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Karthik Mellechervu
Cleveland State University

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MODELING OF TRANSIENT THRUST OF A TWO PHASE PROPELLANT TANK

KARTHIK MELLECHERVU

Bachelor of Science in Mechanical Engineering
Osmania University, Hyderabad, AP, India
May, 2005

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This thesis has been approved
for the department of MECHANICAL ENGINEERING
and the College of Graduate Studies by:

Thesis Chairperson, Hanz Richter, Ph.D.

Department & Date

Asquo Ebiana, Ph.D.

Department & Date

Rama S Gorla, Ph.D.

Department & Date

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MODELING OF TRANSIENT THRUST OF A TWO PHASE PROPELLANT TANK

KARTHIK MELLECHERVU

ABSTRACT

This thesis develops a model to predict various time-varying quantities associated with the evacuation of a pressurized container through a nozzle. The container is part of a satellite propulsion prototype used to study attitude control. Variables of interest include the thrust and mass of fluid in the tank as functions of time. A mathematical model based on transient expressions for mass and energy conservation is derived for a two-phase system representative of the conditions of the fluid inside the tank. The resulting system of differential equations is implemented in Matlab/Simulink, allowing a user to generate thrust and mass histories for various initial conditions.

TABLE OF CONTENTS

LIST OF FIGURES

CHAPTER I

INTRODUCTION

Pressurization and evacuation of propellant tanks are complex processes. This thesis deals with the expulsion of gas from a propellant tank to produce thrust. The present chapter describes the motivation behind this thesis and the overview of the work done in the thesis along with the structure of the report.

1.1 Motivation

The motivation behind this thesis comes from the need to calculate the transient thrust profile as a result of the expulsion of propellant. Some of the examples of these propellant tanks are

- activation of airborne components
- pressurizing personnel and equipment compartments
- pressurizing structures for ground handling
- providing thrust for stage separation
- cylinders used in fire extinguishers
- paint ball guns

- propellant tanks which are used in launching rockets

In future, NASA missions involve pressurization and expulsion of high pressure cryogenic tanks in the space orbits or on low gravity environments. When the nozzle is released, gas will be emitted out at high pressure with certain thrust. The present thesis involves with the calculation of change in thrust along with time.

1.2 Pressurization Systems

Pressurization systems are generally used to control the gas pressure in the the pressurized tanks. The space where the gas is located is called as ullage. It is generally designed to maintain this ullage at a preselected pressure history bounded by the propellant and the tank structural requirements. In pressurization systems the thrust coming out of the nozzle from the tank is directly proportional to the pressure in the control volume of the tank. These systems are a source of high pressurized gas and hence can be used in wide variety of applications. Some of the major applications include

- usage in missile systems
- activation of airborne components
- providing thrust for vehicle vernier attitude correction
- pressurizing personnel and equipment compartments
- pressurizing structures for ground handling
- providing thrust for stage separation

1.3 Types of Pressurization Systems

There are different types of pressurization systems and a designer can choose any one of them depending on his requirement. Any method of producing and controlling gas flow is a potential pressurization system. The most important choice in pressurization system design is the selection of the type of system to be used. The different types of pressurization systems and their advantages and disadvantages can be given as[1]

1.3.1 Stored Gas Systems

This is the most common type of pressurization system that is used currently and the gas that is generally used in storing is helium at cryogenic temperatures. It has a high pressure gas stored in a container. The other two containers act as a fuel storage container and as oxidizer. When the fuel is released from the fuel cylinder, the pressure in the ullage is decreased which is detected by the pressure sensors. This decrease in pressure causes the valves at the fuel container and oxidizer open, thus releasing the pressure through the gas line to the ullage. Vent and relief valves are used in the propellant tanks, so that they can be closed from venting while the gas is getting filled up or else they can be used as safety valves in the event of over pressurization. One simple example of stored gas systems in shown in the following diagram.

1.3.2 The Autogenous System

This type of pressurization system use gas generator products to pressurize the fuel tank to produce the required pressurization or combination of both. This is a combination of gas generator system and the evaporated propellant system. Those are explained in the following sections.

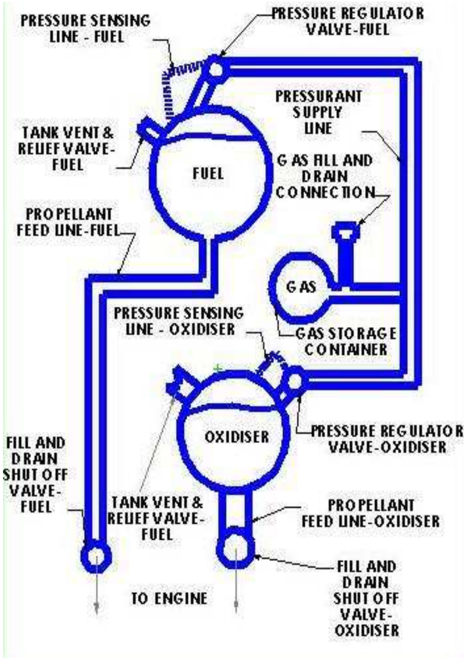

Figure 1.1: Stored Gas Pressurized Systems

1.3.3 The Gas Generator System

In this the gas generator of a pump fed engine is used in obtaining a pressurized gas. The products of gas generator are of the order of 1130 k. They are rich in fuel and they have low molecular weight which is less than 15. Therefore, the products obtained should undergo refining process which include lowering the temperature and conversion of rich fuel mixture to an oxidizer rich gas with an introduction of gas flow prior to use in the oxidizer tank. A typical gas generator system is shown in the following schematic.

1.3.4 Evaporated Propellant System

Some of the propellants can be directly heated and those vapors are directly sent to the propellant tank for pressurization. Many fuels tend to decompose violently under heating, it is chiefly the oxidizers, oxygen and nitrogen tetroxide are considered and

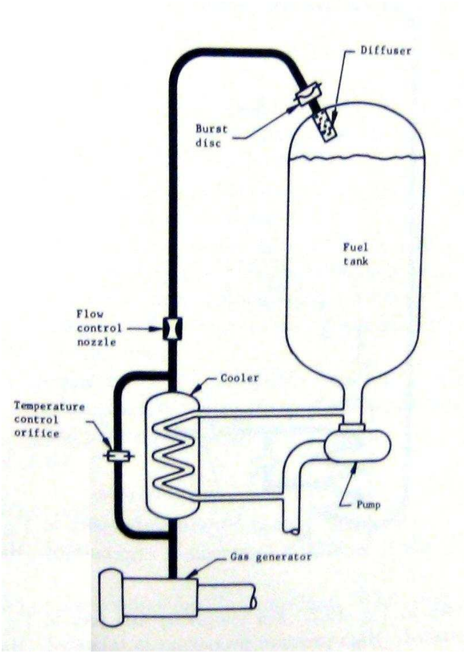

Figure 1.2: Gas Generator Pressurization Systems

hydrogen can also be used. One main advantage of this type of system is the molecular weight of the vapor is decreased which decreases the weight after dissociation.

1.3.5 Stored Liquid System

In this the pressurizing gas that is used for the pressurization can be placed as a liquid in the tank itself to save the weight of the system. The pressurized liquid is stored by submerging the propellant. When there is an out flow from the propeller there will be a change of pressure in the ullage, causing a decay in the pressurizing liquid and the heat of vaporization is absorbed by the propellant causing it get back to its original pressure. The main advantage of this system is it can reduce lot of weight.

1.3.6 Main Tank Injection System

This describes a pressurization process in which a chemical is injected into the propellant tank which causes a low temperature reaction to generate the pressurizing gas. Since the system can be injected with either a solid, liquid or a gas reagent , a variety of system configurations are possible. One of the simplest configuration would be injection of the main propellant rather than a separate reagent.This type is shown in the following figure for a low pressure pump fed system.

1.3.7 Polytropic Expansion Systems

This system is nothing but a pre pressurization of the propellant tank ullage to a sufficient pressure level to assure satisfactory propellant outflow throughout the engine operation. No pressurant is added or removed during the outflow, except for propellant vaporization or pressurant condensation which may occur. Propellants with high vapor pressures, such as cryogenics, will boil vigorously during outflow from a closed container. The pressure in the space above these propellants will adjust to the vapor pressure corresponding to the temperature of the adjacent liquid. This phenomenon can be useful for pressurizing the propellant tanks.

1.4 Desirable Properties of Pressurants

It should be recognized that an optimum pressurization system requires high density pressurant storage and a low molecular weight gas. Some of the most desirable properties of the pressurants can be given as

- The weight of the gas should be directly proportional to the ratio of molecular weight to the compressibility factor$\left(\frac{M}{Z}\right)$
- Low specific heat ratio
- Low solubility
- Low or controlled reactivity

- High density storage
- Low boiling point

1.5 Methods of Tank Pressure Control

In order to obtain a desired pressure profile, the designer has a choice of several alternatives. Following are some of them

- Variable mass flow (pressure regulator)
- Decreasing mass flow(orifice blow-down system)
- Constant mass flow (regulator and fixed orifice)
- No mass flow (polytropic expansion)

1.6 Ullage and its Importance

In a liquid gas pressurization system the area where the gas is located is called as Ullage. Ullage plays a very important role in any vapor-liquid fluid mixture. One example is, A Trans World Airlines Boeing 747 was tragically exploded on July 17, 1996 during the mid flight and fell into the Atlantic Ocean near New York. The reason for this tragedy was later identified as an explosion of the center wing tank by the US National Transportation Safety Board (NTSB)[2]. This resulted to a combined sponsored research of US Federal Aviation Administration and NTSB on various characteristics on vapor-liquid mixture in a center wing tank for various in flight conditions. Previous studies on the air fuel tank explosions can be obtained from ref[3]

Ullage is a necessary and most important thing in a pressurization system. The reasons for this are

- it provides a space for the expansion and compression of the pressurizing gas during the response period of the pressurization system.
- it provides required NPSH (net positive suction head) to the propellant tanks and maintains structural integrity of the propellant tanks during an engine start transient.
- it provides space for the propellant expansion during the increase of ambient temperatures.
- it provides space for the accidental overloading of the tanks.
- it provides space for the ejection of dissolved gases or products reaction with in the propellants.

Ullage size in a regulated pressure system does not greatly affect the pressure regulation, but it does substantially affect the response of all the pressure - control devices. Changes in the size of the ullage can be used in changing the affects of pressure rise rates and varying regulator flow rates.

Ullage composition plays a very important role in the design of pressure regulating system. It has an effect on the response characteristics of the pressure - regulating components and therefore actual mixtures should be used in tests which is useful in determining the final configuration. The effects of ullage on autogenous system is negligible.

Ullage temperature has a direct effect on the overpressure characteristics of a pressurization system. Pressurization of an ullage with fairly high flow rates during pressurization will result in pressure rise rates that are much higher for a warm ullage than for a cold ullage. The difference in rise rates is a result of heat balance between the warm pressurizing gas, cold propellant mass and ullage gas above the propellant.

1.7 Scope of Thesis

The thesis deals with the understanding and study of a pressurized cylinder with single phase and two phase gas media. Calculation of transient thrust is done for both the cases with single and two phase gas media. Differential equations are obtained for the change in density and temperature for both cases. The differential equations are modeled in Simulink with an input of density and temperature as initial value conditions for both cases. The models are then simulated with various time periods. After the simulations, graphs are plotted for various transient properties which include mass flow rate, pressure, mach number, exit velocity, temperature, density and internal energy. Finally the transient thrust is plotted in a graph which shows the change in the profile of the thrust with the change in time.

1.8 Structure of the report

It has 5 chapters. The second chapter deals with the application of first law of thermodynamics to the control volume. The third chapter deals with the single phase gas mediums and the required equations are derived for modeling in Simulink to perform required simulations. Fourth chapter deals with the two- phase gas medium and equations are derived similar to the single phase and a model is created in Simulink and required simulations are made. The fifth chapter offers conclusions, results and recommendations.

CHAPTER II

FIRST LAW APPLIED TO CONTROL VOLUME

2.1 First Law of Thermodynamics

According to the First law of thermodynamics ”In any process, the total energy of the universe remains constant”. In other words it can be stated that Energy can neither be created nor destroyed rather, the amount of energy lost in a steady state or an unsteady state process cannot be greater than the amount of energy gained.

This can also be called as conservation of energy for a thermodynamic system. The increase in the internal energy of a system is equal to the amount of energy added by heating the system, minus the amount lost as a result of the work done by the system on its surroundings.

$$dU = \delta Q - \delta W \tag{2.1}$$

For a reversible process the total amount of heat that can be added to the system can be expressed as $Q = Tds$, where ’s’ is the entropy of the system. Entropy can be defined as the measure of the system’s unavailability to do work.

2.2 First Law Applied to Control Volume

In this section, First Law of thermodynamics is applied to a control volume. Let us assume that a thermodynamics process is undergoing from state 1 to state 2 for a time interval t. Now applying first law to this thermodynamic process, following equation is obtained.

$$\frac{\delta Q}{\delta t} = \frac{E_2 - E_1}{dt} + \frac{\delta W}{\delta t} \tag{2.2}$$

Let us consider a control volume with a mass of m_t and initial energy E_t at time t. The control volume as shown has an inlet and an outlet with areas A_i and A_e respectively. Now, the system consists of the mass in the control volume and the initial mass m_i. When a mass enters into the control volume, there will be a change in the system during this time interval δt. During this δt a mass of m_i enter into the control volume at pressure, temperature, volume, specific energy of P_i, T_i, V_i, e_i respectively and a mass of m_o leaves the control volume at pressure, temperature, volume, specific energy of P_o, T_o, V_o, e_o respectively. This process is schematically shown in figure 2.1.

During this process it has been assumed as the increase and decrease in mass has the uniform properties. As there is a flow of mass across the boundary of the control volume, a certain amount of work is done to induce this process. This work is represented as work done in the control volume as W_{cv}. Let us consider that, this work includes all the work done by the rotating shaft, shear forces, expansion, contraction of the control volume and all the other electric, magnetic or surface effects. During this process, an amount of heat is also crossed across the control volume. Let us represent that δQ amount of heat flows from the control volume to outside in δt time. This can be represented in the following diagram

Rewriting (2.3)

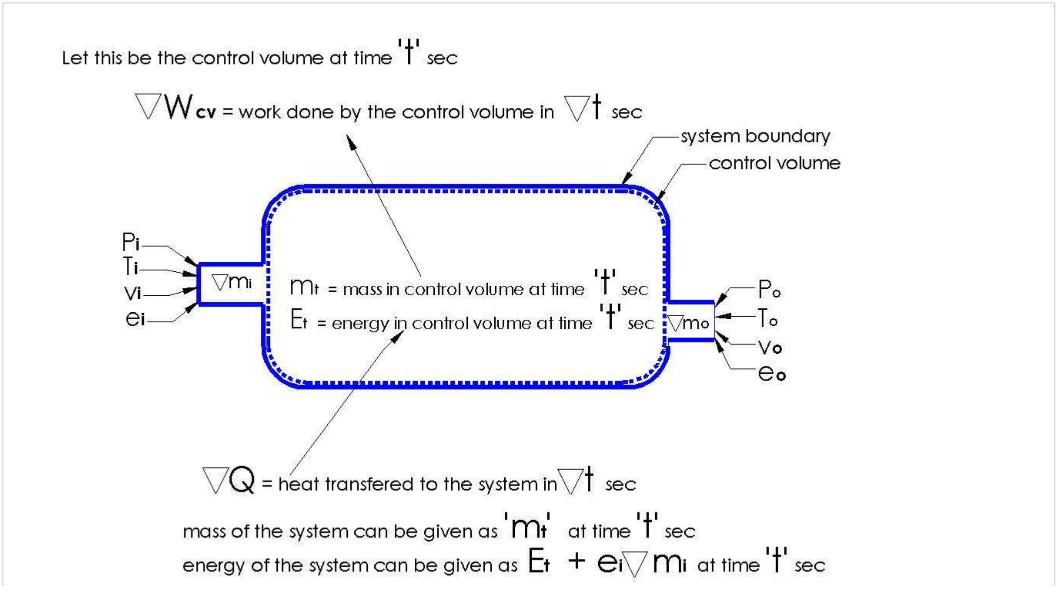

Figure 2.1: Flow of fluid through control volume at time 't'

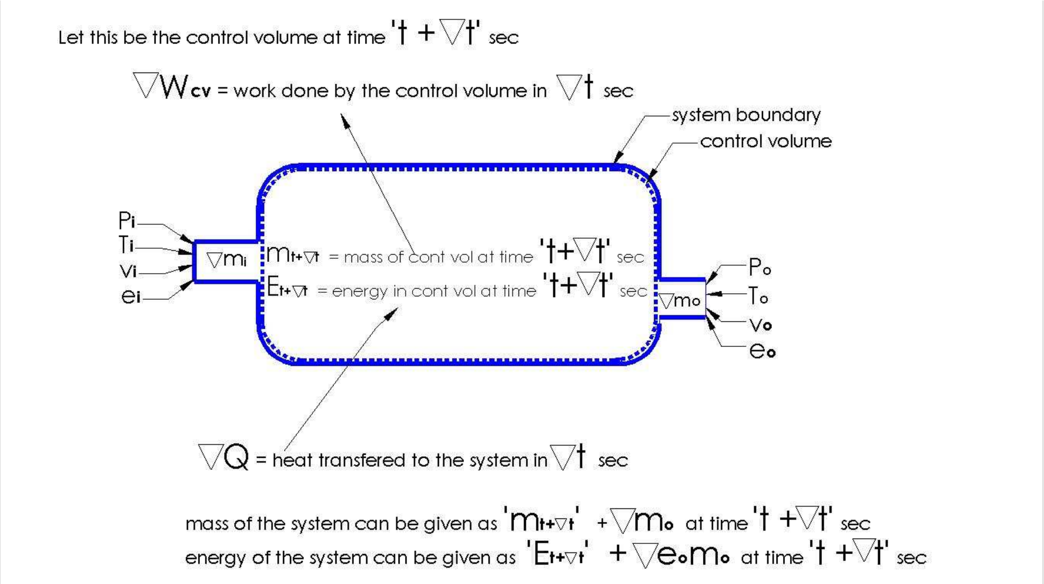

Figure 2.2: Flow of Fluid through control volume at 't+δt'

$$\frac{\delta Q}{\delta t} = \frac{E_2 - E_1}{dt} + \frac{\delta W}{\delta t} \tag{2.3}$$

Now by comparing the process, that we have discussed now and analyzing the above equation, we can transform each term of the above equation into an equivalent equation with equivalent terms with our terminology.

As we have discussed there is a flow of work and energy in the control volume. Let us assume that, E_t is the amount of energy that the control volume has at time t and $E_{t+\delta t}$ is the amount of energy that the control volume has at time $t + \delta t$. When we compare it with the above equation we can replace the terms E_2 and E_1 with the following

$$E_1 = E_t + e_i \delta m_i \tag{2.4}$$

$$E_2 = E_{t+\delta t} + e_o \delta m_o \tag{2.5}$$

Therefore

$$E_2 - E_1 = E_{t+\delta t} + e_o \delta m_o - E_t - e_i \delta m_i \tag{2.6}$$

Rearranging the above equation results to

$$E_2 - E_1 = (E_{t+\delta t} - E_t) + (e_o \delta m_o - e_i \delta m_i) \tag{2.7}$$

In the above equation, the second part of equation $(e_o \delta m_o - e_i \delta m_i)$ is the amount of net flow of energy that is used in crossing the mass from control volume during the time δt.

In order to relate the pressure to the work done on the system, the following theory is usually adopted. The mass of gas that is going into the control volume and coming out of the control volume are δm_i and δm_o respectively. Work is done by the normal force acting at the area of crossing of the mass at the inlet and the outlet. The normal force can be obtained by the product of normal tensile stress and area of the cross section of the inlet or outlet. Therefore the work done is given by the following equation

$$W = -\sigma_n A dl = -\sigma_n \delta V = -\sigma_n v \delta m \tag{2.8}$$

It has been assumed that the normal stress at the point of application is equal to the static pressure at that point and hence the work done can be represented as the following equation

$$W = -PV_e \delta m \tag{2.9}$$

Now this work is calculated at the inlet and the outlet of the control volume during time δt and is given as

$$\delta W = \delta W_{cv} + (P_o V_o \delta m_o - P_i V_i \delta m_i) \tag{2.10}$$

Now rewriting (2.7) and (2.10) per unit time(i.e by dividing each term with δt) we get

$$\frac{E_2 - E_1}{\delta t} = \frac{(E_{t+\delta t} - E_t)}{\delta t} + \frac{e_e \delta m_o}{\delta t} - \frac{e_i \delta m_i}{\delta t} \tag{2.11}$$

$$\frac{\delta W}{\delta t} = \frac{\delta W_{cv}}{\delta t} + \frac{(P_o V_o \delta m_o - P_i V_i \delta m_i)}{\delta t} \tag{2.12}$$

combining terms and rearranging,

$$\frac{\delta Q}{\delta t} + \frac{\delta m_i}{\delta t}(e_i + P_i V_i) = \frac{(E_{t+\delta t} - E_t)}{\delta t} + \frac{\delta m_o}{\delta t}(e_o + P_o V_o) + \frac{\delta W_{cv}}{\delta t} \tag{2.13}$$

In the above equation the term e refers to energy of the system. This implies the summation of internal energy, kinetic energy and potential energy. Therefore, the term e can be written as

$$e = U + \frac{v^2}{2} + gZ \tag{2.14}$$

Now substituting the above term in (2.13) and replacing the terms $U + PV$ with h as enthalpy, the final equation results to

$$\frac{\delta Q}{\delta t} + \frac{\delta m_i}{\delta t}(h_i + \frac{v_i^2}{2} + gZ_i) = \frac{(E_{t+\delta t} - E_t)}{\delta t} + \frac{\delta m_o}{\delta t}(h_o + \frac{v_o^2}{2} + gZ_o) + \frac{\delta W_{cv}}{\delta t} \tag{2.15}$$

Now applying limits to the above equation where δt approaches zero and assuming that there are more number of flow streams entering and leaving the system, the above equation yields to

$$\frac{dQ}{dt} + \sum \frac{dm_i}{dt}(h_i + \frac{v_i^2}{2} + gZ_i) = \frac{(E_{t+dt} - E_t)}{dt} + \sum \frac{dm_o}{dt}(h_o + \frac{v_o^2}{2} + gZ_o) + \frac{dW_{cv}}{dt} \tag{2.16}$$

This equation is the basic equation[4] that is used in deriving the rate of change of temperature and rate of change of density for the single phase and the two phase tanks in the chapters 3 and 4 respectively. This equation is generally used in getting further equations for the steady state and unsteady state.

2.3 Problems of Unsteady state

Some of the problems of the unsteady state flow process are given below:

- inflating of tires and balloons
- charging of rigid vessels from supply lines

- discharging a fluid from a pressurized vessel
- cooking with an ordinary pressure cooker

These problems can be solved by the USUF model, which is called as Uniform state Uniform flow model which is discussed in the next section.

2.4 Uniform State Uniform Flow Process(USUF)

This is the process where there will be an unsteady flow like filling closed tanks with a fluid or discharging a fluid from a cylinder. These things can be explained by a simplified model which is called as USUF model which means Uniform State Uniform Flow model. Some of the basic assumptions in this model are

- The control volume always remains constant with relative to the coordinate frame
- The state of mass with in the control volume may change with time but at any instant of time the state is uniform throughout the entire control volume
- The state of the mass crossing each of the areas of flow on the surface is constant with time although the mass flow rates may be time varying.

This thesis is related to the unsteady state flow and this model has been used in deriving equations for various parameters in chapter 3 and 4.

CHAPTER III

SINGLE-PHASE PROPELLANT TANKS

3.1 Introduction

Single phase single gas medium contains only gas at a certain pressure and the transient thrust along with other parameters are calculated and a simulation is shown in this chapter. The state space equations for the single phase single gas medium are calculated using the thermodynamic laws and a simulation is made using simulink to show the plots of pressure, mass flow rate, temperature and more importantly thrust with the change in time.

3.2 Derivation of Equations

A cylinder is considered with a gas at a certain pressure. The aim is to calculate the transient thrust for the whole length of time, while the gas is expelled completely.

The parameters that change in the cylinder when the nozzle of the cylinder is open are density, internal energy, mass, pressure and temperature. The parameter that is constant throughout the process is volume.

If m_i and m_e are mass going in and out of the system respectively at a given point of time t, then considering the law of conservation of mass,

$$m_t + \delta m_i = m_{t+\delta t} + \delta m_e \tag{3.1}$$

In the present case, there is no gas coming into the cylinder and if m_{cv} is mass of control volume and m_o is the mass of gas expelling out of cylinder, then

$$m_i = 0 \tag{3.2}$$

$$m_e = m_o \tag{3.3}$$

$$m_{t+\delta t} - m_t = m_{cv} \tag{3.4}$$

At a given point of time the mass flowing into the system is equal to the sum of mass stored in the control volume and the mass expelling out of the system.

$$\frac{\delta m_i}{\delta t} = \frac{(m_{t+\delta t} - m_t)}{\delta t} + \frac{\delta m_o}{\delta t} \tag{3.5}$$

Substituting equations (3.2), (3.3) and (3.4) in (3.5)

$$\frac{dm_i}{dt} = \frac{dm_{cv}}{dt} + \frac{dm_o}{dt} \tag{3.6}$$

since there is no mass flowing into the control volume $m_i = 0$,the above equation results to

$$\frac{dm_{cv}}{dt} = -\frac{dm_o}{dt} \tag{3.7}$$

According to the first law of thermodynamics

$$dQ = dU + dW \tag{3.8}$$

Let Q and W_{cv} are the total heat energy and work done in the system and v_i, v_o, Z_i, Z_o are the velocities and heights before going to the system and after coming from the system. Applying first law of thermodynamics to the control volume, from (2.16), the equation results to

$$\frac{dQ}{dt} + \sum \frac{dm_i}{dt}(h_i + \frac{v_i^2}{2} + gZ_i) = \frac{(E_{t+dt} - E_t)}{dt} + \sum \frac{dm_o}{dt}(h_o + \frac{v_o^2}{2} + gZ_o) + \frac{dW_{cv}}{dt} \tag{3.9}$$

The process that is undergoing in the cylinder is considered to be an adiabatic process, which means there is no transfer of heat energy. Therefore

$$\frac{dQ}{dt} = 0 \tag{3.10}$$

Now substituting equations (3.2), (3.3) and (3.10) in (3.9), we get

$$\frac{dU_{cv}}{dt} = -\frac{dm_o}{dt}\left(h_o + \frac{v_o^2}{2} + gZ_e\right) \tag{3.11}$$

Neglecting potential energy the above equation results to

$$\frac{dU_{cv}}{dt} = -\frac{dm_o}{dt}\left(h_o + \frac{v_o^2}{2}\right) \tag{3.12}$$

If m_{cv} is the mass of the control volume and u_{cv} is the internal energy per unit mass of the control volume, then the total internal energy of the system can be given by

$$U_{cv} = m_{cv}u_{cv} \tag{3.13}$$

Now differentiating equation (3.13) with respect to time

$$\frac{dU_{cv}}{dt} = u_{cv}\frac{dm_{cv}}{dt} + m_{cv}\frac{du_{cv}}{dt} \tag{3.14}$$

From equation (3.12) we get,

$$-\frac{dm_o}{dt}\left(h_o+\frac{v_o^2}{2}\right)=u_{cv}\frac{dm_{cv}}{dt}+m_{cv}\frac{du_{cv}}{dt} \tag{3.15}$$

Substituting equation (3.7) in (3.15)

$$-\frac{dm_o}{dt}\left(h_o+\frac{v_o^2}{2}\right)=-u_{cv}\frac{dm_o}{dt}+m_{cv}\frac{du_{cv}}{dt} \tag{3.16}$$

Hence the rate of change of internal energy can be obtained from the above equation

$$\frac{du_{cv}}{dt}=\frac{1}{m_{cv}}\frac{dm_o}{dt}\left(u_{cv}-h_o-\frac{v_o^2}{2}\right) \tag{3.17}$$

Now the rate of change of density is calculated with respect to time.

Density can be defined as the ratio of mass and volume. Hence,

$$\rho_{cv}=\frac{m_{cv}}{V} \tag{3.18}$$

Differentiating equation (3.18) with respect to time

$$\frac{d\rho_{cv}}{dt}=\frac{1}{V}\frac{dm_{cv}}{dt} \tag{3.19}$$

Substituting Equation (3.7) in (3.19)

$$\frac{d\rho_{cv}}{dt}=-\frac{1}{V}\frac{dm_o}{dt} \tag{3.20}$$

Hence the two required equations, the rate of change of internal energy with respect to time and rate of change of density with respect to time are obtained. Both the equations contain m_o which is mass flow rate from the cylinder.

$$\frac{du_{cv}}{dt}=\frac{1}{m_{cv}}\frac{dm_o}{dt}\left(u_{cv}-h_o-\frac{v_o^2}{2}\right) \tag{3.21}$$

$$\frac{d\rho_{cv}}{dt} = -\frac{1}{V}\frac{dm_o}{dt} \tag{3.22}$$

'u_{cv}' = internal energy

'm_{cv}' = mass of the control volume

'm_o' = mass of the fluid expelling out from the nozzle

'h_o' = enthalpy of the fluid at the throat in the nozzle

'v_o' = velocity of fluid coming out of nozzle

'V' = volume of fluid in the cylinder

'ρ_{cv}' = density of fluid in the control volume

The internal energy is a function of density(ρ) and temperature(T).

$$u = f(\rho, T) \tag{3.23}$$

Using the chain rule,

$$\frac{du}{dt} = \frac{\partial u}{\partial \rho}\frac{\partial \rho}{\partial t} + \frac{\partial u}{\partial T}\frac{\partial T}{\partial t} \tag{3.24}$$

if δ is very small the above equation can be written as

$$\frac{du}{dt} = \frac{\partial u}{\partial \rho}\frac{d\rho}{dt} + \frac{\partial u}{\partial T}\frac{dT}{dt} \tag{3.25}$$

substituting (3.20), (3.17) in (3.25) and solving for $\frac{dT}{dt}$, we get

$$\frac{dT}{dt} = \frac{1}{\frac{\partial u}{\partial T}}\left(\frac{du}{dt} + \frac{\partial u}{\partial \rho}\frac{1}{V}\frac{dm_o}{dt}\right) \tag{3.26}$$

substituting for $\frac{du}{dt}$ in (3.26)

$$\frac{dT}{dt} = \frac{1}{V}\frac{dm_o}{dt}\frac{1}{\frac{\partial u}{\partial T}}\left(\frac{u - h_o}{\rho} + \frac{\partial u}{\partial \rho}\right) \tag{3.27}$$

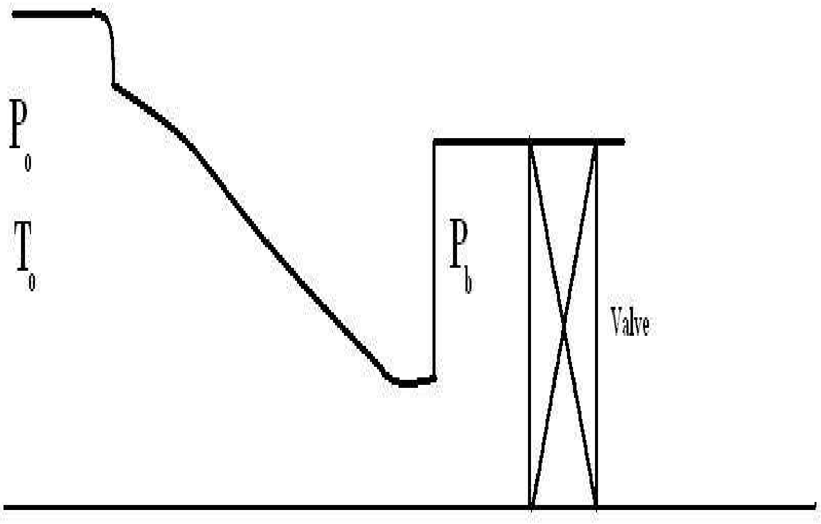

Figure 3.1: Flow of Fluid through Nozzle

In the above equations terms $\frac{dm_o}{dt}$, h_o gives the mass of the gas going out of the cylinder and the enthalpy in the nozzle respectively. This can be obtained from the following section.

3.2.1 Concept of Mass Flow Under Choking Conditions

The mass flow rate in each case is calculated until the flow gets choked (M = 1). The flow exists until the pressure inside control volume reaches the absolute pressure.

A convergent nozzle is used to get the flow choked under given conditions. Following is a schematic diagram which shows a convergent nozzle with a valve.

P_b is the back pressure near the valve and P_o is the stagnation pressure in the cylinder[5]. A flow occurs when the stagnation pressure is greater than the back pressure. When the valve is kept open, there will be a decrease in the back pressure at the valve and this will result in a flow and the flow rate is increased for further decrease in back pressure. The flow rate is increased till it reaches choking conditions(M=1). The condition to reach choking is

$$\frac{p_b}{p_o} < 0.528 \tag{3.28}$$

Exhaust Gas Velocity

When a fluid enters a nozzle it travels at subsonic velocities and when it reaches the throat, as the cross section area of the nozzle comes down, the velocity of the fluid increases and reaches sonic and when it goes further, the area of cross section of the nozzle once again increases , which results in the expansion of gas and becomes more super sonic. Therefore, if T_{cv}, k, p_a and p_{cv} are the temperature of control volume, adiabatic index, ambient pressure and control volume pressure respectively, then, the exhaust velocity of the gas coming out of the pressure end of the nozzle can be given by the following equation[6]

$$V_o = \sqrt{T_{cv}R\left(\frac{2k}{k-1}\right)\left(1-\frac{p_a}{p_{cv}}\right)^{\frac{k-1}{k}}} \tag{3.29}$$

The mass flow rate from the nozzle[7] can be obtained by the following equation,

$$\frac{dm}{dt} = \frac{P_{cv}M}{\left(1+\left(\frac{k-1}{2}\right)M^2\right)^{\frac{k+1}{2(k-1)}}}\sqrt{\frac{k}{T_oR}} \tag{3.30}$$

The Mach number in the above equation can be obtained from the following equation[8]

$$M = \sqrt{\frac{2}{k-1}\left(\left(\frac{P_{cv}}{P_a}\right)^{\frac{k-1}{k}}-1\right)} \tag{3.31}$$

where

P_a is the ambient pressure

The term h_o in equation (3.17) is the enthalpy at the throat in the nozzle and it can be obtained from the following calculations

$$h_o = C_p T_o \tag{3.32}$$

C_p and T_o are the specific heat and temperature at throat[9]. This can be obtained by the following equations

$$C_p = \frac{5}{2}R + R + f_v(t) + f_e(t) \tag{3.33}$$

where

$f_v(t)$ is called as vibrational contribution and is given by

$$f_v(t) = R \sum_{i=1}^{3a-5} \left(\frac{{x_i}^2 {e_i}^x}{({e_i}^x - 1)^2} \right) \tag{3.34}$$

$$x_i = \theta_i / T \tag{3.35}$$

θ_i is an experimental observed constant, expressed in temperature units which represents the stiffness of a vibration. In the case of monoatomic gases $f_e(t)$ is called as electronic contribution and is usually small and is taken as zero.

$f_e(t)$ is very small and hence eliminated

and

$$T_o = \frac{T_{cv}}{1 + \frac{(k-1)}{2} M^2} \tag{3.36}$$

where

T_o is the temperature at the throat in the nozzle

T_{cv} is the temperature of the control volume

A model is created in the simulink with the two state space equations and their dependent variables. Following are the equations which are used to create the simulink model.

$$\frac{dT}{dt} = \frac{1}{V}\frac{dm_o}{dt}\frac{1}{\frac{\partial u}{\partial T}}\left(\frac{u-h_o}{\rho} + \frac{\partial u}{\partial \rho}\right) \tag{3.37}$$

$$\frac{d\rho_{cv}}{dt} = -\frac{1}{V}\frac{dm_o}{dt} \tag{3.38}$$

$$\frac{dm}{dt} = \frac{P_{cv}M}{\left(1+\left(\frac{k-1}{2}\right)M^2\right)^{\frac{k+1}{2(k-1)}}}\sqrt{\frac{k}{T_oR}} \tag{3.39}$$

$$M = \sqrt{\frac{2}{k-1}\left(\left(\frac{P_{cv}}{P_a}\right)^{\frac{k-1}{k}} - 1\right)} \tag{3.40}$$

$$V_o = \sqrt{T_{cv}R\left(\frac{2k}{k-1}\right)\left(1-\frac{p_a}{p_{cv}}\right)} \tag{3.41}$$

$$h_o = C_pT_o \tag{3.42}$$

$$C_p = \frac{5}{2}R + R + f_v(t) + f_e(t) \tag{3.43}$$

$$f_v(t) = R\sum_{i=1}^{3a-5}\left(\frac{{x_i}^2{e_i}^x}{({e_i}^x-1)^2}\right) \tag{3.44}$$

$$T_o = \frac{T_{cv}}{1+\frac{(k-1)}{2}M^2} \tag{3.45}$$

3.3 Simulating the model

The model is made in Simulink and it is simulated with four sets of different time steps and different initial parameters of the density and temperature along with different variables for the area of cross section of the nozzle. The first and second parameters have the same initial conditions except for density and temperature. The third and fourth parameters have same initial conditions except for the area of cross section of the nozzle. The model is shown in the following page.

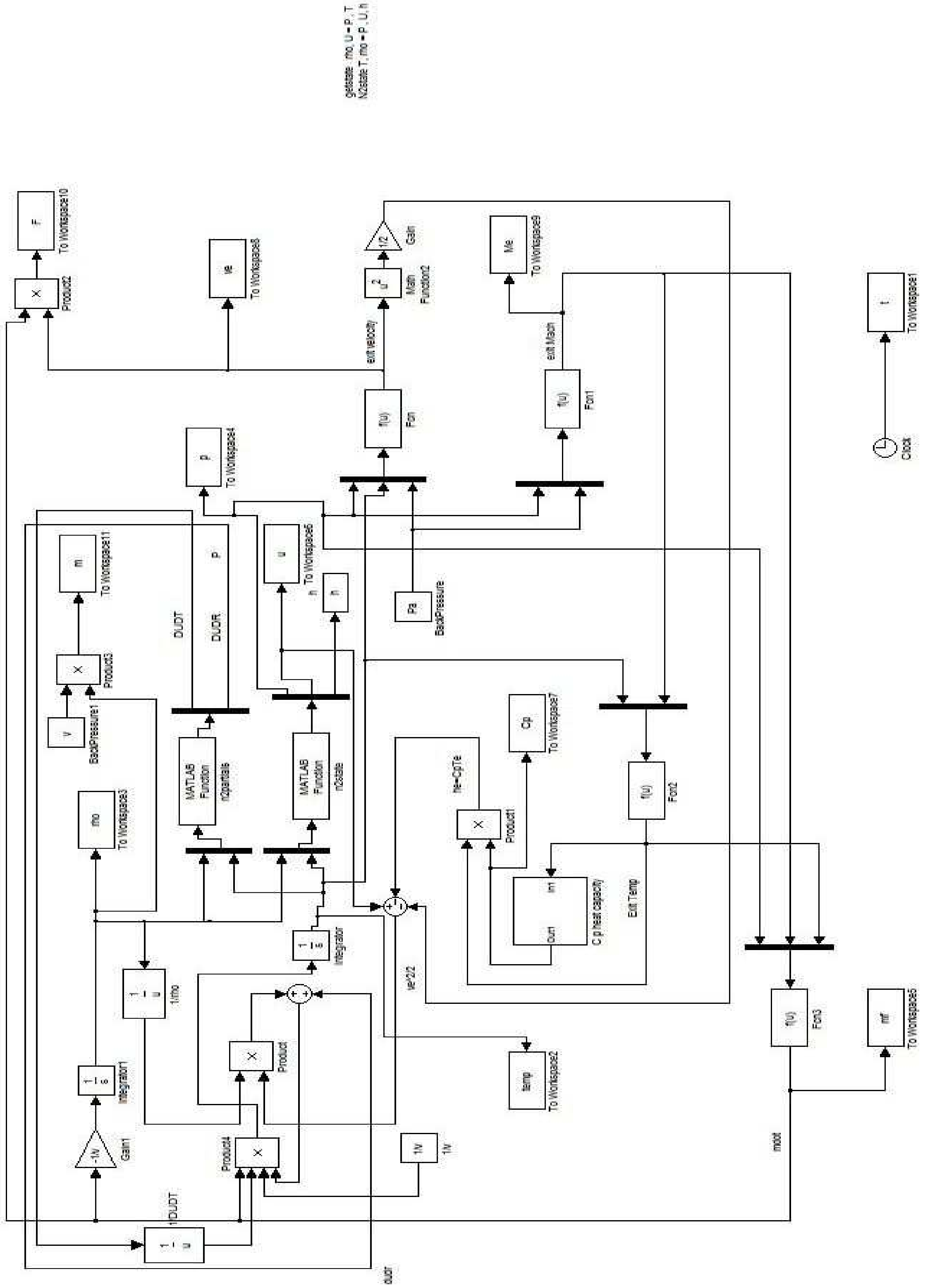

Figure 3.2: Single phase Model

3.3.1 Simulation 1

Following are the values chosen for the temperature and density of gas in the cylinder,

$T = 336.7K \ \rho = 2.00kg/m^3$

The initial values are now given to the model and the model is made to run for a limited time step until the flow gets choked and the following plots are obtained. The different parameters that are chosen for the first simulation are as follows

Area of cross-section of nozzle = *1 X* 10^{-4} m^2

Adiabatic Index = *1.4*

Gas Const for nitrogen = *296.7* $Kj/kg - K$

Density of gas = *2* kg/m^3

Volume of tank = $\pi r^2 h$ = *0.0011* m^3

Ambient Pressure = *1 X* 10^5 *pa*

The simulation time for the above parameters is very small and is about 0.019 sec. The reason for very small time can be attributed to large area of nozzle resulting to the fast expulsion of gas. Following are the different plots obtained for this simulation.

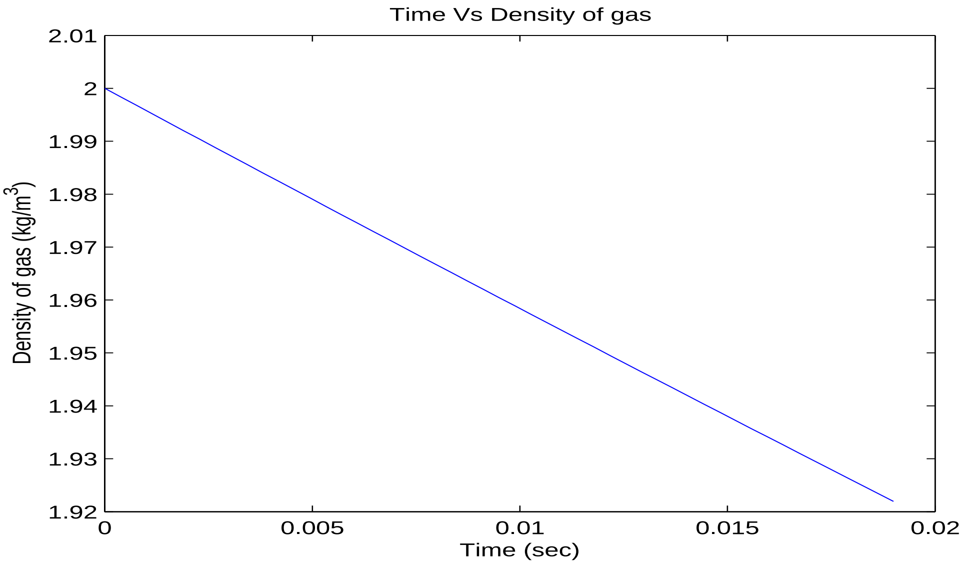

Figure 3.3: Time Vs Density

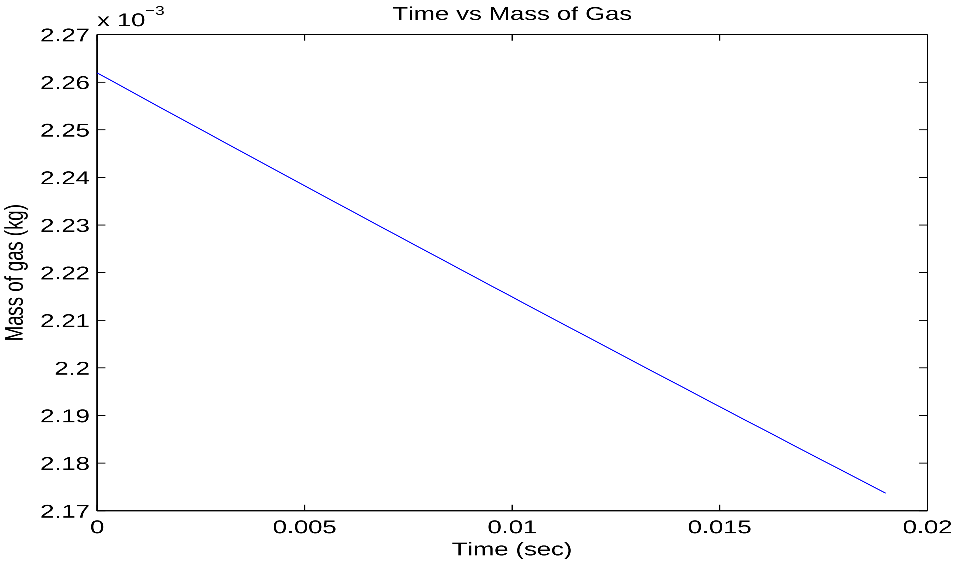

Figure 3.4: Time Vs Mass of gas in Cylinder

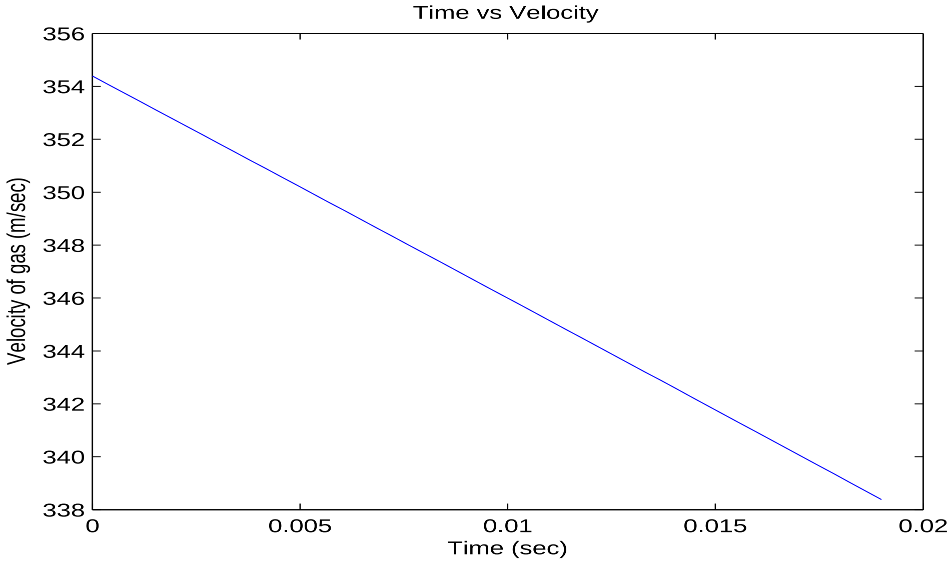

Figure 3.5: Time Vs Velocity

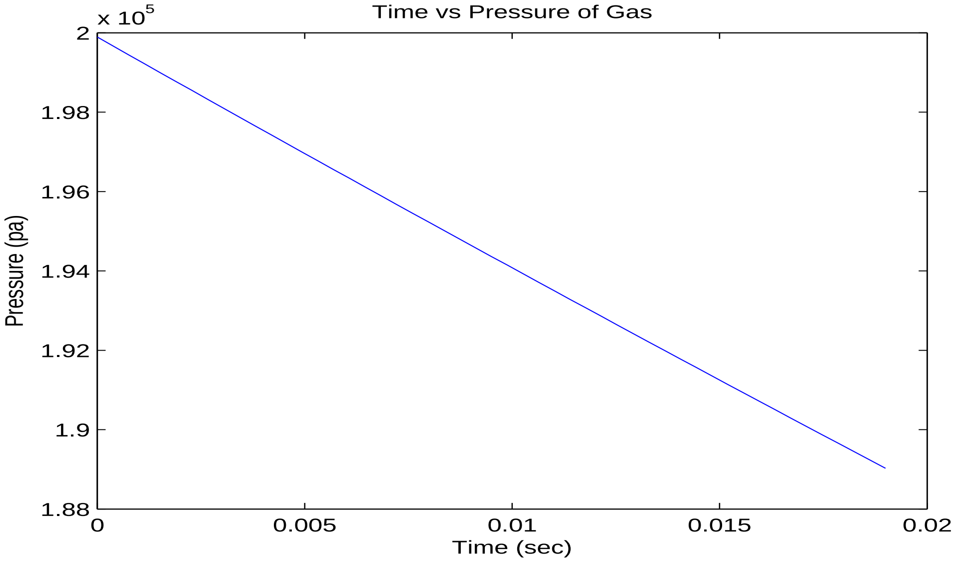

Figure 3.6: Time Vs Pressure

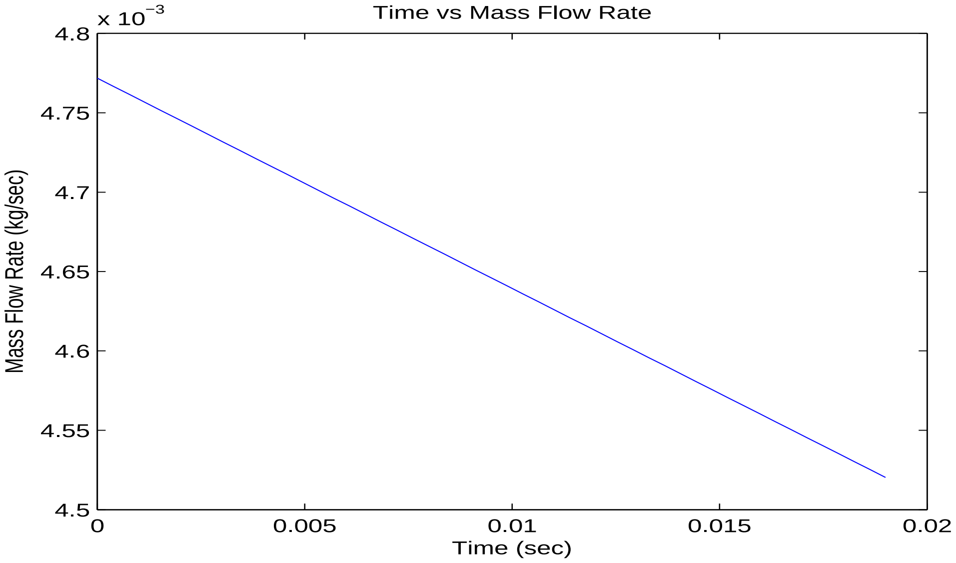

Figure 3.7: Time Vs Mass Flow Rate

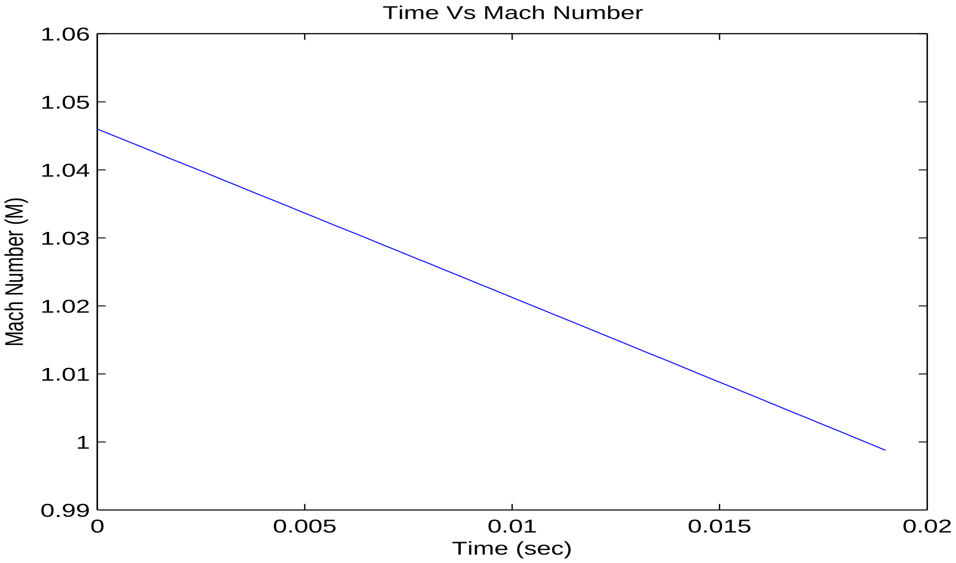

Figure 3.8: Time Vs Mach Number

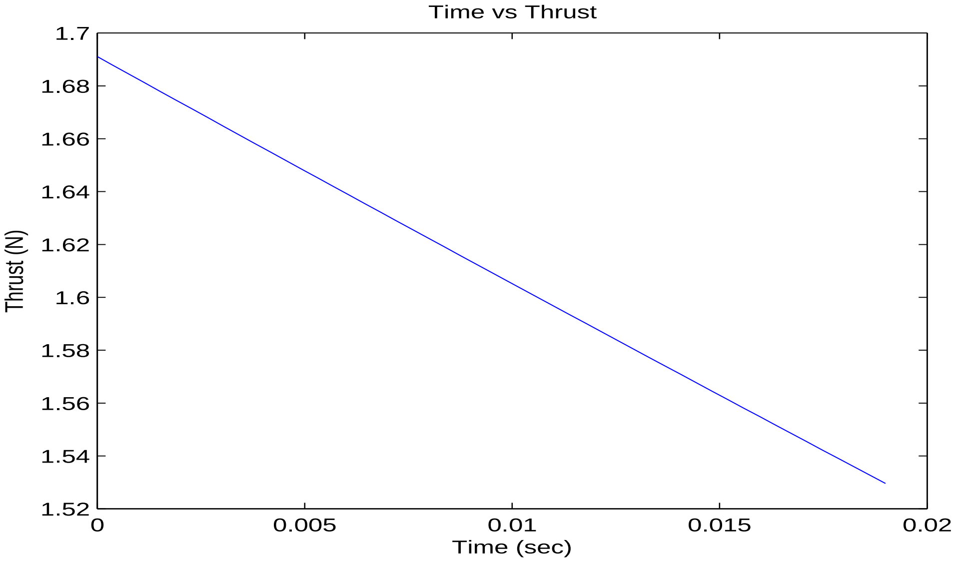

Figure 3.9: Time Vs Thrust

3.3.2 Simulation 2

Following are the values chosen for the temperature and density of gas in the cylinder

$T = 144.3K\ \rho = 7.00kg/m^3$

The initial values are now given to the model and the model is made to run for a limited time step until the flow gets choked and the following plots are obtained. The different parameters that are chosen for this simulation are as follows

Area of cross-section of nozzle = *1 X* $10^{-4}\ m^2$

Adiabatic Index = *1.4*

Gas Const for nitrogen = *296.7* $Kj/kg - K$

Density of gas = *7* kg/m^3

Volume of tank = $\pi r^2 h$ = *0.0011* m^3

Ambient Pressure = *1 X* 10^5 *pa*

The simulation time for the above parameters is increased by a small percentage is about 0.24 sec. The simulation time has been increased because of the increase

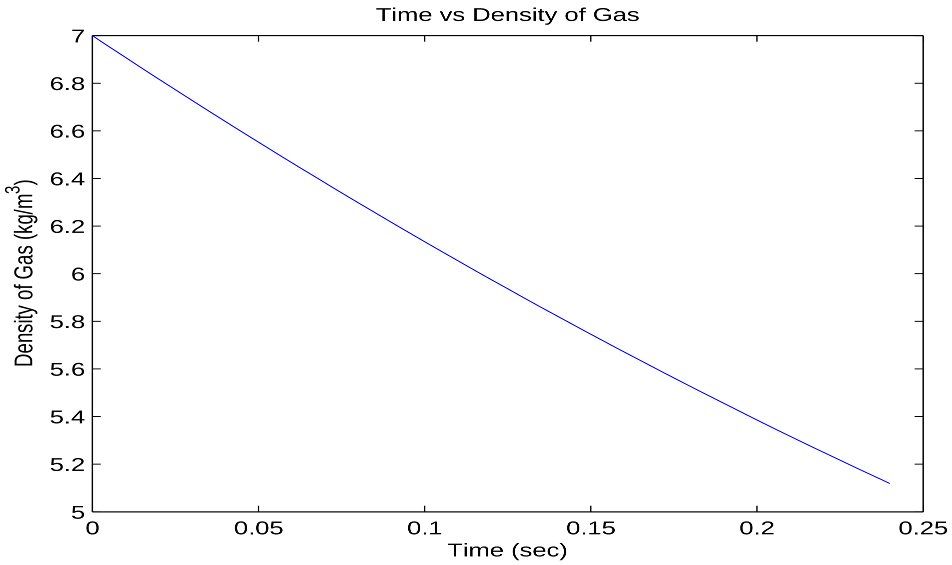

Figure 3.10: Time Vs Density

in density of gas which means, there is more mass of gas in the cylinder than the previous simulation which results to an increase in time for the expulsion of gas.

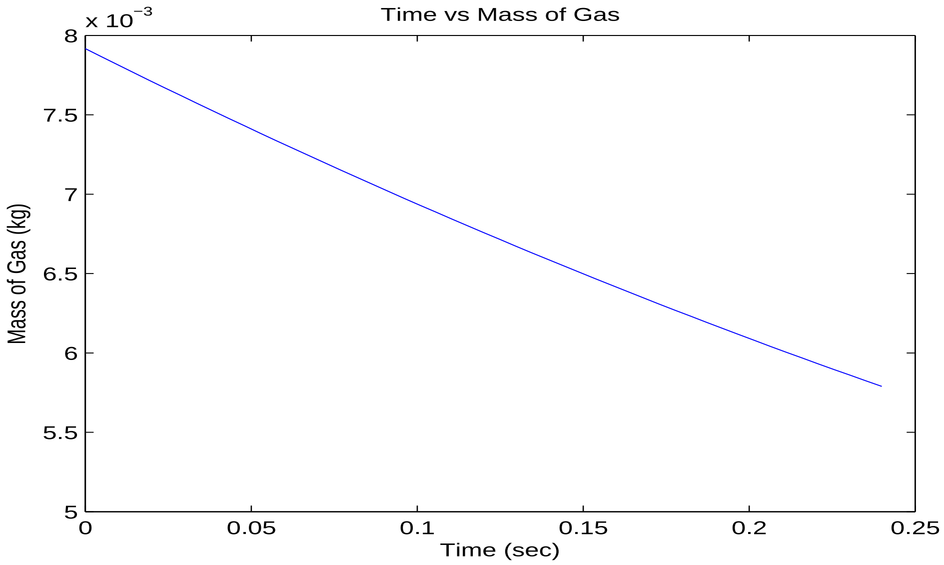

Figure 3.11: Time Vs Mass of gas in Cylinder

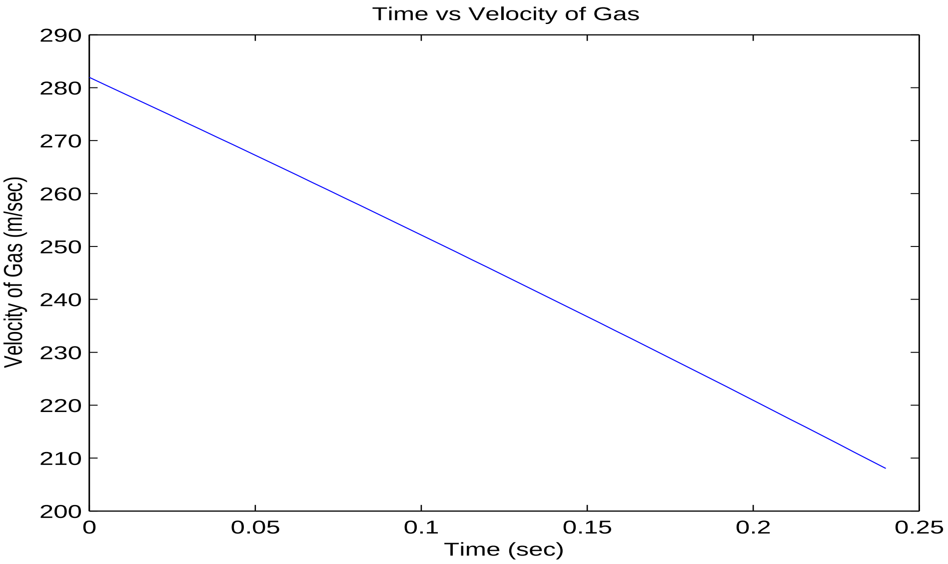

Figure 3.12: Time Vs Velocity

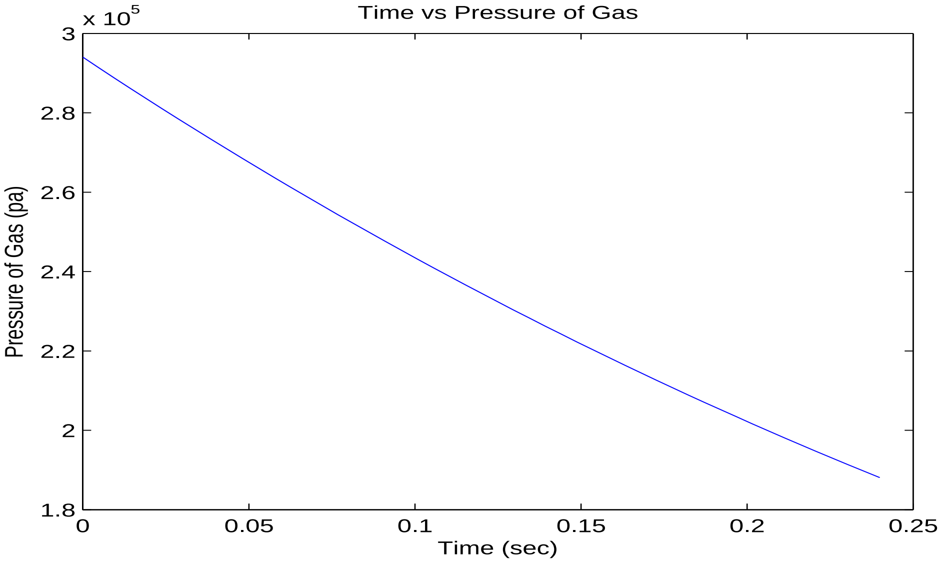

Figure 3.13: Time Vs Pressure

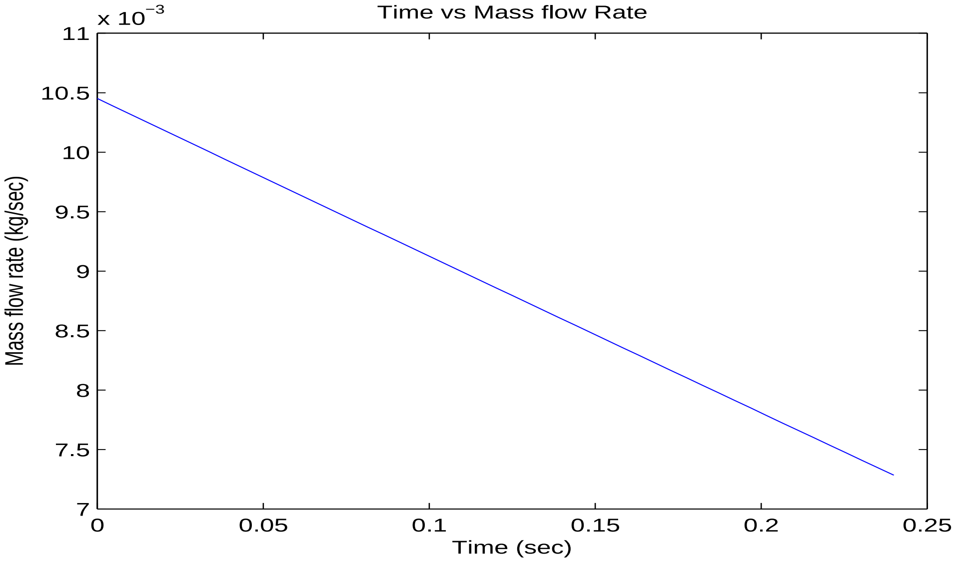

Figure 3.14: Time Vs Mass Flow Rate

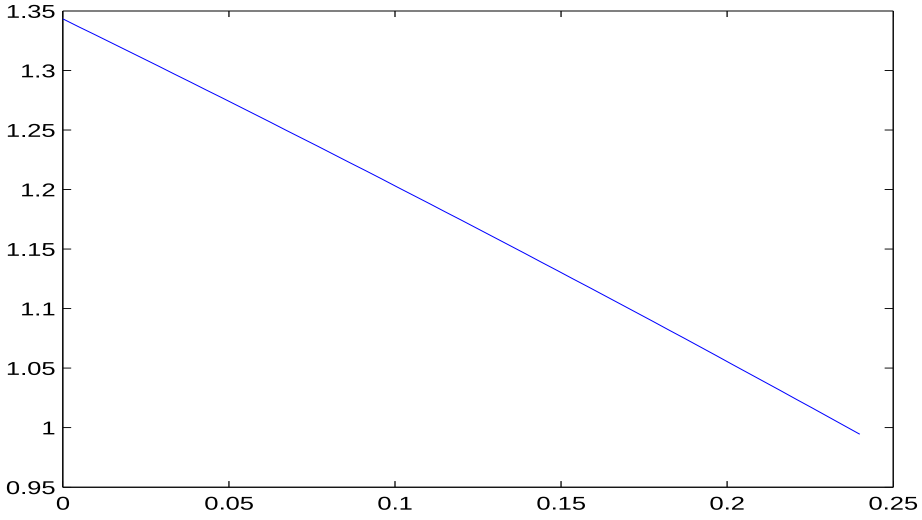

Figure 3.15: Time Vs Mach Number

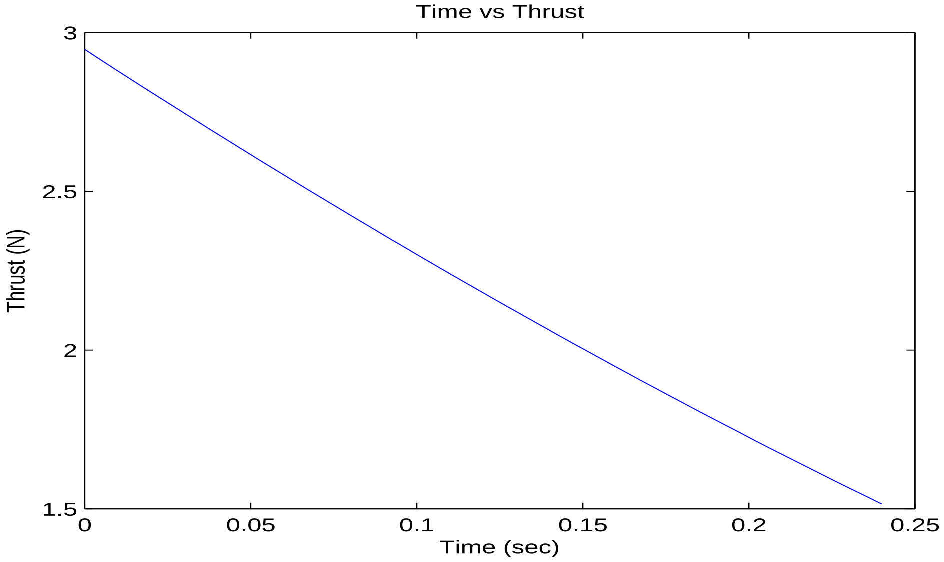

Figure 3.16: Time Vs Thrust

3.3.3 Simulation 3

Following are the values chosen for the temperature and density of gas in the cylinder

$T = 112.23K \ \rho = 12.00kg/m^3$

The initial values are now given to the model and the model is made to run for a limited time step until the flow gets choked and the following plots are obtained. The different parameters that are chosen for this simulation are as follows

Area of cross-section of nozzle = 1 X 10^{-5} m^2

Adiabatic Index = 1.4

Gas Const for nitrogen = 296.7 $Kj/kg - K$

Density of gas = 12 kg/m^3

Volume of tank = $\pi r^2 h$ *= 0.0011* m^3

Ambient Pressure = 1 X 10^5 *pa*

The simulation time for the above parameters is about 4.45 sec. Further increase in the density of gas results to increase in the simulation which should be logically understandable.

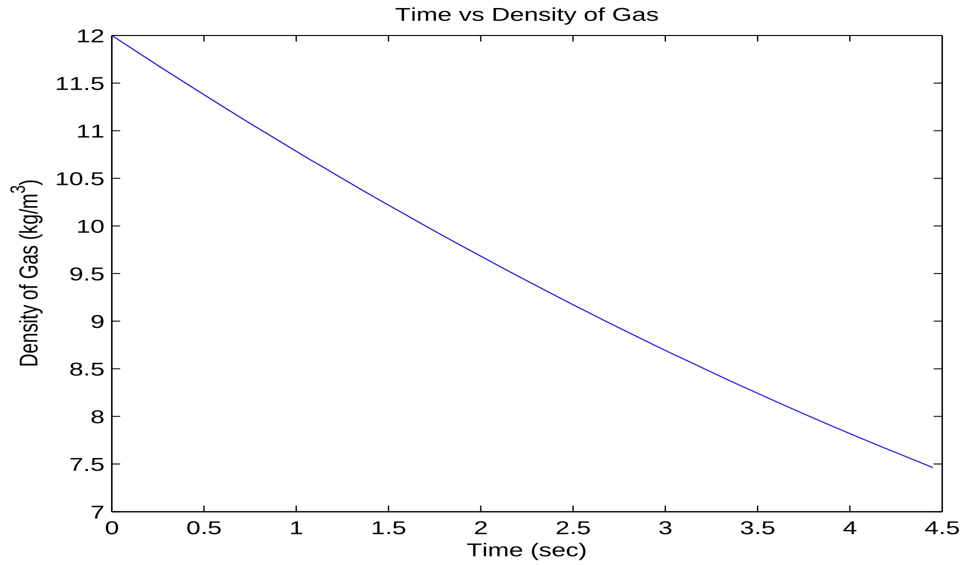

Figure 3.17: Time Vs Density

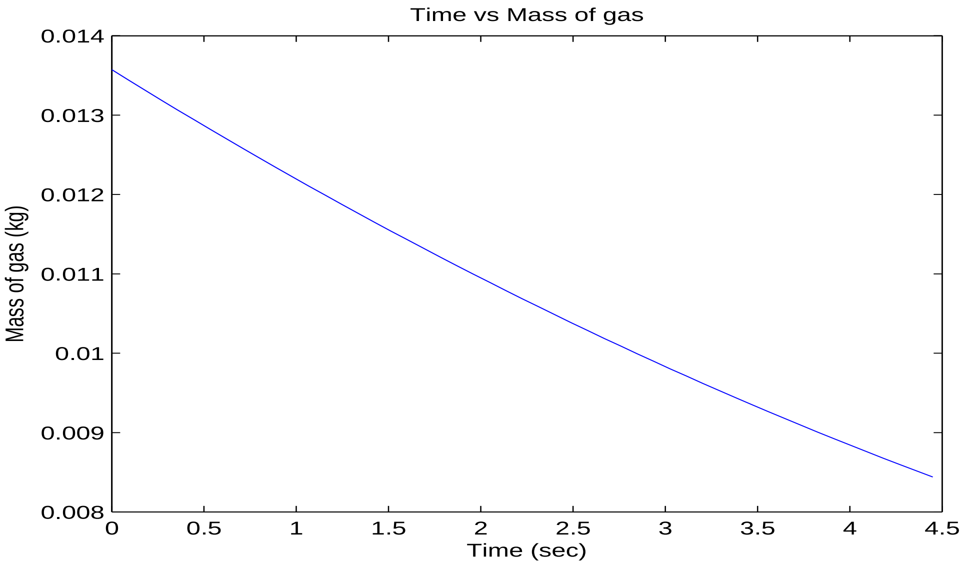

Figure 3.18: Time Vs Mass of gas in Cylinder

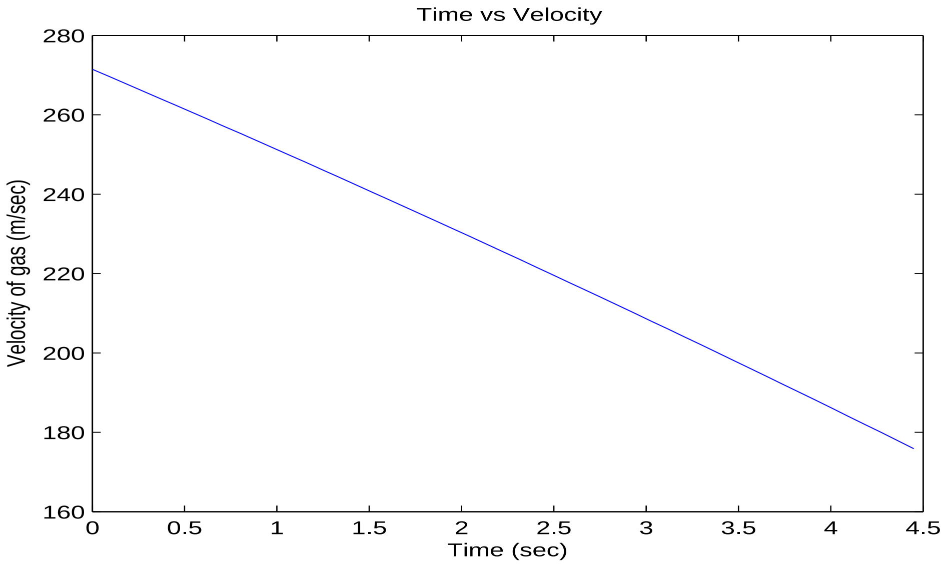

Figure 3.19: Time Vs Velocity

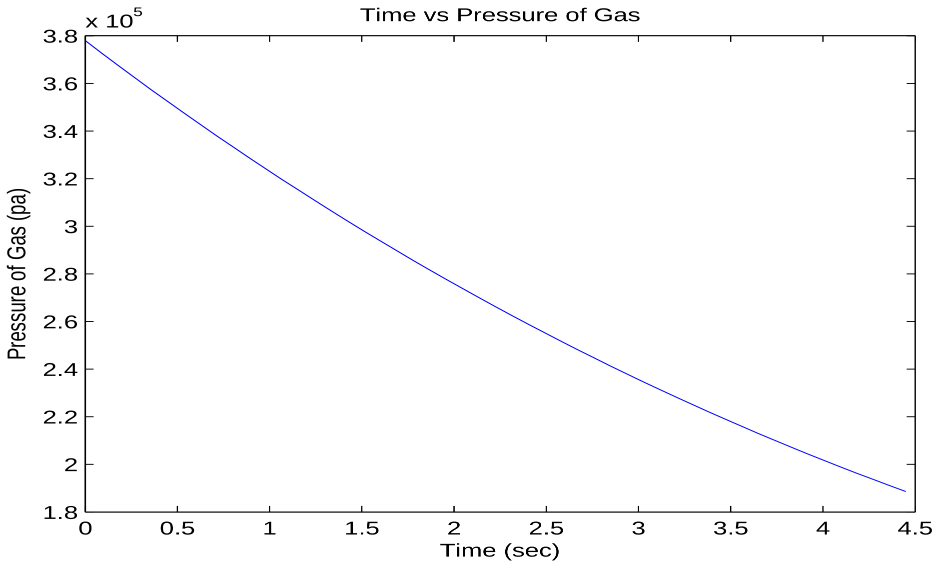

Figure 3.20: Time Vs Pressure

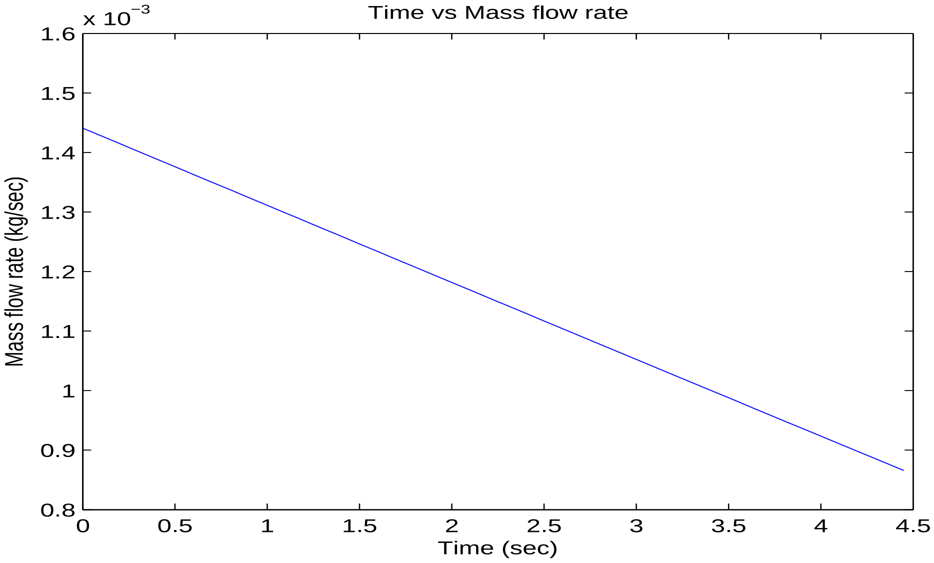

Figure 3.21: Time Vs Mass Flow Rate

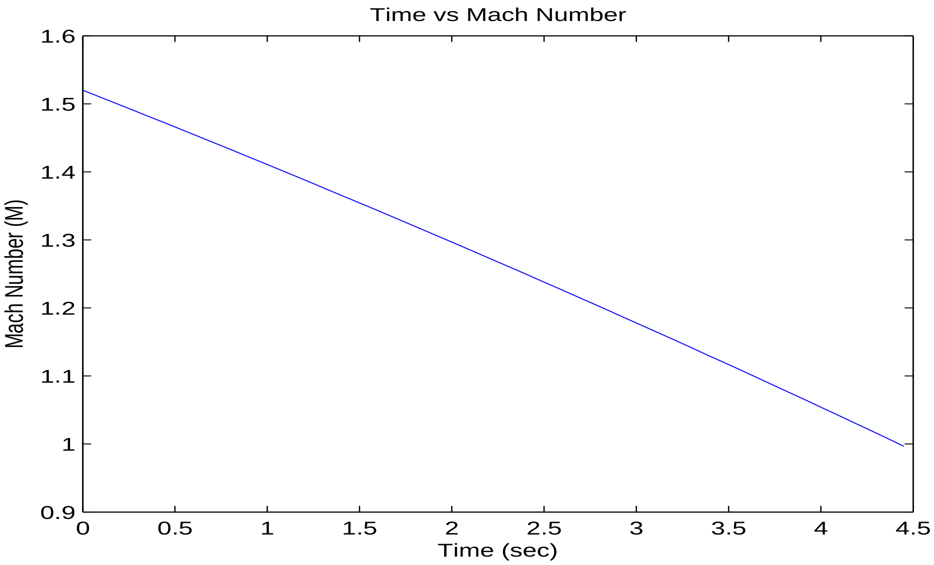

Figure 3.22: Time Vs Mach Number

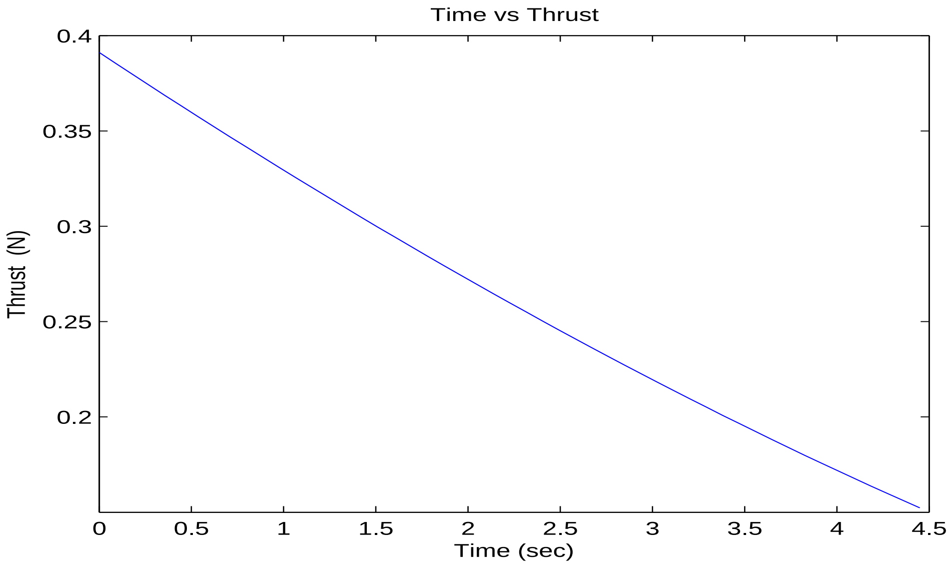

Figure 3.23: Time Vs Thrust

3.3.4 Simulation 4

Following are the values chosen for the temperature and density of gas in the cylinder

$T = 112.23K \ \rho = 12.00 kg/m^3$

The initial values are now given to the model and the model is made to run for a limited time step until the flow gets choked and the following plots are obtained. These are the same initial parameters which are taken in the simulation 3, except that the area of the cross section of the nozzle is further decreased by 10 times. This simulation is demonstrated to show the effect of decrease in the area of the cross section of the nozzle increasing the time for the expulsion of the gas.

The different parameters that are chosen for this simulation are as follows

Area of cross-section of nozzle = 1 X 10^{-6} m^2

Adiabatic Index = 1.4

Gas Const for nitrogen = 296.7 $Kj/kg - K$

Density of gas = 17 kg/m^3

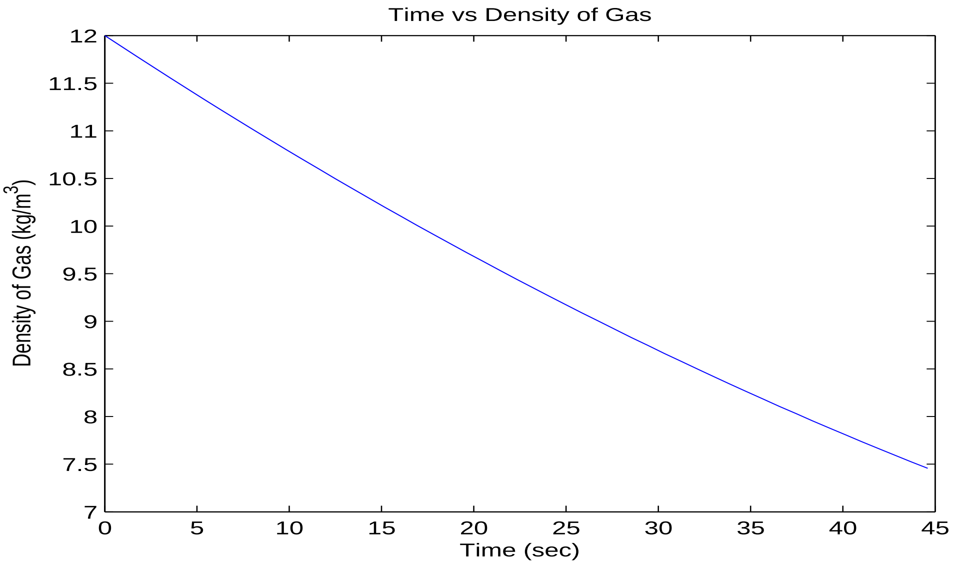

Figure 3.24: Time Vs Density

$Volume\ of\ tank = \pi r^2 h\ =\ 0.0011\ m^3$

$Ambient\ Pressure\ =\ 1\ X\ 10^5\ pa$

The simulation time for the above parameters is about 44.6 sec. This increase in simulation time by a large percentage is because of the combined change of the density and the area of cross section of the nozzle. The area of the cross section of the nozzle is decreased by 10 times and the density is also increased which means more mass of gas in the cylinder. As the area of the cross section of the nozzle is decreased, the amount of time for the expulsion and to get choked has increased considerably.

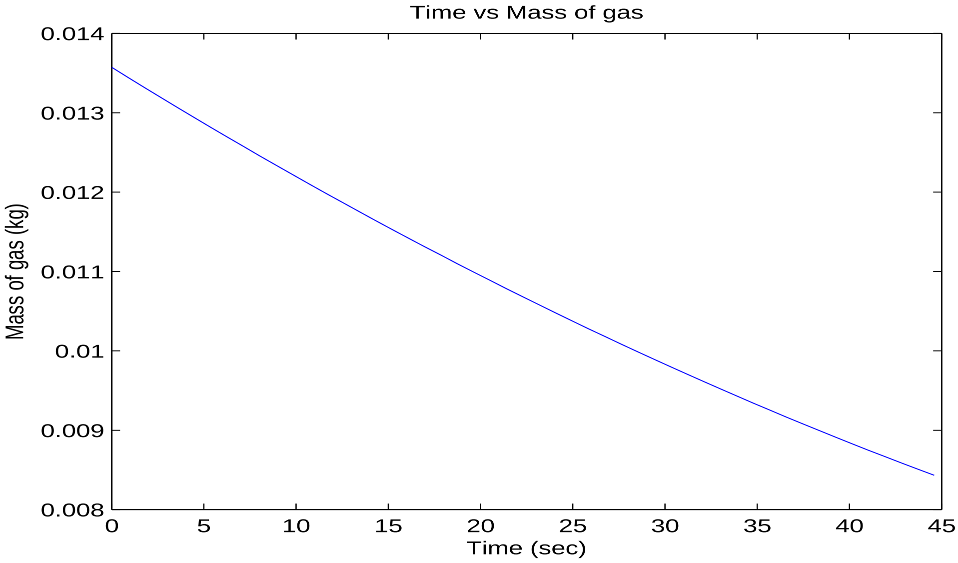

Figure 3.25: Time Vs Mass of gas in Cylinder

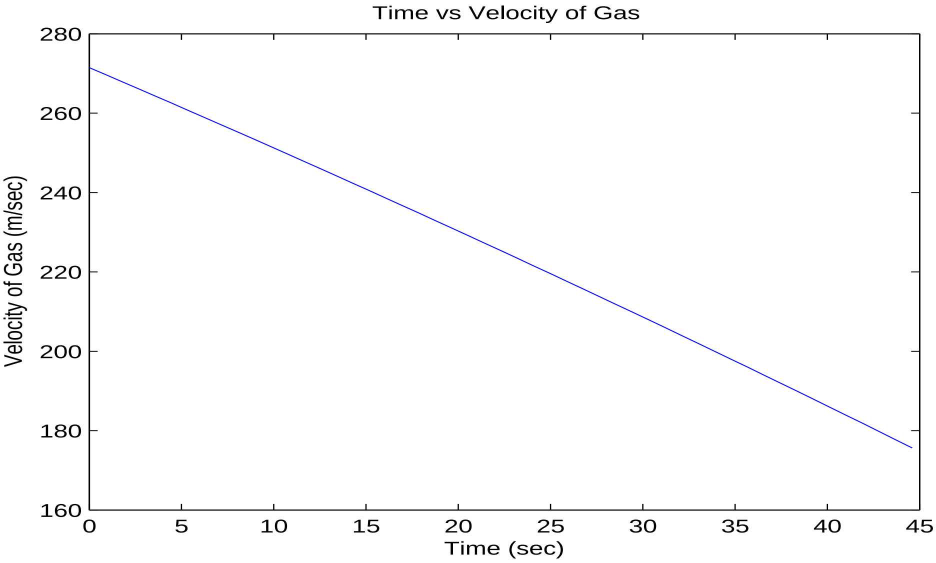

Figure 3.26: Time Vs Velocity

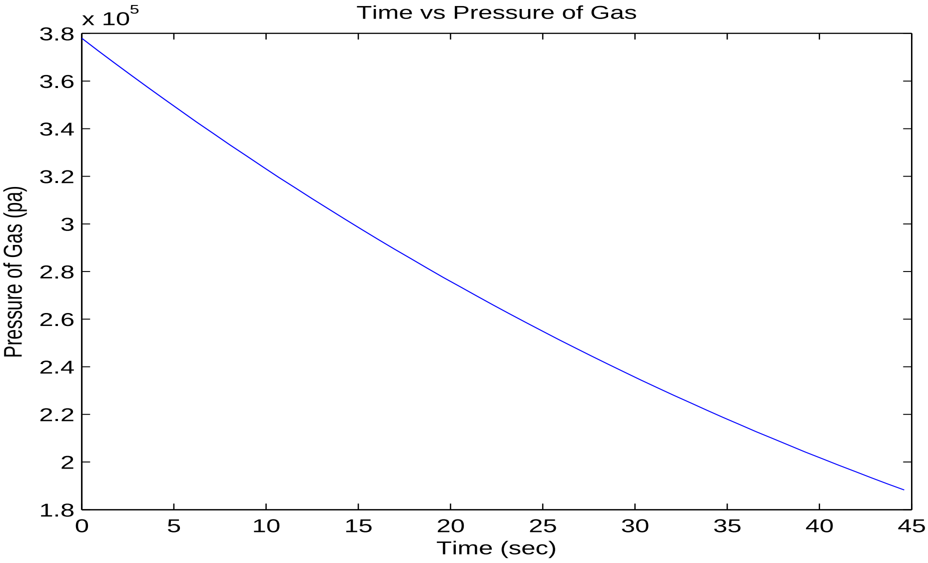

Figure 3.27: Time Vs Pressure

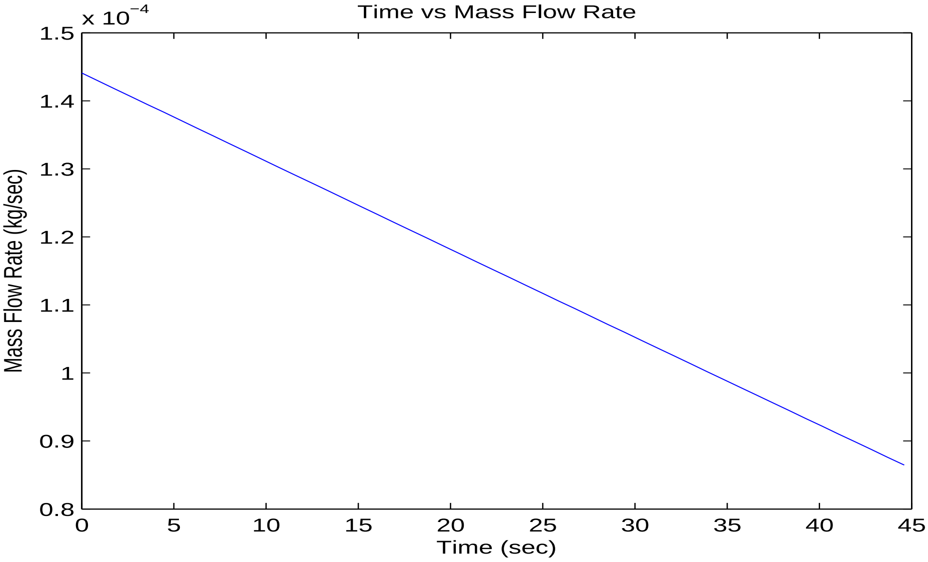

Figure 3.28: Time Vs Mass Flow Rate

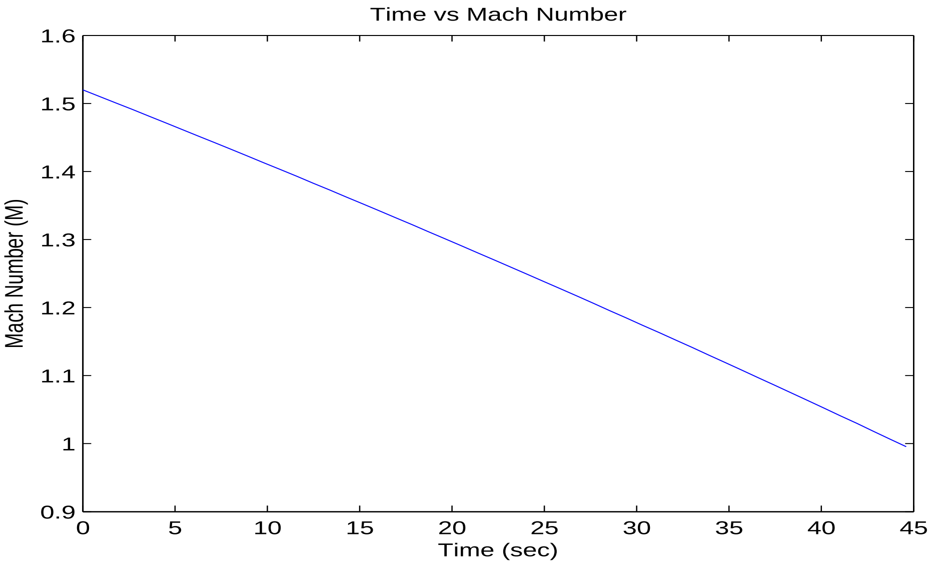

Figure 3.29: Time Vs Mach Number

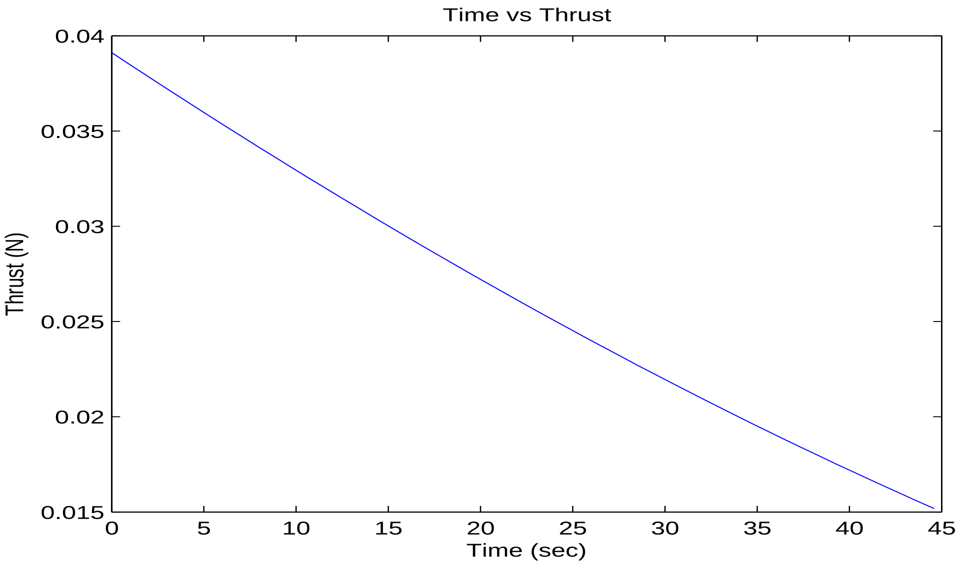

Figure 3.30: Time Vs Thrust

CHAPTER IV

TWO-PHASE PROPELLANT TANKS

4.1 Introduction

In this chapter, the two phase liquified compressed gas containers are described. The required equations for creating the model are derived and a model is made to calculate the transient thrust and other thermodynamic properties.

4.2 Overview

Liquified compressed gases is one of the example of a two phase gas medium. A liquefied compressed gas can be defined as a gas, which when compressed in a container, becomes a liquid at ordinary temperatures and at pressures ranging from 25 to 2500 psi. Liquefied gases have boiling points that range from -130 to 30$^{\circ}F$. At 70$^{\circ}F$ (21.1$^{\circ}C$) the cylinder contains both liquid and gas. Cylinder pressure, or the 'vapor pressure' of the gas, is directly affected by ambient temperature. Increases or decreases in the temperature will cause the vapor pressure to increase or decrease, respectively.

Liquefied gases are packaged under their own vapor pressure and are shipped

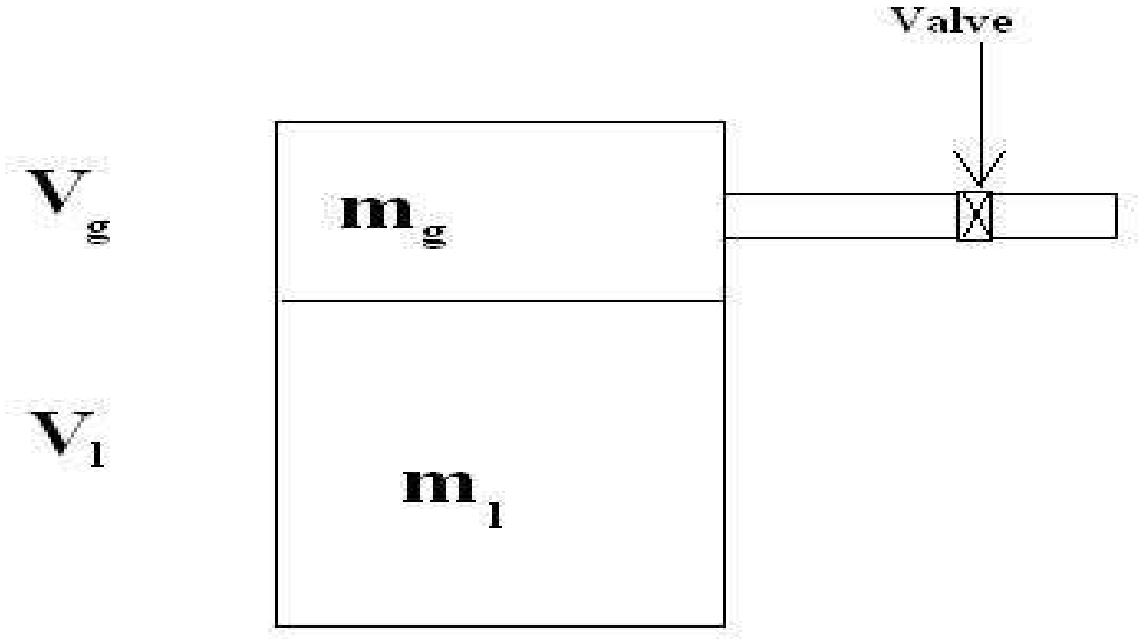

Figure 4.1: Liquid and Gaseous phase in a Cylinder

under rules that limit the maximum amount that can be put into a container to allow for liquid expansion with rising temperatures.

4.3 Derivation of Equations

The equations for the two phase gas medium can be obtained similar to the equations obtained in the single phase gas medium. In this case a cylinder is filled with a high pressure gas which will be in the state of liquid till some level and will be in the form of gas on top of liquid as shown in the figure.

In this case, the liquid medium and gas medium are calculated separately and the equations are obtained. The following section illustrates the derivation of equations for the liquid medium.

4.3.1 Deriving equations for the liquid medium

The mass of liquid in control volume is the difference between mass coming into the control volume and mass going out of the control volume which means evaporation of liquid to gas.

$$\frac{dm_l}{dt} = \frac{dm_i}{dt} - \frac{dm_o}{dt} \tag{4.1}$$

In the above equations the mass coming into the system is zero and hence the above equation can be reduced to

$$\frac{dm_l}{dt} = -\frac{dm_{lg}}{dt} \tag{4.2}$$

Applying first law of thermodynamics to the control volume for which the equation has already been derived in the second chapter. Rewriting the equation (2.16),

$$\frac{dQ}{dt} + \sum \frac{dm_i}{dt}(h_i + \frac{v_i^2}{2} + gZ_i) = \frac{(E_{t+dt} - E_t)}{dt} + \sum \frac{dm_o}{dt}(h_o + \frac{v_e^2}{2} + gZ_o) + \frac{dW_{cv}}{dt} \tag{4.3}$$

The evaporation of liquid to gas is because of the heat energy and it is given as

$$\frac{dQ}{dt} = h_v \frac{dm_{lg}}{dt} \tag{4.4}$$

There is no mass entering into the system and hence $\frac{dm_i}{dt} = 0$ and the inlet velocity and the exit velocities are zeros. The heights are neglected. Now, substituting the required values in the (4.3), the equation is reduced to

$$h_v \frac{dm_{lg}}{dt} = h_0 \frac{dm_{lg}}{dt} + \frac{dU_l}{dt} \tag{4.5}$$

The above equation results to

$$\frac{dU_l}{dt} = (h_v - h_o)\frac{dm_{lg}}{dt} \tag{4.6}$$

If m_l is the mass of the control volume of the liquid and u_l is the internal energy per unit mass of the liquid, then the total internal energy of the control volume can be given as

$$U_l = m_l u_l \tag{4.7}$$

differentiating the above equation,

$$\frac{dU_l}{dt} = m_l\frac{du_l}{dt} + u_l\frac{dm_l}{dt} \tag{4.8}$$

Now equating the equations (4.6), (4.11), the specific internal energy of the system can be found out.

$$m_l\frac{du_l}{dt} + u_l\frac{dm_l}{dt} = (h_v - h_o)\frac{dm_{lg}}{dt} \tag{4.9}$$

substituting (4.2) in the above equation

$$m_l\frac{du_l}{dt} - u_l\frac{dm_{lg}}{dt} = (h_v - h_o)\frac{dm_{lg}}{dt} \tag{4.10}$$

Therefore the rate of change of specific internal energy can be obtained as

$$\frac{du_l}{dt} = \frac{(h_v - h_o + u_l)}{m_l}\frac{dm_{lg}}{dt} \tag{4.11}$$

Now the rate of change of density is calculated with the following equation. The density of the liquid medium is given as

$$\rho_l = \frac{m_l}{v_l} \tag{4.12}$$

The equation is now differentiated with time which is obtained as

$$\frac{d\rho_l}{dt} = \frac{1}{V_l}\left(\frac{dm_l}{dt}\right) - \frac{1}{V_l^2}\left(m_l\frac{dV_l}{dt}\right) \tag{4.13}$$

Now substituting equation (4.2) in the above equation, the equation results to

$$\frac{d\rho_l}{dt} = -\frac{1}{V_l}\left(\frac{dm_{lg}}{dt}\right) - \frac{1}{V_l^2}\left(m_l\frac{dV_l}{dt}\right) \tag{4.14}$$

The internal energy of the liquid medium is a function of density(ρ) and temperature(T) of the liquid medium.

$$u_l = f(\rho_l, T_l) \tag{4.15}$$

Using the chain rule,

$$\frac{du_l}{dt} = \frac{\partial u_l}{\partial \rho_l}\frac{\partial \rho_l}{\partial t} + \frac{\partial u_l}{\partial T}\frac{\partial T_l}{\partial t} \tag{4.16}$$

if δ is very small the above equation can be written as

$$\frac{du_l}{dt} = \frac{\partial u_l}{\partial \rho_l}\frac{d\rho_l}{dt} + \frac{\partial u_l}{\partial T}\frac{dT_l}{dt} \tag{4.17}$$

Now Substituting (4.14) and (4.11) in (4.17), we get the rate of change of temperature in the control volume which is given by the following equations,

$$\frac{dT_l}{dt} = \frac{1}{\frac{\partial u_l}{\partial T_l}}\left(\frac{1}{\rho_l v_l}(h_v - h_o + u_l)\frac{dm_{lg}}{dt}\right) \tag{4.18}$$

Therefore the two equations that are used to create a model are obtained and are rewritten as follows

$$\frac{dT_l}{dt} = \frac{1}{\frac{\partial u_l}{\partial T_l}}\left(\frac{1}{\rho_l v_l}(h_v - h_o + u_l)\frac{dm_{lg}}{dt}\right) \tag{4.19}$$

$$\frac{d\rho_l}{dt} = -\frac{1}{V_l}\left(\frac{dm_{lg}}{dt}\right) - \frac{1}{V_l^2}\left(m_l\frac{dV_l}{dt}\right) \tag{4.20}$$

where

h_v is heat of vaporization, which is obtained from the function 'hvdata' based on the thermodynamic properties of nitrogen from pressures 12.5 *kpa* to 3397.8 *kpa*.

h_o is the enthalpy at the throat. This can be obtained from the following equation

$$h_o = C_p T_o \tag{4.21}$$

C_p and T_o are the specific heat and temperature of the nitrogen at the evaporated phase from liquid. This can be obtained by the following equations

$$C_p = \frac{5}{2}R + R + f_v(t) + f_e(t) \tag{4.22}$$

where

$$f_v(t) = R \sum_{i=1}^{3a-5} \left(\frac{{x_i}^2 {e_i}^x}{({e_i}^x - 1)^2} \right) \tag{4.23}$$

$f_e(t)$ is very small and hence eliminated

$$T_o = \frac{T_l}{1 + \frac{(k-1)}{2} M^2} \tag{4.24}$$

where

T_o is the temperature of the evaporated phase of the liquid

T_l is the temperature of the liquid medium

4.3.2 Deriving equations for the gaseous medium

In this section, the equations for the rate of change of internal energy and density with time are calculated similar to the above section with some small changes. These are obtained as below.

In this case the control volume of the gas is considered where there is an input and output in this case. The input is from the liquid control volume and the output is to the nozzle.

The mass flow rate in the control volume can be given by the following equation

$$\frac{dm_g}{dt} = \frac{dm_i}{dt} - \frac{dm_o}{dt} \tag{4.25}$$

From the above equation, the mass input to the system is obtained from the liquid medium and the mass going out is from the nozzle.

$$m_i = m_{lg}; m_o = m_g \tag{4.26}$$

substituting the above equations in (4.25) the equation is reduced to

$$\frac{dm_g}{dt} = \frac{dm_{lg}}{dt} - \frac{dm_g}{dt} \tag{4.27}$$

Applying first law of thermodynamics to the control volume and this equation has already been derived in the second chapter. Rewriting the equation (2.16) and neglecting the potential and kinetic energies the equation is resulted to

$$\frac{dQ}{dt} + \frac{dm_i}{dt}h_i = \frac{dm_e}{dt}(h_o + \frac{v_o^2}{2}) + \frac{dW_l}{dt} \tag{4.28}$$

In this case, there is no heat energy. Therefore,

$$Q = 0 \tag{4.29}$$

The work done is zero. Therefore,

$$W = 0 \tag{4.30}$$

We know that the internal energy in the cylinder can be given by $U_g = u_g m_g$ and differentiating this equation results to

$$\frac{dU_g}{dt} = u_g\frac{dm_g}{dt} + m_g\frac{du_g}{dt} \tag{4.31}$$

substituting (4.27), (4.26), (4.29), (4.30), (4.31) in (4.28)

$$\frac{du_g}{dt} = \frac{1}{m_g}\left((h_i - u_g)\frac{dm_{lg}}{dt} - (h_o + \frac{{v_e}^2}{2} + u_g)\frac{dm_g}{dt}\right) \tag{4.32}$$

Now the rate of change of density can be calculated with the following equations

$$\rho_g = \frac{m_g}{v_g} \tag{4.33}$$

Differentiating equation (4.4) with respect to time

$$\frac{d\rho_g}{dt} = \frac{1}{V_g}\left(\frac{dm_g}{dt}\right) - \frac{1}{{V_g}^2}\left(m_g\frac{dV_g}{dt}\right) \tag{4.34}$$

Now substituting (4.27) in (4.41), the equation is reduced to

$$\frac{d\rho_g}{dt} = \frac{1}{V_g}\left(\frac{dm_{lg}}{dt} - \frac{dm_g}{dt}\right) - \frac{1}{V_g^2}\left(m_g\frac{dV_g}{dt}\right) \tag{4.35}$$

The internal energy of the gas medium is a function of density(ρ) and temperature(T) of the gas medium.

$$u_g = f(\rho_g, T_g) \tag{4.36}$$

Using the chain rule,

$$\frac{du_g}{dt} = \frac{\partial u_g}{\partial \rho_g}\frac{\partial \rho_g}{\partial t} + \frac{\partial u_g}{\partial T}\frac{\partial T_g}{\partial t} \tag{4.37}$$

if δ is very small the above equation can be written as

$$\frac{du_g}{dt} = \frac{\partial u_g}{\partial \rho_g}\frac{d\rho_g}{dt} + \frac{\partial u_g}{\partial T}\frac{dT_g}{dt} \tag{4.38}$$

Now substituting (4.32) and (4.41) in (4.38), we get the rate of change of temperature in the gas medium and it is given as,

$$\frac{dT_g}{dt} = \left(\frac{1}{\frac{\partial u_g}{\partial T_g}}\right)\left(\frac{1}{m_g}\left((h_i - u_g)\frac{dm_{lg}}{dt} - (h_o + \frac{{v_0}^2}{2} + u_g)\frac{dm_g}{dt}\right) - \frac{\partial u_g}{\partial \rho_g}\left(\frac{1}{v_g}\left(\frac{dm_{lg}}{dt} - \frac{dm_g}{dt}\right) - \frac{\rho_g}{v_g}\frac{dv_g}{dt}\right)\right) \tag{4.39}$$

Therefore the two equations that are used in creating a model in simulink are rewritten as

$$\frac{dT_g}{dt} = \left(\frac{1}{\frac{\partial u_g}{\partial T_g}}\right)\left(\frac{1}{m_g}\left((h_i - u_g)\frac{dm_{lg}}{dt} - (h_o + \frac{{v_0}^2}{2} + u_g)\frac{dm_g}{dt}\right) - \frac{\partial u_g}{\partial \rho_g}\left(\frac{1}{v_g}\left(\frac{dm_{lg}}{dt} - \frac{dm_g}{dt}\right) - \frac{\rho_g}{v_g}\frac{dv_g}{dt}\right)\right) \tag{4.40}$$

$$\frac{d\rho_g}{dt} = \frac{1}{V_g}\left(\frac{dm_{lg}}{dt} - \frac{dm_g}{dt}\right) - \frac{1}{V_g^2}\left(m_g\frac{dV_g}{dt}\right) \tag{4.41}$$

where

h_i is the enthalpy between liquid to gas medium

h_o is the outside enthalpy at throat and this can be obtained as in the liquid medium.

$$h_o = C_pT_e \tag{4.42}$$

C_p and T_e are the specific heat and temperature at throat. This can be obtained by the following equations

$$C_p = \frac{5}{2}R + R + f_v(t) + f_e(t) \tag{4.43}$$

where

$$f_v(t) = R\sum_{i=1}^{3a-5}\left(\frac{{x_i}^2{e_i}^x}{({e_i}^x - 1)^2}\right) \tag{4.44}$$

$f_e(t)$ is very small and hence eliminated

$$T_e = \frac{T_g}{1 + \frac{(k-1)}{2} M^2} \tag{4.45}$$

The other parameters like mass flow rate of gas, exit velocity and mach number can be obtained from the following equations

$$\frac{dm_o}{dt} = \frac{P_{cv} M}{\left(1 + \left(\frac{k-1}{2}\right) M^2\right)^{\frac{k+1}{2(k-1)}}} \sqrt{\frac{k}{T_o R}} \tag{4.46}$$

$$M = \sqrt{\frac{2}{k-1}\left(\left(\frac{P_{cv}}{P_a}\right)^{\frac{k-1}{k}} - 1\right)} \tag{4.47}$$

$$v_e = \sqrt{T_{cv} R \left(\frac{2k}{k-1}\right)\left(1 - \frac{p_a}{p_{cv}}\right)} \tag{4.48}$$

The initial mach number can be obtained from the following equation,

$$\frac{A}{A^*} = \frac{1}{M}\left(\left(\frac{2}{(k+1)}\right)\left(1 + \frac{(k-1)}{2} M^2\right)\right)^{\frac{(k+1)}{2(k-1)}} \tag{4.49}$$

where

A = Area of the cross-section of the nozzle,

A^* = Area of the cross-section of the nozzle where $M = 1$ (at choking)

M = Mach Number

k = Adiabatic Index

In the Present case, as I could not measure the area of the cross section of the nozzle at the point of choking, I assumed the area of the choking of the nozzle is three times smaller than the area of the cross section of the flow of the nozzle. Hence,

$$\frac{A}{A^*} = 3 \tag{4.50}$$

and substituting the adibatic constant $k = 1.3$, we get the mach number as 0.1996, which is used in the simulink model, for intial mach number.

The next important step is to obtain an equation for the rate of evaporation of liquid to gas from liquid to gas medium. The term $\frac{dm_{lg}}{dt}$ has to be found out to make a relation between the liquid phase and gaseous phase of the two phase gas medium. This equation can be found out in the following section.

4.3.3 Calculating the rate of Evaporation

The four differential equations for the liquid and gaseous states need one additional equation to establish a relation ship. The equation which establishes the relationship gives the rate of evaporation from liquid medium to gaseous medium[10]. This can be obtained from the pressure compatibility equation which is given as follows

$$p_l(\rho_l, T_l) = p_g(\rho_g, T_g). \tag{4.51}$$

Differentiating partially equation (4.51)

$$\frac{\partial p_l}{\partial \rho_l}\frac{\delta \rho_l}{\delta t} + \frac{\partial p_l}{\partial T_l}\frac{\delta T_l}{\delta t} = \frac{\partial p_g}{\partial \rho_g}\frac{\delta \rho_g}{\delta t} + \frac{\partial p_g}{\partial T_g}\frac{\delta T_g}{\delta t} \tag{4.52}$$

if δ is very small the above equation can be written as

$$\frac{\partial p_l}{\partial \rho_l}\frac{d\rho_l}{dt} + \frac{\partial p_l}{\partial T_l}\frac{dT_l}{dt} = \frac{\partial p_g}{\partial \rho_g}\frac{d\rho_g}{dt} + \frac{\partial p_g}{\partial T_g}\frac{dT_g}{dt} \tag{4.53}$$

substituting the the rate of change of temperature and density for both the liquid and gas mediums in (4.53), the rate of evaporation is obtained as follows

$$\frac{dm_{lg}}{dt} = \frac{\frac{1}{v_g}\frac{\partial p_g}{\partial \rho_g}\frac{dm_g}{dt} + \frac{\rho_g}{v_g}\frac{\partial p_g}{\partial \rho_g}\frac{dv_g}{dt} - \frac{\partial p_g}{\partial T_g}\frac{1}{\frac{\partial u_g}{\partial T_g}}\frac{1}{\rho_g v_g}\frac{dm_g}{dt}(h_o + \frac{{v_o}^2}{2} + u_g) - \frac{\partial u_g}{\partial \rho_g}\frac{\partial p_g}{\partial\ T_g}\frac{1}{\frac{\partial u_g}{\partial T_g}}\frac{\rho_g}{v_g}\frac{dv_g}{dt}}{\frac{\partial p_g}{\partial\ T_g}\frac{1}{\frac{\partial u_g}{\partial T_g}}\frac{(h_i - u_g)}{\rho_g v_g} + \frac{\partial p_g}{\partial \rho_g}\frac{1}{v_g} - \frac{\partial u_g}{\partial \rho_g}\frac{\partial p_g}{\partial T_g}\frac{1}{\frac{\partial u_g}{\partial T_g}}\frac{1}{v_g} - \frac{\partial p_l}{\partial T_l}\frac{1}{\frac{\partial u_l}{\partial T_l}}\frac{1}{\rho_l v_l}(h_v - h_o + u_l)} \tag{4.54}$$

From the above equation, it can be found that, the term for the rate of change of volume of gas is not defined any where in the two phase section. This can be obtained from the rate of change of density of liquid equation, which has been assumed as zero. Rewriting the equation,

$$\frac{d\rho_l}{dt} = -\frac{1}{V_l}\left(\frac{dm_{lg}}{dt}\right) - \frac{1}{V_l^2}\left(m_l\frac{dV_l}{dt}\right) = 0 \tag{4.55}$$

Rearranging the above equation

$$\frac{1}{V_l}\left(\frac{dm_{lg}}{dt}\right) = -\frac{1}{V_l^2}\left(m_l\frac{dV_l}{dt}\right) \tag{4.56}$$

This results to

$$\frac{dV_l}{dt} = -\frac{1}{\rho_l}\frac{dm_{lg}}{dt} \tag{4.57}$$

4.4 Change in Ullage

The Propellant is discharged to the engine which results to the decrease in resident propellant volume and subsequent increase in ullage volume. [11]We know that, In a control volume, the total volume is always constant and

$$\frac{dV_g}{dt} = -\frac{dV_l}{dt} \tag{4.58}$$

and

$$V = V_g + V_l \tag{4.59}$$

Therefore, the rate of change of volume can be given as

$$\frac{dV_g}{dt} = \frac{(V_g - V)}{m_l}\frac{dm_{lg}}{dt} \tag{4.60}$$

4.5 Creating the Model

A simulink model has been created using the above derived state space equations. The model can be shown in the following figure. The model is then simulated in simulink until the flow gets choked. The initial parameters that are given to the model are the temperature of liquid and gas along with the density of liquid and gas.

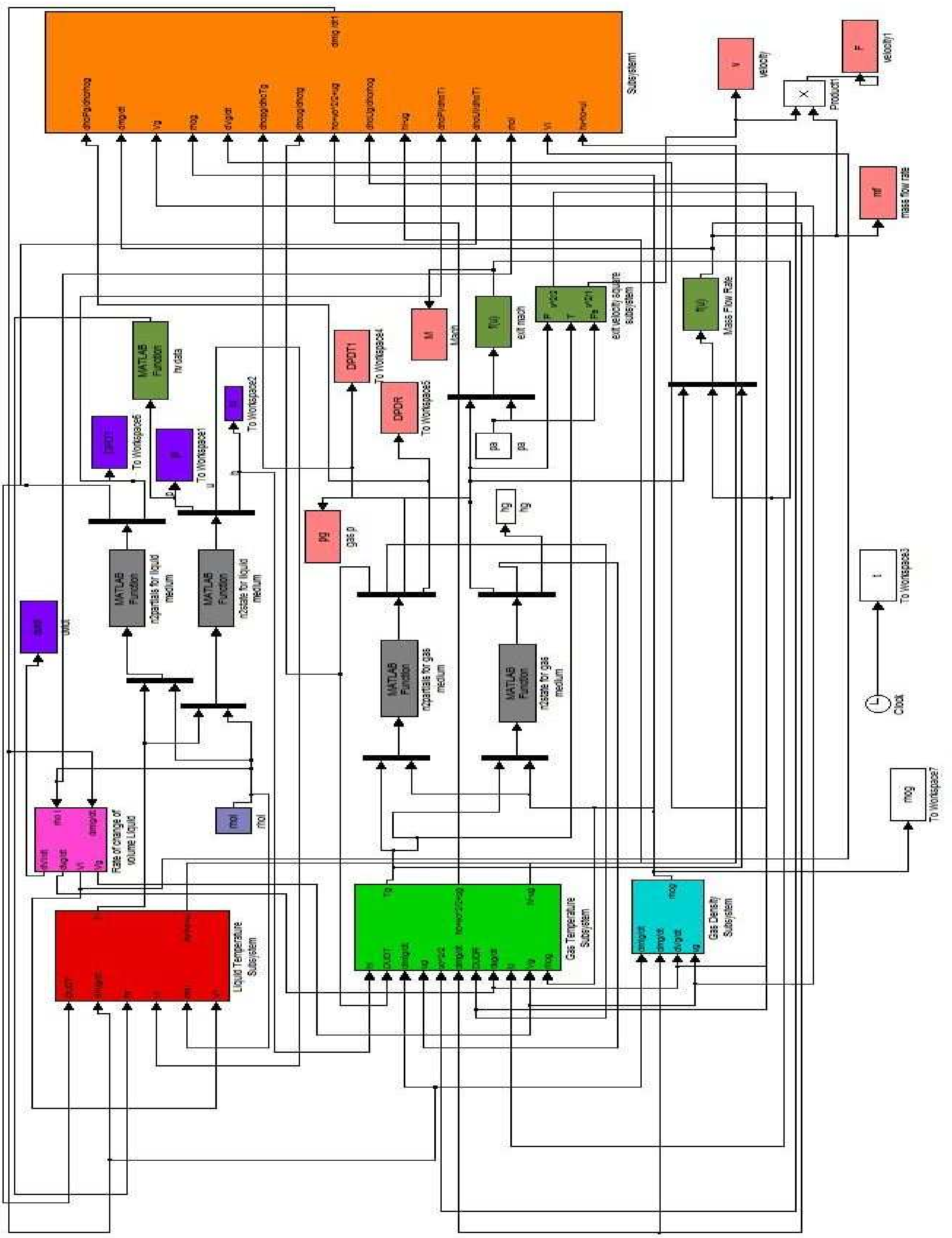

Figure 4.2: Two Phase Simulink Model

4.6 Functions used in the Model

The simulink model that is used has basically one function *n2state.m* and two derived functions *n2partialsl.m* and *n2partialsg.m*. *n2state.m* function uses temperature and density as an input and derives pressure, internal energy and enthalpy as an output. *n2partialsl.m* function uses the temperature and density of liquid as an input and derives, $\frac{\partial U_l}{\partial t}$ and $\frac{\partial P_l}{\partial t}$. *n2partialsg.m* function uses temperature and density of gas as an input and derives $\frac{\partial U_g}{\partial \rho_g}$, $\frac{\partial P_g}{\partial t}$ and $\frac{\partial P_g}{\partial \rho_g}$.

4.7 Significance of the blocks in the model

There are four major blocks which are equivalent to the four equations that are derived for the liquid and gas medium in this section. The red and violet blocks which are named as block 1 and block 2 uses the two equations that are derived for the liquid medium. The green and blue blocks which are named as block 3 and block 4 uses the two equations that are derived for the gas medium. The magnenta block which is block 5 is used in calculating the rate of change of volume of liquid which is obtained by considering the density of liquid as constant. The four grey blocks, blocks 6,7,8,9 are the matlab functions that are explained in the above section. The dark blue blocks gives the output of the variables obtained from the liquid medium. The pink blocks gives the output of the variables obtained from the gas medium. The orange block calculates the rate of evaporation of the liquid to the gas medium which is obtained from the pressure compatibility equation.

4.8 Simulating the model

Before initializing the model, the model needs initial conditions which includes

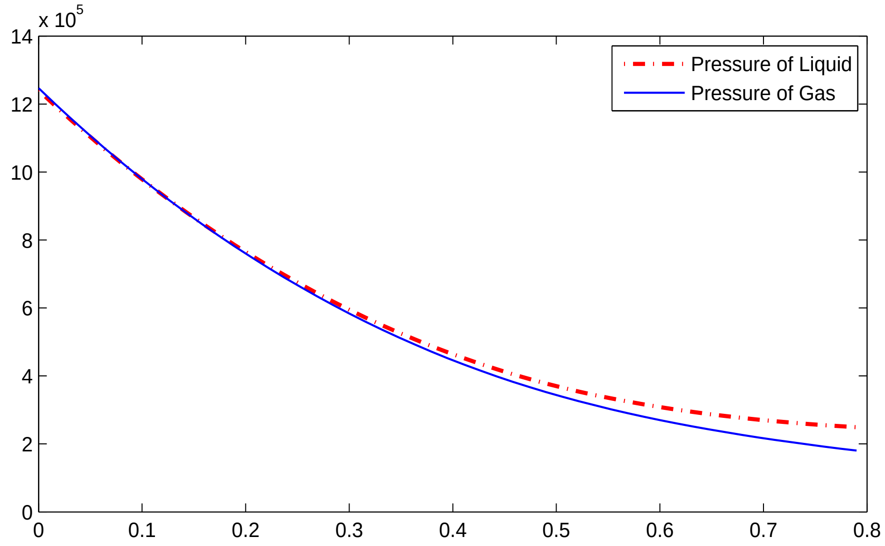

Figure 4.3: Time Vs Pressure

- Temperature of liquid, $T_l = 120K$
- Temperature of gas, $T_g = 151K$
- Density of liquid, $\rho_l = 400kg/m^3$
- Density of gas, $\rho_g = 30kg/m^3$
- Gas constant for Nitrogen, $R = 296.8Kj/Kg - K$
- Area of the Cross-section of the Nozzle, $Ae = 0.00005m^2$
- Atmospheric pressure, $Pa = 100000pa$
- Adiabatic index, $k = 1.3$
- Mach number, $M = 0.1996$

The model is run for a sample time of 0.79*seconds* until the flow gets choked.

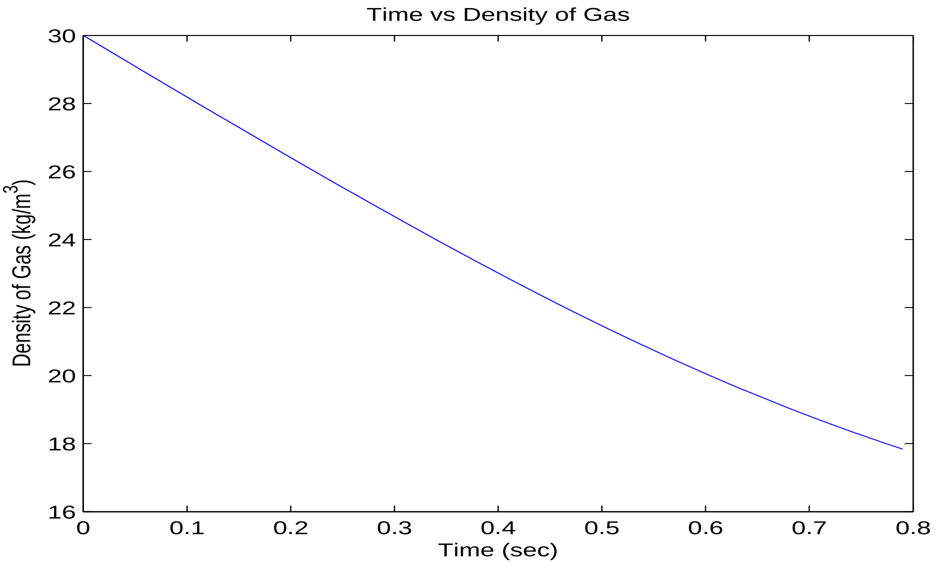

Figure 4.4: Time Vs Gas Density

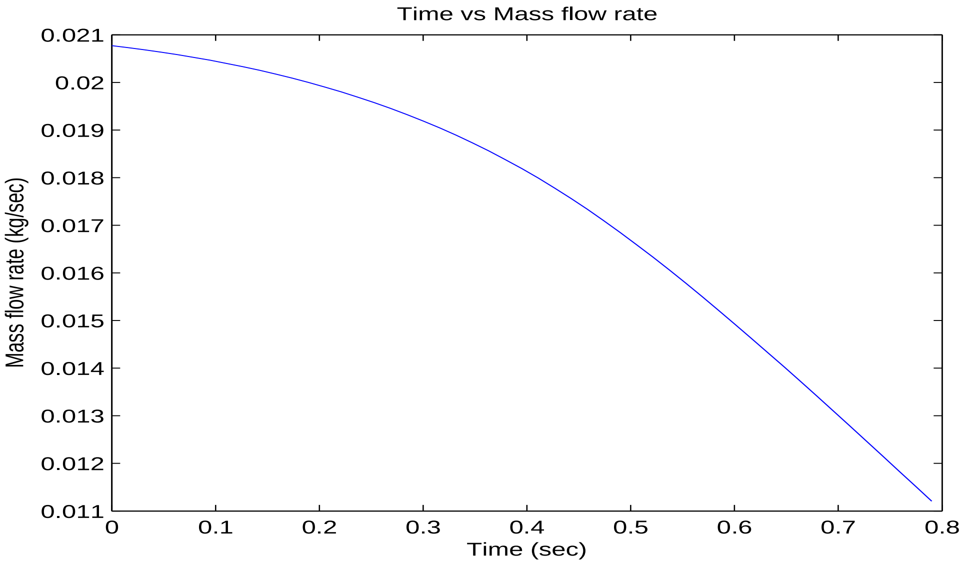

Figure 4.5: Time Vs Mass flow

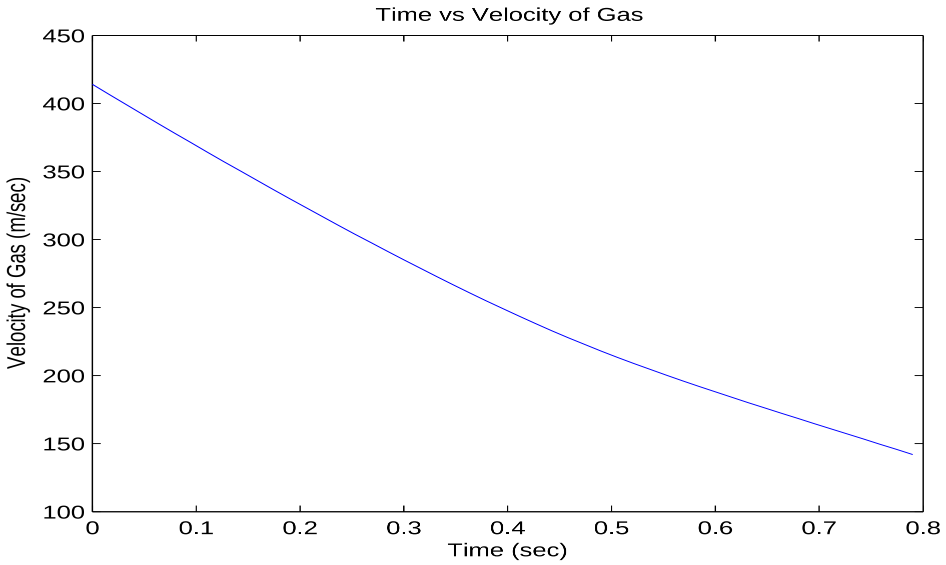

Figure 4.6: Time Vs Velocity

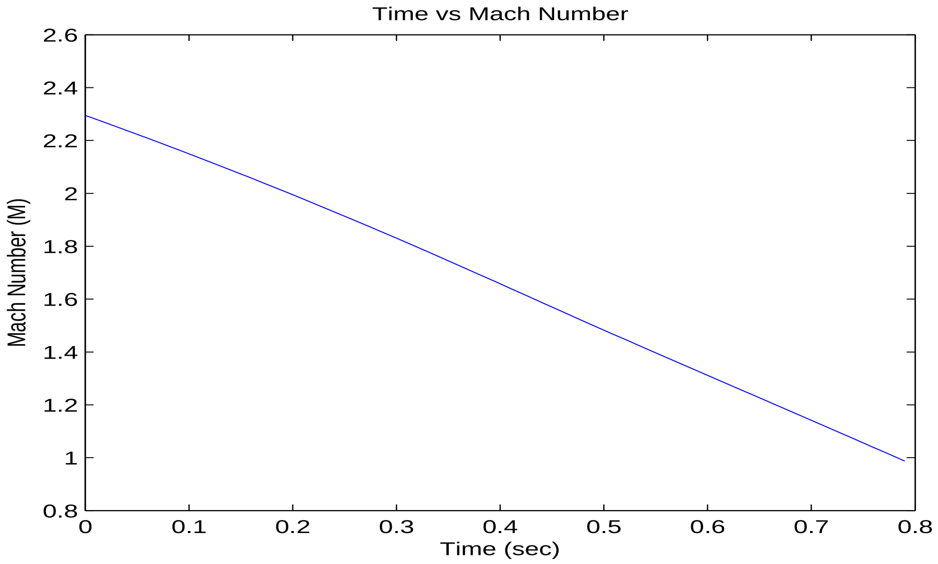

Figure 4.7: Time Vs Mach Number

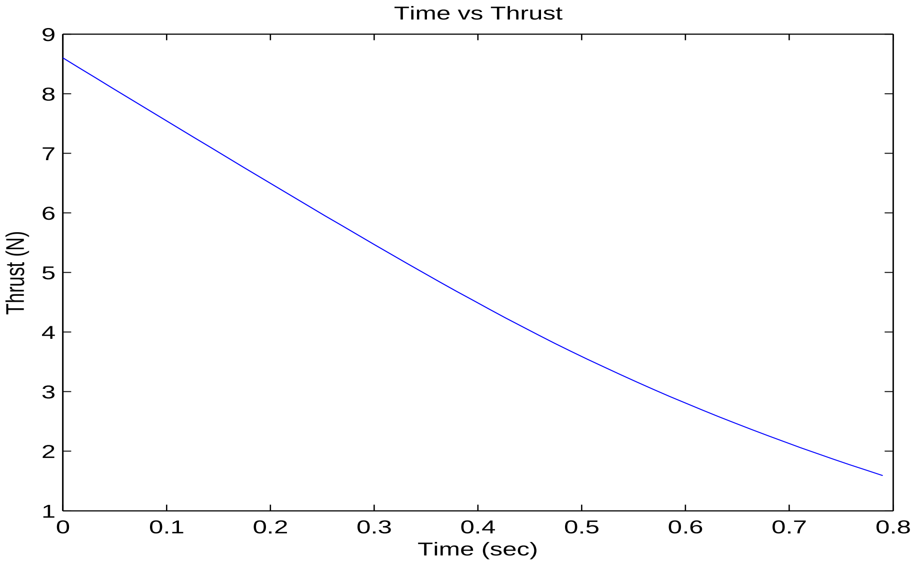

Figure 4.8: Time Vs Thrust

CHAPTER V

CONCLUSIONS AND FUTURE WORK

5.1 Conclusions

The profile of thrust is calculated for both the single phase and two phase propellant tanks. Considering the results of the single phase, the maximum thrust obtained for the first two simulations is given by 1.69 N and 2.85 N respectively. If i take the first two simulations, both nozzles have the same cross-sectional area. The reason for the increase in thrust can be attributed to the greater mass flow rate from the nozzle. If i compare the third and fourth simulations both have the same parameters except for a different cross-sectional area of the nozzle. The resultant thrust obtained for the third and fourth simulations is be given by 0.4 N and 0.04 N. The decrease in the thrust can be attributed to the decrease in the area of the cross section of the nozzle.

The thrust obtained for the two phase propellant tank is given as 8.6 N. In addition to the thrust various thermodynamic properties are also obtained for each time step. The different trajectories for the pressure of liquid and pressure of gas is because of the first order approximation from the pressure compatibility equation, which has been used to calculate the rate at which the liquid is converted to gas. The velocity of the gas, the mass flow rate also started with 410 m/sec and .02075 kg/sec and decreased as the simulation reached choked conditions.

This model can be used in building a control system for a propellant tank which is used in launching various space vehicles.

5.2 Future Improvements

The present work can be extended in future in various directions. Some of the ideas for future recommendations would include the consideration of heat flow and frictional resistance. The other recommendation can be considering the geometrical cross section of the nozzle and if necessary a separate sub system can be made for the cross section of the nozzle. The present model has neglected potential energy while deriving equations. So this model can be extended by considering potential energy for deriving equations where two pipes are placed at two different locations i.e at two different heights which makes the inclusion of potential energy in the model. An experiment can be practically performed with the given set of experimental parameters and the obtained results can be compared to the present work. This model is valid until the flow gets choked. It can be further extended after choking, even though the mass flow rate is constant after choking other thermodynamic properties can be studied. The present model is made by considering density of the liquid nitrogen as constant because, the change of liquid density is really slow. This model can be further developed by taking the density of liquid as not a constant factor and including the rate of change of density of liquid nitrogen.

BIBLIOGRAPHY

[1] *Rocket Propellant and Pressurization Systems.* Prentice-Hall Space Technology Series, 1964.

[2] Marcia L. Huber and Jiann C. Yang. A thermodynamic analysis of fuel vapor characteristics in an aircraft fuel tank ullage. *Fire Safety Journal*, 2002.

[3] Gerstein M Kosvic TC, Zung LB. Analysis of aircraft fuel tank fire and explosion hazards. *Technical Report AFAPL-TR-71-7, Wright Patterson Air Force Base*, 1971.

[4] Gordon J. Van wylen Richard E. Sonntag. *Fundamentals of Classical Thermodynamics.* John Wiley & Sons, 1976.

[5] *Introduction to Fluid Dynamics.* John Wiley & Sons, Inc, 1992.

[6] Wikipedia the free encyclopedia. De laval nozzle article. *http://en.wikipedia.org/wiki/De Laval nozzle*, 2006.

[7] National Aeronautics and Space Administration. Mass flow choking. *http://www.grc.nasa.gov/WWW/K-12/airplane/mflchk.html*, 2006.

[8] Wikipedia the free encyclopedia. Mach number article. *http://en.wikipedia.org/wiki/Mach number*, 2006.

[9] *Fundamentals of Thermodynamics.* John Wiley & Sons, Inc, 1998.

[10] Randolf F. Follett. Ctf dynamic fluid flow modeling program. *NASA John C. Stennis Space Center*, 1996.

[11] Alok Majumdar and Todd Steadman. Numerical modeling of pressurization of a propellant tank. *American Institute of Aeronautics and Astronautics*, 1999.

APPENDICES

APPENDIX A

MATLAB PROGRAMS

Unsteady State Conditions

n2state.m

```
%N2state(T,rho,units)
%This function returns thermodynamic properties of N2 given the
temperature and density.
% units can be 'SI' or 'UK'
% when 'SI' is chosen, the temperature is taken to be in degrees Kelvin
and the density in kg/m^3. The answers are given in Pa, J, K
% when 'UK' is chosen, the temperature is taken to be in degrees Rankine,
and the density in lbm/ft^3. The answers are given in psia, Btu, R
% Programmer: Hanz Richter, NASA, Winter 2004.
% Correlation taken from Span et. al., "A Reference Equation of State for
the Thermodynamic Properties of
% Nitrogen for Temperatures from 63.151 to 1000K and Pressures to
2200 MPa", Journal of Physical and Chemical
```

```
% Reference Data", Vol 29, N.6, 2000, pp. 1361-ff.",

function props=n2state(T,rho,units)


Pc=3.3958e6;                %critical pressure in Pa
Tc=126.1920;                %critical temperature in K
factor=1;


rhoc=11.1839;              %critical density in mol/dm^3
M=0.02801348;            %molecular weight, kg/mol
rhoc=rhoc*M/0.1^3;     %critical density in kg/m^3
Tc=126.192;                %critical temperature in K
Rm=8.314510;               %gas constant in J/mol/K
R=Rm/M;                      %gas constant in J/kg/K
factor_press=1; factor_rho=1; factor_u=1; N(1:6)=[0.924803575275
-0.492448489428 0.661883336938 -0.192902649201e1 -0.622469309629e-1
0.349943957581]; N(7:12)=[0.564857472498 -0.161720005987e1
-0.481395031883 0.421150636384 -0.161962230825e-1 0.172100994165];
N(13:18)=[0.735448924933e-2 0.168077305479e-1 -0.107626664179e-2
-0.137318088513e-1 0.635466899859e-3 0.304432279419e-2];
N(19:24)=[-0.435762336045e-1 -0.723174889316e-1 0.389644315272e-1
-0.212201363910e-1 0.408822981509e-2 -0.551990017984e-4];
N(25:30)=[-0.462016716479e-1 -0.300311716011e-2 0.368825891208e-1
-0.255856846220e-2 0.896915264558e-2 -0.441513370350e-2];
N(31:36)=[0.133722924858e-2 0.264832491957e-3 0.196688194015e2
```

```
-0.209115600730e2 0.167788306989e-1 0.262767566274e4];

ik=[1 1 2 2 3 3 1 1 1 3 3 4 6 6 7 7 8 8 1 2 3 4 5 8 4 5 5 8 3 5 6 9
1 1 3 2]; jk=[0.25 0.875 0.5 0.875 0.375 0.75 0.5 0.75 2 1.25 3.5 1
0.5 3 0 2.75 0.75 2.5 4 6 6 3 3 6 16 11 15 12 12 7 4 16 0 1 2 3];
lk=[0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 2 2 2 2 2 2 3 3 3 3 4 4 4 4
2 2 2 2];

a=[2.5 -12.76952708 -0.00784163 -1.934819e-4 -1.247742e-5
6.678326e-8 1.012941 26.65788];

phi=[20 20 15 25]; bet=[325 325 300 275];
gam=[1.16 1.16 1.13 1.25];  %careful: these are indexed starting with 33 in
                             the formulas. Note index shift in the summations
                             below.

if (strcmp(units,'UK'))
    factor_rho=1/6.24279605761e-2;      %factor to go from lbm/ft^3 to kg/m^3
    T=((T-459.67)-32)*5/9+273.15;       %convert to Kelvin for entering formula
    rhoc=rhoc/factor_rho;               %convert critical density to lbm/ft^3
    factor_press=1.450377377e-4;        %conversion factor to from Pa to psi
    factor_u=4.29922613929e-4;          %conversion factor from J/kg to Btu/lbm
end

del=rho/rhoc;   %nondimensional
tau=Tc/T;         %nondimensional (note: it's the reciprocal reduced temp! I
                                    spent half a day trying to find an error,
```

```
                                   I had T/Tc)



%Computation of partial derivatives of Helmholtz free energy:


sum1=0; sum2=0; sum3=0; sum4=0; sum5=0; sum6=0; for i=1:36,
    sum1=sum1+(i<=6)*ik(i)*N(i)*del^ik(i)*tau^jk(i);
    sum2=sum2+(i>=7 & i<=32)*N(i)*del^ik(i)*tau^jk(i)*exp(-del^lk(i))*(ik(i)
         -lk(i)*del^lk(i));
    if i>=33,
      sum3=sum3+N(i)*del^ik(i)*tau^jk(i)*exp(-phi(i-32)*(del-1)^2-bet(i-32)
      *(tau-gam(i-32))^2)*(ik(i)-2*del*phi(i-32)*(del-1));
      sum6=sum6+N(i)*del^ik(i)*tau^jk(i)*exp(-phi(i-32)*(del-1)^2-bet(i-32)
      *(tau-gam(i-32))^2)*(jk(i)-2*tau*phi(i-32)*(tau-gam(i-32)));
    end
    sum4=sum4+(i<=6)*jk(i)*N(i)*del^ik(i)*tau^jk(i);
    sum5=sum5+(i>=7 & i<=32)*jk(i)*N(i)*del^ik(i)*tau^jk(i)*exp(-del^lk(i));
end del_Dalpha_rDdel=sum1+sum2+sum3;
tau_Dalpha_rDtau=sum4+sum5+sum6;
tau_Dalpha_idDtau=a(1)+a(3)*tau-a(4)/tau-2*a(5)*tau^(-2)-3*a(6)*tau^(-3)
                  +a(7)*a(8)*tau/(exp(a(8)*tau)-1);


%Pressure:
press=factor_press*rho*factor_rho*R*T*(1+del_Dalpha_rDdel);


%Internal Energy: (including shifting)
u=factor_u*(R*T*(tau_Dalpha_idDtau+tau_Dalpha_rDtau)+0.21175922182738);
```

```
%Enthalpy:  (including shifting)
h=factor_u*(R*T*((tau_Dalpha_idDtau+tau_Dalpha_rDtau)+del_Dalpha_rDdel+1)
  +0.21175922182738);

props=[press u h];
```

n2partials.m

```
% returns the DUDT(dhoU/dhoT) and DUDR(dhoU/dhoR) for an input of
%temperature(t) and density(rho)

function n2partials = n2partials(T,rho);
delta = 0.02;
ut1=n2state(T-delta,rho,'SI');
ut2 = n2state(T+delta,rho,'SI');
x=ut1(2);
y=ut2(2);
DUDT=(y-x)/2/delta;
ur1 = n2state(T,rho-delta,'SI');
ur2 = n2state(T,rho+delta,'SI');
a=ur1(2);
b=ur2(2);
DUDR=(b-a)/2/delta;
n2partials = [DUDT,DUDR];
```

n2partialsl.m

```
%returns DPDT(dhoP/dhoT) and DUDT(dhoU/dhoT) for a given
%input of temperature and density



function n2partials = n2partials(t,rho);
delta = 0.02;
ut1=n2state(t-delta,rho,'SI');
ut2 = n2state(t+delta,rho,'SI');
x=ut1(1);
y=ut2(1);
DPDT=(y-x)/2/delta;
a=ut1(2);
b=ut2(2);
DUDT=(b-a)/2/delta;


n2partials = [DUDT,DPDT];
```

n2partialsg.m

```
%returns DUDT(dhoU/dhoT), DUDR(dhoU/dhoR), DPDT(dhoP/dhoT),DPDR(dhoP/dhoR)
%for a given input of temperature and density


function n2partials = n2partials(t,rho);
delta = 0.02;
ut1 = n2state(t-delta,rho,'SI');
ut2 = n2state(t+delta,rho,'SI');
x=ut1(1);
y=ut2(1);
DPDT=(y-x)/2/delta;
a=ut1(2);
b=ut2(2);
DUDT=(b-a)/2/delta;

ur1 = n2state(t,rho-delta,'SI');
ur2 = n2state(t,rho+delta,'SI');
c=ur1(1);
d=ur2(1);
DPDR=(d-c)/2/delta;
e=ur1(2);
f=ur2(2);
DUDR=(f-e)/2/delta;
n2partials = [DUDT,DUDR,DPDT,DPDR];
```