	0.449999	2.451952
B yx 304	0.435407	0.435407	0.435407	0.435407	0.435407	2.449843
B yy 304	6.147122	6.147122	6.147122	6.147122	6.147122	8.786275

Table 8. Stiffness and damping coefficients with journal bearing oscillation frequency of 304 rads/s and maximum bearing velocity of 0.304 m/s.

Appendix B

Sample Calculation

The solution of the mathematical model inthis work is obtained using the symbolic computational software 'MAPLE'. A sample session file from the program is given in this appendix. Since Maple has some pre-defined names, all of the names described in the nomenclature can not be used. Therefore, some alternative names are used in the program, such as Rey instead of Re for Reynolds number. All the names used in the program are given below.

Bxx, Byy	Dimensionless direct damping coefficients
Bxy, Byx	Dimensionless cross coupled damping coefficients
C	Radial clearance
fx, fy	Dimensionless fluid force components
G	Turbulence coefficient
h	Dimensionless film thickness
J	Bearing diameter
Kxx, Kyy	Dimensionless direct stiffness coefficients
Kxy, Kyx	Dimensionless cross coupled stiffness coefficients
L	Bearing length
p	Dimensionless oil film pressure

po	Steady state dimensionless oil film pressure
R	Bearing radius
Rey	Reynolds number
V	Maximum bearing velocity
x, y, z	Dimensionless coordinates
xo, yo	Steady state dimensionless coordinates of the journal center
θ	Angular coordinate
α	Starting position of the pressure bearing film
μ	Dynamic viscosity
ρ	Density
σ	Modified Sommerfeld number
τ	Dimensionless time
ϕ	Bearing oscillation frequency
ω	Journal angular velocity

> with(DEtools) :

> h := 1-x(τ)·cos(θ)-y(τ)·sin(θ);

$$h := 1 - x(\tau)\cos(\theta) - y(\tau)\sin(\theta) \quad (1)$$

The term 'ϰ' is a component of the pressure due to rotation used to simplify data entry

> $\varkappa := \frac{Rey\cdot C}{R\cdot h}\Big(4\cdot\text{'}diff(h,\tau,\theta)\text{'}+4\cdot\text{'}diff(h,\tau,\tau)\text{'}+\text{'}diff(h,\theta,\theta)\text{'}-\frac{2}{h}\big((\text{'}diff(h,\theta)\text{'})^2 + 4\cdot\text{'}diff(h,\theta)\text{'}\cdot\text{'}diff(h,\tau)\text{'}+4\cdot(\text{'}diff(h,\tau)\text{'})^2\big)\Big)+\frac{24\cdot G}{h^3}(\text{'}diff(h,\theta)\text{'}+2\cdot\text{'}diff(h,\tau)\text{'});$

$$\varkappa := \frac{1}{R\left(1 - x(\tau)\cos(\theta) - y(\tau)\sin(\theta)\right)}\left(Rey\,C\left(4\left(\frac{\partial^2}{\partial\theta\,\partial\tau}h\right) + 4\left(\frac{\partial^2}{\partial\tau^2}h\right) + \frac{\partial^2}{\partial\theta^2}h - \frac{2\left(\left(\frac{\partial}{\partial\theta}h\right)^2 + 4\left(\frac{\partial}{\partial\theta}h\right)\left(\frac{\partial}{\partial\tau}h\right) + 4\left(\frac{\partial}{\partial\tau}h\right)^2\right)}{1 - x(\tau)\cos(\theta) - y(\tau)\sin(\theta)}\right)\right) + \frac{24\,G\left(\frac{\partial}{\partial\theta}h + 2\left(\frac{\partial}{\partial\tau}h\right)\right)}{\left(1 - x(\tau)\cos(\theta) - y(\tau)\sin(\theta)\right)^3} \quad (2)$$

κ is the pressure generated by the rotation of the journal

> $\kappa := \frac{\varkappa}{2}\cdot z^2 - \frac{\varkappa\cdot L^2}{8};$

$$\kappa := \frac{1}{2}\left(\frac{1}{R\left(1 - x(\tau)\cos(\theta) - y(\tau)\sin(\theta)\right)}\left(Rey\,C\left(4\left(\frac{d}{d\tau}x(\tau)\right)\sin(\theta) - 4\left(\frac{d}{d\tau}y(\tau)\right)\cos(\theta) - 4\left(\frac{d^2}{d\tau^2}x(\tau)\right)\cos(\theta) - 4\left(\frac{d^2}{d\tau^2}y(\tau)\right)\sin(\theta)\right.\right.\right. \quad (3)$$

$$+x(\tau)\cos(\theta)+y(\tau)\sin(\theta)$$

$$-\frac{1}{1-x(\tau)\cos(\theta)-y(\tau)\sin(\theta)}\Bigg(2\Bigg((x(\tau)\sin(\theta)-y(\tau)\cos(\theta))^2$$

$$+4\,(x(\tau)\sin(\theta)-y(\tau)\cos(\theta))\left(-\left(\frac{d}{d\tau}x(\tau)\right)\cos(\theta)\right.$$

$$\left.-\left(\frac{d}{d\tau}y(\tau)\right)\sin(\theta)\right)+4\left(-\left(\frac{d}{d\tau}x(\tau)\right)\cos(\theta)\right.$$

$$\left.-\left(\frac{d}{d\tau}y(\tau)\right)\sin(\theta)\right)^2\Bigg)\Bigg)\Bigg)$$

$$+\frac{1}{(1-x(\tau)\cos(\theta)-y(\tau)\sin(\theta))^3}\Bigg(24\,G\Bigg(x(\tau)\sin(\theta)-y(\tau)\cos(\theta)$$

$$-2\left(\frac{d}{d\tau}x(\tau)\right)\cos(\theta)-2\left(\frac{d}{d\tau}y(\tau)\right)\sin(\theta)\Bigg)\Bigg)\Bigg)z^2$$

$$-\frac{1}{8}\Bigg(\frac{1}{R\,(1-x(\tau)\cos(\theta)-y(\tau)\sin(\theta))}\Bigg(Rey\,C\Bigg(4\left(\frac{d}{d\tau}x(\tau)\right)\sin(\theta)$$

$$-4\left(\frac{d}{d\tau}y(\tau)\right)\cos(\theta)-4\left(\frac{d^2}{d\tau^2}x(\tau)\right)\cos(\theta)-4\left(\frac{d^2}{d\tau^2}y(\tau)\right)\sin(\theta)$$

$$+x(\tau)\cos(\theta)+y(\tau)\sin(\theta)$$

$$-\frac{1}{1-x(\tau)\cos(\theta)-y(\tau)\sin(\theta)}\Bigg(2\Bigg((x(\tau)\sin(\theta)-y(\tau)\cos(\theta))^2$$

$$+4\,(x(\tau)\sin(\theta)-y(\tau)\cos(\theta))\left(-\left(\frac{d}{d\tau}x(\tau)\right)\cos(\theta)\right.$$

$$\left.-\left(\frac{d}{d\tau}y(\tau)\right)\sin(\theta)\right)+4\left(-\left(\frac{d}{d\tau}x(\tau)\right)\cos(\theta)\right.$$

$$\left.-\left(\frac{d}{d\tau}y(\tau)\right)\sin(\theta)\right)^2\Bigg)\Bigg)\Bigg)$$

$$+\frac{1}{(1-x(\tau)\cos(\theta)-y(\tau)\sin(\theta))^3}\left(24\,G\left(x(\tau)\sin(\theta)-y(\tau)\cos(\theta)\right.\right.$$
$$\left.\left.\left.-2\left(\frac{d}{d\tau}x(\tau)\right)\cos(\theta)-2\left(\frac{d}{d\tau}y(\tau)\right)\sin(\theta)\right)\right)\right)L^2$$

The term 'ϵ' is a component of the pressure due to rotation used to simplify data entry

$$> \epsilon := \frac{Rey\cdot C}{R}\cdot\frac{-1\cdot J}{L\cdot h}\frac{V}{R\cdot\omega}\left(h\cdot\phi\cdot\cos(\phi\cdot\tau)+\frac{V}{R\cdot\omega}\cdot\sin(\phi\cdot\tau)\frac{\partial}{\partial\tau}h+\frac{1}{2}\cdot\frac{V}{R\cdot\omega}\cdot\sin(\phi\cdot\tau)\cdot\frac{\partial}{\partial\theta}h\right)-\frac{J}{L\cdot h}\cdot\frac{12\cdot G\cdot V}{h\cdot R\cdot\omega}\cdot\sin(\phi\cdot\tau);$$

$$\epsilon := -\left(Rey\,C\,J\,V\left((1-x(\tau)\cos(\theta)-y(\tau)\sin(\theta))\,\phi\cos(\phi\,\tau)\right.\right.$$
$$+\frac{V\sin(\phi\,\tau)\left(-\left(\frac{d}{d\tau}x(\tau)\right)\cos(\theta)-\left(\frac{d}{d\tau}y(\tau)\right)\sin(\theta)\right)}{R\,\omega}$$
$$\left.\left.+\frac{1}{2}\frac{V\sin(\phi\,\tau)\,(x(\tau)\sin(\theta)-y(\tau)\cos(\theta))}{R\,\omega}\right)\right)\Big/\left(R^2L\,(1-x(\tau)\cos(\theta)-y(\tau)\sin(\theta))\,\omega\right)$$
$$-\frac{12\,J\,G\,V\sin(\phi\,\tau)}{L\,(1-x(\tau)\cos(\theta)-y(\tau)\sin(\theta))^2R\,\omega} \quad \textbf{(4)}$$

ξ is the pressure generated by the oscillation of the bearing when n=0,2,4,6....

$$> \xi := \epsilon\cdot z+\frac{\epsilon\cdot L}{2};$$

$$\xi := \left(-\left(Rey\,C\,J\,V\left((1-x(\tau)\cos(\theta)-y(\tau)\sin(\theta))\,\phi\cos(\phi\,\tau)\right.\right.\right.$$
$$+\frac{V\sin(\phi\,\tau)\left(-\left(\frac{d}{d\tau}x(\tau)\right)\cos(\theta)-\left(\frac{d}{d\tau}y(\tau)\right)\sin(\theta)\right)}{R\,\omega}$$
$$\left.\left.+\frac{1}{2}\frac{V\sin(\phi\,\tau)\,(x(\tau)\sin(\theta)-y(\tau)\cos(\theta))}{R\,\omega}\right)\right)\Big/\left(R^2L\,(1-x(\tau)\cos(\theta)-y(\tau)\sin(\theta))\,\omega\right) \quad \textbf{(5)}$$

$$- \frac{12 J G V \sin(\phi \tau)}{L (1 - x(\tau) \cos(\theta) - y(\tau) \sin(\theta))^2 R \omega} \Bigg) z + \frac{1}{2} \Bigg(- \Bigg(Rey\, C\, J\, V \Bigg((1$$

$$- x(\tau) \cos(\theta) - y(\tau) \sin(\theta)) \phi \cos(\phi \tau)$$

$$+ \frac{V \sin(\phi \tau) \left(-\left(\frac{d}{d\tau} x(\tau) \right) \cos(\theta) - \left(\frac{d}{d\tau} y(\tau) \right) \sin(\theta) \right)}{R \omega}$$

$$+ \frac{1}{2} \frac{V \sin(\phi \tau) (x(\tau) \sin(\theta) - y(\tau) \cos(\theta))}{R \omega} \Bigg)\Bigg) \Bigg/ (R^2 L (1$$

$$- x(\tau) \cos(\theta) - y(\tau) \sin(\theta)) \omega)$$

$$- \frac{12 J G V \sin(\phi \tau)}{L (1 - x(\tau) \cos(\theta) - y(\tau) \sin(\theta))^2 R \omega} \Bigg) L$$

The pressure due to rotating and oscillating is summed.

> p := κ + ξ

$$p := \frac{1}{2} \Bigg(\frac{1}{R (1 - x(\tau) \cos(\theta) - y(\tau) \sin(\theta))} \Bigg(Rey\, C \Bigg(4 \left(\frac{d}{d\tau} x(\tau) \right) \sin(\theta) \tag{6}$$

$$- 4 \left(\frac{d}{d\tau} y(\tau) \right) \cos(\theta) - 4 \left(\frac{d^2}{d\tau^2} x(\tau) \right) \cos(\theta) - 4 \left(\frac{d^2}{d\tau^2} y(\tau) \right) \sin(\theta)$$

$$+ x(\tau) \cos(\theta) + y(\tau) \sin(\theta)$$

$$- \frac{1}{1 - x(\tau) \cos(\theta) - y(\tau) \sin(\theta)} \Bigg(2 \Big((x(\tau) \sin(\theta) - y(\tau) \cos(\theta))^2$$

$$+ 4 (x(\tau) \sin(\theta) - y(\tau) \cos(\theta)) \Bigg(-\left(\frac{d}{d\tau} x(\tau) \right) \cos(\theta)$$

$$- \left(\frac{d}{d\tau} y(\tau) \right) \sin(\theta) \Bigg) + 4 \Bigg(-\left(\frac{d}{d\tau} x(\tau) \right) \cos(\theta)$$

$$-\left(\left(\frac{d}{d\tau}y(\tau)\right)\sin(\theta)\right)^2\Bigg)\Bigg)\Bigg)\Bigg)$$

$$+\frac{1}{(1-x(\tau)\cos(\theta)-y(\tau)\sin(\theta))^3}\Bigg(24\,G\Bigg(x(\tau)\sin(\theta)-y(\tau)\cos(\theta)$$

$$-2\left(\frac{d}{d\tau}x(\tau)\right)\cos(\theta)-2\left(\frac{d}{d\tau}y(\tau)\right)\sin(\theta)\Bigg)\Bigg)\Bigg)z^2$$

$$-\frac{1}{8}\Bigg(\frac{1}{R\,(1-x(\tau)\cos(\theta)-y(\tau)\sin(\theta))}\Bigg(Rey\,C\Bigg(4\left(\frac{d}{d\tau}x(\tau)\right)\sin(\theta)$$

$$-4\left(\frac{d}{d\tau}y(\tau)\right)\cos(\theta)-4\left(\frac{d^2}{d\tau^2}x(\tau)\right)\cos(\theta)-4\left(\frac{d^2}{d\tau^2}y(\tau)\right)\sin(\theta)$$

$$+x(\tau)\cos(\theta)+y(\tau)\sin(\theta)$$

$$-\frac{1}{1-x(\tau)\cos(\theta)-y(\tau)\sin(\theta)}\Bigg(2\Big((x(\tau)\sin(\theta)-y(\tau)\cos(\theta))^2$$

$$+4\,(x(\tau)\sin(\theta)-y(\tau)\cos(\theta))\left(-\left(\frac{d}{d\tau}x(\tau)\right)\cos(\theta)\right.$$

$$\left.-\left(\frac{d}{d\tau}y(\tau)\right)\sin(\theta)\right)+4\left(-\left(\frac{d}{d\tau}x(\tau)\right)\cos(\theta)\right.$$

$$\left.-\left(\frac{d}{d\tau}y(\tau)\right)\sin(\theta)\right)^2\Big)\Bigg)\Bigg)\Bigg)$$

$$+\frac{1}{(1-x(\tau)\cos(\theta)-y(\tau)\sin(\theta))^3}\Bigg(24\,G\Bigg(x(\tau)\sin(\theta)-y(\tau)\cos(\theta)$$

$$-2\left(\frac{d}{d\tau}x(\tau)\right)\cos(\theta)-2\left(\frac{d}{d\tau}y(\tau)\right)\sin(\theta)\Bigg)\Bigg)\Bigg)L^2+\Bigg($$

$$-\Bigg(Rey\,C\,J\,V\Bigg((1-x(\tau)\cos(\theta)-y(\tau)\sin(\theta))\,\phi\cos(\phi\,\tau)$$

$$+\frac{V\sin(\phi\,\tau)\left(-\left(\frac{d}{d\tau}x(\tau)\right)\cos(\theta)-\left(\frac{d}{d\tau}y(\tau)\right)\sin(\theta)\right)}{R\,\omega}$$

$$+\frac{1}{2}\,\frac{V\sin(\phi\,\tau)\,(x(\tau)\sin(\theta)-y(\tau)\cos(\theta))}{R\,\omega}\Bigg)\Bigg)\Bigg/\Big(R^2\,L\,(1$$

$$-x(\tau)\cos(\theta)-y(\tau)\sin(\theta))\,\omega\Big)$$

$$-\frac{12\,J\,G\,V\sin(\phi\,\tau)}{L\,(1-x(\tau)\cos(\theta)-y(\tau)\sin(\theta))^2\,R\,\omega}\Bigg)z+\frac{1}{2}\Bigg(\Bigg(-\Bigg(Rey\,C\,J\,V\Bigg((1$$

$$-x(\tau)\cos(\theta)-y(\tau)\sin(\theta))\,\phi\cos(\phi\,\tau)$$

$$+\frac{V\sin(\phi\,\tau)\left(-\left(\frac{d}{d\tau}x(\tau)\right)\cos(\theta)-\left(\frac{d}{d\tau}y(\tau)\right)\sin(\theta)\right)}{R\,\omega}$$

$$+\frac{1}{2}\,\frac{V\sin(\phi\,\tau)\,(x(\tau)\sin(\theta)-y(\tau)\cos(\theta))}{R\,\omega}\Bigg)\Bigg)\Bigg/\Big(R^2\,L\,(1$$

$$-x(\tau)\cos(\theta)-y(\tau)\sin(\theta))\,\omega\Big)$$

$$-\frac{12\,J\,G\,V\sin(\phi\,\tau)}{L\,(1-x(\tau)\cos(\theta)-y(\tau)\sin(\theta))^2\,R\,\omega}\Bigg)L$$

More manageable terms are substituted int to the pressure equation here.

```
> p := subs(diff(x(τ), τ, τ) = x11, diff(y(τ), τ, τ) = y11, diff(x(τ), τ) = x1,
      diff(y(τ), τ) = y1, x(τ) = x, y(τ) = y, p);
```

$$p:=\frac{1}{2}\Bigg(\frac{1}{R\,(1-x\cos(\theta)-y\sin(\theta))}\Bigg(Rey\,C\Bigg(4\,x1\sin(\theta)-4\,y1\cos(\theta)\tag{7}$$

$$-4\,x11\cos(\theta)-4\,y11\sin(\theta)+x\cos(\theta)+y\sin(\theta)$$

$$-\frac{1}{1-x\cos(\theta)-y\sin(\theta)}\big(2\,((x\sin(\theta)-y\cos(\theta))^2+4\,(x\sin(\theta)$$

$$-y\cos(\theta))\,(-x1\cos(\theta)-y1\sin(\theta))+4\,(-x1\cos(\theta)-y1\sin(\theta))^2)\big)\Bigg)$$

$$\Bigg)+\frac{24\,G\,(x\sin(\theta)-y\cos(\theta)-2\,x1\cos(\theta)-2\,y1\sin(\theta))}{(1-x\cos(\theta)-y\sin(\theta))^3}\Bigg)z^2$$

$$-\frac{1}{8}\left(\frac{1}{R\,(1-x\cos(\theta)-y\sin(\theta))}\left(Rey\,C\left(4\,x1\sin(\theta)-4\,y1\cos(\theta)\right.\right.\right.$$

$$-4\,x11\cos(\theta)-4\,y11\sin(\theta)+x\cos(\theta)+y\sin(\theta)$$

$$-\frac{1}{1-x\cos(\theta)-y\sin(\theta)}\left(2\left((x\sin(\theta)-y\cos(\theta))^2+4\,(x\sin(\theta)\right.\right.$$

$$\left.\left.\left.\left.-y\cos(\theta))\,(-x1\cos(\theta)-y1\sin(\theta))+4\,(-x1\cos(\theta)-y1\sin(\theta))^2\right)\right)\right)$$

$$\left.\right)+\frac{24\,G\,(x\sin(\theta)-y\cos(\theta)-2\,x1\cos(\theta)-2\,y1\sin(\theta))}{(1-x\cos(\theta)-y\sin(\theta))^3}\Bigg)L^2+\Bigg($$

$$-\frac{1}{R^2\,L\,(1-x\cos(\theta)-y\sin(\theta))\,\omega}\left(Rey\,C\,J\,V\left((1-x\cos(\theta)\right.\right.$$

$$-y\sin(\theta))\,\phi\cos(\phi\,\tau)+\frac{V\sin(\phi\,\tau)\,(-x1\cos(\theta)-y1\sin(\theta))}{R\,\omega}$$

$$\left.\left.+\frac{1}{2}\,\frac{V\sin(\phi\,\tau)\,(x\sin(\theta)-y\cos(\theta))}{R\,\omega}\right)\right)$$

$$-\frac{12\,J\,G\,V\sin(\phi\,\tau)}{L\,(1-x\cos(\theta)-y\sin(\theta))^2\,R\,\omega}\Bigg)z+\frac{1}{2}\Bigg($$

$$-\frac{1}{R^2\,L\,(1-x\cos(\theta)-y\sin(\theta))\,\omega}\left(Rey\,C\,J\,V\left((1-x\cos(\theta)\right.\right.$$

$$-y\sin(\theta))\,\phi\cos(\phi\,\tau)+\frac{V\sin(\phi\,\tau)\,(-x1\cos(\theta)-y1\sin(\theta))}{R\,\omega}$$

$$\left.\left.+\frac{1}{2}\,\frac{V\sin(\phi\,\tau)\,(x\sin(\theta)-y\cos(\theta))}{R\,\omega}\right)\right)$$

$$-\frac{12\,J\,G\,V\sin(\phi\,\tau)}{L\,(1-x\cos(\theta)-y\sin(\theta))^2\,R\,\omega}\Bigg)L$$

x and y are defined as time variable values

$> h := subs(x(\tau)=x, y(\tau)=y, h);$

$h := 1-x\cos(\theta)-y\sin(\theta)$ **(8)**

The turbuklence coefficient, G, is defined.

$> G := 1+\delta\cdot s\cdot 'h'\cdot Rey^{0.96}$

$G := 1+\delta\,s\,h\,Rey^{0.96}$ **(9)**

The constant, δ, is defined

$> \delta := 3.47\cdot 10^{-4}$

$\delta := 0.0003470000000$ **(10)**

s is defined

$$> s := \frac{1}{\pi}\cdot\arctan\left(\left(\frac{2}{\pi}\right)\cdot\left(\frac{(ReyT-ReyL)}{ReyT+ReyL}\cdot\frac{\frac{Rey}{ReyA}}{1-\frac{Rey}{ReyA}}\right)\right)$$

$$s := \frac{\arctan\left(\frac{2\,(ReyT-ReyL)\,Rey}{\pi\,(ReyT+ReyL)\,ReyA\left(1-\frac{Rey}{ReyA}\right)}\right)}{\pi} \quad (11)$$

The Reynolds number for the part of the flow which is turbulent is defined

$$> ReyT := 41.2\cdot\left(\frac{\left(\frac{R}{C}\right)}{(1-ep)^3+\frac{4}{3}\cdot\left(\frac{L}{J}\right)^2\cdot ep1^2\cdot(1-ep)}\right)^{0.5}$$

$$ReyT := 41.2\left(\frac{R}{C\left((1-ep)^3+\frac{4}{3}\,\frac{L^2\,ep1^2\,(1-ep)}{J^2}\right)}\right)^{0.5} \quad (12)$$

The Reynolds number for the part of the flow which is laminar is defined

$$> ReyL := 41.2\cdot\left(\frac{\left(\frac{R}{C}\right)}{(1+ep)^3+\frac{4}{3}\cdot\left(\frac{L}{J}\right)^2\cdot ep1^2\cdot(1+ep)}\right)^{0.5}$$

$$ReyL := 41.2\left(\frac{R}{C\left((1+ep)^3+\frac{4}{3}\,\frac{L^2\,ep1^2\,(1+ep)}{J^2}\right)}\right)^{0.5} \quad (13)$$

The average of ReyT and ReyL is defined

$$> ReyA :='\left(\frac{ReyT}{2}+\frac{ReyL}{2}\right)';$$

$$ReyA := \frac{1}{2}\,ReyT+\frac{1}{2}\,ReyL \quad (14)$$

$$> ep := (x^2+y^2)^{0.5};$$

$$ep := (x^2+y^2)^{0.5} \quad (15)$$

$$> ep1 := \frac{(x\cdot x1+y\cdot y1)}{(x^2+y^2)^{0.5}};$$

$$ep1 := \frac{x\,x1+y\,y1}{(x^2+y^2)^{0.5}} \quad (16)$$

ReyT is evaluated

$> ReyT;$

$$41.2\left(\frac{R}{C\left(\left(1-(x^2+y^2)^{0.5}\right)^3+\frac{4}{3}\,\frac{L^2\,(x\,x1+y\,y1)^2\left(1-(x^2+y^2)^{0.5}\right)}{J^2\,(x^2+y^2)^{1.0}}\right)}\right)^{0.5} \quad (17)$$

ReyL is evaluated

> *ReyL;*

$$41.2\left(\frac{R}{C\left(\left(1+\left(x^2+y^2\right)^{0.5}\right)^3+\frac{4}{3}\,\frac{L^2\,(x\,x1+y\,y1)^2\left(1+\left(x^2+y^2\right)^{0.5}\right)}{J^2\left(x^2+y^2\right)^{1.0}}\right)}\right)^{0.5} \tag{18}$$

ReyA is evaluated

> *ReyA;*

$$20.60000000\left(\frac{R}{C\left(\left(1-\left(x^2+y^2\right)^{0.5}\right)^3+\frac{4}{3}\,\frac{L^2\,(x\,x1+y\,y1)^2\left(1-\left(x^2+y^2\right)^{0.5}\right)}{J^2\left(x^2+y^2\right)^{1.0}}\right)}\right)^{0.5} + 20.60000000\left(\frac{R}{C\left(\left(1+\left(x^2+y^2\right)^{0.5}\right)^3+\frac{4}{3}\,\frac{L^2\,(x\,x1+y\,y1)^2\left(1+\left(x^2+y^2\right)^{0.5}\right)}{J^2\left(x^2+y^2\right)^{1.0}}\right)}\right)^{0.5} \tag{19}$$

s is evaluated

> *s;*

$$\frac{1}{\pi}\left(\arctan\left(\left(2\left(41.2\left(\frac{R}{C\left(\left(1-\left(x^2+y^2\right)^{0.5}\right)^3+\frac{4}{3}\,\frac{L^2\,(x\,x1+y\,y1)^2\left(1-\left(x^2+y^2\right)^{0.5}\right)}{J^2\left(x^2+y^2\right)^{1.0}}\right)}\right)^{0.5} - 41.2\left(\frac{R}{C\left(\left(1+\left(x^2+y^2\right)^{0.5}\right)^3+\frac{4}{3}\,\frac{L^2\,(x\,x1+y\,y1)^2\left(1+\left(x^2+y^2\right)^{0.5}\right)}{J^2\left(x^2+y^2\right)^{1.0}}\right)}\right)^{0.5}\right)\right) \Big/ Rey\right) \right. \tag{20}$$

$$\Bigg(\pi \Bigg(41.2$$

$$\left(\frac{R}{C\left(\left(1-\left(x^2+y^2\right)^{0.5}\right)^3+\frac{4}{3}\ \frac{L^2\left(x\,x1+y\,y1\right)^2\left(1-\left(x^2+y^2\right)^{0.5}\right)}{J^2\left(x^2+y^2\right)^{1.0}}\right)}\right)^{0.5}$$

$$+\,41.2$$

$$\left(\frac{R}{C\left(\left(1+\left(x^2+y^2\right)^{0.5}\right)^3+\frac{4}{3}\ \frac{L^2\left(x\,x1+y\,y1\right)^2\left(1+\left(x^2+y^2\right)^{0.5}\right)}{J^2\left(x^2+y^2\right)^{1.0}}\right)}\right)^{0.5}\Bigg)$$

$$\Bigg(20.60000000$$

$$\left(\frac{R}{C\left(\left(1-\left(x^2+y^2\right)^{0.5}\right)^3+\frac{4}{3}\ \frac{L^2\left(x\,x1+y\,y1\right)^2\left(1-\left(x^2+y^2\right)^{0.5}\right)}{J^2\left(x^2+y^2\right)^{1.0}}\right)}\right)^{0.5}$$

$$+\,20.60000000$$

$$\left(\frac{R}{C\left(\left(1+\left(x^2+y^2\right)^{0.5}\right)^3+\frac{4}{3}\ \frac{L^2\left(x\,x1+y\,y1\right)^2\left(1+\left(x^2+y^2\right)^{0.5}\right)}{J^2\left(x^2+y^2\right)^{1.0}}\right)}\right)^{0.5}\Bigg)$$

$$\Bigg(1-Rey\ \Big/$$

$$\left(20.60000000\right.$$

$$\left(\frac{R}{C\left(\left(1-(x^2+y^2)^{0.5}\right)^3+\frac{4}{3}\,\frac{L^2\,(x\,x1+y\,y1)^2\,\left(1-(x^2+y^2)^{0.5}\right)}{J^2\,(x^2+y^2)^{1.0}}\right)}\right)^{0.5}$$

$$+\,20.60000000$$

$$\left.\left(\frac{R}{C\left(\left(1+(x^2+y^2)^{0.5}\right)^3+\frac{4}{3}\,\frac{L^2\,(x\,x1+y\,y1)^2\,\left(1+(x^2+y^2)^{0.5}\right)}{J^2\,(x^2+y^2)^{1.0}}\right)}\right)^{0.5}\right)$$

$$\left.\left.\left.\left.\right)\right)\right)\right)$$

G is evaluated

> *G*;

1 **(21)**

$$+\,\frac{1}{\pi}\left(0.0003470000000\arctan\left(\left(2\left(41.2\right.\right.\right.\right.$$

$$\left(\frac{R}{C\left(\left(1-(x^2+y^2)^{0.5}\right)^3+\frac{4}{3}\,\frac{L^2\,(x\,x1+y\,y1)^2\,\left(1-(x^2+y^2)^{0.5}\right)}{J^2\,(x^2+y^2)^{1.0}}\right)}\right)^{0.5}$$

$$-\,41.2$$

$$\left.\left(\frac{R}{C\left(\left(1+(x^2+y^2)^{0.5}\right)^3+\frac{4}{3}\,\frac{L^2\,(x\,x1+y\,y1)^2\,\left(1+(x^2+y^2)^{0.5}\right)}{J^2\,(x^2+y^2)^{1.0}}\right)}\right)^{0.5}\right)$$

$$Rey\Bigg) \Bigg/$$

$$\Bigg(\pi\Bigg(41.2$$

$$\left(\frac{R}{C\left(\left(1-\left(x^2+y^2\right)^{0.5}\right)^3+\frac{4}{3}\,\frac{L^2\left(x\,x1+y\,y1\right)^2\left(1-\left(x^2+y^2\right)^{0.5}\right)}{J^2\left(x^2+y^2\right)^{1.0}}\right)}\right)^{0.5}$$

$$+\,41.2$$

$$\left(\frac{R}{C\left(\left(1+\left(x^2+y^2\right)^{0.5}\right)^3+\frac{4}{3}\,\frac{L^2\left(x\,x1+y\,y1\right)^2\left(1+\left(x^2+y^2\right)^{0.5}\right)}{J^2\left(x^2+y^2\right)^{1.0}}\right)}\right)^{0.5}\Bigg)$$

$$\Bigg(20.60000000$$

$$\left(\frac{R}{C\left(\left(1-\left(x^2+y^2\right)^{0.5}\right)^3+\frac{4}{3}\,\frac{L^2\left(x\,x1+y\,y1\right)^2\left(1-\left(x^2+y^2\right)^{0.5}\right)}{J^2\left(x^2+y^2\right)^{1.0}}\right)}\right)^{0.5}$$

$$+\,20.60000000$$

$$\left(\frac{R}{C\left(\left(1+\left(x^2+y^2\right)^{0.5}\right)^3+\frac{4}{3}\frac{L^2\left(x\,x1+y\,y1\right)^2\left(1+\left(x^2+y^2\right)^{0.5}\right)}{J^2\left(x^2+y^2\right)^{1.0}}\right)}\right)^{0.5}\Bigg)$$

$$\Bigg(1-Rey\Bigg/$$

$$\Bigg(20.60000000$$

$$\left(\frac{R}{C\left(\left(1-\left(x^2+y^2\right)^{0.5}\right)^3+\frac{4}{3}\frac{L^2\left(x\,x1+y\,y1\right)^2\left(1-\left(x^2+y^2\right)^{0.5}\right)}{J^2\left(x^2+y^2\right)^{1.0}}\right)}\right)^{0.5}$$

$$+\,20.60000000$$

$$\left(\frac{R}{C\left(\left(1+\left(x^2+y^2\right)^{0.5}\right)^3+\frac{4}{3}\frac{L^2\left(x\,x1+y\,y1\right)^2\left(1+\left(x^2+y^2\right)^{0.5}\right)}{J^2\left(x^2+y^2\right)^{1.0}}\right)}\right)^{0.5}\Bigg)$$

$$\Bigg)\Bigg)\Bigg)\Bigg(\left(1-x\cos(\theta)-y\sin(\theta)\right)Rey^{0.96}\Bigg)$$

p is evaluated

> p;

$$\frac{1}{2}\left(\frac{1}{R\left(1-x\cos(\theta)-y\sin(\theta)\right)}\left(Rey\,C\left(4\,x1\sin(\theta)-4\,y1\cos(\theta)\right.\right.\right. \qquad \textbf{(22)}$$

$$-4\,x11\cos(\theta)-4\,y11\sin(\theta)+x\cos(\theta)+y\sin(\theta)$$

$$-\frac{1}{1-x\cos(\theta)-y\sin(\theta)}\left(2\left(\left(x\sin(\theta)-y\cos(\theta)\right)^{2}+4\left(x\sin(\theta)\right.\right.\right.$$

$$\left.\left.\left.-y\cos(\theta)\right)\left(-x1\cos(\theta)-y1\sin(\theta)\right)+4\left(-x1\cos(\theta)-y1\sin(\theta)\right)^{2}\right)\right)$$

$$\Bigg)+\frac{1}{\left(1-x\cos(\theta)-y\sin(\theta)\right)^{3}}\left(24\left(1\right.\right.$$

$$+\frac{1}{\pi}\left(0.0003470000000\arctan\left(\left(2\left(41.2\right.\right.\right.\right.$$

$$\left(\frac{R}{C\left(\left(1-\left(x^2+y^2\right)^{0.5}\right)^3+\frac{4}{3}\,\frac{L^2\left(x\,x1+y\,y1\right)^2\left(1-\left(x^2+y^2\right)^{0.5}\right)}{J^2\left(x^2+y^2\right)^{1.0}}\right)}\right)^{0.5}$$

$$-41.2$$

$$\left.\left(\frac{R}{C\left(\left(1+\left(x^2+y^2\right)^{0.5}\right)^3+\frac{4}{3}\,\frac{L^2\left(x\,x1+y\,y1\right)^2\left(1+\left(x^2+y^2\right)^{0.5}\right)}{J^2\left(x^2+y^2\right)^{1.0}}\right)}\right)^{0.5}\right)$$

$$\left.Rey\right)\Big/$$

$$\left(\pi\left(41.2\right.\right.$$

$$\left(\frac{R}{C\left(\left(1-\left(x^2+y^2\right)^{0.5}\right)^3+\frac{4}{3}\,\frac{L^2\left(x\,x1+y\,y1\right)^2\left(1-\left(x^2+y^2\right)^{0.5}\right)}{J^2\left(x^2+y^2\right)^{1.0}}\right)}\right)^{0.5}$$

$$+41.2$$

$$\left(\frac{R}{C\left(\left(1+\left(x^2+y^2\right)^{0.5}\right)^3+\frac{4}{3}\frac{L^2\left(x\,x1+y\,y1\right)^2\left(1+\left(x^2+y^2\right)^{0.5}\right)}{J^2\left(x^2+y^2\right)^{1.0}}\right)}\right)^{0.5}\Bigg)$$

$$\Bigg(20.60000000$$

$$\left(\frac{R}{C\left(\left(1-\left(x^2+y^2\right)^{0.5}\right)^3+\frac{4}{3}\frac{L^2\left(x\,x1+y\,y1\right)^2\left(1-\left(x^2+y^2\right)^{0.5}\right)}{J^2\left(x^2+y^2\right)^{1.0}}\right)}\right)^{0.5}$$

$$+\,20.60000000$$

$$\left(\frac{R}{C\left(\left(1+\left(x^2+y^2\right)^{0.5}\right)^3+\frac{4}{3}\frac{L^2\left(x\,x1+y\,y1\right)^2\left(1+\left(x^2+y^2\right)^{0.5}\right)}{J^2\left(x^2+y^2\right)^{1.0}}\right)}\right)^{0.5}\Bigg)$$

$$\Bigg(1-Rey\Bigg/$$

$$\Bigg(20.60000000$$

$$\left(\frac{R}{C\left(\left(1-\left(x^2+y^2\right)^{0.5}\right)^3+\frac{4}{3}\frac{L^2\left(x\,x1+y\,y1\right)^2\left(1-\left(x^2+y^2\right)^{0.5}\right)}{J^2\left(x^2+y^2\right)^{1.0}}\right)}\right)^{0.5}$$

$$+\,20.60000000$$

$$\left(\frac{R}{C\left(\left(1+\left(x^2+y^2\right)^{0.5}\right)^3+\frac{4}{3}\frac{L^2\left(x\,x1+y\,y1\right)^2\left(1+\left(x^2+y^2\right)^{0.5}\right)}{J^2\left(x^2+y^2\right)^{1.0}}\right)}\right)^{0.5}\Bigg)$$

$$\left.\left.\left.\left.\left.\left.\right)\right)\right)\left(1-x\cos(\theta)-y\sin(\theta)\right)Rey^{0.96}\right)\right)\left(x\sin(\theta)-y\cos(\theta)\right.$$

$$\left.\left.\left.-2\,x1\cos(\theta)-2\,y1\sin(\theta)\right)\right)\right)z^{2}$$

$$-\frac{1}{8}\left(\frac{1}{R\left(1-x\cos(\theta)-y\sin(\theta)\right)}\left(Rey\,C\left(4\,x1\sin(\theta)-4\,y1\cos(\theta)\right.\right.\right.$$

$$-4\,x11\cos(\theta)-4\,y11\sin(\theta)+x\cos(\theta)+y\sin(\theta)$$

$$-\frac{1}{1-x\cos(\theta)-y\sin(\theta)}\left(2\left(\left(x\sin(\theta)-y\cos(\theta)\right)^{2}+4\left(x\sin(\theta)\right.\right.\right.$$

$$-y\cos(\theta))\,(-x1\cos(\theta)-y1\sin(\theta))+4\,(-x1\cos(\theta)-y1\sin(\theta))^2)))\Big)$$

$$\Big)+\frac{1}{(1-x\cos(\theta)-y\sin(\theta))^3}\Bigg(24\Bigg(1$$

$$+\frac{1}{\pi}\Bigg(0.0003470000000\arctan\Bigg(\Bigg(2\Bigg(41.2$$

$$\left(\frac{R}{C\left(\left(1-(x^2+y^2)^{0.5}\right)^3+\frac{4}{3}\,\frac{L^2\,(x\,x1+y\,y1)^2\left(1-(x^2+y^2)^{0.5}\right)}{J^2\,(x^2+y^2)^{1.0}}\right)}\right)^{0.5}$$

$$-41.2$$

$$\left(\frac{R}{C\left(\left(1+(x^2+y^2)^{0.5}\right)^3+\frac{4}{3}\,\frac{L^2\,(x\,x1+y\,y1)^2\left(1+(x^2+y^2)^{0.5}\right)}{J^2\,(x^2+y^2)^{1.0}}\right)}\right)^{0.5}\Bigg)$$

$$Rey \Bigg) \Bigg/$$

$$\Bigg(\pi \Bigg(41.2$$

$$\left(\frac{R}{C \left(\left(1 - (x^2 + y^2)^{0.5}\right)^3 + \frac{4}{3} \frac{L^2 (x\,x1 + y\,y1)^2 \left(1 - (x^2 + y^2)^{0.5}\right)}{J^2 (x^2 + y^2)^{1.0}} \right)} \right)^{0.5}$$

$$+ 41.2$$

$$\left(\frac{R}{C \left(\left(1 + (x^2 + y^2)^{0.5}\right)^3 + \frac{4}{3} \frac{L^2 (x\,x1 + y\,y1)^2 \left(1 + (x^2 + y^2)^{0.5}\right)}{J^2 (x^2 + y^2)^{1.0}} \right)} \right)^{0.5} \Bigg)$$

$$\Bigg(20.60000000$$

$$\left(\frac{R}{C \left(\left(1 - (x^2 + y^2)^{0.5}\right)^3 + \frac{4}{3} \frac{L^2 (x\,x1 + y\,y1)^2 \left(1 - (x^2 + y^2)^{0.5}\right)}{J^2 (x^2 + y^2)^{1.0}} \right)} \right)^{0.5}$$

$$+ 20.60000000$$

$$\left(\frac{R}{C\left(\left(1+\left(x^2+y^2\right)^{0.5}\right)^3+\frac{4}{3}\frac{L^2\left(x\,x1+y\,y1\right)^2\left(1+\left(x^2+y^2\right)^{0.5}\right)}{J^2\left(x^2+y^2\right)^{1.0}}\right)}\right)^{0.5}\Bigg)$$

$$\Bigg(1-Rey\Bigg/$$

$$\Bigg(20.60000000$$

$$\left(\frac{R}{C\left(\left(1-\left(x^2+y^2\right)^{0.5}\right)^3+\frac{4}{3}\frac{L^2\left(x\,x1+y\,y1\right)^2\left(1-\left(x^2+y^2\right)^{0.5}\right)}{J^2\left(x^2+y^2\right)^{1.0}}\right)}\right)^{0.5}$$

$$+\,20.60000000$$

$$\left(\frac{R}{C\left(\left(1+\left(x^2+y^2\right)^{0.5}\right)^3+\frac{4}{3}\frac{L^2\left(x\,x1+y\,y1\right)^2\left(1+\left(x^2+y^2\right)^{0.5}\right)}{J^2\left(x^2+y^2\right)^{1.0}}\right)}\right)^{0.5}\Bigg)$$

$$\Bigg)\Bigg)\Bigg)\left(1-x\cos(\theta)-y\sin(\theta)\right)Rey^{0.96}\Bigg)\Bigg)\left(x\sin(\theta)-y\cos(\theta)\right.$$

$$-\,2\,x1\cos(\theta)-2\,y1\sin(\theta)\,)\Bigg)\Bigg)\,L^2+\Bigg($$

$$- \frac{1}{R^2 L \left(1 - x\cos(\theta) - y\sin(\theta)\right)\omega} \left(Rey\, C\, J\, V \left(\left(1 - x\cos(\theta)\right.\right.\right.$$

$$\left. - y\sin(\theta)\right)\phi\cos(\phi\,\tau) + \frac{V\sin(\phi\,\tau)\left(-x1\cos(\theta) - y1\sin(\theta)\right)}{R\,\omega}$$

$$\left.\left. + \frac{1}{2}\,\frac{V\sin(\phi\,\tau)\left(x\sin(\theta) - y\cos(\theta)\right)}{R\,\omega}\right)\right)$$

$$- \frac{1}{L\left(1 - x\cos(\theta) - y\sin(\theta)\right)^2 R\,\omega} \left(12\, J \left(1 \right.\right.$$

$$+\frac{1}{\pi}\left(0.0003470000000\arctan\left(\left(2\left(41.2\right.\right.\right.\right.$$

$$\left(\frac{R}{C\left(\left(1-\left(x^2+y^2\right)^{0.5}\right)^3+\frac{4}{3}\frac{L^2\left(x\,x1+y\,y1\right)^2\left(1-\left(x^2+y^2\right)^{0.5}\right)}{J^2\left(x^2+y^2\right)^{1.0}}\right)}\right)^{0.5}$$

$$-41.2$$

$$\left.\left(\frac{R}{C\left(\left(1+\left(x^2+y^2\right)^{0.5}\right)^3+\frac{4}{3}\frac{L^2\left(x\,x1+y\,y1\right)^2\left(1+\left(x^2+y^2\right)^{0.5}\right)}{J^2\left(x^2+y^2\right)^{1.0}}\right)}\right)^{0.5}\right)$$

$$\left.Rey\right) \Bigg/$$

$$\left(\pi\left(41.2\right.\right.$$

$$\left(\frac{R}{C\left(\left(1-\left(x^2+y^2\right)^{0.5}\right)^3+\frac{4}{3}\frac{L^2\left(x\,x1+y\,y1\right)^2\left(1-\left(x^2+y^2\right)^{0.5}\right)}{J^2\left(x^2+y^2\right)^{1.0}}\right)}\right)^{0.5}$$

$$+41.2$$

$$\left(\frac{R}{C\left(\left(1+\left(x^2+y^2\right)^{0.5}\right)^3+\frac{4}{3}\,\frac{L^2\,(x\,x1+y\,y1)^2\left(1+\left(x^2+y^2\right)^{0.5}\right)}{J^2\left(x^2+y^2\right)^{1.0}}\right)}\right)^{0.5}\Bigg)$$

$$\Bigg(20.60000000$$

$$\left(\frac{R}{C\left(\left(1-\left(x^2+y^2\right)^{0.5}\right)^3+\frac{4}{3}\,\frac{L^2\,(x\,x1+y\,y1)^2\left(1-\left(x^2+y^2\right)^{0.5}\right)}{J^2\left(x^2+y^2\right)^{1.0}}\right)}\right)^{0.5}$$

$$+\,20.60000000$$

$$\left(\frac{R}{C\left(\left(1+\left(x^2+y^2\right)^{0.5}\right)^3+\frac{4}{3}\,\frac{L^2\,(x\,x1+y\,y1)^2\left(1+\left(x^2+y^2\right)^{0.5}\right)}{J^2\left(x^2+y^2\right)^{1.0}}\right)}\right)^{0.5}\Bigg)$$

$$\Bigg(1-Rey\Bigg/$$

$$\Bigg(20.60000000$$

$$\left(\frac{R}{C\left(\left(1-\left(x^2+y^2\right)^{0.5}\right)^3+\frac{4}{3}\,\frac{L^2\,(x\,x1+y\,y1)^2\left(1-\left(x^2+y^2\right)^{0.5}\right)}{J^2\left(x^2+y^2\right)^{1.0}}\right)}\right)^{0.5}$$

$$+\,20.60000000$$

$$\left(\frac{R}{C\left(\left(1+\left(x^2+y^2\right)^{0.5}\right)^3+\frac{4}{3}\,\frac{L^2\,(x\,x1+y\,y1)^2\left(1+\left(x^2+y^2\right)^{0.5}\right)}{J^2\left(x^2+y^2\right)^{1.0}}\right)}\right)^{0.5}\Bigg)$$

$$\left.\left.\left.\left.\left.\left.\left.\right)\right)\right)\left(1-x\cos(\theta)-y\sin(\theta)\right) Rey^{0.96}\right)\right) V\sin(\phi\,\tau)\right)\right) z+\frac{1}{2}\left(\right.$$

$$-\frac{1}{R^2 L\left(1-x\cos(\theta)-y\sin(\theta)\right)\omega}\left(Rey\,C\,J\,V\left(\left(1-x\cos(\theta)\right.\right.\right.$$

$$\left.-y\sin(\theta)\right)\phi\cos(\phi\,\tau)+\frac{V\sin(\phi\,\tau)\left(-x1\cos(\theta)-y1\sin(\theta)\right)}{R\,\omega}$$

$$+\frac{1}{2}\frac{V\sin(\phi\,\tau)\,(x\sin(\theta)-y\cos(\theta))}{R\,\omega}\Bigg)\Bigg)$$

$$-\frac{1}{L\,(1-x\cos(\theta)-y\sin(\theta))^2\,R\,\omega}\Bigg(12\,J\,\Bigg(1$$

$$+\frac{1}{\pi}\Bigg(0.0003470000000\arctan\Bigg(\Bigg(2\,\Bigg(41.2$$

$$\left(\frac{R}{C\left(\left(1-(x^2+y^2)^{0.5}\right)^3+\frac{4}{3}\,\frac{L^2\,(x\,x1+y\,y1)^2\,\left(1-(x^2+y^2)^{0.5}\right)}{J^2\,(x^2+y^2)^{1.0}}\right)}\right)^{0.5}$$

$$-41.2$$

$$\left(\frac{R}{C\left(\left(1+(x^2+y^2)^{0.5}\right)^3+\frac{4}{3}\,\frac{L^2\,(x\,x1+y\,y1)^2\,\left(1+(x^2+y^2)^{0.5}\right)}{J^2\,(x^2+y^2)^{1.0}}\right)}\right)^{0.5}\Bigg)$$

$$Rey\Bigg)\Bigg/$$

$$\left(\pi\left(41.2\right.\right.$$

$$\left(\frac{R}{C\left(\left(1-\left(x^2+y^2\right)^{0.5}\right)^3+\frac{4}{3}\,\frac{L^2\,(x\,x1+y\,y1)^2\left(1-\left(x^2+y^2\right)^{0.5}\right)}{J^2\left(x^2+y^2\right)^{1.0}}\right)}\right)^{0.5}$$

$$+\,41.2$$

$$\left.\left(\frac{R}{C\left(\left(1+\left(x^2+y^2\right)^{0.5}\right)^3+\frac{4}{3}\,\frac{L^2\,(x\,x1+y\,y1)^2\left(1+\left(x^2+y^2\right)^{0.5}\right)}{J^2\left(x^2+y^2\right)^{1.0}}\right)}\right)^{0.5}\right)$$

$$\left(20.60000000\right.$$

$$\left(\frac{R}{C\left(\left(1-\left(x^2+y^2\right)^{0.5}\right)^3+\frac{4}{3}\,\frac{L^2\,(x\,x1+y\,y1)^2\left(1-\left(x^2+y^2\right)^{0.5}\right)}{J^2\left(x^2+y^2\right)^{1.0}}\right)}\right)^{0.5}$$

$$+\,20.60000000$$

$$\left.\left(\frac{R}{C\left(\left(1+\left(x^2+y^2\right)^{0.5}\right)^3+\frac{4}{3}\,\frac{L^2\,(x\,x1+y\,y1)^2\left(1+\left(x^2+y^2\right)^{0.5}\right)}{J^2\left(x^2+y^2\right)^{1.0}}\right)}\right)^{0.5}\right)$$

$$\left(1-Rey\right. \Big/$$

$$\left(20.60000000\right.$$

$$\left(\frac{R}{C\left(\left(1-(x^2+y^2)^{0.5}\right)^3+\frac{4}{3}\frac{L^2\,(x\,x1+y\,y1)^2\left(1-(x^2+y^2)^{0.5}\right)}{J^2\,(x^2+y^2)^{1.0}}\right)}\right)^{0.5}$$

$$+\,20.60000000$$

$$\left.\left(\frac{R}{C\left(\left(1+(x^2+y^2)^{0.5}\right)^3+\frac{4}{3}\frac{L^2\,(x\,x1+y\,y1)^2\left(1+(x^2+y^2)^{0.5}\right)}{J^2\,(x^2+y^2)^{1.0}}\right)}\right)^{0.5}\right)$$

$$\left.\left.\left.\right)\right)\right)\,\left(1-x\cos(\theta)-y\sin(\theta)\right)\,Rey^{0.96}\Big)\Big)\;V\sin(\phi\,\tau)\Big)\Big)\;L$$

The derivatives of apposite functions defined by Capone et al. are defined as F1-F6 and then evaluated

$$> F1 := \left(\left(\frac{2}{(1\text{-}x^2\text{-}y^2)^{0.5}}\right)\right)\cdot\left(\left(\frac{\pi}{2}+\arctan\left(\frac{(y\cdot\cos(\alpha)\text{-}x\cdot\sin(\alpha))}{(1\text{-}x^2\text{-}y^2)^{0.5}}\right)\right)\right)$$

$$F1 := \frac{2\left(\frac{1}{2}\,\pi+\arctan\left(\frac{y\cos(\alpha)-x\sin(\alpha)}{(1-x^2-y^2)^{0.5}}\right)\right)}{(1-x^2-y^2)^{0.5}} \tag{23}$$

$$> F2 := \left(\frac{1}{(x^2+y^2)}\right)\cdot\left(y\cdot('F1'\text{-}\pi)+x\cdot\ln\left(\frac{(1+x\cdot\cos(\alpha)+y\cdot\sin(\alpha))}{(1\text{-}x\cdot\cos(\alpha)\text{-}y\cdot\sin(\alpha))}\right)\right)$$

$$F2 := \frac{y\,(F1-\pi)+x\ln\left(\frac{1+x\cos(\alpha)+y\sin(\alpha)}{1-x\cos(\alpha)-y\sin(\alpha)}\right)}{x^2+y^2} \tag{24}$$

$$> F3 := \left(\frac{1}{(x^2+y^2)}\right)\cdot\left(x\cdot('F1'\text{-}\pi)\text{-}y\cdot\ln\left(\frac{(1+x\cdot\cos(\alpha)+y\cdot\sin(\alpha))}{(1\text{-}x\cdot\cos(\alpha)\text{-}y\cdot\sin(\alpha))}\right)\right)$$

$$F3 := \frac{x\,(F1-\pi)-y\ln\left(\frac{1+x\cos(\alpha)+y\sin(\alpha)}{1-x\cos(\alpha)-y\sin(\alpha)}\right)}{x^2+y^2} \tag{25}$$

$$> F4 := \text{-}\frac{x\cdot y}{(x^2+y^2)}\cdot 'F1'+\left(\frac{2\cdot x\cdot y}{(x^2+y^2)^2}\right)\cdot('F1'\text{-}\pi\text{-}2\cdot y\cdot\cos(\alpha)+2\cdot x\cdot\sin(\alpha))$$

$$+\left(\frac{(x^2\text{-}y^2)}{(x^2+y^2)^2}\right)\cdot\left(\ln\left(\frac{(1+x\cdot\cos(\alpha)+y\cdot\sin(\alpha))}{(1\text{-}x\cdot\cos(\alpha)\text{-}y\cdot\sin(\alpha))}\right)\text{-}2\cdot(x\cdot\cos(\alpha)+y\right.$$

$$\cdot \sin(\alpha)) \Big)$$

$$F4 := -\frac{x\,y\,F1}{x^2+y^2} + \frac{2\,x\,y\,(F1-\pi-2\,y\cos(\alpha)+2\,x\sin(\alpha))}{(x^2+y^2)^2} + \frac{(x^2-y^2)\left(\ln\left(\frac{1+x\cos(\alpha)+y\sin(\alpha)}{1-x\cos(\alpha)-y\sin(\alpha)}\right)-2\,x\cos(\alpha)-2\,y\sin(\alpha)\right)}{(x^2+y^2)^2} \quad (26)$$

$$> F5 := \frac{x\cdot x\cdot 'F1'}{x^2+y^2} - \frac{(x^2-y^2)}{(x^2+y^2)^2}('F1'-\pi-2\cdot y\cdot\cos(\alpha)+2\cdot\sin(\alpha)) + \frac{2\cdot x\cdot y}{(x^2+y^2)^2}\cdot\left(\ln\left(\frac{(1+x\cdot\cos(\alpha)+y\cdot\sin(\alpha))}{(1-x\cdot\cos(\alpha)-y\cdot\sin(\alpha))}\right)-2\cdot(x\cdot\cos(\alpha)+y\cdot\sin(\alpha))\right)$$

$$F5 := \frac{x^2\,F1}{x^2+y^2} - \frac{(x^2-y^2)\,(F1-\pi-2\,y\cos(\alpha)+2\sin(\alpha))}{(x^2+y^2)^2} + \frac{2\,x\,y\left(\ln\left(\frac{1+x\cos(\alpha)+y\sin(\alpha)}{1-x\cos(\alpha)-y\sin(\alpha)}\right)-2\,x\cos(\alpha)-2\,y\sin(\alpha)\right)}{(x^2+y^2)^2} \quad (27)$$

$$> F6 := \frac{y\cdot y\cdot 'F1'}{x^2+y^2} + \frac{(x^2-y^2)}{(x^2+y^2)^2}('F1'-\pi-2\cdot y\cdot\cos(\alpha)+2\cdot\sin(\alpha)) - \left(\frac{2\cdot x\cdot y}{(x^2+y^2)^2}\cdot\left(\ln\left(\frac{(1+x\cdot\cos(\alpha)+y\cdot\sin(\alpha))}{(1-x\cdot\cos(\alpha)-y\cdot\sin(\alpha))}\right)-2\cdot(x\cdot\cos(\alpha)+y\cdot\sin(\alpha))\right)\right)$$

$$F6 := \frac{y^2\,F1}{x^2+y^2} + \frac{(x^2-y^2)\,(F1-\pi-2\,y\cos(\alpha)+2\sin(\alpha))}{(x^2+y^2)^2} - \frac{2\,x\,y\left(\ln\left(\frac{1+x\cos(\alpha)+y\sin(\alpha)}{1-x\cos(\alpha)-y\sin(\alpha)}\right)-2\,x\cos(\alpha)-2\,y\sin(\alpha)\right)}{(x^2+y^2)^2} \quad (28)$$

$$> F2;$$

$$\frac{1}{x^2+y^2}\left(y\left(\frac{2\left(\frac{1}{2}\pi+\arctan\left(\frac{y\cos(\alpha)-x\sin(\alpha)}{(1-x^2-y^2)^{0.5}}\right)\right)}{(1-x^2-y^2)^{0.5}}-\pi\right)+x\ln\left(\frac{1+x\cos(\alpha)+y\sin(\alpha)}{1-x\cos(\alpha)-y\sin(\alpha)}\right)\right) \quad (29)$$

$$> F3;$$

$$\frac{1}{x^2+y^2}\left(x\left(\frac{2\left(\frac{1}{2}\pi+\arctan\left(\frac{y\cos(\alpha)-x\sin(\alpha)}{(1-x^2-y^2)^{0.5}}\right)\right)}{(1-x^2-y^2)^{0.5}}-\pi\right)\right. \quad (30)$$

$$-y\ln\left(\frac{1+x\cos(\alpha)+y\sin(\alpha)}{1-x\cos(\alpha)-y\sin(\alpha)}\right)\Bigg)$$

> F4;

$$-\frac{2\,x\,y\left(\frac{1}{2}\pi+\arctan\left(\frac{y\cos(\alpha)-x\sin(\alpha)}{(1-x^2-y^2)^{0.5}}\right)\right)}{(x^2+y^2)\,(1-x^2-y^2)^{0.5}} \tag{31}$$

$$+\frac{1}{(x^2+y^2)^2}\left(2\,x\,y\left(\frac{2\left(\frac{1}{2}\pi+\arctan\left(\frac{y\cos(\alpha)-x\sin(\alpha)}{(1-x^2-y^2)^{0.5}}\right)\right)}{(1-x^2-y^2)^{0.5}}-\pi\right.\right.$$

$$\left.\left.-2\,y\cos(\alpha)+2\,x\sin(\alpha)\right)\right)$$

$$+\frac{(x^2-y^2)\left(\ln\left(\frac{1+x\cos(\alpha)+y\sin(\alpha)}{1-x\cos(\alpha)-y\sin(\alpha)}\right)-2\,x\cos(\alpha)-2\,y\sin(\alpha)\right)}{(x^2+y^2)^2}$$

> F5;

$$\frac{2\,x^2\left(\frac{1}{2}\pi+\arctan\left(\frac{y\cos(\alpha)-x\sin(\alpha)}{(1-x^2-y^2)^{0.5}}\right)\right)}{(1-x^2-y^2)^{0.5}\,(x^2+y^2)}-\frac{1}{(x^2+y^2)^2}\Bigg((x^2 \tag{32}$$

$$-y^2)\left(\frac{2\left(\frac{1}{2}\pi+\arctan\left(\frac{y\cos(\alpha)-x\sin(\alpha)}{(1-x^2-y^2)^{0.5}}\right)\right)}{(1-x^2-y^2)^{0.5}}-\pi-2\,y\cos(\alpha)\right.$$

$$\left.+2\sin(\alpha)\right)\Bigg)$$

$$+\frac{2\,x\,y\left(\ln\left(\frac{1+x\cos(\alpha)+y\sin(\alpha)}{1-x\cos(\alpha)-y\sin(\alpha)}\right)-2\,x\cos(\alpha)-2\,y\sin(\alpha)\right)}{(x^2+y^2)^2}$$

> F6;

$$\frac{2\,y^2\left(\frac{1}{2}\pi+\arctan\left(\frac{y\cos(\alpha)-x\sin(\alpha)}{(1-x^2-y^2)^{0.5}}\right)\right)}{(1-x^2-y^2)^{0.5}\,(x^2+y^2)}+\frac{1}{(x^2+y^2)^2}\Bigg((x^2 \tag{33}$$

$$-y^2)\left(\frac{2\left(\frac{1}{2}\pi+\arctan\left(\frac{y\cos(\alpha)-x\sin(\alpha)}{(1-x^2-y^2)^{0.5}}\right)\right)}{(1-x^2-y^2)^{0.5}}-\pi-2\,y\cos(\alpha)\right.$$

$$+2\sin(\alpha)\Bigg)\Bigg)$$

$$-\frac{2xy\left(\ln\left(\frac{1+x\cos(\alpha)+y\sin(\alpha)}{1-x\cos(\alpha)-y\sin(\alpha)}\right)-2x\cos(\alpha)-2y\sin(\alpha)\right)}{\left(x^2+y^2\right)^2}$$

The derivatives of apposite functions defined by Capone et al. are used to determine the unsteady state fluid force components.

$$> fx := '((x-2\cdot y1)\cdot diff(F1, y, x)-(y+2\cdot x1)\cdot diff(F1, x, x))+2\cdot\delta\cdot s\cdot Rey^{0.96}\cdot((x-2\cdot y1)\cdot diff(F2, x)-(y+2\cdot x1)\cdot diff(F3, x))+\frac{1}{6}\cdot Rey\cdot\left(\frac{C}{R}\right)\cdot((2\cdot x-4\cdot y1)\cdot(y+2\cdot x1)\cdot diff(F4, x)-(x-2\cdot y1)^2\cdot diff(F5, x)-(y+2\cdot x1)^2\cdot diff(F6, x))+\frac{1}{6}\cdot Rey\cdot\left(\frac{C}{R}\right)\cdot\left(\left(\frac{1}{2}\cdot x-2\cdot y1-2\cdot x11\right)\cdot F6+\left(\frac{1}{2}y+2\cdot x1-2\cdot y11\right)\cdot F4\right)';$$

$$fx := (x-2\,y1)\left(\frac{\partial^2}{\partial x\,\partial y}F1\right)-(y+2\,x1)\left(\frac{\partial^2}{\partial x^2}F1\right)+2\,\delta\,s\,Rey^{0.96}\left((x-2\,y1)\left(\frac{\partial}{\partial x}F2\right)-(y+2\,x1)\left(\frac{\partial}{\partial x}F3\right)\right)+\frac{1}{6}\,\frac{1}{R}\left(Rey\,C\left((2\,x-4\,y1)\,(y+2\,x1)\left(\frac{\partial}{\partial x}F4\right)-(x-2\,y1)^2\left(\frac{\partial}{\partial x}F5\right)-(y+2\,x1)^2\left(\frac{\partial}{\partial x}F6\right)\right)\right)+\frac{1}{6}\,\frac{Rey\,C\left(\left(\frac{1}{2}\,x-2\,y1-2\,x11\right)F6+\left(\frac{1}{2}\,y+2\,x1-2\,y11\right)F4\right)}{R} \tag{34}$$

$$> fy := '((x-2\cdot y1)\cdot diff(F1, y, y)-(y+2\cdot x1)\cdot diff(F1, x, y))+2\cdot\delta\cdot s\cdot Rey^{0.96}\cdot((x-2\cdot y1)\cdot diff(F2, y)-(y+2\cdot x1)\cdot diff(F3, y))+\frac{1}{6}\cdot Rey\cdot\left(\frac{C}{R}\right)\cdot((2\cdot x-4\cdot y1)\cdot(y+2\cdot x1)\cdot diff(F4, y)-(x-2\cdot y1)^2\cdot diff(F5, y)-(y+2\cdot x1)^2\cdot diff(F6, y))+\frac{1}{6}\cdot Rey\cdot\left(\frac{C}{R}\right)\cdot\left(\left(\frac{1}{2}\cdot x-2\cdot y1-2\cdot x11\right)\cdot F4+\left(\frac{1}{2}y+2\cdot x1-2\cdot y11\right)\cdot F5\right)';$$

$$fy := (x-2\,y1)\left(\frac{\partial^2}{\partial y^2}F1\right)-(y+2\,x1)\left(\frac{\partial^2}{\partial y\,\partial x}F1\right)+2\,\delta\,s\,Rey^{0.96}\left((x-2\,y1)\left(\frac{\partial}{\partial y}F2\right)-(y+2\,x1)\left(\frac{\partial}{\partial y}F3\right)\right)+\frac{1}{6}\,\frac{1}{R}\left(Rey\,C\left((2\,x-4\,y1)\,(y+2\,x1)\left(\frac{\partial}{\partial y}F4\right)-(x-2\,y1)^2\left(\frac{\partial}{\partial y}F5\right)-(y+2\,x1)^2\left(\frac{\partial}{\partial y}F6\right)\right)\right)+\frac{1}{6}\,\frac{Rey\,C\left(\left(\frac{1}{2}\,x-2\,y1-2\,x11\right)F4+\left(\frac{1}{2}\,y+2\,x1-2\,y11\right)F5\right)}{R} \tag{35}$$

The steady state pressure is evaluated

```
> po := subs(x1 = 0, x11 = 0, y1 = 0, y11 = 0, z = 0, θ = α + π, x = xo, y = yo, p);
```

$$po := -\frac{1}{8}\left(\frac{1}{R\,(1 - xo\cos(\alpha+\pi) - yo\sin(\alpha+\pi))}\left(Rey\,C\left(xo\cos(\alpha+\pi)\right.\right.\right. \tag{36}$$

$$\left.\left.+ yo\sin(\alpha+\pi) - \frac{2\,(xo\sin(\alpha+\pi) - yo\cos(\alpha+\pi))^2}{1 - xo\cos(\alpha+\pi) - yo\sin(\alpha+\pi)}\right)\right)$$

$$+ \frac{1}{(1 - xo\cos(\alpha+\pi) - yo\sin(\alpha+\pi))^3}\left(24\left(1\right.\right.$$

$$+ \frac{1}{\pi}\left(0.0003470000000\arctan\left(\left(2\left(41.2\left(\frac{R}{C\left(1-(xo^2+yo^2)^{0.5}\right)^3}\right)^{0.5}\right.\right.\right.\right.$$

$$\left.\left. - 41.2\left(\frac{R}{C\left(1+(xo^2+yo^2)^{0.5}\right)^3}\right)^{0.5}\right)Rey\right) \Big/$$

$$\left(\pi\left(41.2\left(\frac{R}{C\left(1-(xo^2+yo^2)^{0.5}\right)^3}\right)^{0.5}\right.\right.$$

$$\left.\left. + 41.2\left(\frac{R}{C\left(1+(xo^2+yo^2)^{0.5}\right)^3}\right)^{0.5}\right)\right.$$

$$\left(20.60000000\left(\frac{R}{C\left(1-(xo^2+yo^2)^{0.5}\right)^3}\right)^{0.5}\right.$$

$$+ 20.60000000 \left(\frac{R}{C \left(1 + \left(xo^2 + yo^2\right)^{0.5}\right)^3} \right)^{0.5} \Bigg) \Bigg(1 - Rey \Big/$$

$$\Bigg(20.60000000 \left(\frac{R}{C \left(1 - \left(xo^2 + yo^2\right)^{0.5}\right)^3} \right)^{0.5}$$

$$+ 20.60000000 \left(\frac{R}{C \left(1 + \left(xo^2 + yo^2\right)^{0.5}\right)^3} \right)^{0.5} \Bigg)\Bigg)\Bigg)\Bigg) \left(1 - xo \cos(\alpha + \pi)\right.$$

$$\left. - yo \sin(\alpha + \pi)\right) Rey^{0.96} \Bigg)\Bigg) \left(xo \sin(\alpha + \pi) - yo \cos(\alpha + \pi)\right) \Bigg)\Bigg) L^2 + \frac{1}{2} \Bigg($$

$$- \Bigg(Rey\, C\, J\, V \Bigg(\left(1 - xo \cos(\alpha + \pi) - yo \sin(\alpha + \pi)\right) \phi \cos(\phi \tau)$$

$$+ \frac{1}{2} \frac{V \sin(\phi \tau) \left(xo \sin(\alpha + \pi) - yo \cos(\alpha + \pi)\right)}{R\, \omega} \Bigg)\Bigg) \Big/ \left(R^2 L \left(1\right.\right.$$

$$- xo\cos(\alpha + \pi) - yo\sin(\alpha + \pi))\ \omega) - \Bigg(12\,J\Bigg(1$$

$$+ \frac{1}{\pi}\Bigg(0.0003470000000\ \arctan\Bigg(\Bigg(2\left(41.2\left(\frac{R}{C\left(1-\left(xo^2+yo^2\right)^{0.5}\right)^3}\right)^{0.5}\right.$$

$$\left. - 41.2\left(\frac{R}{C\left(1+\left(xo^2+yo^2\right)^{0.5}\right)^3}\right)^{0.5}\right)Rey\Bigg) \Bigg/$$

$$\Bigg(\pi\left(41.2\left(\frac{R}{C\left(1-\left(xo^2+yo^2\right)^{0.5}\right)^3}\right)^{0.5}\right.$$

$$+ 41.2 \left(\frac{R}{C \left(1 + \left(xo^2 + yo^2\right)^{0.5}\right)^3} \right)^{0.5} \Bigg)$$

$$\Bigg(20.60000000 \left(\frac{R}{C \left(1 - \left(xo^2 + yo^2\right)^{0.5}\right)^3} \right)^{0.5}$$

$$+ 20.60000000 \left(\frac{R}{C \left(1 + \left(xo^2 + yo^2\right)^{0.5}\right)^3} \right)^{0.5} \Bigg) \Bigg(1 - Rey \Bigg/$$

$$\Bigg(20.60000000 \left(\frac{R}{C \left(1 - \left(xo^2 + yo^2\right)^{0.5}\right)^3} \right)^{0.5}$$

$$+ 20.60000000 \left(\frac{R}{C \left(1 + \left(xo^2 + yo^2\right)^{0.5}\right)^3} \right)^{0.5} \Bigg) \Bigg) \Bigg) \Bigg) \left(1 - xo \cos(\alpha + \pi)\right)$$

$$- yo \sin(\alpha + \pi)) \, Rey^{0.96} \Big) \Big) \, V \sin(\phi \tau) \Big) \Big/ \quad \Big(L \, (1 - xo \cos(\alpha$$

$$+ \pi) - yo \sin(\alpha + \pi))^2 \, R \, \omega \Big) \, L \Bigg)$$

Steady state fluid force components are evaluated.

> *fox* := *subs*(*x* = *xo*, *y* = *yo*, *x1* = 0, *x11* = 0, *y1* = 0, *y11* = 0, *fx*);

$$fox := xo \left(\frac{6.00 \left(\frac{1}{2} \pi + \arctan\left(\frac{yo \cos(\alpha) - xo \sin(\alpha)}{(1 - xo^2 - yo^2)^{0.5}} \right) \right) yo \, xo}{(1 - xo^2 - yo^2)^{2.5}} \right.$$

$$+ \frac{2.0 \left(- \frac{\sin(\alpha)}{(1 - xo^2 - yo^2)^{0.5}} + \frac{1.0 \, (yo \cos(\alpha) - xo \sin(\alpha)) \, xo}{(1 - xo^2 - yo^2)^{1.5}} \right) yo}{(1 - xo^2 - yo^2)^{1.5} \left(1 + \frac{(yo \cos(\alpha) - xo \sin(\alpha))^2}{(1 - xo^2 - yo^2)^{1.0}} \right)}$$

$$+ \frac{2.0 \left(\frac{\cos(\alpha)}{(1 - xo^2 - yo^2)^{0.5}} + \frac{1.0 \, (yo \cos(\alpha) - xo \sin(\alpha)) \, yo}{(1 - xo^2 - yo^2)^{1.5}} \right) xo}{(1 - xo^2 - yo^2)^{1.5} \left(1 + \frac{(yo \cos(\alpha) - xo \sin(\alpha))^2}{(1 - xo^2 - yo^2)^{1.0}} \right)}$$

$$+ \left(2 \left(\frac{1.0 \cos(\alpha) \, xo}{(1 - xo^2 - yo^2)^{1.5}} - \frac{1.0 \sin(\alpha) \, yo}{(1 - xo^2 - yo^2)^{1.5}} + \frac{3.00 \, (yo \cos(\alpha) - xo \sin(\alpha)) \, yo \, xo}{(1 - xo^2 - yo^2)^{2.5}} \right) \right) \Big/$$

$$+ \frac{(yo \cos(\alpha) - xo \sin(\alpha))^2}{(1 - xo^2 - yo^2)^{1.0}} \Big) \Big) - \left(2 \left(\frac{\cos(\alpha)}{(1 - xo^2 - yo^2)^{0.5}} \right. \right.$$

$$+ \frac{1.0\,(yo\cos(\alpha) - xo\sin(\alpha))\,yo}{(1 - xo^2 - yo^2)^{1.5}}\Bigg)\Bigg($$

$$- \frac{2\,(yo\cos(\alpha) - xo\sin(\alpha))\,\sin(\alpha)}{(1 - xo^2 - yo^2)^{1.0}} + \frac{2.0\,(yo\cos(\alpha) - xo\sin(\alpha))^2\,xo}{(1 - xo^2 - yo^2)^{2.0}}\Bigg)$$

$$\Bigg)\Bigg/\left((1 - xo^2 - yo^2)^{0.5}\left(1 + \frac{(yo\cos(\alpha) - xo\sin(\alpha))^2}{(1 - xo^2 - yo^2)^{1.0}}\right)^2\right)$$

$$- yo\left(\frac{6.00\left(\frac{1}{2}\pi + \arctan\left(\frac{yo\cos(\alpha) - xo\sin(\alpha)}{(1 - xo^2 - yo^2)^{0.5}}\right)\right)xo^2}{(1 - xo^2 - yo^2)^{2.5}}\right.$$

$$+ \frac{4.0\left(-\frac{\sin(\alpha)}{(1 - xo^2 - yo^2)^{0.5}} + \frac{1.0\,(yo\cos(\alpha) - xo\sin(\alpha))\,xo}{(1 - xo^2 - yo^2)^{1.5}}\right)xo}{(1 - xo^2 - yo^2)^{1.5}\left(1 + \frac{(yo\cos(\alpha) - xo\sin(\alpha))^2}{(1 - xo^2 - yo^2)^{1.0}}\right)}$$

$$+ \frac{2.0\left(\frac{1}{2}\pi + \arctan\left(\frac{yo\cos(\alpha) - xo\sin(\alpha)}{(1 - xo^2 - yo^2)^{0.5}}\right)\right)}{(1 - xo^2 - yo^2)^{1.5}} + \Bigg(2\Bigg($$

$$- \frac{2.0\sin(\alpha)\,xo}{(1 - xo^2 - yo^2)^{1.5}} + \frac{3.00\,(yo\cos(\alpha) - xo\sin(\alpha))\,xo^2}{(1 - xo^2 - yo^2)^{2.5}}$$

$$+ \frac{1.0\,(yo\cos(\alpha) - xo\sin(\alpha))}{(1 - xo^2 - yo^2)^{1.5}}\Bigg)\Bigg)\Bigg/\Bigg((1 - xo^2 - yo^2)^{0.5}\Bigg(1$$

$$+ \frac{(yo\cos(\alpha) - xo\sin(\alpha))^2}{(1 - xo^2 - yo^2)^{1.0}}\Bigg)\Bigg) - \Bigg(2\Bigg(-\frac{\sin(\alpha)}{(1 - xo^2 - yo^2)^{0.5}}$$

$$+ \frac{1.0\,(yo\cos(\alpha) - xo\sin(\alpha))\,xo}{(1 - xo^2 - yo^2)^{1.5}}\Bigg)\Bigg($$

$$- \frac{2\,(yo\cos(\alpha) - xo\sin(\alpha))\,\sin(\alpha)}{(1 - xo^2 - yo^2)^{1.0}} + \frac{2.0\,(yo\cos(\alpha) - xo\sin(\alpha))^2\,xo}{(1 - xo^2 - yo^2)^{2.0}}\Bigg)$$

$$\Bigg) \Bigg/ \left(\left(1 - xo^2 - yo^2\right)^{0.5} \left(1 + \frac{\left(yo \cos(\alpha) - xo \sin(\alpha)\right)^2}{\left(1 - xo^2 - yo^2\right)^{1.0}}\right)^2 \right) \Bigg)$$

$$+ \frac{1}{\pi} \left(0.0006940000000 \arctan\left(\left(2 \left(41.2 \left(\frac{R}{C \left(1 - \left(xo^2 + yo^2\right)^{0.5}\right)^3} \right)^{0.5} \right. \right. \right. \right.$$

$$\left. - 41.2 \left(\frac{R}{C \left(1 + \left(xo^2 + yo^2\right)^{0.5}\right)^3} \right)^{0.5} \right) Rey \Bigg) \Bigg/$$

$$\left(\pi \left(41.2 \left(\frac{R}{C \left(1 - \left(xo^2 + yo^2\right)^{0.5}\right)^3} \right)^{0.5} \right. \right.$$

$$\left. + 41.2 \left(\frac{R}{C \left(1 + \left(xo^2 + yo^2\right)^{0.5}\right)^3} \right)^{0.5} \right)$$

$$\left(20.60000000 \left(\frac{R}{C \left(1 - \left(xo^2 + yo^2\right)^{0.5}\right)^3} \right)^{0.5} \right.$$

$$+ 20.60000000 \left(\frac{R}{C \left(1 + \left(xo^2 + yo^2\right)^{0.5}\right)^3} \right)^{0.5} \Bigg) \Bigg(1 - Rey \Big/$$

$$\Bigg(20.60000000 \left(\frac{R}{C \left(1 - \left(xo^2 + yo^2\right)^{0.5}\right)^3} \right)^{0.5}$$

$$+ 20.60000000 \left(\frac{R}{C \left(1 + \left(xo^2 + yo^2\right)^{0.5}\right)^3} \right)^{0.5} \Bigg)\Bigg)\Bigg)\Bigg) Rey^{0.96} \Bigg(xo \Bigg($$

$$- \frac{1}{\left(xo^2 + yo^2\right)^2} \Bigg(2 \Bigg(yo \Bigg(\frac{2 \left(\frac{1}{2} \pi + \arctan\left(\frac{yo \cos(\alpha) - xo \sin(\alpha)}{\left(1 - xo^2 - yo^2\right)^{0.5}} \right) \right)}{\left(1 - xo^2 - yo^2\right)^{0.5}}$$

$$- \pi \Bigg) + xo \ln\left(\frac{1 + xo \cos(\alpha) + yo \sin(\alpha)}{1 - xo \cos(\alpha) - yo \sin(\alpha)} \right) \Bigg) xo \Bigg)$$

$$+ \frac{1}{xo^2 + yo^2} \Bigg(yo \Bigg(\frac{2.0 \left(\frac{1}{2} \pi + \arctan\left(\frac{yo \cos(\alpha) - xo \sin(\alpha)}{\left(1 - xo^2 - yo^2\right)^{0.5}} \right) \right) xo}{\left(1 - xo^2 - yo^2\right)^{1.5}}$$

$$+\frac{2\left(-\frac{\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}+\frac{1.0\,(yo\cos(\alpha)-xo\sin(\alpha))\,xo}{(1-xo^2-yo^2)^{1.5}}\right)}{(1-xo^2-yo^2)^{0.5}\left(1+\frac{(yo\cos(\alpha)-xo\sin(\alpha))^2}{(1-xo^2-yo^2)^{1.0}}\right)}\Bigg)$$

$$+\ln\left(\frac{1+xo\cos(\alpha)+yo\sin(\alpha)}{1-xo\cos(\alpha)-yo\sin(\alpha)}\right)$$

$$+\frac{1}{1+xo\cos(\alpha)+yo\sin(\alpha)}\Bigg(xo\Bigg(\frac{\cos(\alpha)}{1-xo\cos(\alpha)-yo\sin(\alpha)}$$

$$+\frac{(1+xo\cos(\alpha)+yo\sin(\alpha))\cos(\alpha)}{(1-xo\cos(\alpha)-yo\sin(\alpha))^2}\Bigg)(1-xo\cos(\alpha)-yo\sin(\alpha))\Bigg)\Bigg)$$

$$\Bigg)-yo\Bigg($$

$$-\frac{1}{(xo^2+yo^2)^2}\left(2\left(2\left(xo\left(\frac{2\left(\frac{1}{2}\pi+\arctan\left(\frac{yo\cos(\alpha)-xo\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}\right)\right)}{(1-xo^2-yo^2)^{0.5}}\right.\right.\right.\right.$$

$$- \pi \Bigg) - yo \ln\left(\frac{1 + xo \cos(\alpha) + yo \sin(\alpha)}{1 - xo \cos(\alpha) - yo \sin(\alpha)} \right) \Bigg) xo \Bigg)$$

$$+ \frac{1}{xo^2 + yo^2} \left(\frac{2 \left(\frac{1}{2} \pi + \arctan\left(\frac{yo \cos(\alpha) - xo \sin(\alpha)}{(1 - xo^2 - yo^2)^{0.5}} \right) \right)}{(1 - xo^2 - yo^2)^{0.5}} - \pi \right.$$

$$+ xo \left(\frac{2.0 \left(\frac{1}{2} \pi + \arctan\left(\frac{yo \cos(\alpha) - xo \sin(\alpha)}{(1 - xo^2 - yo^2)^{0.5}} \right) \right) xo}{(1 - xo^2 - yo^2)^{1.5}} \right.$$

$$\left. + \frac{2 \left(- \frac{\sin(\alpha)}{(1 - xo^2 - yo^2)^{0.5}} + \frac{1.0 \, (yo \cos(\alpha) - xo \sin(\alpha)) \, xo}{(1 - xo^2 - yo^2)^{1.5}} \right)}{(1 - xo^2 - yo^2)^{0.5} \left(1 + \frac{(yo \cos(\alpha) - xo \sin(\alpha))^2}{(1 - xo^2 - yo^2)^{1.0}} \right)} \right)$$

$$- \frac{1}{1 + xo \cos(\alpha) + yo \sin(\alpha)} \left(yo \left(\frac{\cos(\alpha)}{1 - xo \cos(\alpha) - yo \sin(\alpha)} \right. \right.$$

$$\left. \left. + \frac{(1 + xo \cos(\alpha) + yo \sin(\alpha)) \cos(\alpha)}{(1 - xo \cos(\alpha) - yo \sin(\alpha))^2} \right) (1 - xo \cos(\alpha) - yo \sin(\alpha)) \right) \Bigg)$$

$$\Bigg) \Bigg) \Bigg) + \frac{1}{6} \frac{1}{R} \Bigg(Rey \, C \Bigg(2 \, xo \, yo \Bigg($$

$$- \frac{2 \, yo \left(\frac{1}{2} \pi + \arctan\left(\frac{yo \cos(\alpha) - xo \sin(\alpha)}{(1 - xo^2 - yo^2)^{0.5}} \right) \right)}{(xo^2 + yo^2) \, (1 - xo^2 - yo^2)^{0.5}}$$

$$+\frac{4\,xo^2\,yo\left(\frac{1}{2}\pi+\arctan\left(\frac{yo\cos(\alpha)-xo\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}\right)\right)}{(xo^2+yo^2)^2\,(1-xo^2-yo^2)^{0.5}}$$

$$-\frac{2.0\,xo^2\,yo\left(\frac{1}{2}\pi+\arctan\left(\frac{yo\cos(\alpha)-xo\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}\right)\right)}{(xo^2+yo^2)\,(1-xo^2-yo^2)^{1.5}}$$

$$-\frac{2\,xo\,yo\left(-\frac{\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}+\frac{1.0\,(yo\cos(\alpha)-xo\sin(\alpha))\,xo}{(1-xo^2-yo^2)^{1.5}}\right)}{(xo^2+yo^2)\,(1-xo^2-yo^2)^{0.5}\left(1+\frac{(yo\cos(\alpha)-xo\sin(\alpha))^2}{(1-xo^2-yo^2)^{1.0}}\right)}$$

$$+\frac{1}{(xo^2+yo^2)^2}\left(2\,yo\left(\frac{2\left(\frac{1}{2}\pi+\arctan\left(\frac{yo\cos(\alpha)-xo\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}\right)\right)}{(1-xo^2-yo^2)^{0.5}}-\pi\right.\right.$$

$$\left.\left.-2\,yo\cos(\alpha)+2\,xo\sin(\alpha)\right)\right)$$

$$- \frac{1}{(xo^2 + yo^2)^3} \left(8\, xo^2\, yo \left(\frac{2 \left(\frac{1}{2}\pi + \arctan\left(\frac{yo\cos(\alpha) - xo\sin(\alpha)}{(1 - xo^2 - yo^2)^{0.5}} \right) \right)}{(1 - xo^2 - yo^2)^{0.5}} \right.\right.$$

$$\left.\left. - \pi - 2\, yo\cos(\alpha) + 2\, xo\sin(\alpha) \right) \right)$$

$$+ \frac{1}{(xo^2 + yo^2)^2} \left(2\, xo\, yo \left(\frac{1}{(1 - xo^2 - yo^2)^{1.5}} \left(2.0 \left(\frac{1}{2}\pi \right.\right.\right.\right.$$

$$\left.\left.\left. + \arctan\left(\frac{yo\cos(\alpha) - xo\sin(\alpha)}{(1 - xo^2 - yo^2)^{0.5}} \right) \right) xo \right)$$

$$+ \frac{2 \left(-\frac{\sin(\alpha)}{(1 - xo^2 - yo^2)^{0.5}} + \frac{1.0\,(yo\cos(\alpha) - xo\sin(\alpha))\, xo}{(1 - xo^2 - yo^2)^{1.5}} \right)}{(1 - xo^2 - yo^2)^{0.5} \left(1 + \frac{(yo\cos(\alpha) - xo\sin(\alpha))^2}{(1 - xo^2 - yo^2)^{1.0}} \right)}$$

$$+2\sin(\alpha)\Bigg)\Bigg)$$

$$+\frac{2\,xo\left(\ln\left(\frac{1+xo\cos(\alpha)+yo\sin(\alpha)}{1-xo\cos(\alpha)-yo\sin(\alpha)}\right)-2\,xo\cos(\alpha)-2\,yo\sin(\alpha)\right)}{(xo^2+yo^2)^2}$$

$$+\frac{1}{(xo^2+yo^2)^2}\Bigg((xo^2$$

$$-yo^2)\left(\frac{1}{1+xo\cos(\alpha)+yo\sin(\alpha)}\Bigg(\Bigg(\frac{\cos(\alpha)}{1-xo\cos(\alpha)-yo\sin(\alpha)}\right.$$

$$+\frac{(1+xo\cos(\alpha)+yo\sin(\alpha))\cos(\alpha)}{(1-xo\cos(\alpha)-yo\sin(\alpha))^2}\Bigg)(1-xo\cos(\alpha)-yo\sin(\alpha))\Bigg)$$

$$-2\cos(\alpha)\Bigg)\Bigg)-\frac{1}{(xo^2+yo^2)^3}\Bigg(4\,(xo^2$$

$$-yo^2)\left(\ln\left(\frac{1+xo\cos(\alpha)+yo\sin(\alpha)}{1-xo\cos(\alpha)-yo\sin(\alpha)}\right)-2\,xo\cos(\alpha)\right.$$

$$\left.-2\,yo\sin(\alpha)\right)xo\Bigg)\Bigg)$$

$$-xo^2\left(\frac{4\,xo\left(\frac{1}{2}\pi+\arctan\left(\frac{yo\cos(\alpha)-xo\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}\right)\right)}{(1-xo^2-yo^2)^{0.5}\,(xo^2+yo^2)}\right.$$

$$+\frac{2.0\,xo^3\left(\frac{1}{2}\pi+\arctan\left(\frac{yo\cos(\alpha)-xo\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}\right)\right)}{(1-xo^2-yo^2)^{1.5}\,(xo^2+yo^2)}$$

$$+\frac{2\,xo^2\left(-\frac{\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}+\frac{1.0\,(yo\cos(\alpha)-xo\sin(\alpha))\,xo}{(1-xo^2-yo^2)^{1.5}}\right)}{(1-xo^2-yo^2)^{0.5}\left(1+\frac{(yo\cos(\alpha)-xo\sin(\alpha))^2}{(1-xo^2-yo^2)^{1.0}}\right)(xo^2+yo^2)}$$

$$-\frac{4\,xo^3\left(\frac{1}{2}\pi+\arctan\left(\frac{yo\cos(\alpha)-xo\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}\right)\right)}{(1-xo^2-yo^2)^{0.5}(xo^2+yo^2)^2}$$

$$-\frac{1}{(xo^2+yo^2)^2}\left(2\,xo\left(\frac{2\left(\frac{1}{2}\pi+\arctan\left(\frac{yo\cos(\alpha)-xo\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}\right)\right)}{(1-xo^2-yo^2)^{0.5}}-\pi\right.\right.$$

$$\left.\left.-2\,yo\cos(\alpha)+2\sin(\alpha)\right)\right)+\frac{1}{(xo^2+yo^2)^3}\left(4\,(xo^2\right.$$

$$-yo^2)\left(\frac{2\left(\frac{1}{2}\pi+\arctan\left(\frac{yo\cos(\alpha)-xo\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}\right)\right)}{(1-xo^2-yo^2)^{0.5}}-\pi-2\,yo\cos(\alpha)\right.$$

$$+ 2\sin(\alpha)\Bigg)\, xo\Bigg) - \frac{1}{(xo^2+yo^2)^2}\Bigg((xo^2$$

$$-yo^2)\left(\frac{2.0\left(\frac{1}{2}\pi + \arctan\left(\frac{yo\cos(\alpha) - xo\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}\right)\right) xo}{(1-xo^2-yo^2)^{1.5}}\right.$$

$$\left.+\frac{2\left(-\frac{\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}} + \frac{1.0\,(yo\cos(\alpha) - xo\sin(\alpha))\,xo}{(1-xo^2-yo^2)^{1.5}}\right)}{(1-xo^2-yo^2)^{0.5}\left(1+\frac{(yo\cos(\alpha)-xo\sin(\alpha))^2}{(1-xo^2-yo^2)^{1.0}}\right)}\right)\Bigg)$$

$$+\frac{2\,yo\left(\ln\left(\frac{1+xo\cos(\alpha)+yo\sin(\alpha)}{1-xo\cos(\alpha)-yo\sin(\alpha)}\right) - 2\,xo\cos(\alpha) - 2\,yo\sin(\alpha)\right)}{(xo^2+yo^2)^2}$$

$$+\frac{1}{(xo^2+yo^2)^2}\Bigg(2\,xo\,yo\left(\frac{1}{1+xo\cos(\alpha)+yo\sin(\alpha)}\Bigg(\Bigg(1\right.$$

$$/ (1 - xo\cos(\alpha) - yo\sin(\alpha))(\cos(\alpha))$$

$$+ \frac{(1 + xo\cos(\alpha) + yo\sin(\alpha))\cos(\alpha)}{(1 - xo\cos(\alpha) - yo\sin(\alpha))^2}\Bigg)(1 - xo\cos(\alpha) - yo\sin(\alpha))\Bigg)$$

$$-2\cos(\alpha)\Bigg)\Bigg)$$

$$- \frac{1}{(xo^2 + yo^2)^3}\Bigg(8\,xo^2\,yo\Bigg(\ln\Bigg(\frac{1 + xo\cos(\alpha) + yo\sin(\alpha)}{1 - xo\cos(\alpha) - yo\sin(\alpha)}\Bigg)$$

$$-2\,xo\cos(\alpha) - 2\,yo\sin(\alpha)\Bigg)\Bigg)\Bigg)$$

$$-yo^2 \left(\frac{2.0\, yo^2 \left(\frac{1}{2}\pi + \arctan\left(\frac{yo\cos(\alpha) - xo\sin(\alpha)}{(1 - xo^2 - yo^2)^{0.5}} \right) \right) xo}{(1 - xo^2 - yo^2)^{1.5} (xo^2 + yo^2)} \right.$$

$$+ \frac{2\, yo^2 \left(-\frac{\sin(\alpha)}{(1 - xo^2 - yo^2)^{0.5}} + \frac{1.0\, (yo\cos(\alpha) - xo\sin(\alpha))\, xo}{(1 - xo^2 - yo^2)^{1.5}} \right)}{(1 - xo^2 - yo^2)^{0.5} \left(1 + \frac{(yo\cos(\alpha) - xo\sin(\alpha))^2}{(1 - xo^2 - yo^2)^{1.0}} \right) (xo^2 + yo^2)}$$

$$- \frac{4\, yo^2 \left(\frac{1}{2}\pi + \arctan\left(\frac{yo\cos(\alpha) - xo\sin(\alpha)}{(1 - xo^2 - yo^2)^{0.5}} \right) \right) xo}{(1 - xo^2 - yo^2)^{0.5} (xo^2 + yo^2)^2}$$

$$+ \frac{1}{(xo^2 + yo^2)^2} \left(2\, xo \left(\frac{2 \left(\frac{1}{2}\pi + \arctan\left(\frac{yo\cos(\alpha) - xo\sin(\alpha)}{(1 - xo^2 - yo^2)^{0.5}} \right) \right)}{(1 - xo^2 - yo^2)^{0.5}} - \pi \right. \right.$$

$$\left. \left. - 2\, yo\cos(\alpha) + 2\sin(\alpha) \right) \right) - \frac{1}{(xo^2 + yo^2)^3} \left(4\, (xo^2 \right.$$

$$- yo^2) \left(\frac{2 \left(\frac{1}{2}\pi + \arctan\left(\frac{yo\cos(\alpha) - xo\sin(\alpha)}{(1 - xo^2 - yo^2)^{0.5}} \right) \right)}{(1 - xo^2 - yo^2)^{0.5}} - \pi - 2\, yo\cos(\alpha) \right.$$

$$+2\sin(\alpha)\Bigg)\,xo\Bigg) + \frac{1}{(xo^2+yo^2)^2}\Bigg((xo^2$$

$$-yo^2)\left(\frac{2.0\left(\frac{1}{2}\pi+\arctan\left(\frac{yo\cos(\alpha)-xo\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}\right)\right)xo}{(1-xo^2-yo^2)^{1.5}}\right.$$

$$\left.+\frac{2\left(-\frac{\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}+\frac{1.0\,(yo\cos(\alpha)-xo\sin(\alpha))\,xo}{(1-xo^2-yo^2)^{1.5}}\right)}{(1-xo^2-yo^2)^{0.5}\left(1+\frac{(yo\cos(\alpha)-xo\sin(\alpha))^2}{(1-xo^2-yo^2)^{1.0}}\right)}\right)\Bigg)$$

$$-\frac{2\,yo\left(\ln\left(\frac{1+xo\cos(\alpha)+yo\sin(\alpha)}{1-xo\cos(\alpha)-yo\sin(\alpha)}\right)-2\,xo\cos(\alpha)-2\,yo\sin(\alpha)\right)}{(xo^2+yo^2)^2}$$

$$-\frac{1}{(xo^2+yo^2)^2}\Bigg(2\,xo\,yo\Bigg(\frac{1}{1+xo\cos(\alpha)+yo\sin(\alpha)}\Bigg(\Bigg(1$$

$$/\,(1-xo\cos(\alpha)-yo\sin(\alpha))\,(\cos(\alpha))$$

$$+\frac{(1+xo\cos(\alpha)+yo\sin(\alpha))\cos(\alpha)}{(1-xo\cos(\alpha)-yo\sin(\alpha))^2}\Bigg)(1-xo\cos(\alpha)-yo\sin(\alpha))\Bigg)$$

$$-2\cos(\alpha)\Bigg)\Bigg)$$

$$+\frac{1}{(xo^2+yo^2)^3}\Bigg(8\,xo^2\,yo\Bigg(\ln\left(\frac{1+xo\cos(\alpha)+yo\sin(\alpha)}{1-xo\cos(\alpha)-yo\sin(\alpha)}\right)$$

$$-2\,xo\cos(\alpha)-2\,yo\sin(\alpha)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)$$

$$+2\sin(\alpha)\Bigg)xo\Bigg)+\frac{1}{(xo^2+yo^2)^2}\Bigg((xo^2$$

$$-yo^2)\left(\frac{2.0\left(\frac{1}{2}\pi+\arctan\left(\frac{yo\cos(\alpha)-xo\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}\right)\right)xo}{(1-xo^2-yo^2)^{1.5}}\right.$$

$$\left.+\frac{2\left(-\frac{\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}+\frac{1.0\,(yo\cos(\alpha)-xo\sin(\alpha))\,xo}{(1-xo^2-yo^2)^{1.5}}\right)}{(1-xo^2-yo^2)^{0.5}\left(1+\frac{(yo\cos(\alpha)-xo\sin(\alpha))^2}{(1-xo^2-yo^2)^{1.0}}\right)}\right)\Bigg)$$

$$-\frac{2\,yo\left(\ln\left(\frac{1+xo\cos(\alpha)+yo\sin(\alpha)}{1-xo\cos(\alpha)-yo\sin(\alpha)}\right)-2\,xo\cos(\alpha)-2\,yo\sin(\alpha)\right)}{(xo^2+yo^2)^2}$$

$$-\frac{1}{(xo^2+yo^2)^2}\Bigg(2\,xo\,yo\Bigg(\frac{1}{1+xo\cos(\alpha)+yo\sin(\alpha)}\Bigg(\Bigg(1$$

$$/(1-xo\cos(\alpha)-yo\sin(\alpha))(\cos(\alpha))$$

$$+\frac{(1+xo\cos(\alpha)+yo\sin(\alpha))\cos(\alpha)}{(1-xo\cos(\alpha)-yo\sin(\alpha))^2}\Bigg)(1-xo\cos(\alpha)-yo\sin(\alpha))\Bigg)$$

$$-2\cos(\alpha)\Bigg)\Bigg)$$

$$+\frac{1}{(xo^2+yo^2)^3}\Bigg(8\,xo^2\,yo\Bigg(\ln\left(\frac{1+xo\cos(\alpha)+yo\sin(\alpha)}{1-xo\cos(\alpha)-yo\sin(\alpha)}\right)$$

$$-2\,xo\cos(\alpha)-2\,yo\sin(\alpha)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)$$

$$+\frac{1}{6}\,\frac{1}{R}\left(Rey\,C\left(\frac{1}{2}\,xo\left(\frac{2\,yo^2\left(\frac{1}{2}\,\pi+\arctan\left(\frac{yo\cos(\alpha)-xo\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}\right)\right)}{(1-xo^2-yo^2)^{0.5}\,(xo^2+yo^2)}\right.\right.\right.$$

$$+\frac{1}{(xo^2+yo^2)^2}\left((xo^2\right.$$

$$\left.-yo^2)\left(\frac{2\left(\frac{1}{2}\,\pi+\arctan\left(\frac{yo\cos(\alpha)-xo\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}\right)\right)}{(1-xo^2-yo^2)^{0.5}}-\pi-2\,yo\cos(\alpha)\right.\right.$$

$$\left.\left.+2\sin(\alpha)\right)\right)$$

$$-\frac{1}{(xo^2+yo^2)^2}\left(2\,xo\,yo\left(\ln\left(\frac{1+xo\cos(\alpha)+yo\sin(\alpha)}{1-xo\cos(\alpha)-yo\sin(\alpha)}\right)\right.\right.$$

$$\left.\left.\left.-2\,xo\cos(\alpha)-2\,yo\sin(\alpha)\right)\right)\right)+\frac{1}{2}\,yo\left(\right.$$

$$-\frac{2\,xo\,yo\left(\frac{1}{2}\,\pi+\arctan\left(\frac{yo\cos(\alpha)-xo\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}\right)\right)}{(xo^2+yo^2)\,(1-xo^2-yo^2)^{0.5}}$$

$$+\frac{1}{(xo^2+yo^2)^2}\left(2\,xo\,yo\left(\frac{2\left(\frac{1}{2}\,\pi+\arctan\left(\frac{yo\cos(\alpha)-xo\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}\right)\right)}{(1-xo^2-yo^2)^{0.5}}\right.\right.$$

$$\left.\left.-\pi-2\,yo\cos(\alpha)+2\,xo\sin(\alpha)\right)\right)$$

$$+\frac{1}{(xo^2+yo^2)^2}\left((xo^2-yo^2)\left(\ln\left(\frac{1+xo\cos(\alpha)+yo\sin(\alpha)}{1-xo\cos(\alpha)-yo\sin(\alpha)}\right)\right.\right.$$

$$\left.\left.-2\,xo\cos(\alpha)-2\,yo\sin(\alpha)\right)\right)\Bigg)\Bigg)\Bigg)$$

> *foy* := *subs*(*x* = *xo*, *y* = *yo*, *x1* = 0, *x11* = 0, *y1* = 0, *y11* = 0, *fy*);

$$foy := xo\left(\frac{6.00\left(\frac{1}{2}\pi+\arctan\left(\frac{yo\cos(\alpha)-xo\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}\right)\right)yo^2}{(1-xo^2-yo^2)^{2.5}}\right.$$

$$+\frac{4.0\left(\frac{\cos(\alpha)}{(1-xo^2-yo^2)^{0.5}}+\frac{1.0\,(yo\cos(\alpha)-xo\sin(\alpha))\,yo}{(1-xo^2-yo^2)^{1.5}}\right)yo}{(1-xo^2-yo^2)^{1.5}\left(1+\frac{(yo\cos(\alpha)-xo\sin(\alpha))^2}{(1-xo^2-yo^2)^{1.0}}\right)}$$

$$+\frac{2.0\left(\frac{1}{2}\pi+\arctan\left(\frac{yo\cos(\alpha)-xo\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}\right)\right)}{(1-xo^2-yo^2)^{1.5}}$$

$$+\left(2\left(\frac{2.0\cos(\alpha)\,yo}{(1-xo^2-yo^2)^{1.5}}+\frac{3.00\,(yo\cos(\alpha)-xo\sin(\alpha))\,yo^2}{(1-xo^2-yo^2)^{2.5}}+\frac{1.0\,(yo\cos(\alpha)-xo\sin(\alpha}{(1-xo^2-yo^2)^{1.5}}\right.\right.$$

$$\left.\left.+\frac{(yo\cos(\alpha)-xo\sin(\alpha))^2}{(1-xo^2-yo^2)^{1.0}}\right)\right)-\left(2\left(\frac{\cos(\alpha)}{(1-xo^2-yo^2)^{0.5}}\right.\right.$$

$$\left.+\frac{1.0\,(yo\cos(\alpha)-xo\sin(\alpha))\,yo}{(1-xo^2-yo^2)^{1.5}}\right)\left(\frac{2\,(yo\cos(\alpha)-xo\sin(\alpha))\cos(\alpha)}{(1-xo^2-yo^2)^{1.0}}\right.$$

$$\left.\left.+\frac{2.0\,(yo\cos(\alpha)-xo\sin(\alpha))^2\,yo}{(1-xo^2-yo^2)^{2.0}}\right)\right)\Bigg/\left((1-xo^2-yo^2)^{0.5}\left(1\right.\right.$$

$$\left.\left.\left.+\frac{(yo\cos(\alpha)-xo\sin(\alpha))^2}{(1-xo^2-yo^2)^{1.0}}\right)^2\right)\right)$$

$$- yo \left(\frac{6.00 \left(\frac{1}{2} \pi + \arctan\left(\frac{yo \cos(\alpha) - xo \sin(\alpha)}{(1 - xo^2 - yo^2)^{0.5}} \right) \right) yo\, xo}{(1 - xo^2 - yo^2)^{2.5}} \right.$$

$$+ \frac{2.0 \left(\frac{\cos(\alpha)}{(1 - xo^2 - yo^2)^{0.5}} + \frac{1.0\, (yo \cos(\alpha) - xo \sin(\alpha))\, yo}{(1 - xo^2 - yo^2)^{1.5}} \right) xo}{(1 - xo^2 - yo^2)^{1.5} \left(1 + \frac{(yo \cos(\alpha) - xo \sin(\alpha))^2}{(1 - xo^2 - yo^2)^{1.0}} \right)}$$

$$+ \frac{2.0 \left(- \frac{\sin(\alpha)}{(1 - xo^2 - yo^2)^{0.5}} + \frac{1.0\, (yo \cos(\alpha) - xo \sin(\alpha))\, xo}{(1 - xo^2 - yo^2)^{1.5}} \right) yo}{(1 - xo^2 - yo^2)^{1.5} \left(1 + \frac{(yo \cos(\alpha) - xo \sin(\alpha))^2}{(1 - xo^2 - yo^2)^{1.0}} \right)}$$

$$+ \left(2 \left(\frac{1.0 \cos(\alpha)\, xo}{(1 - xo^2 - yo^2)^{1.5}} - \frac{1.0 \sin(\alpha)\, yo}{(1 - xo^2 - yo^2)^{1.5}} \right.\right.$$

$$\left.\left.\left. + \frac{3.00\, (yo \cos(\alpha) - xo \sin(\alpha))\, yo\, xo}{(1 - xo^2 - yo^2)^{2.5}} \right) \right) \middle/ \left((1 - xo^2 - yo^2)^{0.5} \left(1 \right.\right.\right.$$

$$\left.\left. + \frac{(yo \cos(\alpha) - xo \sin(\alpha))^2}{(1 - xo^2 - yo^2)^{1.0}} \right) \right) - \left(2 \left(- \frac{\sin(\alpha)}{(1 - xo^2 - yo^2)^{0.5}} \right.\right.$$

$$\left. + \frac{1.0\, (yo \cos(\alpha) - xo \sin(\alpha))\, xo}{(1 - xo^2 - yo^2)^{1.5}} \right) \left(\frac{2\, (yo \cos(\alpha) - xo \sin(\alpha))\, \cos(\alpha)}{(1 - xo^2 - yo^2)^{1.0}} \right.$$

$$\left.\left. + \frac{2.0\, (yo \cos(\alpha) - xo \sin(\alpha))^2\, yo}{(1 - xo^2 - yo^2)^{2.0}} \right) \right) \middle/ \left((1 - xo^2 - yo^2)^{0.5} \left(1 \right.\right.$$

$$\left.\left.\left. + \frac{(yo \cos(\alpha) - xo \sin(\alpha))^2}{(1 - xo^2 - yo^2)^{1.0}} \right)^2 \right) \right)$$

$$+\frac{1}{\pi}\Bigg(0.0006940000000 \arctan\Bigg(\Bigg(2\Bigg(41.2\left(\frac{R}{C\left(1-\left(xo^2+yo^2\right)^{0.5}\right)^3}\right)^{0.5}$$

$$-41.2\left(\frac{R}{C\left(1+\left(xo^2+yo^2\right)^{0.5}\right)^3}\right)^{0.5}\Bigg) Rey\Bigg) \Bigg/$$

$$\Bigg(\pi\Bigg(41.2\left(\frac{R}{C\left(1-\left(xo^2+yo^2\right)^{0.5}\right)^3}\right)^{0.5}$$

$$+41.2\left(\frac{R}{C\left(1+\left(xo^2+yo^2\right)^{0.5}\right)^3}\right)^{0.5}\Bigg)$$

$$\Bigg(20.60000000\left(\frac{R}{C\left(1-\left(xo^2+yo^2\right)^{0.5}\right)^3}\right)^{0.5}$$

$$+20.60000000\left(\frac{R}{C\left(1+\left(xo^2+yo^2\right)^{0.5}\right)^3}\right)^{0.5}\Bigg)\Bigg(1-Rey\Big/$$

$$\Bigg(20.60000000\left(\frac{R}{C\left(1-\left(xo^2+yo^2\right)^{0.5}\right)^3}\right)^{0.5}$$

$$+20.60000000\left(\frac{R}{C\left(1+\left(xo^2+yo^2\right)^{0.5}\right)^3}\right)^{0.5}\Bigg)\Bigg)\Bigg)\Bigg)\,Rey^{0.96}\Bigg(xo\Bigg($$

$$-\frac{1}{\left(xo^2+yo^2\right)^2}\Bigg(2\Bigg(yo\Bigg(\frac{2\left(\frac{1}{2}\pi+\arctan\left(\frac{yo\cos(\alpha)-xo\sin(\alpha)}{\left(1-xo^2-yo^2\right)^{0.5}}\right)\right)}{\left(1-xo^2-yo^2\right)^{0.5}}$$

$$-\pi\Bigg)+xo\ln\left(\frac{1+xo\cos(\alpha)+yo\sin(\alpha)}{1-xo\cos(\alpha)-yo\sin(\alpha)}\right)\Bigg)yo\Bigg)$$

$$+\frac{1}{xo^2+yo^2}\Bigg(\frac{2\left(\frac{1}{2}\pi+\arctan\left(\frac{yo\cos(\alpha)-xo\sin(\alpha)}{\left(1-xo^2-yo^2\right)^{0.5}}\right)\right)}{\left(1-xo^2-yo^2\right)^{0.5}}-\pi$$

$$+yo\left(\frac{2.0\left(\frac{1}{2}\pi+\arctan\left(\frac{yo\cos(\alpha)-xo\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}\right)\right)yo}{(1-xo^2-yo^2)^{1.5}}\right.$$

$$\left.+\frac{2\left(\frac{\cos(\alpha)}{(1-xo^2-yo^2)^{0.5}}+\frac{1.0\,(yo\cos(\alpha)-xo\sin(\alpha))\,yo}{(1-xo^2-yo^2)^{1.5}}\right)}{(1-xo^2-yo^2)^{0.5}\left(1+\frac{(yo\cos(\alpha)-xo\sin(\alpha))^2}{(1-xo^2-yo^2)^{1.0}}\right)}\right)$$

$$+\frac{1}{1+xo\cos(\alpha)+yo\sin(\alpha)}\left(xo\left(\frac{\sin(\alpha)}{1-xo\cos(\alpha)-yo\sin(\alpha)}\right.\right.$$

$$\left.\left.+\frac{(1+xo\cos(\alpha)+yo\sin(\alpha))\sin(\alpha)}{(1-xo\cos(\alpha)-yo\sin(\alpha))^2}\right)(1-xo\cos(\alpha)-yo\sin(\alpha))\right)$$

$$\Bigg)-yo\Bigg($$

$$-\frac{1}{(xo^2+yo^2)^2}\left(2\left(xo\left(\frac{2\left(\frac{1}{2}\pi+\arctan\left(\frac{yo\cos(\alpha)-xo\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}\right)\right)}{(1-xo^2-yo^2)^{0.5}}\right.\right.\right.$$

$$
\left.\left.\left. -\pi \right) - yo \ln\left(\frac{1 + xo\cos(\alpha) + yo\sin(\alpha)}{1 - xo\cos(\alpha) - yo\sin(\alpha)} \right) \right) yo \right)
$$

$$
+ \frac{1}{xo^2 + yo^2} \left(xo \left(\frac{2.0 \left(\frac{1}{2}\pi + \arctan\left(\frac{yo\cos(\alpha) - xo\sin(\alpha)}{(1 - xo^2 - yo^2)^{0.5}} \right) \right) yo}{(1 - xo^2 - yo^2)^{1.5}} \right.\right.
$$

$$
\left. + \frac{2 \left(\frac{\cos(\alpha)}{(1 - xo^2 - yo^2)^{0.5}} + \frac{1.0\,(yo\cos(\alpha) - xo\sin(\alpha))\,yo}{(1 - xo^2 - yo^2)^{1.5}} \right)}{(1 - xo^2 - yo^2)^{0.5} \left(1 + \frac{(yo\cos(\alpha) - xo\sin(\alpha))^2}{(1 - xo^2 - yo^2)^{1.0}} \right)} \right)
$$

$$
- \ln\left(\frac{1 + xo\cos(\alpha) + yo\sin(\alpha)}{1 - xo\cos(\alpha) - yo\sin(\alpha)} \right)
$$

$$
- \frac{1}{1 + xo\cos(\alpha) + yo\sin(\alpha)} \left(yo \left(\frac{\sin(\alpha)}{1 - xo\cos(\alpha) - yo\sin(\alpha)} \right.\right.
$$

$$
\left.\left. + \frac{(1 + xo\cos(\alpha) + yo\sin(\alpha))\sin(\alpha)}{(1 - xo\cos(\alpha) - yo\sin(\alpha))^2} \right) (1 - xo\cos(\alpha) - yo\sin(\alpha)) \right) \right)
$$

$$
\left.\left.\left. \right)\right)\right) + \frac{1}{6}\,\frac{1}{R} \left(Rey\,C \left(2\,xo\,yo \left(\right.\right.\right.
$$

$$
- \frac{2\,xo \left(\frac{1}{2}\pi + \arctan\left(\frac{yo\cos(\alpha) - xo\sin(\alpha)}{(1 - xo^2 - yo^2)^{0.5}} \right) \right)}{(1 - xo^2 - yo^2)^{0.5}\,(xo^2 + yo^2)}
$$

$$+\frac{4\,yo^2\left(\frac{1}{2}\,\pi+\arctan\left(\frac{yo\cos(\alpha)-xo\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}\right)\right)xo}{(1-xo^2-yo^2)^{0.5}\,(xo^2+yo^2)^2}$$

$$-\frac{2.0\,yo^2\left(\frac{1}{2}\,\pi+\arctan\left(\frac{yo\cos(\alpha)-xo\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}\right)\right)xo}{(1-xo^2-yo^2)^{1.5}\,(xo^2+yo^2)}$$

$$-\frac{2\,xo\,yo\left(\frac{\cos(\alpha)}{(1-xo^2-yo^2)^{0.5}}+\frac{1.0\,(yo\cos(\alpha)-xo\sin(\alpha))\,yo}{(1-xo^2-yo^2)^{1.5}}\right)}{(xo^2+yo^2)\,(1-xo^2-yo^2)^{0.5}\left(1+\frac{(yo\cos(\alpha)-xo\sin(\alpha))^2}{(1-xo^2-yo^2)^{1.0}}\right)}$$

$$+\frac{1}{(xo^2+yo^2)^2}\left(2\,xo\left(\frac{2\left(\frac{1}{2}\,\pi+\arctan\left(\frac{yo\cos(\alpha)-xo\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}\right)\right)}{(1-xo^2-yo^2)^{0.5}}-\pi\right.\right.$$

$$\left.\left.-2\,yo\cos(\alpha)+2\,xo\sin(\alpha)\right)\right)$$

$$-\frac{1}{(xo^2+yo^2)^3}\left(8\,xo\,yo^2\left(\frac{2\left(\frac{1}{2}\pi+\arctan\left(\frac{yo\cos(\alpha)-xo\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}\right)\right)}{(1-xo^2-yo^2)^{0.5}}\right.\right.$$

$$\left.\left.-\pi-2\,yo\cos(\alpha)+2\,xo\sin(\alpha)\right)\right)$$

$$+\frac{1}{(xo^2+yo^2)^2}\left(2\,xo\,yo\left(\frac{1}{(1-xo^2-yo^2)^{1.5}}\left(2.0\left(\frac{1}{2}\pi\right.\right.\right.\right.$$

$$\left.\left.\left.+\arctan\left(\frac{yo\cos(\alpha)-xo\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}\right)\right)yo\right)$$

$$+\frac{2\left(\frac{\cos(\alpha)}{(1-xo^2-yo^2)^{0.5}}+\frac{1.0\,(yo\cos(\alpha)-xo\sin(\alpha))\,yo}{(1-xo^2-yo^2)^{1.5}}\right)}{(1-xo^2-yo^2)^{0.5}\left(1+\frac{(yo\cos(\alpha)-xo\sin(\alpha))^2}{(1-xo^2-yo^2)^{1.0}}\right)}$$

$$-2\cos(\alpha)\Bigg)\Bigg)$$

$$-\frac{2\,yo\left(\ln\left(\frac{1+xo\cos(\alpha)+yo\sin(\alpha)}{1-xo\cos(\alpha)-yo\sin(\alpha)}\right)-2\,xo\cos(\alpha)-2\,yo\sin(\alpha)\right)}{(xo^2+yo^2)^2}$$

$$+\frac{1}{(xo^2+yo^2)^2}\Bigg((xo^2$$

$$-yo^2)\left(\frac{1}{1+xo\cos(\alpha)+yo\sin(\alpha)}\Bigg(\Bigg(\frac{\sin(\alpha)}{1-xo\cos(\alpha)-yo\sin(\alpha)}\right.$$

$$+\frac{(1+xo\cos(\alpha)+yo\sin(\alpha))\sin(\alpha)}{(1-xo\cos(\alpha)-yo\sin(\alpha))^2}\Bigg)(1-xo\cos(\alpha)-yo\sin(\alpha))\Bigg)$$

$$-2\sin(\alpha)\Big)\Big)-\frac{1}{(xo^2+yo^2)^3}\Big(4\,(xo^2$$

$$-yo^2)\left(\ln\left(\frac{1+xo\cos(\alpha)+yo\sin(\alpha)}{1-xo\cos(\alpha)-yo\sin(\alpha)}\right)-2\,xo\cos(\alpha)\right.$$

$$\left.-2\,yo\sin(\alpha)\right)yo\Big)\Big)$$

$$-xo^2\left(\frac{2.0\,xo^2\,yo\left(\frac{1}{2}\pi+\arctan\left(\frac{yo\cos(\alpha)-xo\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}\right)\right)}{(xo^2+yo^2)\,(1-xo^2-yo^2)^{1.5}}\right.$$

$$+\frac{2\,xo^2\left(\frac{\cos(\alpha)}{(1-xo^2-yo^2)^{0.5}}+\frac{1.0\,(yo\cos(\alpha)-xo\sin(\alpha))\,yo}{(1-xo^2-yo^2)^{1.5}}\right)}{(1-xo^2-yo^2)^{0.5}\left(1+\frac{(yo\cos(\alpha)-xo\sin(\alpha))^2}{(1-xo^2-yo^2)^{1.0}}\right)(xo^2+yo^2)}$$

$$-\frac{4\,xo^2\,yo\left(\frac{1}{2}\,\pi+\arctan\left(\frac{yo\cos(\alpha)-xo\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}\right)\right)}{(xo^2+yo^2)^2\,(1-xo^2-yo^2)^{0.5}}$$

$$+\frac{1}{(xo^2+yo^2)^2}\left(2\,yo\left(\frac{2\left(\frac{1}{2}\,\pi+\arctan\left(\frac{yo\cos(\alpha)-xo\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}\right)\right)}{(1-xo^2-yo^2)^{0.5}}-\pi\right.\right.$$

$$\left.\left.-2\,yo\cos(\alpha)+2\sin(\alpha)\right)\right)+\frac{1}{(xo^2+yo^2)^3}\left(4\,(xo^2\right.$$

$$\left.-yo^2)\left(\frac{2\left(\frac{1}{2}\,\pi+\arctan\left(\frac{yo\cos(\alpha)-xo\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}\right)\right)}{(1-xo^2-yo^2)^{0.5}}-\pi-2\,yo\cos(\alpha)\right.\right.$$

$$\left.\left.+2\sin(\alpha)\right)yo\right)-\frac{1}{(xo^2+yo^2)^2}\left((xo^2\right.$$

$$- yo^2) \left(\frac{2.0 \left(\frac{1}{2} \pi + \arctan\left(\frac{yo \cos(\alpha) - xo \sin(\alpha)}{(1 - xo^2 - yo^2)^{0.5}} \right) \right) yo}{(1 - xo^2 - yo^2)^{1.5}} \right.$$

$$+ \frac{2 \left(\frac{\cos(\alpha)}{(1 - xo^2 - yo^2)^{0.5}} + \frac{1.0 \, (yo \cos(\alpha) - xo \sin(\alpha)) \, yo}{(1 - xo^2 - yo^2)^{1.5}} \right)}{(1 - xo^2 - yo^2)^{0.5} \left(1 + \frac{(yo \cos(\alpha) - xo \sin(\alpha))^2}{(1 - xo^2 - yo^2)^{1.0}} \right)}$$

$$\left. \left. - 2 \cos(\alpha) \right) \right)$$

$$+ \frac{2 \, xo \left(\ln\left(\frac{1 + xo \cos(\alpha) + yo \sin(\alpha)}{1 - xo \cos(\alpha) - yo \sin(\alpha)} \right) - 2 \, xo \cos(\alpha) - 2 \, yo \sin(\alpha) \right)}{(xo^2 + yo^2)^2}$$

$$+ \frac{1}{(xo^2 + yo^2)^2} \left(2 \, xo \, yo \left(\frac{1}{1 + xo \cos(\alpha) + yo \sin(\alpha)} \left(\left(1 \right.\right.\right.\right.$$

$$/ (1 - xo\cos(\alpha) - yo\sin(\alpha))(\sin(\alpha))$$

$$+ \frac{(1 + xo\cos(\alpha) + yo\sin(\alpha))\sin(\alpha)}{(1 - xo\cos(\alpha) - yo\sin(\alpha))^2}\Bigg) (1 - xo\cos(\alpha) - yo\sin(\alpha))\Bigg)$$

$$- 2\sin(\alpha)\Bigg)\Bigg)$$

$$- \frac{1}{(xo^2 + yo^2)^3}\Bigg(8\,xo\,yo^2\Bigg(\ln\Bigg(\frac{1 + xo\cos(\alpha) + yo\sin(\alpha)}{1 - xo\cos(\alpha) - yo\sin(\alpha)}\Bigg)$$

$$- 2\,xo\cos(\alpha) - 2\,yo\sin(\alpha)\Bigg)\Bigg)\Bigg)$$

$$-yo^2\left(\frac{4\,yo\left(\frac{1}{2}\,\pi+\arctan\left(\frac{yo\cos(\alpha)-xo\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}\right)\right)}{(xo^2+yo^2)\,(1-xo^2-yo^2)^{0.5}}\right.$$

$$+\frac{2.0\,yo^3\left(\frac{1}{2}\,\pi+\arctan\left(\frac{yo\cos(\alpha)-xo\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}\right)\right)}{(1-xo^2-yo^2)^{1.5}\,(xo^2+yo^2)}$$

$$+\frac{2\,yo^2\left(\frac{\cos(\alpha)}{(1-xo^2-yo^2)^{0.5}}+\frac{1.0\,(yo\cos(\alpha)-xo\sin(\alpha))\,yo}{(1-xo^2-yo^2)^{1.5}}\right)}{(1-xo^2-yo^2)^{0.5}\left(1+\frac{(yo\cos(\alpha)-xo\sin(\alpha))^2}{(1-xo^2-yo^2)^{1.0}}\right)(xo^2+yo^2)}$$

$$-\frac{4\,yo^3\left(\frac{1}{2}\,\pi+\arctan\left(\frac{yo\cos(\alpha)-xo\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}\right)\right)}{(1-xo^2-yo^2)^{0.5}\,(xo^2+yo^2)^2}$$

$$-\frac{1}{(xo^2+yo^2)^2}\left(2\,yo\left(\frac{2\left(\frac{1}{2}\,\pi+\arctan\left(\frac{yo\cos(\alpha)-xo\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}\right)\right)}{(1-xo^2-yo^2)^{0.5}}-\pi\right.\right.$$

$$\left.\left.-2\,yo\cos(\alpha)+2\sin(\alpha)\right)\right)-\frac{1}{(xo^2+yo^2)^3}\left(4\,(xo^2\right.$$

$$-yo^2)\left(\frac{2\left(\frac{1}{2}\pi+\arctan\left(\frac{yo\cos(\alpha)-xo\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}\right)\right)}{(1-xo^2-yo^2)^{0.5}}-\pi-2\,yo\cos(\alpha)\right.$$

$$\left.\left.+2\sin(\alpha)\right)yo\right)+\frac{1}{(xo^2+yo^2)^2}\left((xo^2\right.$$

$$-yo^2)\left(\frac{2.0\left(\frac{1}{2}\pi+\arctan\left(\frac{yo\cos(\alpha)-xo\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}\right)\right)yo}{(1-xo^2-yo^2)^{1.5}}\right.$$

$$+\frac{2\left(\frac{\cos(\alpha)}{(1-xo^2-yo^2)^{0.5}}+\frac{1.0\,(yo\cos(\alpha)-xo\sin(\alpha))\,yo}{(1-xo^2-yo^2)^{1.5}}\right)}{(1-xo^2-yo^2)^{0.5}\left(1+\frac{(yo\cos(\alpha)-xo\sin(\alpha))^2}{(1-xo^2-yo^2)^{1.0}}\right)}$$

$$\left.\left.-2\cos(\alpha)\right)\right)$$

$$-\frac{2\,xo\left(\ln\left(\frac{1+xo\cos(\alpha)+yo\sin(\alpha)}{1-xo\cos(\alpha)-yo\sin(\alpha)}\right)-2\,xo\cos(\alpha)-2\,yo\sin(\alpha)\right)}{(xo^2+yo^2)^2}$$

$$-\frac{1}{(xo^2+yo^2)^2}\left(2\,xo\,yo\left(\frac{1}{1+xo\cos(\alpha)+yo\sin(\alpha)}\left(\left((1\right.\right.\right.\right.$$

$$/(1 - xo\cos(\alpha) - yo\sin(\alpha))(\sin(\alpha))$$

$$+ \frac{(1 + xo\cos(\alpha) + yo\sin(\alpha))\sin(\alpha)}{(1 - xo\cos(\alpha) - yo\sin(\alpha))^2}\Bigg)(1 - xo\cos(\alpha) - yo\sin(\alpha))\Bigg)$$

$$-2\sin(\alpha)\Bigg)\Bigg)$$

$$+ \frac{1}{(xo^2 + yo^2)^3}\Bigg(8\,xo\,yo^2\Bigg(\ln\left(\frac{1 + xo\cos(\alpha) + yo\sin(\alpha)}{1 - xo\cos(\alpha) - yo\sin(\alpha)}\right)$$

$$-2\,xo\cos(\alpha) - 2\,yo\sin(\alpha)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg) + \frac{1}{6}\,\frac{1}{R}\Bigg(Rey\,C\Bigg(\frac{1}{2}\,xo\Bigg($$

$$-\frac{2\,xo\,yo\left(\frac{1}{2}\pi + \arctan\left(\frac{yo\cos(\alpha) - xo\sin(\alpha)}{(1 - xo^2 - yo^2)^{0.5}}\right)\right)}{(xo^2 + yo^2)(1 - xo^2 - yo^2)^{0.5}}$$

$$+ \frac{1}{(xo^2 + yo^2)^2}\left(2\,xo\,yo\left(\frac{2\left(\frac{1}{2}\pi + \arctan\left(\frac{yo\cos(\alpha) - xo\sin(\alpha)}{(1 - xo^2 - yo^2)^{0.5}}\right)\right)}{(1 - xo^2 - yo^2)^{0.5}}\right.\right.$$

$$\left.\left.-\pi - 2\,yo\cos(\alpha) + 2\,xo\sin(\alpha)\right)\right)$$

$$+ \frac{1}{(xo^2 + yo^2)^2}\Bigg((xo^2 - yo^2)\Bigg(\ln\left(\frac{1 + xo\cos(\alpha) + yo\sin(\alpha)}{1 - xo\cos(\alpha) - yo\sin(\alpha)}\right)$$

$$-2\,xo\cos(\alpha) - 2\,yo\sin(\alpha)\Bigg)\Bigg)\Bigg)$$

$$+\frac{1}{2}\,yo\left(\frac{2\,xo^2\left(\frac{1}{2}\,\pi+\arctan\left(\frac{yo\cos(\alpha)-xo\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}\right)\right)}{(1-xo^2-yo^2)^{0.5}\,(xo^2+yo^2)}\right.$$

$$-\frac{1}{(xo^2+yo^2)^2}\left((xo^2\right.$$

$$\left.-yo^2)\left(\frac{2\left(\frac{1}{2}\,\pi+\arctan\left(\frac{yo\cos(\alpha)-xo\sin(\alpha)}{(1-xo^2-yo^2)^{0.5}}\right)\right)}{(1-xo^2-yo^2)^{0.5}}-\pi-2\,yo\cos(\alpha)\right.\right.$$

$$\left.\left.+2\sin(\alpha)\right)\right)$$

$$+\frac{1}{(xo^2+yo^2)^2}\left(2\,xo\,yo\left(\ln\left(\frac{1+xo\cos(\alpha)+yo\sin(\alpha)}{1-xo\cos(\alpha)-yo\sin(\alpha)}\right)\right.\right.$$

$$\left.\left.\left.\left.\left.-2\,xo\cos(\alpha)-2\,yo\sin(\alpha)\right)\right)\right)\right)\right)$$

Simpler terms for using the steady state pressure and fluid force components are defined.

> *eqn1* := 'fox';

$eqn1 := fox$ **(39)**

> $eqn2 := 'foy - \frac{1}{\sigma}'$;

$eqn2 := foy - \frac{1}{\sigma}$ **(40)**

> *eqn3* := 'po';

$eqn3 := po$ **(41)**

The Reynolds number is defined

> $Rey := \frac{\rho \cdot \omega \cdot R \cdot C}{\mu}$;

$Rey := \frac{\rho\,\omega\,R\,C}{\mu}$ **(42)**

Journal bearing parameters are defined.

ω := journal angularvelocity$\left(\frac{rads}{s}\right)$; R := bearing radius(meters); J := bearing diameter(meters); L := bearing length(meters); C := bearing clearance(meters); V := maximum bearing velocity; ϕ := bearing oscillation frequency;

> ω := 94.2; R := .01275; J := .0254; L := .0254; C := .0000254; V := 0.304; ϕ := 304.0; $\tau := \frac{3 \cdot \pi}{4}$;

$\omega := 94.2$
$R := 0.01275$
$J := 0.0254$
$L := 0.0254$
$C := 0.0000254$
$V := 0.304$
$\phi := 304.0$
$$\tau := \frac{3}{4}\pi \tag{43}$$

Lubricant parameters are defined. For this case we will use oil with a density of 850 kg per cubic meter and dynamic viscosity of 0.05 Pa*s.

> $\mu := 0.05;\ \rho := 850;$

$\mu := 0.05$
$$\rho := 850 \tag{44}$$

> $Rey := Rey;$

$$Rey := 0.5186133900 \tag{45}$$

The modified Sommerfeld number is defined. If all else remains constant, this will vary with the load on the bearing.

> $\sigma := 1$

$$\sigma := 1 \tag{46}$$

Newton-Raphson method
Solving for xo, yo, and alpha with eqn1, eqn2, and eqn3 with an intial estimate

> $with\ (linalg)$:

Initial estimates are made for xo, yo, and alpha

> $x_1 := 0.5;\ y_1 := 0.0;\ \chi := 3.144;$

$x_1 := 0.5$
$y_1 := 0.$
$$\chi := 3.144 \tag{47}$$

Eqn1, eqn2, and eqn3 are evaluated using the initial estimates and reported below

> $e1 := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn1))$:

> $e2 := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn2))$:

> $e3 := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn3))$:

> *Initial values*

$$Initial\ values \tag{48}$$

> $xo, yo, \alpha, x_1, y_1, \chi;$

$$xo, yo, \alpha, 0.5, 0., 3.144 \tag{49}$$

> 'eqn1, eqn2, eqn3', e1, e2, e3;

$$eqn1, eqn2, eqn3, -1.777691988, 1.418450146, -0.001028338560 \tag{50}$$

Here four matrices are defined. The determinants of these matrices will be used to determine what adjustment to the value of xo, yo, and alpha is to be.

> $d := matrix(3, 3, 0)$:

> $n1 := matrix(3, 3, 0)$:

> $n2 := matrix(3, 3, 0)$:

```
> n3 := matrix(3,3,0) :
> eqn11 := diff(eqn1, xo) :
> eqn12 := diff(eqn1, yo) :
> eqn13 := diff(eqn1, α) :
> eqn14 := diff(eqn2, xo) :
> eqn15 := diff(eqn2, yo) :
> eqn16 := diff(eqn2, α) :
> eqn17 := diff(eqn3, xo) :
> eqn18 := diff(eqn3, yo) :
> eqn19 := diff(eqn3, α) :
> d[1,1] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn11)) :
> d[1,2] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn12)) :
> d[1,3] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn13)) :
> d[2,1] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn14)) :
> d[2,2] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn15)) :
> d[2,3] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn16)) :
> d[3,1] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn17)) :
> d[3,2] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn18)) :
> d[3,3] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn19)) :
> evalm(d) :
> detd := d[1,1]·d[2,2]·d[3,3]-d[1,1]·d[2,3]·d[3,2]-d[1,2]·d[2,1]·d[3,3]
      + d[1,3]·d[2,1]·d[3,2] + d[1,2]·d[2,3]·d[3,1]-d[1,3]·d[2,2]·d[3,1]:
> n1[1,1] := -e1:
> n1[1,2] := d[1,2]:
> n1[1,3] := d[1,3]:
> n1[2,1] := -e2:
> n1[2,2] := d[2,2]:
> n1[2,3] := d[2,3]:
> n1[3,1] := -e3:
> n1[3,2] := d[3,2]:
> n1[3,3] := d[3,3]:
> evalm(n1) :
> detn1 := n1[1,1]·n1[2,2]·n1[3,3]-n1[1,1]·n1[2,3]·n1[3,2]-n1[1,2]·n1[2,1]
      ·n1[3,3] + n1[1,3]·n1[2,1]·n1[3,2] + n1[1,2]·n1[2,3]·n1[3,1]-n1[1,3]
      ·n1[2,2]·n1[3,1]:
> n2[1,1] := d[1,1]:
> n2[1,2] := -e1:
```

```
> n2[1,3] := d[1,3]:
> n2[2,1] := d[2,1]:
> n2[2,2] := -e2:
> n2[2,3] := d[2,3]:
> n2[3,1] := d[3,1]:
> n2[3,2] := -e3:
> n2[3,3] := d[3,3]:
> evalm(n2) :
> detn2 := n2[1,1]·n2[2,2]·n2[3,3]−n2[1,1]·n2[2,3]·n2[3,2]−n2[1,2]·n2[2,1]
    ·n2[3,3] + n2[1,3]·n2[2,1]·n2[3,2] + n2[1,2]·n2[2,3]·n2[3,1]−n2[1,3]
    ·n2[2,2]·n2[3,1]:
> n3[1,1] := d[1,1]:
> n3[1,2] := d[1,2]:
> n3[1,3] := -e1:
> n3[2,1] := d[2,1]:
> n3[2,2] := d[2,2]:
> n3[2,3] := -e2:
> n3[3,1] := d[3,1]:
> n3[3,2] := d[3,2]:
> n3[3,3] := -e3:
> evalm(n3) :
> detn3 := n3[1,1]·n3[2,2]·n3[3,3]−n3[1,1]·n3[2,3]·n3[3,2]−n3[1,2]·n3[2,1]
    ·n3[3,3] + n3[1,3]·n3[2,1]·n3[3,2] + n3[1,2]·n3[2,3]·n3[3,1]-n3[1,3]
    ·n3[2,2]·n3[3,1]:
```

Here the change in the values to be used for xo, yo, and alpha are defined.

$> x_{del} := \dfrac{detn1}{detd}$:

$> y_{del} := \dfrac{detn2}{detd}$:

$> \psi := \dfrac{detn3}{detd}$:

Here the values to be used for xo, yo, and alpha are re-defined.

$> x_1 := x_1 + x_{del}$:

$> y_1 := y_1 + y_{del}$:

$> \chi := \chi + \psi$:

Eqn1, eqn2, and eqn3 are re-evaluated using the new values of xo, yo, and alpha

$> e1 := evalf\left(subs\left(xo = x_1, yo = y_1, \alpha = \chi, eqn1\right)\right)$:

$> e2 := evalf\left(subs\left(xo = x_1, yo = y_1, \alpha = \chi, eqn2\right)\right)$:

```
> e3 := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn3)) :
```
Fifteen subsequent iterations are performed here
```
> Iteration Number One
```
Iteration Number One **(51)**

Current values of xo, yo, and alpha are reported. This is repeated for each iteration.
```
> 'xo, yo, α', x_1, y_1, χ;
```
xo, yo, α, 0.3516752415, -0.004619924457, 3.004791933 **(52)**

Current values of eqn1, eqn2, and eqn3 are reported. This is repeated for each iteration.
```
> 'eqn1, eqn2, eqn3', e1, e2, e3;
```
eqn1, eqn2, eqn3, -0.6093917823, 0.3536569534, -0.0007052916032 **(53)**
```
>
> d[1, 1] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn11)) :
> d[1, 2] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn12)) :
> d[1, 3] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn13)) :
> d[2, 1] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn14)) :
> d[2, 2] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn15)) :
> d[2, 3] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn16)) :
> d[3, 1] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn17)) :
> d[3, 2] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn18)) :
> d[3, 3] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn19)) :
> evalm(d) :
> detd := d[1, 1]·d[2, 2]·d[3, 3]-d[1, 1]·d[2, 3]·d[3, 2]-d[1, 2]·d[2, 1]·d[3, 3]
      + d[1, 3]·d[2, 1]·d[3, 2] + d[1, 2]·d[2, 3]·d[3, 1]-d[1, 3]·d[2, 2]·d[3, 1]:
> n1[1, 1] :=-e1 :
> n1[1, 2] := d[1, 2] :
> n1[1, 3] := d[1, 3] :
> n1[2, 1] :=-e2 :
> n1[2, 2] := d[2, 2] :
> n1[2, 3] := d[2, 3] :
> n1[3, 1] :=-e3 :
> n1[3, 2] := d[3, 2] :
> n1[3, 3] := d[3, 3] :
> evalm(n1) :
> detn1 := n1[1, 1]·n1[2, 2]·n1[3, 3]-n1[1, 1]·n1[2, 3]·n1[3, 2]-n1[1, 2]·n1[2, 1]
      ·n1[3, 3] + n1[1, 3]·n1[2, 1]·n1[3, 2] + n1[1, 2]·n1[2, 3]·n1[3, 1]-n1[1, 3]
      ·n1[2, 2]·n1[3, 1] :
> n2[1, 1] := d[1, 1] :
```

```
> n2[1,2] := -e1 :
> n2[1,3] := d[1,3] :
> n2[2,1] := d[2,1] :
> n2[2,2] := -e2 :
> n2[2,3] := d[2,3] :
> n2[3,1] := d[3,1] :
> n2[3,2] := -e3 :
> n2[3,3] := d[3,3] :
> evalm(n2) :
> detn2 := n2[1,1]·n2[2,2]·n2[3,3]-n2[1,1]·n2[2,3]·n2[3,2]-n2[1,2]·n2[2,1]
      ·n2[3,3] + n2[1,3]·n2[2,1]·n2[3,2] + n2[1,2]·n2[2,3]·n2[3,1]-n2[1,3]
      ·n2[2,2]·n2[3,1] :
> n3[1,1] := d[1,1] :
> n3[1,2] := d[1,2] :
> n3[1,3] := -e1 :
> n3[2,1] := d[2,1] :
> n3[2,2] := d[2,2] :
> n3[2,3] := -e2 :
> n3[3,1] := d[3,1] :
> n3[3,2] := d[3,2] :
> n3[3,3] := -e3 :
> evalm(n3) :
> detn3 := n3[1,1]·n3[2,2]·n3[3,3]-n3[1,1]·n3[2,3]·n3[3,2]-n3[1,2]·n3[2,1]
      ·n3[3,3] + n3[1,3]·n3[2,1]·n3[3,2] + n3[1,2]·n3[2,3]·n3[3,1]-n3[1,3]
      ·n3[2,2]·n3[3,1] :
```

$> x_{del} := \dfrac{detn1}{detd}$:

$> y_{del} := \dfrac{detn2}{detd}$:

$> \psi := \dfrac{detn3}{detd}$:

$> x_1 := x_1 + x_{del}$:

$> y_1 := y_1 + y_{del}$:

$> \chi := \chi + \psi$:

$> e1 := evalf\left(subs\left(xo = x_1, yo = y_1, \alpha = \chi, eqn1\right)\right)$:

$> e2 := evalf\left(subs\left(xo = x_1, yo = y_1, \alpha = \chi, eqn2\right)\right)$:

$> e3 := evalf\left(subs\left(xo = x_1, yo = y_1, \alpha = \chi, eqn3\right)\right)$:

```
> Iteration Number Two
Iteration Number Two                                                        (54)
> 'xo, yo, α', x_1, y_1, χ;
xo, yo, α, 0.2754453539, -0.04765131968, 2.512722548                        (55)
> 'eqn1, eqn2, eqn3', e1, e2, e3;
eqn1, eqn2, eqn3, -0.0906870439, -0.0116577356, -0.0004413190293            (56)
> d[1,1] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn11)) :
> d[1,2] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn12)) :
> d[1,3] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn13)) :
> d[2,1] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn14)) :
> d[2,2] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn15)) :
> d[2,3] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn16)) :
> d[3,1] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn17)) :
> d[3,2] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn18)) :
> d[3,3] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn19)) :
> evalm(d) :
> detd := d[1,1]·d[2,2]·d[3,3]-d[1,1]·d[2,3]·d[3,2]-d[1,2]·d[2,1]·d[3,3]
      + d[1,3]·d[2,1]·d[3,2] + d[1,2]·d[2,3]·d[3,1]-d[1,3]·d[2,2]·d[3,1]:
> n1[1,1] := -e1 :
> n1[1,2] := d[1,2]:
> n1[1,3] := d[1,3]:
> n1[2,1] := -e2 :
> n1[2,2] := d[2,2]:
> n1[2,3] := d[2,3]:
> n1[3,1] := -e3 :
> n1[3,2] := d[3,2]:
> n1[3,3] := d[3,3]:
> evalm(n1) :
> detn1 := n1[1,1]·n1[2,2]·n1[3,3]-n1[1,1]·n1[2,3]·n1[3,2]-n1[1,2]·n1[2,1]
      ·n1[3,3] + n1[1,3]·n1[2,1]·n1[3,2] + n1[1,2]·n1[2,3]·n1[3,1]-n1[1,3]
      ·n1[2,2]·n1[3,1]:
> n2[1,1] := d[1,1]:
> n2[1,2] := -e1 :
> n2[1,3] := d[1,3]:
> n2[2,1] := d[2,1]:
> n2[2,2] := -e2 :
> n2[2,3] := d[2,3]:
```

```
> n2[3,1] := d[3,1]:
> n2[3,2] := -e3:
> n2[3,3] := d[3,3]:
> evalm(n2) :
> detn2 := n2[1,1]·n2[2,2]·n2[3,3]-n2[1,1]·n2[2,3]·n2[3,2]-n2[1,2]·n2[2,1]
        ·n2[3,3] + n2[1,3]·n2[2,1]·n2[3,2] + n2[1,2]·n2[2,3]·n2[3,1]-n2[1,3]
        ·n2[2,2]·n2[3,1]:
> n3[1,1] := d[1,1]:
> n3[1,2] := d[1,2]:
> n3[1,3] := -e1:
> n3[2,1] := d[2,1]:
> n3[2,2] := d[2,2]:
> n3[2,3] := -e2:
> n3[3,1] := d[3,1]:
> n3[3,2] := d[3,2]:
> n3[3,3] := -e3:
> evalm(n3) :
> detn3 := n3[1,1]·n3[2,2]·n3[3,3]-n3[1,1]·n3[2,3]·n3[3,2]-n3[1,2]·n3[2,1]
        ·n3[3,3] + n3[1,3]·n3[2,1]·n3[3,2] + n3[1,2]·n3[2,3]·n3[3,1]-n3[1,3]
        ·n3[2,2]·n3[3,1]:
```

$> x_{del} := \dfrac{detn1}{detd}$:

$> y_{del} := \dfrac{detn2}{detd}$:

$> \psi := \dfrac{detn3}{detd}$:

$> x_1 := x_1 + x_{del}$:

$> y_1 := y_1 + y_{del}$:

$> \chi := \chi + \psi$:

$> e1 := evalf\left(subs\left(xo = x_1, yo = y_1, \alpha = \chi, eqn1\right)\right)$:

$> e2 := evalf\left(subs\left(xo = x_1, yo = y_1, \alpha = \chi, eqn2\right)\right)$:

$> e3 := evalf\left(subs\left(xo = x_1, yo = y_1, \alpha = \chi, eqn3\right)\right)$:

> Iteration Number Three

Iteration Number Three **(57)**

> 'xo, yo, α', x_1, y_1, χ;

$xo, yo, \alpha, 0.3021316320, -0.04760521380, 2.146395989$ **(58)**

> 'eqn1, eqn2, eqn3', e1, e2, e3;

(59)

$eqn1, eqn2, eqn3, 0.02237052751, -0.069404395, -0.0001351545174$ **(59)**

```
> d[1,1] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn11)) :
> d[1,2] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn12)) :
> d[1,3] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn13)) :
> d[2,1] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn14)) :
> d[2,2] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn15)) :
> d[2,3] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn16)) :
> d[3,1] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn17)) :
> d[3,2] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn18)) :
> d[3,3] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn19)) :
> evalm(d) :
> detd := d[1,1]·d[2,2]·d[3,3]-d[1,1]·d[2,3]·d[3,2]-d[1,2]·d[2,1]·d[3,3]
      + d[1,3]·d[2,1]·d[3,2] + d[1,2]·d[2,3]·d[3,1]-d[1,3]·d[2,2]·d[3,1]:
> n1[1,1] :=-e1:
> n1[1,2] := d[1,2]:
> n1[1,3] := d[1,3]:
> n1[2,1] :=-e2:
> n1[2,2] := d[2,2]:
> n1[2,3] := d[2,3]:
> n1[3,1] :=-e3:
> n1[3,2] := d[3,2]:
> n1[3,3] := d[3,3]:
> evalm(n1) :
> detn1 := n1[1,1]·n1[2,2]·n1[3,3]-n1[1,1]·n1[2,3]·n1[3,2]-n1[1,2]·n1[2,1]
      ·n1[3,3] + n1[1,3]·n1[2,1]·n1[3,2] + n1[1,2]·n1[2,3]·n1[3,1]-n1[1,3]
      ·n1[2,2]·n1[3,1]:
> n2[1,1] := d[1,1]:
> n2[1,2] :=-e1:
> n2[1,3] := d[1,3]:
> n2[2,1] := d[2,1]:
> n2[2,2] := -e2:
> n2[2,3] := d[2,3]:
> n2[3,1] := d[3,1]:
> n2[3,2] := -e3:
> n2[3,3] := d[3,3]:
> evalm(n2) :
> detn2 := n2[1,1]·n2[2,2]·n2[3,3]-n2[1,1]·n2[2,3]·n2[3,2]-n2[1,2]·n2[2,1]
```

```
·n2[3,3] + n2[1,3]·n2[2,1]·n2[3,2] + n2[1,2]·n2[2,3]·n2[3,1]-n2[1,3]
    ·n2[2,2]·n2[3,1]:
> n3[1,1] := d[1,1]:
> n3[1,2] := d[1,2]:
> n3[1,3] := -e1:
> n3[2,1] := d[2,1]:
> n3[2,2] := d[2,2]:
> n3[2,3] := -e2:
> n3[3,1] := d[3,1]:
> n3[3,2] := d[3,2]:
> n3[3,3] := -e3:
> evalm(n3) :
> detn3 := n3[1,1]·n3[2,2]·n3[3,3]-n3[1,1]·n3[2,3]·n3[3,2]-n3[1,2]·n3[2,1]
    ·n3[3,3] + n3[1,3]·n3[2,1]·n3[3,2] + n3[1,2]·n3[2,3]·n3[3,1]-n3[1,3]
    ·n3[2,2]·n3[3,1]:
```

$> x_{del} := \frac{detn1}{detd}$:

$> y_{del} := \frac{detn2}{detd}$:

$> \psi := \frac{detn3}{detd}$:

$> x_1 := x_1 + x_{del}$:

$> y_1 := y_1 + y_{del}$:

$> \chi := \chi + \psi$:

$> e1 := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn1))$:

$> e2 := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn2))$:

$> e3 := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn3))$:

> Iteration Number Four

Iteration Number Four **(60)**

> 'xo, yo, α', x_1, y_1, χ;

xo, *yo*, α, 0.3295687793, -0.04339643264, 2.138290329 **(61)**

> 'eqn1, eqn2, eqn3', e1, e2, e3;

eqn1, *eqn2*, *eqn3*, 0.0003488167, -0.0009637335, 0.000004114622 **(62)**

$> d[1,1] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn11))$:

$> d[1,2] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn12))$:

$> d[1,3] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn13))$:

```
> d[2,1] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn14)) :
> d[2,2] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn15)) :
> d[2,3] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn16)) :
> d[3,1] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn17)) :
> d[3,2] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn18)) :
> d[3,3] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn19)) :
> evalm(d) :
> detd := d[1,1]·d[2,2]·d[3,3]-d[1,1]·d[2,3]·d[3,2]-d[1,2]·d[2,1]·d[3,3]
      + d[1,3]·d[2,1]·d[3,2] + d[1,2]·d[2,3]·d[3,1]-d[1,3]·d[2,2]·d[3,1] :
> n1[1,1] :=-e1 :
> n1[1,2] := d[1,2] :
> n1[1,3] := d[1,3] :
> n1[2,1] :=-e2 :
> n1[2,2] := d[2,2] :
> n1[2,3] := d[2,3] :
> n1[3,1] :=-e3 :
> n1[3,2] := d[3,2] :
> n1[3,3] := d[3,3] :
> evalm(n1) :
> detn1 := n1[1,1]·n1[2,2]·n1[3,3]-n1[1,1]·n1[2,3]·n1[3,2]-n1[1,2]·n1[2,1]
     ·n1[3,3] + n1[1,3]·n1[2,1]·n1[3,2] + n1[1,2]·n1[2,3]·n1[3,1]-n1[1,3]
     ·n1[2,2]·n1[3,1] :
> n2[1,1] := d[1,1] :
> n2[1,2] :=-e1 :
> n2[1,3] := d[1,3] :
> n2[2,1] := d[2,1] :
> n2[2,2] := -e2 :
> n2[2,3] := d[2,3] :
> n2[3,1] := d[3,1] :
> n2[3,2] := -e3 :
> n2[3,3] := d[3,3] :
> evalm(n2) :
> detn2 := n2[1,1]·n2[2,2]·n2[3,3]-n2[1,1]·n2[2,3]·n2[3,2]-n2[1,2]·n2[2,1]
     ·n2[3,3] + n2[1,3]·n2[2,1]·n2[3,2] + n2[1,2]·n2[2,3]·n2[3,1]-n2[1,3]
     ·n2[2,2]·n2[3,1] :
> n3[1,1] := d[1,1] :
> n3[1,2] := d[1,2] :
> n3[1,3] := -e1 :
```

```
> n3[2,1] := d[2,1]:
> n3[2,2] := d[2,2]:
> n3[2,3] := -e2:
> n3[3,1] := d[3,1]:
> n3[3,2] := d[3,2]:
> n3[3,3] := -e3:
> evalm(n3) :
> detn3 := n3[1,1]·n3[2,2]·n3[3,3]-n3[1,1]·n3[2,3]·n3[3,2]-n3[1,2]·n3[2,1]
      ·n3[3,3] + n3[1,3]·n3[2,1]·n3[3,2] + n3[1,2]·n3[2,3]·n3[3,1]-n3[1,3]
      ·n3[2,2]·n3[3,1]:
```

> $x_{del} := \dfrac{detn1}{detd}$:

> $y_{del} := \dfrac{detn2}{detd}$:

> $\psi := \dfrac{detn3}{detd}$:

> $x_1 := x_1 + x_{del}$:

> $y_1 := y_1 + y_{del}$:

> $\chi := \chi + \psi$:

> $e1 := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn1))$:

> $e2 := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn2))$:

> $e3 := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn3))$:

> *Iteration Number Five*

Iteration Number Five **(63)**

> 'xo, yo, α', x_1, y_1, χ;

xo, *yo*, α, 0.3286114175, -0.04385778636, 2.142583170 **(64)**

> 'eqn1, eqn2, eqn3', e1, e2, e3;

eqn1, *eqn2*, *eqn3*, 0.00000130769, -0.000014612, $5.2088\ 10^{-8}$ **(65)**

> $d[1,1] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn11))$:

> $d[1,2] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn12))$:

> $d[1,3] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn13))$:

> $d[2,1] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn14))$:

> $d[2,2] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn15))$:

> $d[2,3] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn16))$:

```
> d[3,1] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn17)) :
> d[3,2] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn18)) :
> d[3,3] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn19)) :
> evalm(d) :
> detd := d[1,1]·d[2,2]·d[3,3]-d[1,1]·d[2,3]·d[3,2]-d[1,2]·d[2,1]·d[3,3]
      + d[1,3]·d[2,1]·d[3,2] + d[1,2]·d[2,3]·d[3,1]-d[1,3]·d[2,2]·d[3,1] :
> n1[1,1] := -e1 :
> n1[1,2] := d[1,2] :
> n1[1,3] := d[1,3] :
> n1[2,1] := -e2 :
> n1[2,2] := d[2,2] :
> n1[2,3] := d[2,3] :
> n1[3,1] := -e3 :
> n1[3,2] := d[3,2] :
> n1[3,3] := d[3,3] :
> evalm(n1) :
> detn1 := n1[1,1]·n1[2,2]·n1[3,3]-n1[1,1]·n1[2,3]·n1[3,2]-n1[1,2]·n1[2,1]
      ·n1[3,3] + n1[1,3]·n1[2,1]·n1[3,2] + n1[1,2]·n1[2,3]·n1[3,1]-n1[1,3]
      ·n1[2,2]·n1[3,1] :
> n2[1,1] := d[1,1] :
> n2[1,2] := -e1 :
> n2[1,3] := d[1,3] :
> n2[2,1] := d[2,1] :
> n2[2,2] := -e2 :
> n2[2,3] := d[2,3] :
> n2[3,1] := d[3,1] :
> n2[3,2] := -e3 :
> n2[3,3] := d[3,3] :
> evalm(n2) :
> detn2 := n2[1,1]·n2[2,2]·n2[3,3]-n2[1,1]·n2[2,3]·n2[3,2]-n2[1,2]·n2[2,1]
      ·n2[3,3] + n2[1,3]·n2[2,1]·n2[3,2] + n2[1,2]·n2[2,3]·n2[3,1]-n2[1,3]
      ·n2[2,2]·n2[3,1] :
> n3[1,1] := d[1,1] :
> n3[1,2] := d[1,2] :
> n3[1,3] := -e1 :
> n3[2,1] := d[2,1] :
> n3[2,2] := d[2,2] :
> n3[2,3] := -e2 :
> n3[3,1] := d[3,1] :
```

```
> n3[3,2] := d[3,2]:
> n3[3,3] := -e3:
> evalm(n3) :
> detn3 := n3[1,1]·n3[2,2]·n3[3,3]-n3[1,1]·n3[2,3]·n3[3,2]-n3[1,2]·n3[2,1]
    ·n3[3,3] + n3[1,3]·n3[2,1]·n3[3,2] + n3[1,2]·n3[2,3]·n3[3,1]-n3[1,3]
    ·n3[2,2]·n3[3,1]:
```

> $x_{del} := \dfrac{detn1}{detd}$:

> $y_{del} := \dfrac{detn2}{detd}$:

> $\psi := \dfrac{detn3}{detd}$:

> $x_1 := x_1 + x_{del}$:

> $y_1 := y_1 + y_{del}$:

> $\chi := \chi + \psi$:

> $e1 := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn1))$:

> $e2 := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn2))$:

> $e3 := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn3))$:

> *Iteration Number Six*

Iteration Number Six **(66)**

> 'xo, yo, α', x_1, y_1, χ;

xo, *yo*, α, 0.3286217246, -0.04385617879, 2.142563120 **(67)**

> 'eqn1, eqn2, eqn3', e1, e2, e3;

eqn1, *eqn2*, *eqn3*, $1.\ 10^{-10}$, $1.4\ 10^{-12}$, $2.\ 10^{-12}$ **(68)**

> $d[1,1] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn11))$:

> $d[1,2] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn12))$:

> $d[1,3] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn13))$:

> $d[2,1] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn14))$:

> $d[2,2] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn15))$:

> $d[2,3] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn16))$:

> $d[3,1] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn17))$:

> $d[3,2] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn18))$:

> $d[3,3] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn19))$:

> evalm(d) :

```
> detd := d[1,1]·d[2,2]·d[3,3]-d[1,1]·d[2,3]·d[3,2]-d[1,2]·d[2,1]·d[3,3]
     + d[1,3]·d[2,1]·d[3,2] + d[1,2]·d[2,3]·d[3,1]-d[1,3]·d[2,2]·d[3,1]:
> n1[1,1] := -e1:
> n1[1,2] := d[1,2]:
> n1[1,3] := d[1,3]:
> n1[2,1] := -e2:
> n1[2,2] := d[2,2]:
> n1[2,3] := d[2,3]:
> n1[3,1] := -e3:
> n1[3,2] := d[3,2]:
> n1[3,3] := d[3,3]:
> evalm(n1) :
> detn1 := n1[1,1]·n1[2,2]·n1[3,3]-n1[1,1]·n1[2,3]·n1[3,2]-n1[1,2]·n1[2,1]
     ·n1[3,3] + n1[1,3]·n1[2,1]·n1[3,2] + n1[1,2]·n1[2,3]·n1[3,1]-n1[1,3]
     ·n1[2,2]·n1[3,1]:
> n2[1,1] := d[1,1]:
> n2[1,2] := -e1:
> n2[1,3] := d[1,3]:
> n2[2,1] := d[2,1]:
> n2[2,2] := -e2:
> n2[2,3] := d[2,3]:
> n2[3,1] := d[3,1]:
> n2[3,2] := -e3:
> n2[3,3] := d[3,3]:
> evalm(n2) :
> detn2 := n2[1,1]·n2[2,2]·n2[3,3]-n2[1,1]·n2[2,3]·n2[3,2]-n2[1,2]·n2[2,1]
     ·n2[3,3] + n2[1,3]·n2[2,1]·n2[3,2] + n2[1,2]·n2[2,3]·n2[3,1]-n2[1,3]
     ·n2[2,2]·n2[3,1]:
> n3[1,1] := d[1,1]:
> n3[1,2] := d[1,2]:
> n3[1,3] := -e1:
> n3[2,1] := d[2,1]:
> n3[2,2] := d[2,2]:
> n3[2,3] := -e2:
> n3[3,1] := d[3,1]:
> n3[3,2] := d[3,2]:
> n3[3,3] := -e3:
> evalm(n3) :
> detn3 := n3[1,1]·n3[2,2]·n3[3,3]-n3[1,1]·n3[2,3]·n3[3,2]-n3[1,2]·n3[2,1]
     ·n3[3,3] + n3[1,3]·n3[2,1]·n3[3,2] + n3[1,2]·n3[2,3]·n3[3,1]-n3[1,3]
     ·n3[2,2]·n3[3,1]:
```

$> x_{del} := \dfrac{detn1}{detd}$:

$> y_{del} := \dfrac{detn2}{detd}$:

$> \psi := \dfrac{detn3}{detd}$:

$> x_1 := x_1 + x_{del}$:

$> y_1 := y_1 + y_{del}$:

$> \chi := \chi + \psi$:

$> e1 := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn1))$:

$> e2 := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn2))$:

$> e3 := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn3))$:

> *Iteration Number Seven*

Iteration Number Seven (69)

$>$ 'xo, yo, α', x_1, y_1, χ;

xo, *yo*, α, 0.3286217250, -0.04385617859, 2.142563119 (70)

$>$ 'eqn1, eqn2, eqn3', e1, e2, e3;

eqn1, *eqn2*, *eqn3*, 0., 0., $1.\ 10^{-12}$ (71)

$> d[1,1] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn11))$:

$> d[1,2] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn12))$:

$> d[1,3] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn13))$:

$> d[2,1] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn14))$:

$> d[2,2] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn15))$:

$> d[2,3] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn16))$:

$> d[3,1] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn17))$:

$> d[3,2] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn18))$:

$> d[3,3] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn19))$:

$> evalm(d)$:

$$> detd := d[1,1]\cdot d[2,2]\cdot d[3,3] - d[1,1]\cdot d[2,3]\cdot d[3,2] - d[1,2]\cdot d[2,1]\cdot d[3,3] + d[1,3]\cdot d[2,1]\cdot d[3,2] + d[1,2]\cdot d[2,3]\cdot d[3,1] - d[1,3]\cdot d[2,2]\cdot d[3,1]:$$

$> n1[1,1] := -e1$:

$> n1[1,2] := d[1,2]$:

$> n1[1,3] := d[1,3]$:

$> n1[2,1] := -e2$:

```
> n1[2,2] := d[2,2]:
> n1[2,3] := d[2,3]:
> n1[3,1] := -e3:
> n1[3,2] := d[3,2]:
> n1[3,3] := d[3,3]:
> evalm(n1) :
> detn1 := n1[1,1]·n1[2,2]·n1[3,3]-n1[1,1]·n1[2,3]·n1[3,2]-n1[1,2]·n1[2,1]
      ·n1[3,3] + n1[1,3]·n1[2,1]·n1[3,2] + n1[1,2]·n1[2,3]·n1[3,1]-n1[1,3]
      ·n1[2,2]·n1[3,1]:
> n2[1,1] := d[1,1]:
> n2[1,2] := -e1:
> n2[1,3] := d[1,3]:
> n2[2,1] := d[2,1]:
> n2[2,2] := -e2:
> n2[2,3] := d[2,3]:
> n2[3,1] := d[3,1]:
> n2[3,2] := -e3:
> n2[3,3] := d[3,3]:
> evalm(n2) :
> detn2 := n2[1,1]·n2[2,2]·n2[3,3]-n2[1,1]·n2[2,3]·n2[3,2]-n2[1,2]·n2[2,1]
      ·n2[3,3] + n2[1,3]·n2[2,1]·n2[3,2] + n2[1,2]·n2[2,3]·n2[3,1]-n2[1,3]
      ·n2[2,2]·n2[3,1]:
> n3[1,1] := d[1,1]:
> n3[1,2] := d[1,2]:
> n3[1,3] := -e1:
> n3[2,1] := d[2,1]:
> n3[2,2] := d[2,2]:
> n3[2,3] := -e2:
> n3[3,1] := d[3,1]:
> n3[3,2] := d[3,2]:
> n3[3,3] := -e3:
> evalm(n3) :
> detn3 := n3[1,1]·n3[2,2]·n3[3,3]-n3[1,1]·n3[2,3]·n3[3,2]-n3[1,2]·n3[2,1]
      ·n3[3,3] + n3[1,3]·n3[2,1]·n3[3,2] + n3[1,2]·n3[2,3]·n3[3,1]-n3[1,3]
      ·n3[2,2]·n3[3,1]:
```

$> x_{del} := \dfrac{detn1}{detd} :$

$> y_{del} := \dfrac{detn2}{detd} :$

```
> ψ := detn3/detd :
> x_1 := x_1 + x_del :
> y_1 := y_1 + y_del :
> χ := χ + ψ :
> e1 := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn1)) :
> e2 := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn2)) :
> e3 := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn3)) :
> Iteration Number Eight
```

Iteration Number Eight **(72)**

```
> 'xo, yo, α', x_1, y_1, χ;
```

xo, *yo*, α, 0.3286217248, -0.04385617868, 2.142563120 **(73)**

```
> 'eqn1, eqn2, eqn3', e1, e2, e3;
```

eqn1, *eqn2*, *eqn3*, $2.\ 10^{-10}$, $3.526341652\ 10^{-14}$, 0. **(74)**

```
> d[1,1] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn11)) :
> d[1,2] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn12)) :
> d[1,3] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn13)) :
> d[2,1] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn14)) :
> d[2,2] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn15)) :
> d[2,3] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn16)) :
> d[3,1] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn17)) :
> d[3,2] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn18)) :
> d[3,3] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn19)) :
> evalm(d) :
> detd := d[1,1]·d[2,2]·d[3,3]−d[1,1]·d[2,3]·d[3,2]−d[1,2]·d[2,1]·d[3,3]
      + d[1,3]·d[2,1]·d[3,2] + d[1,2]·d[2,3]·d[3,1]−d[1,3]·d[2,2]·d[3,1] :
> n1[1,1] := −e1 :
> n1[1,2] := d[1,2] :
> n1[1,3] := d[1,3] :
> n1[2,1] := −e2 :
> n1[2,2] := d[2,2] :
> n1[2,3] := d[2,3] :
> n1[3,1] := −e3 :
> n1[3,2] := d[3,2] :
```

```
> n1[3,3] := d[3,3]:
> evalm(n1) :
> detn1 := n1[1,1]·n1[2,2]·n1[3,3]-n1[1,1]·n1[2,3]·n1[3,2]-n1[1,2]·n1[2,1]
      ·n1[3,3] + n1[1,3]·n1[2,1]·n1[3,2] + n1[1,2]·n1[2,3]·n1[3,1]-n1[1,3]
      ·n1[2,2]·n1[3,1]:
> n2[1,1] := d[1,1]:
> n2[1,2] :=-e1:
> n2[1,3] := d[1,3]:
> n2[2,1] := d[2,1]:
> n2[2,2] := -e2:
> n2[2,3] := d[2,3]:
> n2[3,1] := d[3,1]:
> n2[3,2] := -e3:
> n2[3,3] := d[3,3]:
> evalm(n2) :
> detn2 := n2[1,1]·n2[2,2]·n2[3,3]-n2[1,1]·n2[2,3]·n2[3,2]-n2[1,2]·n2[2,1]
      ·n2[3,3] + n2[1,3]·n2[2,1]·n2[3,2] + n2[1,2]·n2[2,3]·n2[3,1]-n2[1,3]
      ·n2[2,2]·n2[3,1]:
> n3[1,1] := d[1,1]:
> n3[1,2] := d[1,2]:
> n3[1,3] := -e1:
> n3[2,1] := d[2,1]:
> n3[2,2] := d[2,2]:
> n3[2,3] := -e2:
> n3[3,1] := d[3,1]:
> n3[3,2] := d[3,2]:
> n3[3,3] := -e3:
> evalm(n3) :
> detn3 := n3[1,1]·n3[2,2]·n3[3,3]-n3[1,1]·n3[2,3]·n3[3,2]-n3[1,2]·n3[2,1]
      ·n3[3,3] + n3[1,3]·n3[2,1]·n3[3,2] + n3[1,2]·n3[2,3]·n3[3,1]-n3[1,3]
      ·n3[2,2]·n3[3,1]:
```

$> x_{del} := \dfrac{detn1}{detd}$:

$> y_{del} := \dfrac{detn2}{detd}$:

$> \psi := \dfrac{detn3}{detd}$:

$> x_1 := x_1 + x_{del}$:

$> y_1 := y_1 + y_{del}$:

```
> χ := χ + ψ :
> e1 := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn1)) :
> e2 := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn2)) :
> e3 := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn3)) :
> Iteration Number Nine
```

Iteration Number Nine (75)

```
> 'xo, yo, α', x_1, y_1, χ;
```

xo, yo, α, 0.3286217248, -0.04385617861, 2.142563120 (76)

```
> 'eqn1, eqn2, eqn3', e1, e2, e3;
```

eqn1, eqn2, eqn3, $2.\ 10^{-11}$, $1.\ 10^{-9}$, $1.\ 10^{-12}$ (77)

```
> d[1, 1] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn11)) :
> d[1, 2] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn12)) :
> d[1, 3] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn13)) :
> d[2, 1] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn14)) :
> d[2, 2] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn15)) :
> d[2, 3] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn16)) :
> d[3, 1] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn17)) :
> d[3, 2] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn18)) :
> d[3, 3] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn19)) :
> evalm(d) :
> detd := d[1, 1]·d[2, 2]·d[3, 3]-d[1, 1]·d[2, 3]·d[3, 2]-d[1, 2]·d[2, 1]·d[3, 3]
      + d[1, 3]·d[2, 1]·d[3, 2] + d[1, 2]·d[2, 3]·d[3, 1]-d[1, 3]·d[2, 2]·d[3, 1] :
> n1[1, 1] := -e1 :
> n1[1, 2] := d[1, 2] :
> n1[1, 3] := d[1, 3] :
> n1[2, 1] := -e2 :
> n1[2, 2] := d[2, 2] :
> n1[2, 3] := d[2, 3] :
> n1[3, 1] := -e3 :
> n1[3, 2] := d[3, 2] :
> n1[3, 3] := d[3, 3] :
> evalm(n1) :
> detn1 := n1[1, 1]·n1[2, 2]·n1[3, 3]-n1[1, 1]·n1[2, 3]·n1[3, 2]-n1[1, 2]·n1[2, 1]
      ·n1[3, 3] + n1[1, 3]·n1[2, 1]·n1[3, 2] + n1[1, 2]·n1[2, 3]·n1[3, 1]-n1[1, 3]
      ·n1[2, 2]·n1[3, 1] :
> n2[1, 1] := d[1, 1] :
```

```
> n2[1,2] := -e1 :
> n2[1,3] := d[1,3] :
> n2[2,1] := d[2,1] :
> n2[2,2] := -e2 :
> n2[2,3] := d[2,3] :
> n2[3,1] := d[3,1] :
> n2[3,2] := -e3 :
> n2[3,3] := d[3,3] :
> evalm(n2) :
> detn2 := n2[1,1]·n2[2,2]·n2[3,3]-n2[1,1]·n2[2,3]·n2[3,2]-n2[1,2]·n2[2,1]
    ·n2[3,3] + n2[1,3]·n2[2,1]·n2[3,2] + n2[1,2]·n2[2,3]·n2[3,1]-n2[1,3]
    ·n2[2,2]·n2[3,1] :
> n3[1,1] := d[1,1] :
> n3[1,2] := d[1,2] :
> n3[1,3] := -e1 :
> n3[2,1] := d[2,1] :
> n3[2,2] := d[2,2] :
> n3[2,3] := -e2 :
> n3[3,1] := d[3,1] :
> n3[3,2] := d[3,2] :
> n3[3,3] := -e3 :
> evalm(n3) :
> detn3 := n3[1,1]·n3[2,2]·n3[3,3]-n3[1,1]·n3[2,3]·n3[3,2]-n3[1,2]·n3[2,1]
    ·n3[3,3] + n3[1,3]·n3[2,1]·n3[3,2] + n3[1,2]·n3[2,3]·n3[3,1]-n3[1,3]
    ·n3[2,2]·n3[3,1] :
```

$> x_{del} := \frac{detn1}{detd} :$

$> y_{del} := \frac{detn2}{detd} :$

$> \psi := \frac{detn3}{detd} :$

$> x_1 := x_1 + x_{del} :$

$> y_1 := y_1 + y_{del} :$

$> \chi := \chi + \psi :$

$> e1 := evalf\left(subs\left(xo = x_1, yo = y_1, \alpha = \chi, eqn1\right)\right) :$

$> e2 := evalf\left(subs\left(xo = x_1, yo = y_1, \alpha = \chi, eqn2\right)\right) :$

$> e3 := evalf\left(subs\left(xo = x_1, yo = y_1, \alpha = \chi, eqn3\right)\right) :$

```
> Iteration Number Ten
Iteration Number Ten                                                        (78)
> 'xo, yo, α', x_1, y_1, χ;
xo, yo, α, 0.3286217249, -0.04385617876, 2.142563120                        (79)
> 'eqn1, eqn2, eqn3', e1, e2, e3;
eqn1, eqn2, eqn3, 2. 10^-10, 0., 1. 10^-12                                  (80)
> d[1,1] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn11)) :
> d[1,2] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn12)) :
> d[1,3] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn13)) :
> d[2,1] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn14)) :
> d[2,2] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn15)) :
> d[2,3] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn16)) :
> d[3,1] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn17)) :
> d[3,2] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn18)) :
> d[3,3] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn19)) :
> evalm(d) :
> detd := d[1,1]·d[2,2]·d[3,3]-d[1,1]·d[2,3]·d[3,2]-d[1,2]·d[2,1]·d[3,3]
      + d[1,3]·d[2,1]·d[3,2] + d[1,2]·d[2,3]·d[3,1]-d[1,3]·d[2,2]·d[3,1]:
> n1[1,1] := -e1:
> n1[1,2] := d[1,2]:
> n1[1,3] := d[1,3]:
> n1[2,1] := -e2:
> n1[2,2] := d[2,2]:
> n1[2,3] := d[2,3]:
> n1[3,1] := -e3:
> n1[3,2] := d[3,2]:
> n1[3,3] := d[3,3]:
> evalm(n1) :
> detn1 := n1[1,1]·n1[2,2]·n1[3,3]-n1[1,1]·n1[2,3]·n1[3,2]-n1[1,2]·n1[2,1]
      ·n1[3,3] + n1[1,3]·n1[2,1]·n1[3,2] + n1[1,2]·n1[2,3]·n1[3,1]-n1[1,3]
      ·n1[2,2]·n1[3,1]:
> n2[1,1] := d[1,1]:
> n2[1,2] := -e1:
> n2[1,3] := d[1,3]:
> n2[2,1] := d[2,1]:
> n2[2,2] := -e2:
> n2[2,3] := d[2,3]:
```

```
> n2[3,1] := d[3,1]:
> n2[3,2] := -e3:
> n2[3,3] := d[3,3]:
> evalm(n2) :
> detn2 := n2[1,1]·n2[2,2]·n2[3,3]-n2[1,1]·n2[2,3]·n2[3,2]-n2[1,2]·n2[2,1]
      ·n2[3,3] + n2[1,3]·n2[2,1]·n2[3,2] + n2[1,2]·n2[2,3]·n2[3,1]-n2[1,3]
      ·n2[2,2]·n2[3,1]:
> n3[1,1] := d[1,1]:
> n3[1,2] := d[1,2]:
> n3[1,3] := -e1:
> n3[2,1] := d[2,1]:
> n3[2,2] := d[2,2]:
> n3[2,3] := -e2:
> n3[3,1] := d[3,1]:
> n3[3,2] := d[3,2]:
> n3[3,3] := -e3:
> evalm(n3) :
> detn3 := n3[1,1]·n3[2,2]·n3[3,3]-n3[1,1]·n3[2,3]·n3[3,2]-n3[1,2]·n3[2,1]
      ·n3[3,3] + n3[1,3]·n3[2,1]·n3[3,2] + n3[1,2]·n3[2,3]·n3[3,1]-n3[1,3]
      ·n3[2,2]·n3[3,1]:
```

$> x_{del} := \dfrac{detn1}{detd}$:

$> y_{del} := \dfrac{detn2}{detd}$:

$> \psi := \dfrac{detn3}{detd}$:

$> x_1 := x_1 + x_{del}$:

$> y_1 := y_1 + y_{del}$:

$> \chi := \chi + \psi$:

$> e1 := evalf\left(subs\left(xo = x_1, yo = y_1, \alpha = \chi, eqn1\right)\right)$:

$> e2 := evalf\left(subs\left(xo = x_1, yo = y_1, \alpha = \chi, eqn2\right)\right)$:

$> e3 := evalf\left(subs\left(xo = x_1, yo = y_1, \alpha = \chi, eqn3\right)\right)$:

> Iteration Number Eleven

Iteration Number Eleven (81)

$>$ 'xo, yo, α', x_1, y_1, χ;

$xo, yo, \alpha, 0.3286217247, -0.04385617878, 2.142563121$ (82)

> 'eqn1, eqn2, eqn3', e1, e2, e3;

$eqn1, eqn2, eqn3, 5.\ 10^{-10}, 2.\ 10^{-9}, 0.$ (83)

```
> d[1,1] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn11)) :
> d[1,2] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn12)) :
> d[1,3] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn13)) :
> d[2,1] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn14)) :
> d[2,2] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn15)) :
> d[2,3] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn16)) :
> d[3,1] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn17)) :
> d[3,2] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn18)) :
> d[3,3] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn19)) :
> evalm(d) :
> detd := d[1,1]·d[2,2]·d[3,3]-d[1,1]·d[2,3]·d[3,2]-d[1,2]·d[2,1]·d[3,3]
     + d[1,3]·d[2,1]·d[3,2] + d[1,2]·d[2,3]·d[3,1]-d[1,3]·d[2,2]·d[3,1] :
> n1[1,1] := -e1 :
> n1[1,2] := d[1,2] :
> n1[1,3] := d[1,3] :
> n1[2,1] := -e2 :
> n1[2,2] := d[2,2] :
> n1[2,3] := d[2,3] :
> n1[3,1] := -e3 :
> n1[3,2] := d[3,2] :
> n1[3,3] := d[3,3] :
> evalm(n1) :
> detn1 := n1[1,1]·n1[2,2]·n1[3,3]-n1[1,1]·n1[2,3]·n1[3,2]-n1[1,2]·n1[2,1]
     ·n1[3,3] + n1[1,3]·n1[2,1]·n1[3,2] + n1[1,2]·n1[2,3]·n1[3,1]-n1[1,3]
     ·n1[2,2]·n1[3,1] :
> n2[1,1] := d[1,1] :
> n2[1,2] := -e1 :
> n2[1,3] := d[1,3] :
> n2[2,1] := d[2,1] :
> n2[2,2] := -e2 :
> n2[2,3] := d[2,3] :
> n2[3,1] := d[3,1] :
> n2[3,2] := -e3 :
> n2[3,3] := d[3,3] :
> evalm(n2) :
> detn2 := n2[1,1]·n2[2,2]·n2[3,3]-n2[1,1]·n2[2,3]·n2[3,2]-n2[1,2]·n2[2,1]
```

```
    ·n2[3,3] + n2[1,3]·n2[2,1]·n2[3,2] + n2[1,2]·n2[2,3]·n2[3,1]-n2[1,3]
    ·n2[2,2]·n2[3,1]:
> n3[1,1] := d[1,1]:
> n3[1,2] := d[1,2]:
> n3[1,3] := -e1:
> n3[2,1] := d[2,1]:
> n3[2,2] := d[2,2]:
> n3[2,3] := -e2:
> n3[3,1] := d[3,1]:
> n3[3,2] := d[3,2]:
> n3[3,3] := -e3:
> evalm(n3) :
> detn3 := n3[1,1]·n3[2,2]·n3[3,3]-n3[1,1]·n3[2,3]·n3[3,2]-n3[1,2]·n3[2,1]
    ·n3[3,3] + n3[1,3]·n3[2,1]·n3[3,2] + n3[1,2]·n3[2,3]·n3[3,1]-n3[1,3]
    ·n3[2,2]·n3[3,1]:
```

$> x_{del} := \dfrac{detn1}{detd}$:

$> y_{del} := \dfrac{detn2}{detd}$:

$> \psi := \dfrac{detn3}{detd}$:

$> x_1 := x_1 + x_{del}$:

$> y_1 := y_1 + y_{del}$:

$> \chi := \chi + \psi$:

$> e1 := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn1))$:

$> e2 := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn2))$:

$> e3 := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn3))$:

> Iteration Number Twelve

Iteration Number Twelve **(84)**

$>$ 'xo, yo, α', x_1, y_1, χ;

xo, *yo*, α, 0.3286217248, -0.04385617849, 2.142563118 **(85)**

> 'eqn1, eqn2, eqn3', e1, e2, e3;

eqn1, *eqn2*, *eqn3*, $5.2\ 10^{-10}$, $1.8\ 10^{-9}$, 0. **(86)**

$> d[1,1] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn11))$:

$> d[1,2] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn12))$:

$> d[1,3] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn13))$:

```
> d[2,1] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn14)) :
> d[2,2] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn15)) :
> d[2,3] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn16)) :
> d[3,1] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn17)) :
> d[3,2] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn18)) :
> d[3,3] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn19)) :
> evalm(d) :
> detd := d[1,1]·d[2,2]·d[3,3]-d[1,1]·d[2,3]·d[3,2]-d[1,2]·d[2,1]·d[3,3]
      + d[1,3]·d[2,1]·d[3,2] + d[1,2]·d[2,3]·d[3,1]-d[1,3]·d[2,2]·d[3,1]:
> n1[1,1] :=-e1 :
> n1[1,2] := d[1,2]:
> n1[1,3] := d[1,3]:
> n1[2,1] :=-e2 :
> n1[2,2] := d[2,2]:
> n1[2,3] := d[2,3]:
> n1[3,1] :=-e3 :
> n1[3,2] := d[3,2]:
> n1[3,3] := d[3,3]:
> evalm(n1) :
> detn1 := n1[1,1]·n1[2,2]·n1[3,3]-n1[1,1]·n1[2,3]·n1[3,2]-n1[1,2]·n1[2,1]
      ·n1[3,3] + n1[1,3]·n1[2,1]·n1[3,2] + n1[1,2]·n1[2,3]·n1[3,1]-n1[1,3]
      ·n1[2,2]·n1[3,1]:
> n2[1,1] := d[1,1]:
> n2[1,2] :=-e1 :
> n2[1,3] := d[1,3]:
> n2[2,1] := d[2,1]:
> n2[2,2] := -e2 :
> n2[2,3] := d[2,3]:
> n2[3,1] := d[3,1]:
> n2[3,2] := -e3 :
> n2[3,3] := d[3,3]:
> evalm(n2) :
> detn2 := n2[1,1]·n2[2,2]·n2[3,3]-n2[1,1]·n2[2,3]·n2[3,2]-n2[1,2]·n2[2,1]
      ·n2[3,3] + n2[1,3]·n2[2,1]·n2[3,2] + n2[1,2]·n2[2,3]·n2[3,1]-n2[1,3]
      ·n2[2,2]·n2[3,1]:
> n3[1,1] := d[1,1]:
> n3[1,2] := d[1,2]:
> n3[1,3] := -e1 :
```

```
> n3[2,1] := d[2,1] :
> n3[2,2] := d[2,2] :
> n3[2,3] := -e2 :
> n3[3,1] := d[3,1] :
> n3[3,2] := d[3,2] :
> n3[3,3] := -e3 :
> evalm(n3) :
> detn3 := n3[1,1]·n3[2,2]·n3[3,3]-n3[1,1]·n3[2,3]·n3[3,2]-n3[1,2]·n3[2,1]
    ·n3[3,3] + n3[1,3]·n3[2,1]·n3[3,2] + n3[1,2]·n3[2,3]·n3[3,1]-n3[1,3]
    ·n3[2,2]·n3[3,1] :
```

$> x_{del} := \frac{detn1}{detd}$:

$> y_{del} := \frac{detn2}{detd}$:

$> \psi := \frac{detn3}{detd}$:

$> x_1 := x_1 + x_{del}$:

$> y_1 := y_1 + y_{del}$:

$> \chi := \chi + \psi$:

$> e1 := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn1))$:

$> e2 := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn2))$:

$> e3 := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn3))$:

> Iteration Number Thirteen

Iteration Number Thirteen **(87)**

$> 'xo, yo, \alpha', x_1, y_1, \chi;$

$xo, yo, \alpha, 0.3286217247, -0.04385617873, 2.142563120$ **(88)**

> 'eqn1, eqn2, eqn3', e1, e2, e3;

$eqn1, eqn2, eqn3, 0., 3.\ 10^{-10}, 1.\ 10^{-12}$ **(89)**

$> d[1,1] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn11))$:

$> d[1,2] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn12))$:

$> d[1,3] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn13))$:

$> d[2,1] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn14))$:

$> d[2,2] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn15))$:

$> d[2,3] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn16))$:

```
> d[3,1] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn17)) :
> d[3,2] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn18)) :
> d[3,3] := evalf(subs(xo = x_1, yo = y_1, α = χ, eqn19)) :
> evalm(d) :
> detd := d[1,1]·d[2,2]·d[3,3]-d[1,1]·d[2,3]·d[3,2]-d[1,2]·d[2,1]·d[3,3]
      + d[1,3]·d[2,1]·d[3,2] + d[1,2]·d[2,3]·d[3,1]-d[1,3]·d[2,2]·d[3,1]:
> n1[1,1] :=-e1:
> n1[1,2] := d[1,2]:
> n1[1,3] := d[1,3]:
> n1[2,1] :=-e2:
> n1[2,2] := d[2,2]:
> n1[2,3] := d[2,3]:
> n1[3,1] :=-e3:
> n1[3,2] := d[3,2]:
> n1[3,3] := d[3,3]:
> evalm(n1) :
> detn1 := n1[1,1]·n1[2,2]·n1[3,3]-n1[1,1]·n1[2,3]·n1[3,2]-n1[1,2]·n1[2,1]
      ·n1[3,3] + n1[1,3]·n1[2,1]·n1[3,2] + n1[1,2]·n1[2,3]·n1[3,1]-n1[1,3]
      ·n1[2,2]·n1[3,1]:
> n2[1,1] := d[1,1]:
> n2[1,2] :=-e1:
> n2[1,3] := d[1,3]:
> n2[2,1] := d[2,1]:
> n2[2,2] := -e2:
> n2[2,3] := d[2,3]:
> n2[3,1] := d[3,1]:
> n2[3,2] := -e3:
> n2[3,3] := d[3,3]:
> evalm(n2) :
> detn2 := n2[1,1]·n2[2,2]·n2[3,3]-n2[1,1]·n2[2,3]·n2[3,2]-n2[1,2]·n2[2,1]
      ·n2[3,3] + n2[1,3]·n2[2,1]·n2[3,2] + n2[1,2]·n2[2,3]·n2[3,1]-n2[1,3]
      ·n2[2,2]·n2[3,1]:
> n3[1,1] := d[1,1]:
> n3[1,2] := d[1,2]:
> n3[1,3] := -e1:
> n3[2,1] := d[2,1]:
> n3[2,2] := d[2,2]:
> n3[2,3] := -e2:
> n3[3,1] := d[3,1]:
```

> $n3[3,2] := d[3,2]$:

> $n3[3,3] := -e3$:

> $evalm(n3)$:

> $detn3 := n3[1,1] \cdot n3[2,2] \cdot n3[3,3] - n3[1,1] \cdot n3[2,3] \cdot n3[3,2] - n3[1,2] \cdot n3[2,1] \cdot n3[3,3] + n3[1,3] \cdot n3[2,1] \cdot n3[3,2] + n3[1,2] \cdot n3[2,3] \cdot n3[3,1] - n3[1,3] \cdot n3[2,2] \cdot n3[3,1]$:

> $x_{del} := \frac{detn1}{detd}$:

> $y_{del} := \frac{detn2}{detd}$:

> $\psi := \frac{detn3}{detd}$:

> $x_1 := x_1 + x_{del}$:

> $y_1 := y_1 + y_{del}$:

> $\chi := \chi + \psi$:

> $e1 := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn1))$:

> $e2 := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn2))$:

> $e3 := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn3))$:

> Iteration Number Fourteen

Iteration Number Fourteen **(90)**

> 'xo, yo, α', x_1, y_1, χ;

$xo, yo, \alpha, 0.3286217249, -0.04385617871, 2.142563120$ **(91)**

> 'eqn1, eqn2, eqn3', e1, e2, e3;

$eqn1, eqn2, eqn3, 1.\ 10^{-10}, 0., 0.$ **(92)**

> $d[1,1] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn11))$:

> $d[1,2] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn12))$:

> $d[1,3] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn13))$:

> $d[2,1] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn14))$:

> $d[2,2] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn15))$:

> $d[2,3] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn16))$:

> $d[3,1] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn17))$:

> $d[3,2] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn18))$:

> $d[3,3] := evalf(subs(xo = x_1, yo = y_1, \alpha = \chi, eqn19))$:

> $evalm(d)$:

```
> detd := d[1,1]·d[2,2]·d[3,3]-d[1,1]·d[2,3]·d[3,2]-d[1,2]·d[2,1]·d[3,3]
      + d[1,3]·d[2,1]·d[3,2] + d[1,2]·d[2,3]·d[3,1]-d[1,3]·d[2,2]·d[3,1]:
> n1[1,1] :=-e1:
> n1[1,2] := d[1,2]:
> n1[1,3] := d[1,3]:
> n1[2,1] :=-e2:
> n1[2,2] := d[2,2]:
> n1[2,3] := d[2,3]:
> n1[3,1] :=-e3:
> n1[3,2] := d[3,2]:
> n1[3,3] := d[3,3]:
> evalm(n1) :
> detn1 := n1[1,1]·n1[2,2]·n1[3,3]-n1[1,1]·n1[2,3]·n1[3,2]-n1[1,2]·n1[2,1]
      ·n1[3,3] + n1[1,3]·n1[2,1]·n1[3,2] + n1[1,2]·n1[2,3]·n1[3,1]-n1[1,3]
      ·n1[2,2]·n1[3,1]:
> n2[1,1] := d[1,1]:
> n2[1,2] :=-e1:
> n2[1,3] := d[1,3]:
> n2[2,1] := d[2,1]:
> n2[2,2] := -e2:
> n2[2,3] := d[2,3]:
> n2[3,1] := d[3,1]:
> n2[3,2] := -e3:
> n2[3,3] := d[3,3]:
> evalm(n2) :
> detn2 := n2[1,1]·n2[2,2]·n2[3,3]-n2[1,1]·n2[2,3]·n2[3,2]-n2[1,2]·n2[2,1]
      ·n2[3,3] + n2[1,3]·n2[2,1]·n2[3,2] + n2[1,2]·n2[2,3]·n2[3,1]-n2[1,3]
      ·n2[2,2]·n2[3,1]:
> n3[1,1] := d[1,1]:
> n3[1,2] := d[1,2]:
> n3[1,3] := -e1:
> n3[2,1] := d[2,1]:
> n3[2,2] := d[2,2]:
> n3[2,3] := -e2:
> n3[3,1] := d[3,1]:
> n3[3,2] := d[3,2]:
> n3[3,3] := -e3:
> evalm(n3) :
> detn3 := n3[1,1]·n3[2,2]·n3[3,3]-n3[1,1]·n3[2,3]·n3[3,2]-n3[1,2]·n3[2,1]
      ·n3[3,3] + n3[1,3]·n3[2,1]·n3[3,2] + n3[1,2]·n3[2,3]·n3[3,1]-n3[1,3]
      ·n3[2,2]·n3[3,1]:
```

$> x_{del} := \frac{detn1}{detd}$:

$> y_{del} := \frac{detn2}{detd}$:

$> \psi := \frac{detn3}{detd}$:

$> x_1 := x_1 + x_{del}$:

$> y_1 := y_1 + y_{del}$:

$> \chi := \chi + \psi$:

$> e1 := evalf\left(subs\left(xo = x_1, yo = y_1, \alpha = \chi, eqn1\right)\right)$:

$> e2 := evalf\left(subs\left(xo = x_1, yo = y_1, \alpha = \chi, eqn2\right)\right)$:

$> e3 := evalf\left(subs\left(xo = x_1, yo = y_1, \alpha = \chi, eqn3\right)\right)$:

> *Iteration Number Fifteen*

Iteration Number Fifteen **(93)**

$>$ 'xo, yo, α', x_1, y_1, χ;

xo, yo, α, 0.3286217249, -0.04385617874, 2.142563120 **(94)**

> 'eqn1, eqn2, eqn3', e1, e2, e3;

$eqn1, eqn2, eqn3, 1.\ 10^{-10}, 1.\ 10^{-9}, 1.\ 10^{-12}$ **(95)**

The stiffness and damping coefficients are presented here.

$> Kxx := evalf\left(subs\left(x = x_1, y = y_1, \alpha = \chi, x1 = 0, y1 = 0, x11 = 0, y11 = 0, -diff(fx, x)\right)\right);$

$Kxx := 0.6609734097$ **(96)**

$> Kxy := evalf\left(subs\left(x = x_1, y = y_1, \alpha = \chi, x1 = 0, y1 = 0, x11 = 0, y11 = 0, -diff(fx, y)\right)\right);$

$Kxy := 1.705501525$ **(97)**

$> Kyx := evalf\left(subs\left(x = x_1, y = y_1, \alpha = \chi, x1 = 0, y1 = 0, x11 = 0, y11 = 0, -diff(fy, x)\right)\right);$

$Kyx := -3.030430594$ **(98)**

$> Kyy := evalf\left(subs\left(x = x_1, y = y_1, \alpha = \chi, x1 = 0, y1 = 0, x11 = 0, y11 = 0, -diff(fy, y)\right)\right);$

$Kyy := 2.359400578$ **(99)**

$> Bxx := evalf\left(subs\left(x = x_1, y = y_1, \alpha = \chi, x1 = 0, y1 = 0, x11 = 0, y11 = 0, -diff(fx, x1)\right)\right);$

$Bxx := 3.326644682$ **(100)**

$> Bxy := evalf\left(subs\left(x = x_1, y = y_1, \alpha = \chi, x1 = 0, y1 = 0, x11 = 0, y11 = 0, diff(fx, y1)\right)\right);$

$Bxy := 0.4499991455$ **(101)**

$> Byx := evalf\left(subs\left(x = x_1, y = y_1, \alpha = \chi, x1 = 0, y1 = 0, x11 = 0, y11 = 0, diff(fy, x1)\right)\right);$

$Byx := 0.4354072719$ **(102)**

$> Byy := evalf\left(subs\left(x = x_1, y = y_1, \alpha = \chi, x1 = 0, y1 = 0, x11 = 0, y11 = 0, -diff(fy, y1)\right)\right);$

$Byy := 6.147121683$ **(103)**

>