A Risk-Based Approach for Examining Vertical Separation Distances in On-Site Wastewater Treatment Systems

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A RISK-BASED APPROACH FOR EXAMINING VERTICAL SEPARATION DISTANCES IN ON-SITE WASTEWATER TREATMENT SYSTEMS

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Bachelor of Science Civil Engineering
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August, 2009

submitted in partial fulfillment of requirements for the degree

MASTER OF SCIENCE IN CIVIL ENGINEERING
at the
CLEVELAND STATE UNIVERSITY
August, 2010
This thesis has been approved
for the Department of CIVIL AND ENVIRONMENTAL ENGINEERING
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ACKNOWLEDGEMENT

I would like to thank my thesis advisor, Dr. Mark Tumeo, for his patience, guidance, and unwavering support over the last few years. I am very fortunate to have had the chance to work with him. This thesis is as much a product of his guidance and support as it is my own effort.

I would also like to thank the other members of my thesis committee, Dr. Duffy and Rebecca Fugitt, for their time and suggestions, which lead to many important improvements to the thesis.

Finally, I would like to thank my wife, Melissa, for her understanding and support over the last few years while I was working on this thesis.
A RISK-BASED APPROACH FOR EXAMINING VERTICAL SEPARATION DISTANCES IN ON-SITE WASTEWATER TREATMENT SYSTEMS

ANTHONY M. JANICEK

ABSTRACT

Regulations regarding the use of on-site wastewater treatment systems in many states lack a sufficient scientific basis, which in many cases restricts the use of on-site systems, drives up cost, and restricts innovation of new treatment technologies. Of particular regulatory concern is the minimum vertical separation distance (VSD) located in the area between the trench bottom of the subsurface soil absorption system and any restricting or limiting layer. The minimum VSD needed for proper effluent treatment is based on many complex and interrelated factors regarding physical, chemical, and biological soil conditions at a particular site. Research has shown that depending on soil type and conditions, VSD between 1.5 feet and 4 feet is enough to adequately treat effluent yet many states use a “one size fits all” approach when setting regulations for on-site treatment systems. A stochastic mathematical model has been developed that provides an estimation of the probability that a contaminant concentration will reach a certain point below the trench bottom of the subsurface soil absorption system. This model has been incorporated into a simple, easy-to-use, Excel® based computer program that allows the user to evaluate the potential range of fecal coliform concentrations that may reach a specified groundwater or surface water location. This model has been developed to aid regulators, land use planners, and designers to quickly evaluate the associated risks of contamination from a specified on-site wastewater treatment system in a specified soil.
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CHAPTER 1
INTRODUCTION

Once considered appropriate only for rural areas where construction of large, centralized systems was not practical, on-site systems are currently being used as an effective tool for wastewater management in rural areas as well as, in both large and small urban centers (Siegrist et al., 2007). Knowledge gained through research and experience as well as, advances in treatment technology, have lead to the widespread use of on-site wastewater treatment systems as a permanent solution to wastewater management (Siegrist et al., 2005; USEPA, 2002). Today, on-site systems, also referred to as decentralized systems, are used to treat waste not only from individual homes but also from small housing developments, or clusters, as well as commercial establishments such as restaurants and hotels.

On-site wastewater treatment is the collection, treatment, and disposal or reuse of wastewater at or near the location in which the waste is generated (Crites & Tchobanoglous, 1998). Approximately 25% of the United States Population and 35% of all new construction are served by on-site wastewater treatment systems, which collect, treat, and release approximately 4 billion gallons of treated effluent per day (Lowe & Siegrist, 2007, USEPA, 2002; US Census Bureau, 1997). When properly designed and
maintained on-site sewage treatment systems provide a high level of treatment efficiency at a low cost while simultaneously protecting public health and environmental quality (Hall, 1990; Siegrist et al., 2001). However, when not properly designed and maintained, these systems can fail creating serious environmental and health risks.

1.1 On-site Wastewater Treatment Systems

Conventional on-site systems typically consist of a biological treatment unit and a subsurface absorption system both of which degrade chemical compounds and remove microorganisms from domestic wastewater through physical, chemical, and biological processes. Approximately 85% of on-site wastewater treatment systems are conventional systems (Davis & Cornwell, 2008). One typical example of an on-site wastewater treatment system is a septic tank and leach field collectively known as a septic system.

The septic tank is the biological treatment unit of the septic system, consisting of a tank (or several tanks), with each tank internally separated into two chambers. The first chamber provides primary treatment of the wastewater and removes most of the settleable solids; fats, oils, and greases; and other floatable organic matter through physical processes such as sedimentation and floatation while also removing settleable organic solids through chemical and biological processes such as anaerobic liquefaction (USEPA, 2002). Anaerobic liquefaction is a process in which acid-forming bacteria partially digest the solids by hydrolyzing the proteins and converting them to volatile fatty acids, most of which are dissolved in the water phase (USEPA, 2002). Any remaining solids that are not dissolved into the water phase of the septic tank effluent are eventually pumped out during routine maintenance. The second chamber of the septic tank allows for further settlement to take place before the septic tank effluent is discharged to a leach
field or, in general terms, a subsurface absorption system.

The subsurface absorption system typically consists of a distribution pipe, which carries STE from the septic tank; a series of trenches filled with porous media such as gravel, sand, or synthetic material; and finally the native soil directly under the trench bottom or infiltrative surface area. The subsurface absorption system provides additional treatment of the septic tank effluent through a combination of additional physical, chemical, and biological processes. The porous material in the trench is typically saturated while the native soil under the trench is unsaturated (called the vadose zone).

The two main components of the subsurface absorption system, the trench and the native soil, are the interface between the engineered portion of the system and the natural portion of the system. Generally, the hydraulic conductivity in the native soil is lower than the porous media of the trench. This drop in permeability from the trench material to the native vadose zone soil causes ponding of STE at the trench bottom. Over time, as a result of ponding, a biological film develops at the interface between the saturated zone of the trench bottom and the unsaturated zone of the native soil just beneath the trench. This biological film consists of decomposing organic material, bacterial biomass, and various minerals all of which act to create a biologically active fine filter layer, also known as the biological layer. The biological layer is only a few centimeters thick but increases pollutant transformation and removal substantially through enhanced sorption, nitrification, biological decay, and bacterial die-off (Siegrist et al., 2005). The STE is purified even further as it passes through the native soil; it is in this final stage of the treatment process, that bacteria and virus as well as organic matter are further removed from the effluent before it reaches the groundwater table.
The septic system described above is just one type of on-site treatment system. Although there are many variations, including a variety of pretreatment options, on-site systems consist of two main components: the biological treatment unit, or septic tank, and the subsurface absorption system. The septic tank is the site of primary wastewater treatment while the subsurface absorption system distributes, or disposes, the septic tank effluent over a large area where further treatment occurs before it is returned to the groundwater table. Site characteristics and soil properties typically dictate which system will be installed in a given area. In areas where site characteristics or soil conditions are unfavorable additional treatment may be required. Additional treatment options are often referred to as pretreatment or alternative treatment depending on the source. Site factors that would necessitate additional treatment are high groundwater tables, shallow limiting layers such as bedrock, very slowly or rapidly permeable soils, close proximity to a water table, and a small lot size (Davis & Cornwell, 2008). Examples of alternative treatment options and the criteria under which a given system would be utilized are provided in Table 1-1.

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Distribution</th>
<th>Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Septic Tank</td>
<td>Leach Field</td>
<td>Large lot,</td>
</tr>
<tr>
<td>Septic Tank</td>
<td>Mound System</td>
<td>High groundwater table or shallow limiting layer, very permeable or very impermeable native soil</td>
</tr>
<tr>
<td>Septic Tank &amp; Aerobic Treatment Unit (pretreatment)</td>
<td>Leach Field and/or Mound System</td>
<td>High groundwater table, very permeable or very impermeable native soil, small lot size</td>
</tr>
</tbody>
</table>
The septic tank effluent from a conventional system is applied to the subsurface absorption system in one of two ways, through intermittent gravity-flow application or periodic (dosing) application (Charbeneau, 2000). Through intermittent gravity-flow application, gravity is used to distribute STE to the subsurface absorption field. In periodic dosing, a pump is used to distribute STE evenly throughout the subsurface absorption system.

Alternative, or pretreatment, systems are typically used in areas where greater wastewater treatment is required or where there is not enough natural soil to effectively treat the wastewater. Examples of alternative systems are aerobic pretreatment units, sand filter systems, and mound systems. Aerobic pretreatment units, similar to biological treatment units, are typically used as a “pretreatment” in conjunction with biological treatment units to enhance effluent treatment. Mound systems and sand filtration systems are typically installed when minimal natural soil is available for distribution and disposal of effluent. Both these systems can also be used to provide increased treatment to the effluent before disposal. There are many different types of systems often having different names depending on the source of information. As a result, it is necessary to specify that in this document, subsurface absorption system refers to the subsurface infiltrative area of the system and septic tank effluent, or STE, refers to the effluent discharged from the primary biological treatment unit, or septic tank.

1.2 Failure of On-site Systems

On-site wastewater treatment systems are considered to have failed when pollutants reach groundwater used for drinking water or nearby surface water used for recreational purposes (USEPA, 2002). When not treated properly, domestic wastewater
can contain many pollutants that pose risks to human health and the environment such as toxic organic compounds, nitrogen, phosphorous, heavy metals, and pathogenic microorganisms. Ingesting toxic organic compounds in contaminated drinking water can result in neurological and developmental problems in humans (USEPA, 2002). Excess nitrogen and phosphorous can result in eutrophication of nearby surface waters. Furthermore, there are several pathogenic microorganisms found in untreated domestic wastewater. A number of bacteria, virus, protozoa, and parasites can cause a wide range of neurological, gastrointestinal, respiratory, renal, and other diseases (USEPA, 2002).

According to the USEPA failure rates of on-site wastewater treatment systems could be as high as 20% (USEPA, 2002; US Census Bureau, 1997). Failure of on-site systems can occur in two ways: the system can fail to filter and biologically degrade the waste before it reaches the groundwater table, or the system becomes overloaded and wastewater rises to the surface (Tumeo & Newland, 2009). If systems fail and wastewater surfaces humans and animals can be directly exposed to pathogens. Similarly, when contaminated wastewater reaches the groundwater table, pathogenic microorganisms (i.e., virus, bacteria, and parasites) and other potentially hazardous compounds can be transported long distances and potentially contaminate drinking water wells in addition to contaminating surface water used for recreation (Canter & Know, 1985; Newland, 2003). With approximately 75% of all U.S. cities relying on groundwater for drinking water, failure of on-site systems pose a serious threat to public health and the environment (US Department of Commerce, 1997). The native soil in the vadose zone that lies underneath the trench in a subsurface absorption system is extremely important with regards to wastewater treatment performance. There are many
complex and interrelated physical, chemical, and biological factors that influence treatment of wastewater in the vadose zone. According to research conducted by Crane & Moore (1985), these factors include but are not limited to:

- soil pH,
- filtration and adsorption capabilities of the soil,
- atmospheric conditions such as temperature and available sunlight,
- texture and soil particle size,
- microbial activity,
- salt concentration,
- organic matter content,
- and hydraulic conditions.

In addition to the physical, chemical, and biological characteristics of the soil, the distance between the trench bottom and any confining or limiting layer also affects treatment performance. This portion of the subsurface absorption system is the final treatment stage of the on-site wastewater treatment system before the septic tank effluent reaches the groundwater table. As a result, there must be adequate distance from the bottom of the trench to the groundwater table (or other limiting layer) in order to allow enough time for wastes to be neutralized and pathogenic microorganisms to be reduced to acceptable levels.

Given the potential risks and continued growth of the use of on-site wastewater treatment systems in the U.S., there is a growing need for regulatory solutions regarding on-site wastewater treatment systems that are effective in protecting public health and preserving water quality.
1.3 Regulation of On-site Wastewater Treatment Systems in Ohio

In general, early laws regarding on-site wastewater “disposal” were largely based on soil percolation tests, local practices, and past experience (US EPA, 2002). As a result, many early codes did not take into consideration the complex physical, chemical, and biological interrelationships among soil conditions, wastewater characteristics, microorganisms, and the atmosphere (US EPA, 2002). Since the first laws regarding wastewater disposal were passed, many older laws have been revised in an effort to increase treatment efficiency by increasing the size of the biological treatment unit and the subsurface absorption system (Kreissl, 1982; Plews, 1977). Minimum trench widths, horizontal setbacks from potable water supplies, minimum vertical separation distances from the trench bottom to limiting layers, and maximum allowable land slopes were specified in an effort to protect public health and the environment (Kriessl, 1982). Although state lawmakers continue to revise codes regarding on-site wastewater treatment systems, few revisions have addressed the fundamental issue of system performance with respect to risk management for both a site and the area in which it is located (US EPA, 2002). To achieve public health and environmental objectives, wastewater management strategies need to focus on system performance, pollutant transport and fate as well as planning, design, siting, installation, and maintenance for on-site systems (US EPA, 2002).

In 2005, Ohio Revised Code (ORC) Chapter 3718 was adopted into state law which required the Ohio Department of Health to adopt rules relating to home sewage treatment systems and small flow on-site sewage treatment systems. The rules adopted by the Ohio Department of Health, Ohio Administrative Code (OAC) Chapter 3701-29,
became effective on January 1, 2007 and contained specifications regarding the siting, design, installation, operation, monitoring, maintenance, and abandonment of household sewage treatment systems. Shortly after the passage of OAC 3701-29 concerns and disagreements arose over the implementation and regulation of the rules. Of primary concern was the economic impact the rules would have on both current and future property owners as well as the potentially adverse impacts on current and future alternative technologies regarding on-site sewage treatment systems. As a result, Amended Substitute House Bill 119 was passed and put into effect January 1, 2007, which suspended portions of ORC chapter 3718 until July 1, 2009 and reinstated laws that had been previously in effect statewide regarding rules relating to household and on-site sewage treatment systems.

House Bill 194 and Senate Bill 100 of the 128th General Assembly of the State of Ohio extended the suspension and temporary provisions governing household and small flow on-site sewage treatment systems to allow the general assembly to enact new requirements pertaining to such systems. In June 2010, the 128th General Assembly enacted new provisions that become effective mid-September 2010 pertaining to household and small flow on-site sewage treatment systems. These rules codified in Sub. Senate Bill 110, allows for rules adopted by local health districts to remain in effect until the Public Health Council adopts new rules for statewide application in January 2012. The events surrounding the passage of laws pertaining to household and small flow on-site sewage treatment systems in Ohio illustrate the need not only to include scientific and technical factors in the regulatory process, but to also include social and political factors as well.
1.4 Vertical Separation Distance

As explained above, of particular regulatory concern with regards to household and small flow on-site sewage treatment systems in Ohio, as well as many other states, is the minimum vertical depth located in the area between the trench bottom of the subsurface absorption system and any restricting or limiting layer (i.e., groundwater table, bedrock, or excessively permeable soils) which allows sufficient time for wastes to be neutralized and pathogenic microorganisms to be reduced to acceptable levels (Bicki & Brown, 1990; Hall, 1990). In addition, other unique characteristics of this area can either increase or decrease the effective treatment ability of this final stage of the on-site sewage treatment system; physical, chemical, and biological factors such as temperature, microbial activity, moisture content, pH, salt concentration, organic matter content, and hydraulic conditions are just some of the controlling factors in effluent treatment (Yates & Yates, 1988). Regulators have termed the minimum allowable vertical depth of this layer as the vertical separation distance (VSD), which is typically controlled by the seasonally high water table. Although dependent upon sufficient soil conditions controlled by complex physical, chemical, and biological factors, studies have shown that a minimum vertical separation distance of at least 18 inches can be enough to properly treat septic tank effluent to acceptable levels (Bohrer & Converse, 2001; Brown et al., 1979; Hagedorn et al., 1981; Otis et al., 1977; Tyler et al., 1977).

The minimum vertical separation distance required by regulatory agencies varies widely from state to state with some states having multiple minimum vertical separation distances being required depending on the region, county, or district in which the subsurface soil absorption system is being constructed. For example, in Ohio, the
minimum VSD is currently set by the board of health within the county that the subsurface soil absorption system is to be constructed as outlined in ORC 3718.041(A) resulting in required minimum vertical separation distances ranging from 1 to 4 feet depending on location. Similarly, minimum required vertical separation distances for all other states in the U.S. typically range from approximately 1 to 4 feet depending on the state (USEPA, 1980a; Crites & Tchobanoglous, 1998; Wayland & Oppelt, 2002). This wide range of separation distances is largely a function of the physical, chemical, and biological conditions of the soil as well as the hydrologic conditions within the vadose zone. With approximately 25% of homes in the U.S. using some form of on-site wastewater treatment and disposal, there is clearly a need to use scientific and technical knowledge as a basis to create policy that will regulate on-site sewage treatment systems to ensure public health and provide for minimal environmental impact (Hall, 1990; Siegrist et al., 2001; Wayland & Oppelt, 2002). Properly sited, designed, installed and maintained on-site sewage treatment systems provide a high level of treatment efficiency at a low cost over long periods of time while simultaneously protecting public health and environmental quality by returning highly treated effluent to the receiving environment (Hall, 1990; Siegrist et al., 2001).
CHAPTER II
LITERATURE/BACKGROUND

The majority of on-site wastewater treatment systems discharge treated water into the subsurface, which ultimately leads to the groundwater table. Subsurface soil consists of interconnected pore spaces, or void spaces, and is referred to as a porous medium. The interconnected pore spaces allow fluids such as water or air to flow through the soil pore spaces. The soil pore spaces may be connected by tortuous pathways, that is indirect or circuitous routes through the porous medium (Dullien, 1979). As a result, flow of water through the subsurface can be very complicated.

2.1 Subsurface Flow in Soils

Subsurface water occurs in two main zones: the saturated zone and the unsaturated zone. The unsaturated zone, also termed the partially saturated zone or vadose zone, is nearest to the earth’s surface and extends from the ground surface to the water table. In the vadose zone the void spaces are filled with water, air, and water vapor. The amount and distribution of water in this zone depends on soil texture, vegetation, and atmospheric conditions (Charbeneau, 2000). In addition, the water
present in the vadose zone is a result of three forces (Roberson et al., 1998; US EPA, 2002):

- capillary forces (water held in the soil by capillary action),
- adsorptive forces (water adhering to the surface of soil particles),
- and gravity (water draining downward through the soil).

Capillary, adsorptive, and gravitational forces complicate unsaturated flow, or flow in the vadose zone, causing vertical or lateral movement of water depending on the permeability of the soil (Roberson et al., 1998). Highly permeable soils can hold as much as 10 to 20% of water saturation while soils with lower permeability can hold as much as 90% of water saturation (Charbeneau, 2000). Furthermore, as a result of these forces and the soil pore spaces, the pressure in the vadose zone is usually negative (less than atmospheric pressure) and the permeability is not constant throughout this zone as it is in the saturated zone. The thickness of the vadose zone, or the distance from the ground surface to the water table, varies with the amount of rainfall. In general unsaturated flow conditions are much slower than saturated flow conditions (Hall, 1990; Roberson et al., 1998).

The groundwater table marks the beginning of the saturated zone. In the saturated zone water occupies all the soil pore spaces and as a result, air is prevented from entering, promoting anaerobic conditions unlike the aerobic conditions of the vadose zone. In soils with coarse texture, flow of water is more rapid whereas saturated flow through finely textured soils is generally much slower (Hall, 1990; Roberson et al., 1998).
2.2 Flow in the Subsurface Absorption System

Fluid transport through the subsurface absorption system occurs through three zones: the trench, the infiltration zone, and the vadose zone. Most of the physical, chemical, and biological treatment that occurs in the subsurface absorption system occurs within the infiltration zone and the vadose zone (US EPA, 2002). As mentioned previously, a biologically active fine filter layer, or biological layer, is formed at the interface of the trench bottom and the vadose zone, or more simply within the infiltration zone. Initially, particulate matter from the wastewater accumulates in the pore spaces of the infiltration zone providing a source of carbon and nutrients for the biologically active microorganisms in the soil. Over time, the accumulation of particulate matter as well as biologically active microorganisms and their by-products result in the formation of a biological layer. As a result of the reduced porosity and permeability caused by the clogging of soil pore space and the subsequent formation of the biological layer, ponding occurs within the trench (Jones & Tyler, 1965; Siegrist, 1987; Tyler et al., 1991).

The rate at which the wastewater infiltrates into the vadose zone is controlled by both the biological layer and the vadose zone soil (Bouma, 1975; Tyler & Kuns, 2000). Over time, decline of the infiltration rate into the vadose zone from the infiltration zone eventually leads to a long-term steady state infiltration rate known as the long-term acceptance rate (LTAR) (Radcliffe et al., 2009). The interface between the infiltration zone and the vadose zone is the transition between saturated flow in the trench and unsaturated flow below the trench.

In the vadose zone, contact time is increased as a result of capillary and adsorptive forces. Furthermore, increased aeration is facilitated by the movement of
wastewater over soil particles and through soil pore spaces. These characteristics are key aspects with regards to wastewater treatment efficiency in this zone. The negative atmospheric pressure, which is ultimately due to the capillary and adsorptive forces, act to draw in water into the finer pore spaces while the larger pore spaces typically remain filled with air. The unsaturated nature of the vadose zone allows oxygen to diffuse into the larger pore spaces creating a suitable environment for aerobic microorganisms, which are partially responsible for treatment performance, to grow on soil particles (Hall, 1990; US EPA, 2002). Provided that there is enough distance between the infiltration zone and the groundwater table, unsaturated, aerobic flow conditions in the vadose zone create an ideal environment in which nutrients and pathogenic microorganisms can be removed from wastewater to acceptable levels.

2.3 Wastewater Characteristics

The composition of domestic wastewater is a complex mixture of organic and inorganic compounds as well as pathogenic microorganisms. Typically, the most abundant compounds found in domestic wastewater are phosphorus and nitrogen.

2.3-1 Nutrients: Both organic and inorganic forms of nitrogen are found in wastewater. The presence of organic nitrogen is due largely to nitrogen containing compounds such as proteins, amino acids, and urea, which are excreted from the human body (Sawyer et al., 2003). Inorganic nitrogen is present in the form of nitrate and nitrite, both of which result from the oxidation of ammonia in a process called nitrification (Holden & Fierer, 2005; Willey et al., 2009). When discharged to receiving waters in excessive amounts,
nitrogen can lead to low dissolved oxygen concentrations as well as excessive plant growth (Carodona, 1998).

Phosphorus in domestic wastewater exists mainly in the form inorganic phosphates, which originate from human urination, agricultural run-off, and household detergents. Excess phosphorus in receiving waters can lead to excessive plant growth (Sawyer et al., 2003). Both nitrogen and phosphorus are used as a source of nutrition for bacteria in wastewater and in soil (Willey et al., 2009). Many inorganic compounds such as sodium, bicarbonates, chlorides, sulfates, calcium, and potassium, to name a few, are present in domestic wastewater. In addition, many other compounds such as fats, oils, greases, and organic matter can also be found in domestic wastewater (Sawyer et al., 2003).

2.3-2 Microorganisms: Domestic wastewater contains a large variety of microorganisms most of which are non-pathogenic. However, humans who are infected or carriers of disease can discharge pathogenic microorganisms into wastewater (Carodona, 1998). Pathogenic microorganisms can be bacteria, viruses, or parasites causing a wide variety of diseases. Fecal coliforms are coliform bacteria that originate in feces and are frequently used as indicator organisms of pathogenic water contamination because they are easy to test for and more numerous than other types of microorganisms (Coradona, 1998; Willey et al., 2009). Septic tank effluent typically contains about $10^6$ to $10^8$ CFU/100 ml fecal coliforms (USEPA, 2002). Due to the fact that fecal coliform bacteria are used as an indicator of water quality, determination of the die-off or inactivation of these organisms in the environment is of critical concern if management practices are to be developed to minimize contamination of drinking water (Crane & Moore, 1985).
2.4 Microorganism Survival in the Subsurface

Microorganism survival, and more specifically bacterial survival, in the subsurface is largely a function of many complex and interrelated physical, chemical, and biological characteristics of the vadose zone soil (Holden & Fierer, 2005). Studies have shown that, in general, temperature, pH, moisture content, nutrient supply, solar radiation, oxygen content, and predation by indigenous soil microflora have the greatest effect on coliform bacteria survival (Burge & March, 1978; Carodona, 1998; Dunlop, 1968; Gerba et al., 1975, Holden & Fierer, 2005; Kibbey et al., 1978). The effect of these stresses reduce bacterial survival time in the subsurface to typically less than 20 days, with longer survival times under certain soil conditions (Pekdeger, 1984; US EPA, 2002).

Temperature extremes, both high and low, are disruptive to survival of bacteria in the vadose due to the relatively stable temperature conditions in the subsurface (Klein & Casida, 1967; Kibbey et al., 1978; Fierer, et al., 2003a). Extremes in pH, both alkaline and acidic, of soil and water greatly increase the die-off and inactivation of bacteria in the vadose zone (Crane & Moore, 1985; Kibbey et al., 1978; US EPA, 2002). Dramatic shifts in soil moisture content can also influence bacteria survival rates. Desiccation resistant indigenous soil bacteria are favored by drier soil conditions while fecal coliform bacteria survival is generally increased in moist soil conditions (Carodona, 1998; Gerba et al., 1975; Holden & Fierer, 2005; Kibbey et al., 1978). In addition, sufficient nutrient supply and organic matter content of soil are necessary for bacterial survival in the subsurface. Nutrient concentrations (carbon, nitrogen, and other nutrients), abundant in surface soil, decrease with increasing depth thereby resulting, in part, to lower microbial
abundances lower in vadose zone soil (Fierer et al., 2003b; Holden & Fierer, 2005; Klein & Cassida, 1967;). Sunlight, or solar radiation, can be effective in reducing bacteria but typically only near the surface (Crane et al., 1980; US EPA, 2002). Aerobic conditions are unfavorable to anaerobic septic bacteria; these conditions also increase predation of larger pathogenic bacteria by aerobic soil bacteria (Carodona, 1998).

In addition to natural die-off, pathogenic microorganisms can be retained in the soil through entrapment or filtering and soil adsorption. As soil pore size decreases, larger microorganisms become trapped in soil pores (Carodona, 1998). Furthermore, conditions typical of the vadose zone such as uniform effluent distribution as a result of the decreased permeability of the biological layer as well as, decreased moisture content, increase adsorption of microorganisms (Carodona, 1998; Reneau et al., 1989).

2.5 Treatment Performance in the Vadose Zone

Treatment in the subsurface soil system is the result of processes by which, disease causing microorganisms, as well as organic and inorganic materials, are removed from the wastewater before being returned to the hydrologic cycle (Hall, 1990). Due to the high degree of treatment that occurs in the unsaturated, or vadose, zone below the trench bottom, allowing for adequate separation distance from groundwater of any other impervious layer that results I saturation is crucial in not only preventing contaminants from reaching the groundwater table but also in transforming or reducing the concentration of contaminants to an acceptable level. During unsaturated flow conditions through the vadose zone, the primary movement of water is in the vertical direction due to gravitational forces. In addition, capillary forces caused by surface tension between
water and air within soil pore spaces act to hold the wastewater in close proximity to soil particles and allow for treatment by interaction with microorganisms on the soil particle surfaces (Charbeneau, 2000). Further treatment is accomplished as a result of increased contact time due to much slower flow rates as well as increased aerobic conditions that occur in the unsaturated zone (Hurst, 1991). The vadose zone, is the final treatment aspect of the on-site wastewater treatment system before the effluent returns to the environment and ultimately into the groundwater table. In conjunction with the biological layer which precedes it, the vadose zone is a major source for the inactivation of pathogenic bacteria, viruses, and other microorganisms.

### 2.6 Vertical Separation Distance in On-site Systems

Allowing for adequate time and distance for effluent to travel vertically through the vadose zone is essential in order to provide effective treatment of wastewater before it reaches the groundwater table. As mentioned previously, the long-term acceptance rate, dictated by the physical characteristics of the biological layer, controls flow into the vadose zone. As a result of this phenomenon, unsaturated flow into the vadose zone is much slower allowing increased contact time and increased aeration, all of which promote faster and more complete treatment of wastewater (Carodona, 1998; Hall, 1990; US EPA, 2002). In addition to vertical movement, horizontal movement can also occur as a result of a confining or limiting layer such as bedrock and will often occur once the effluent reaches the groundwater table (Stewart & Reneau, 1981; Field et al., 2007).
In order to protect public health, occasionally horizontal separation distances are specified to provide protection of wells, springs, and surface waters. However, research has shown that providing adequate vertical separation is much more effective in removing contaminants and protecting public health than horizontal separation; because horizontal flow usually requires saturated conditions, if vertical separation provides adequate treatment before horizontal flow begins, contamination as a result of horizontal flow will be prevented (Carodona, 1998).

Studies have shown that when soil conditions are sufficiently unsaturated, 2 to 3 feet is sufficient to remove nearly all fecal indicator bacteria and viruses (Otis et al., 1977; Tyler et al., 1977). Research conducted by Tyler et al (1977) showed that there was a 3 log reduction (1000 times less) in bacterial numbers within the first foot of soil below the trench bottom and within 2 feet bacterial numbers were reduced to acceptable levels for treated wastewater. Similarly, research conducted by Bohrer and Converse (2001) showed that different sites receiving septic tank effluent with soils ranging from coarse sand to clay loam, within 18-24 inches very low fecal coliform concentrations were detected and that beyond 24 inches there was no detection of fecal coliforms. Furthermore, research has shown that substantial, if not complete, removal of bacterial concentrations occurred within one to two feet below the trench bottom (Hagedorn et al., 1981; Miles et al., 2007; Tyler et al., 1977). In fact, research involving both column and field studies has shown that depending on soil type and conditions, vertical separation between 1 and 4 feet is enough to adequately remove bacteria and viruses (Bohrer & Converse, 2001; Brown et al., 1979; Hansel & Machmeier, 1980; Lance et al., 1976;
2.7 Modeling On-site Wastewater Treatment Systems

Models are used to represent systems in order to examine aspects of a system or to predict future outcomes of a system that under normal circumstances would be difficult or impossible. Models are not reality, but rather representations of reality. There are many different types of models. Models can be of objects, such as model airplanes or scaled structures used by designers and engineers, or models can be mathematical representations of a system, such as a population growth model that uses a mathematical expression to predict population growth at some point in the future. Kloeden (1994) suggests that through modeling we seek to:

1.) describe and understand the dynamical interactions and evolution of a real system;
2.) analyze and simulate a model under conditions that may not be possible or practical in the real system; and
3.) make predictions.

With respect to modeling the risks associated with microbial contamination form on-site wastewater treatment systems, both survival and movement are the two main factors to consider when modeling the fate of microorganisms in the subsurface. Survival is typically modeled using some form of decay, which is defined as the irreversible destruction of a contaminant by chemical, physical, or biological processes. Movement, or transport, of microorganisms in the subsurface is typically modeled using advection, dispersion, and adsorption (Yate & Yates, 1991). Advection, with regards to the
movement of microorganisms, can simply be described as the transport of microorganisms by the flow of water (Gerba et al., 1991). Similarly, dispersion, can be simply described as the mechanical mixing and spreading out of the microorganisms, considered to be in solution, as the water passes over soil particles. Adsorption is the reversible or irreversible chemical binding of microorganisms to the surface of soil particles.

There are several mathematical models available today which model microbial transport in porous media. Typically, microbial movement is modeled as a contaminant using a modified form of the non-linear, partial differential advection-dispersion equation for solute transport, which incorporates the four factors affecting microorganism movement and survival mentioned above (Hurts, 1991; Tufenkji, 2007). However, several problems exist with using the advection-dispersion equation to model bacterial transport in the subsurface.

The advection-dispersion equation assumes that the contaminant is in solution (Raina et al., 2009). However, microorganisms are not transported as contaminants in solution but rather as colloids (Dickinson, 1991; Yates & Yates, 1991). In addition, the decay rate of a microorganism is a highly variable parameter, spanning several orders of magnitude for a single organism, based on several complex and interrelated physical, chemical, and biological factors (Crane & Moore, 1986; Yates & Yates, 1991). Therefore, using a single decay rate or using an average decay rate for a group of microorganisms could result in inaccurate results in the solution to the advection-dispersion model. Furthermore, due to the fact that the decay rate is based on multiple environmental factors, obtaining a decay rate (even experimentally in the lab) relevant to
the particular situation being modeled would be very difficult. In addition, the number of input parameter is very high. Often estimations must be made for many of the unknown parameters because determination would be too costly.

Very little work has been done on bacterial transport specifically in the vadose zone (Kim et al., 2008). Physical characteristics of the vadose zone such as the presence of air further complicate bacterial movement through this zone with the addition of another phase (i.e., air) (Schafer et al., 1998). More specifically, in addition to the factors that affect bacterial movement in the saturated zone (i.e., advection, dispersion, and adsorption), in the vadose zone there is also attachment of bacteria at the air-water interface (Kim et al., 2008). Another type of bacterial flow model is the potential flow model, which assumes microorganisms flow with the water. As a result, only two of the four processes previously described apply to this model, that is, advection and decay (Yates & Yates, 1991). The major limitation to this model is that inactivation, or die-off, is the only removal mechanism included (in the unsaturated zone, adsorption is an important mechanism for removal).

In general, there are four problems associated with adapting existing transport models to describe microbial transport (Dickinson, 1991):

1.) important parameters are not directly measurable and/or are variable in the flow regime,
2.) aquifer hydraulic properties are not measured or reported for many microbial studies,
3.) microorganisms are not transported as contaminants in solution but as colloids, and
4.) important factors in the behavior of microorganisms are not included in the governing equations.
Soil is not the natural habitat of fecal bacteria and other pathogenic microorganisms found in domestic wastewater and although they might experience a period of regrowth in the subsurface, they will eventually die-off or become inactivated (Gerba et al, 1991 via Hurst). A review by Crane & Moore (1986) provides several models for bacterial die-off under multiple environmental conditions. They concluded that “…the simplest model, that of first order die-off kinetics, is the most advantageous…The first order model appears to accurately describe the die-off of bacteria under all conditions…” (pg. 426)

However, as noted previously the die-off rate constant is a highly variable parameter spanning several orders of magnitude for any specific bacterial type.

2.7-1 **Stochastic vs. Deterministic Modeling:** In a deterministic model there is no randomness involved in producing the model result, the output is always the same for a given input. Stochastic systems take into account random variations of processes over time and space, that is, random variation is describable by some probability distribution (Tumeo, 1994). Put simply, a stochastic model, as its output, will provide the probability, or chance, that a value will occur within a given range of values. Stochasticity involves the fact that all natural processes have natural variations as a result of the variations in the input parameters used to describe the natural process (Tumeo, 1994).

To illustrate the difference between stochastic and deterministic models, consider the development of a model for a septic system. A designer of a septic system would like to know the concentration of bacteria at three feet below the bottom of the trench of a
subsurface absorption system. A deterministic model would use a mathematical equation providing one single output to calculate the bacterial concentration three feet below the trench bottom. The output from this deterministic model would provide one single estimate of concentration, according to the assumptions and simplifications of the underlying equations and the specific inputs provided. However, as mentioned previously, there is variation in the input parameters used to describe natural processes. For example, site characteristics, such as water content, porosity, and permeability, are dependent upon soil type, which can vary a great deal throughout the site used for a septic system. Typically, only characteristics of a single soil type are used as the inputs for the advection-dispersion equation. As a result, the output from the deterministic model is not based on the actual soil conditions of the site, rather only one soil type from the site. If compared to real-world data, the deterministic model may correlate some or even most of the time but not all.

In contrast, a stochastic model, which takes into account the natural variation of input parameters, would not provide the exact concentration three feet below the trench bottom as its output but rather it would provide the probability, or chance, that the bacterial concentration would be within a certain range of values.
CHAPTER III

PURPOSE

The purpose of this project was to develop an easy to use stochastic model that will provide the “risk” of a bacterial contamination reaching a point below the subsurface absorption system as the result of specified input parameters. Most of the models that examine fate and transport of contaminants, available and that are widely used today, are both complex and deterministic in nature.

Some of these models can be very complex requiring the solution of multiple, simultaneous, non-linear differential equations regarding hydraulic flow and contaminant transport in unsaturated porous media. They are often data intensive, with required inputs being highly site specific (Hurst 1991). In addition, many of the inputs (terms in the flow and transport equations relating to the hydraulic and soil characteristics of the site) are not readily available to designers and regulators. Furthermore, in most models, microorganisms are typically modeled as contaminants in solution with regards to transport. However, microorganisms are not transported as contaminants but as colloids (Hurst, 1991).
As discussed in the Literature review section, deterministic models describe the behavior of a system based on physical laws. In reality, natural systems are never completely deterministic; there are always unpredictable factors present that need to be taken into account. As a result, exact prediction of a system is not possible. However, a model that would predict the probability that particular value will occur at a particular time within a certain confidence interval (i.e., the probability that a contaminant concentration will reach a specified point in the subsurface absorption system) would be useful not only to designers and regulators but to the public as well.

To address the shortcomings of commonly available models, the model developed in this project requires inputs that are site parameters any designer would readily have (e.g. LTAR, trench dimensions). Further, to address “risk”, the model is “stochastic” - that is, it does NOT provide the exact contaminant concentration at a point, but rather it will provide a conservative estimation of the probability of a given concentration at any selected distance below the trench bottom.

This model is intended to be used in several ways: by regulators to aid in educating industry professionals and the public on the actual risks associated with contaminants, bacterial or otherwise, reaching the groundwater table; as a tool for land use planning by municipalities or county officials when developing or updating land use codes as well as a tool that can aid in the decision process of granting variances; and also by designers in determining the most appropriate treatment system for a specific site.
CHAPTER IV
METHODS

4.1 Model Framework

Conceptually, the model is separated into three sections, or layers, corresponding to the three different zones the septic tank effluent will pass through: the trench, the biological layer or infiltration zone, and the native soil in the vadose zone. In addition, a fourth and optional layer is available which represents horizontal transport through the native soil down-gradient from the trench. The input parameters for the model are:

- the daily design flow (gpd),
- the long-term acceptance rate (LTAR) (gpd/ft²),
- the trench material (chamber, extruded polystyrene, gravel, and sand)
- the trench dimensions (i.e., length, width, depth) (ft)
- design pipe size and depth of cover (in, ft),
- the maximum allowable VSD (ft),
- confining or limiting layer depth (ft),
- horizontal distance of concern (if this option is selected) (ft), and the
- sidewall factor (if any) corresponding to the biological layer growth on the sides of the trench. (unitless)
The output of the model is the probability distribution of the contaminant concentration at a specified point corresponding to the user specified desired vertical separation distance. The model provides the “risk”, or probability distribution, of a given contaminant concentration reaching a specified point. This final concentration is NOT the exact contaminant concentration corresponding to the user specified VSD that results from the given input loading rather it is a conservative estimation of the probability that a given contaminant concentration will reach a certain point (the user specified VSD) below the trench bottom of the subsurface soil absorption system.

To model bacterial inactivation through the three zones, Chick’s Law of Disinfection was used. The use of this equation requires the time of travel through each layer as an input. The time of travel for each layer, or zone, is calculated from the flow rate into that zone. Therefore, calculations of the flow rate and the time of travel are specific to a particular zone. As a result, a detailed discussion regarding the flow rate and time of travel through a particular zone will be included below in the section corresponding to that zone.

4.2 Bacterial Inactivation

Determining the inactivation, or die-off, of pathogenic microorganisms in the environment is critical in developing management practices to minimize contamination of surface and groundwater when waste materials are applied to land (Crane & Moore, 1985). There are many factors that influence the inactivation of microorganisms in the environment. However, as mentioned previously, the first order die-off equation appears to accurately model inactivation of microorganisms under many different conditions.
As a result, the driving mathematical equation used in this model to determine inactivation of pathogenic microorganisms is the bacterial decay equation proposed by Chick in 1908. Known as Chick’s Law of Disinfection, this equation is also the model of a simple first order reaction in chemical kinetics (Crane & Moore, 1985):

\[ N = N_0 e^{-kt} \]

where,

\( N \) = the number of microorganisms (CFU/100ml)
\( N_0 \) = the initial number of microorganisms (CFU/100ml)
\( k \) = the inactivation rate constant, or die-off constant (day\(^{-1}\))
\( t \) = contact time (day).

The initial concentration, \( N_0 \), was determined using field data of septic tank effluent over a six-month period. In all, a total of seventy-five septic tank effluent samples containing fecal coliform concentration data were used to determine the mean and standard deviation for the initial input concentration for the model.

The inactivation rate constant was determined for different soil types using field data from a study performed by Bohrer and Converse (2001) in which the dominant soil type under the distribution area of the septic system was determined and fecal coliform concentrations were determined every six inches up to 42 inches (3.5 feet). Using an average saturated hydraulic conductivity for the soil type in the study, the time for the effluent to reach each sampling point was determined using the algebraic expression; distance (d) equals rate (v) * time (t). Rearranging,

\[ t = \frac{d}{v} \]
or in terms of hydraulic conductivity,

\[ t = \frac{d}{k} \]

where,

- \( t \) = time (day)
- \( d \) = distance to each data point (ft)
- \( k \) = average hydraulic conductivity of dominant soil type (ft/day).

The concentrations taken from the Bohrer and Converse study were transformed to fit a bell curve by taking the natural log of each concentration. The transformed fecal coliform concentrations were plotted as a function of time and the slope of the line, which is the inactivation rate constant, was determined (See Appendix A). Finally, depending on the layer, the time parameter in the decay equation was determined (a detailed description is provided below). Using this information, the concentration of microorganisms at the interface of each layer can be determined.

### 4.3 Layer 1: The Trench

In the first layer, the trench, the starting point is defined by the inputs for the model. Due to saturated conditions in the trench and the limited amount of materials used to fill trenches, the time that the effluent will travel through this layer is based on the material used to fill the trench. The corresponding hydraulic conductivity of that material and filtration, if any, can be calculated using the travel time and the first order decay equation.

The trench materials included in this model at this time are: sand, gravel, chambers, and extruded polystyrene. Based on the compact structure and characteristics of the sand matrix, which allows for filtration to occur, the only material in this model in
which filtration in the trench will occur is sand. As a result, when sand is selected as the
trench material in the model, the travel time through the trench is calculated using the
algebraic expression
\[ d = v \times t \]
where the velocity (v) is taken as an average value of hydraulic conductivity for sand and
the distance (d) is the vertical distance from the bottom of the pipe to the trench bottom.
The value calculated for time of travel in the trench is then used in Chick’s Law along
with the initial concentration and the inactivation rate constant (if sand is used) and the
distribution of the concentration of microorganisms at the trench bottom is calculated and
used as an input for the second layer, the biological layer.

When any other trench material is selected (i.e., gravel, chamber, extruded
polystyrene), the travel time through the trench is taken as instantaneous, or zero. As a
result, the value for t in the first order decay equation is zero resulting in no inactivation
of microorganisms in the trench. In this case the input concentration distribution into the
second layer, the infiltration zone, is the same as the input concentration into the trench.

4.4 Layer 2: The Infiltration Zone

It should be noted that the biological layer develops over time as a result of
ponding and is not present initially. The situation when the biological layer is not
presents occurs at system start up and is the point in time where the greatest amount of
“risk” occurs. Therefore "start-up" could be used to present the largest probable
contaminant concentration. However, once the biological layer develops, reduction of
bacterial concentrations, if any, will occur in this layer as a result of filtration, die-off,
and the other numerous factors discussed in the literature review above. The travel time
through this layer is calculated using the infiltrative surface area and the long-term acceptance rate (LTAR). As mentioned previously, the LTAR is dictated by the physical characteristics of the biological layer and is a measure of the permeability of this layer. The infiltrative surface area is the surface area of the biological layer and is assumed to be equal to the surface area of the trench bottom. In reality, the biological layer extends upward a portion of the trench wall due to the ponding that occurs in the trench. Known as the sidewall factor, not all states consider this portion of the biological layer as part of the infiltrative surface area. As a result, there is an optional input parameter in the model that allows the sidewall factor to be taken into account for states that use this factor as part of the infiltrative surface area. The travel time through the biological layer is calculated by first determining the flow rate into the biological layer using the LTAR and the infiltrative surface area (ISA) of the trench:

\[
\text{Flow (Q)} = \text{LTAR} \times \text{ISA}
\]

Then, dividing the volume of the biological layer by the flow:

\[
\text{t} = \frac{V}{Q}
\]

where,

\[
\text{t} = \text{time (day)} \\
\text{Q} = \text{flow (ft}^3/\text{day)} \\
\text{V} = \text{volume of the biological layer (ft}^3). 
\]

The corresponding contaminant concentration at the bottom of the infiltration zone is calculated using the first order decay equation with the time through the zone, the concentration that resulted at the bottom of the trench, and the inactivation rate constant for the biological layer as the inputs to the equation. The distribution of the concentration
of microorganisms at the bottom of the infiltration zone is used as the input initial concentration into the final zone, the vadose zone.

4.5 Layer 3: The Vadose Zone

The resulting flow of the effluent concentration from the biological layer into the third layer, the vadose zone, is controlled by the LTAR. As a result, the volumetric flow into the vadose zone is much slower than that of either of the above layers. Therefore, while the trench and infiltration layers are typically saturated, the natural soil below the trench is unsaturated. Many models dealing with approximating fate and transport of contaminants in unsaturated porous media can be very complex, requiring several input parameters that might not be available to the intended users of this model. As a result, saturated conditions are assumed for this portion of the model. This assumption, while not the most accurate approach, allows for a much more simple yet conservative model to be used. The time through this layer is calculated by dividing the user specified desired vertical separation distance (VSD) by the rate of flow through the layer, controlled by the saturated hydraulic conductivity corresponding to the dominant soil type of the vadose zone soil. This formula is the same algebraic expression mentioned previously; distance (d) equals rate (v) times time (t):

\[ t = \frac{\text{VSD}}{K_{\text{soil}}} \]

where,

\( t = \) time (days)

\( \text{VSD} = \) user specified vertical separation distance (ft)

\( K_{\text{soil}} = \) the saturated hydraulic conductivity of the native soil underlying the trench (\( \text{ft}^3/\text{day}/\text{ft}^2 = \text{ft/day} \))
Using the decay equation with the time through the layer, the concentration of microorganisms at the bottom of the infiltration zone, and the inactivation rate constant corresponding to the dominant soil type in the vadose zone the concentration of microorganisms at the user specified VSD is calculated. As mentioned previously, this is not the exact concentration of microorganisms at the specified point; rather a probability distribution providing the “risk” that a concentration of microorganisms will reach the user specified point. The final output of the model provides a conservative estimation of the probability that a contaminant concentration will reach a certain point below the trench bottom of the subsurface soil absorption system.

4.6 Layer 4 (Optional): Horizontal Flow in the Vadose Zone

As mentioned previously, horizontal flow of septic tank effluent can occur in the vadose zone as a result of any confining or limiting layer such as bedrock and will often occur once the effluent reaches the groundwater table. As a result, a forth section was added to the model to account for this phenomena. There are two additional inputs for this section, the depth to the confining layer and the horizontal distance of concern. Due to the fact that this phenomenon is occurring in the vadose zone, the model assumes that the hydraulic conductivity in the native soil is isotropic. Therefore, the rate of flow is the same as that calculated in the previous section. Similarly, the time of travel to the horizontal distance of concern is calculated using the same equation as that in the previous section, except the distance variable, in this case, is the depth to the confining layer plus the horizontal distance of concern. The time of travel to the horizontal distance of concern is calculated and used in the decay equation to calculate the concentration of microorganisms at the user specified distance.
4.7 Stochasticity in the Model

All natural processes have vary naturally. This natural variation (or "stochasticity") is modeled mathematically by variations in the input parameters used to describe the natural process. As discussed previously, many different environmental factors affect the inactivation of microorganism. Therefore, to account for this variation in the model, the initial concentration and the decay rate "constant" in the first order decay equation, are treated as stochastic variables, meaning they are not "constant" but vary with a set probability around a mean value. In contrast to a deterministic model, which would yield one unique output for each set of "constants" input, this stochastic model yields a range of outputs, or a distribution of probable concentrations, for the distribution of inputs, thereby taking into account the natural variation in the input parameters (i.e., the initial concentration and the decay rate constant).

To accomplish this, the model uses a technique developed by Tumeo and Orlob (1989), in which stochastic differential equations were used to derive the first order decay equation, resulting in a mean concentration with a deviation around the mean as a function of the random variations of the input parameters. In the derivation, all terms, which contain a variation term, are separated from all terms containing only mean values resulting in two equations:

Mean Concentration:

$$\overline{C} = C_0 e^{-kt}$$  \hspace{1cm} (4-1)
Variance of Concentration:

\[ \sigma_C^2 = e^{2k_1t} \left( e^{2t^2\sigma_k^2} - e^{t^2\sigma_k^2} + e^{t^2\sigma_k^2/2} \right) \]  

where,

\[ \sigma_0 = \text{standard deviation of the initial concentration (CFU/100ml)} \]
\[ \sigma_{k1} = \text{standard deviation of } k_1 \text{ (day}^{-1}\text{)} \]
\[ \sigma_C = \text{standard deviation in initial concentration (CFU/100ml)} \]
\[ k_1 = \text{inactivation constant for the layer (day}^{-1}\text{)} \]
\[ t = \text{time through layer, or contact time (days)} \]
\[ C_0 = \text{initial concentration (CFU/100ml)} \].

The first equation is simply the first order decay equation and the second equation is the variance containing only terms that involve a variation. The reader is referred to Tumeo and Orlob (1990; 1989) for a complete discussion of the derivation of these stochastic equations.

These equations were applied to each conceptual layer of the model resulting in a mean concentration and variance for each layer. As mentioned before, the mean concentration that results from one layer is the input, or initial, concentration for the layer that follows. Similarly, the variance that results from one layer is the input variance for the next layer. In addition, as mentioned at the beginning of this chapter, the initial mean concentration and standard deviation (variance is the standard deviation squared) were taken from field data of septic tank effluent. Furthermore, in determining the initial standard deviation the data used were transformed to fit a bell curve by taking the natural log of each data point. As a result, the output is a range, or a probability distribution, of concentrations that reach a user specified point in the subsurface absorption system.
5.1 Model Calibration

Once the model had been developed it was important to calibrate the model using existing field data. Model calibration consists of changing the values of model input parameters in an attempt to match existing field data. During the calibration process, it is important to use field data that accurately characterizes the conditions being modeled. A successful calibration against valid field data will produce a much more reliable model.

Bacterial decay is the main aspect of the model that contains the most variation intrinsically. As a result, decay (more specifically the decay rate constant) was the main parameter of focus in the calibration process. Prior to calibration, there was a large discrepancy between field data and model data when comparing bacterial concentrations in the infiltration zone and just beneath this zone in the first few inches of the vadose zone. Beyond this area, upon visual inspection the data tended to match fairly well. The
reason for the discrepancy between the model data and the field data is that the model, although programmed to accommodate the actions of the infiltration zone, does not. This is due to the lack of data regarding inactivation rate constants for this layer. In the model the inactivation rate constant is taken as zero resulting in no die-off in that region. This discrepancy and corresponding lack of infiltration zone data will be discussed in greater detail below.

In order to account for the discrepancy between field data and model data mentioned previously it was necessary to incorporate into the model program an inactivation rate constant for the infiltration zone. Lacking the necessary data for an inactivation rate constant from the literature, the infiltration zone inactivation rate constant was adjusted and incorporated into the model by calibrating the model to data published by Siegrist et al (2000). This study was used because it not only contained field data that characterized conditions being modeled, but it also contained bacterial concentrations at incremental depths within the first few inches just beneath the infiltration zone. Calibration of the decay constant involved changing the constant in the model incrementally and comparing the output of the model to the actual field data using the percent error calculation (See Table 5-1). When comparing the field data concentration to the model output concentration, it can be seen that there is a slight difference between the two values. This fact can also be seen when observing the percent error between the two values. Although it was possible to reduce this error, it was not practical due to the fact that inactivation rate constants are only reported to three significant figures. As a result the inactivation rate constant at a depth of 0-2 inches (0.144) was incorporated into the model.
Table 5-1: Determination of inactivation rate constant for the infiltration zone to be incorporated into the model.

<table>
<thead>
<tr>
<th>Depth</th>
<th>$C_0$</th>
<th>Concentration Field</th>
<th>Concentration Model</th>
<th>Inactivation Rate Constant</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-2</td>
<td>3.20E+05</td>
<td>1.25E+05</td>
<td>1.26E+05</td>
<td>0.144</td>
<td>0.49</td>
</tr>
</tbody>
</table>

5.2 Model Validation

As part of the model development and to test the underlying assumptions in the model, the model was validated against field data from an existing septic system. Put simply, validation determines how well the model correlates with real-world data when input constants are NOT adjusted (as they are in calibration). Field data was obtained from a study conducted by Bohrer and Converse (2001). No other literature known to this author contained all the information needed to validate the model. In fact, additional information not contained in the study conducted by Bohrer and Converse would have been useful in the validation process, however, a detailed discussion regarding this matter is presented below in the chapter titled “Conclusion & Future Directions”.

5.2-1 Input for Validation: The Bohrer and Converse (2001) study focused on a septic system serving a three bedroom, single-family residence with an average daily water usage of 122 gallons per day. The leaching field consisted of a two trenches, 95 feet in length by 5 ft in width and an overall depth of approximately 15 inches. In addition to concentration data, the study included detailed information regarding site and design characteristics necessary to validate the model. A summary of all input information taken from the study and used as the input to the model is shown in Table 5-2 and Table 5-3.
Table 5-2: Parameters used for validation taken from Bohrer and Converse (2001).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow, Q (gpd)</td>
<td>122</td>
</tr>
<tr>
<td>Loading rate (gpd/ft²)</td>
<td>0.23</td>
</tr>
<tr>
<td>Soil Type</td>
<td>Clay Loam</td>
</tr>
</tbody>
</table>

2 Trenches:

| Length (ft) | 95 |
| Width (ft)  | 5  |
| Depth (inches) | 15 |
| FC Cₐ (CFU/100ml) | 35,000,000 |

Table 5-3: Field and model data after calibration of model with respect to infiltration zone inactivation rate constant.

<table>
<thead>
<tr>
<th>Distance (in)</th>
<th>Concentration (MPN/gram soil)</th>
<th>Water Content (g water/g soil)</th>
<th>Field Data Concentration (CFU/100ml)</th>
<th>Model Concentration (CFU/100ml)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FC</td>
<td>1</td>
<td>19</td>
<td>110870.31</td>
<td>106931.36</td>
</tr>
<tr>
<td>FC</td>
<td>6</td>
<td>19</td>
<td>0.69</td>
<td>13092.72</td>
</tr>
<tr>
<td>FC</td>
<td>12</td>
<td>8</td>
<td>0.32</td>
<td>5587.41</td>
</tr>
<tr>
<td>FC</td>
<td>18</td>
<td>3</td>
<td>0.28</td>
<td>13795.08</td>
</tr>
<tr>
<td>FC</td>
<td>24</td>
<td>1</td>
<td>0.26</td>
<td>2242.65</td>
</tr>
<tr>
<td>FC</td>
<td>30</td>
<td>1</td>
<td>0.23</td>
<td>2723.54</td>
</tr>
<tr>
<td>FC</td>
<td>36</td>
<td>1</td>
<td>0.19</td>
<td>1.00</td>
</tr>
<tr>
<td>FC</td>
<td>42</td>
<td>1</td>
<td>0.18</td>
<td>1.00</td>
</tr>
</tbody>
</table>

5.2-2: Study Results Used for Model Output Comparison: During the Bohrer and Converse study, samples of septic tank effluent were taken and analyzed for fecal coliform concentrations and were reported in CFU/100ml (CFU-colony forming unit). Starting 1 inch below the distribution line, soil samples were taken every 6 inches below the line up to 42 inches (3.5 ft). The samples were analyzed to determine fecal coliform concentrations. The results were reported as median values in MPN/gram of soil (MPN-most probable number).

In order to be used for the model, the incremental concentration data needed to be converted from MPN/gram of soil to CFU/100ml (for sample conversion See Appendix B). First, a conversion was made from the mass of the soil to an equivalent volume of water. This was accomplished using moisture content data, reported in the study as gram
of water per gram of soil, at each sample depth to determine the amount of moisture in 1 gram of soil. Multiplying this information by the density of water provides the volume of water occupied in the gram of soil. In addition to uncertainty resulting from minor variations in experimental procedure, estimating procedures for both MPN and CFU are by nature of the method intrinsically variable. Furthermore, the MPN method is more variable than the CFU method with estimates for the MPN method being slightly higher than CFU estimates (Cho et al., 2010; Gronewold & Wolpert, 2008). A regression model developed by Cho et al. (2010) for converting from MPN to CFU was used to make the final conversion. A plot of field and model concentration data as a function of depth was made and a regression analysis performed to illustrate the degree of correlation between the model and field data (See Figure 5-1 and Appendix B).

Figure 5-1: Plot of concentration vs. depth for both the field and model data sets.
5.2-3 Regression Analysis of Model versus Field Study Results: All input parameters corresponding to the field data were entered into the model program. Using the concentration data, the input variable “minimum allowable VSD” was varied incrementally from 6 inches to 42 inches. The output concentration was recorded for each depth. As mentioned previously, using field and model data, concentration as a function of time was plotted. The resulting graph showed that both sets of data resembled an exponential decay. From Figure 5.1, it can be seen that at higher concentrations the data sets deviate slightly but as the concentrations decrease, the data sets follow an almost identical curve.

Using the transformed field data as the independent variable and the transformed model data as the dependent variable, a linear regression analysis was performed to quantify the ”goodness-of-fit” between the model and field data.. The correlation coefficient \( R^2 \) was calculated to be 0.56. The \( R^2 \)-value is an indication to the extent to which the model data represents the field data. A value of \( R^2 = 0.56 \) indicates that 56% of the variance in the field data is represented by the model.
CHAPTER VI
EXAMPLE MODEL APPLICATION

As stated previously, this model is intended to be used in several ways:

- by regulators to aid in educating industry professionals and the public on the actual risks associated with bacterial contaminants, reaching the groundwater table;
- as a tool for land use planning by municipalities or county officials when developing or updating land use codes as well as a tool that can aid in the decision process of granting variances; and
- by designers in determining the most appropriate treatment system for a specific site.

Further, this model is risk-based approach to examining vertical separation distances in on-site wastewater treatment systems, which means that it will NOT provide the exact concentration at a specified point below the trench bottom of the subsurface absorption system but rather probability that a bacterial concentration will reach a specified point. As such, the model is intended to be used when making risk management decisions. To demonstrate this use, a case study application is provided below.

6.1 Case Study Scenario

To illustrate this point, take a scenario in which county officials want to set a vertical separation distance rule to help ensure that groundwater within their county is not
contaminated from on-site wastewater treatment systems. In order to do this, the officials are considering setting a county wide minimum vertical separation distance between the trench bottom of the subsurface absorption system and the groundwater table within their county. However, if one minimum vertical separation distance is set for the entire county it might restrict development of certain areas where the land might not meet the minimum vertical separation distance. However, the site could contain soils that are suitable to effectively treat septic tank effluent to the minimum distance or the on-site system could be designed to treat wastewater to an acceptable level within the minimum vertical separation distance.

Instead of setting one minimum vertical separation distance for the entire county the officials decide to use this model program to consider the use of a risk-based approach which allows setting the minimum VSD for an on-site wastewater treatment system on a case-by-case basis. Shortly after this decision a new family moves to the county and wants to build a house on a plot of land they own. The home would require an on-site wastewater treatment system. From the preliminary design of the system it has been determined that the groundwater table on the site would be approximately one foot from the trench bottom of the subsurface absorption system which creates a concern of groundwater contamination. The information provided by the designer is used as the input to the model to determine the “risk” or probability that the contaminant concentration will reach a specified distance between the trench bottom and the groundwater table. This will aid in making a decision as to whether or not it would be acceptable to allow the on-site system to be built.
6.1-1: Model Use: To use the Excel© based model, the design parameters are input into the section marked “Input Design Parameters” (see Figure 6-1).

Figure 6-1:
Main page and Close-up graphic of the input section of Excel© based model.
The design of the system will have all of these parameters. The input parameters “Pretreatment”, “Sidewall Factor”, “Confining Layer Depth”, and “Horizontal Dist. Of Concern” may or may not be needed depending on the situation. In this scenario they are not used and therefore left blank (these 4 parameters are not necessary for the model to work). Of particular importance is the “Max. Allowable VSD” input parameter. In this scenario, the groundwater table is located at one foot below the trench bottom therefore the user input could either be one foot or some value more or less than that. For example, one might want to use the model to determine the concentration probability distribution that occurs 6 inches below the trench bottom and therefore 6 inches above the groundwater table.

Once the necessary input data has been entered, the output data will almost instantaneously appear (actual process time will depend on the speed of the computer being used). The output screen is located just below the input screen on the main page of the program. There are two output screens, one for situations in which there is no confining layer below the system and another for situations in which there is a confining layer below the system. In this example there is no confining layer so the first output screen, the one titled “Program Output (no confining layer)”, is the one to be viewed (see Figure 6-2).

The output is the user input vertical separation distance (VSD), the most probable concentration, and the probability that this concentration will reach the user specified VSD. It should be remembered that this is NOT the exact concentration at this point.
The output shows that from the given inputs, based on a 95% confidence level, the probability that the concentration shown in the output of the model (i.e., < 1 CFU/100ml) is between 0.02 to 40.8 CFU/100ml. Put more simply one can be 95% confident that at the level selected the concentration will be between 0.02 and 40.8 CFU/100ml. In reality, it is not possible for a concentration to be a fraction of a colony-forming unit (e.g., .02 CFU/100ml). Therefore, the result would be interpreted as that there is a 95% chance (probability) that the concentration at 6 inches below trench bottom is between 0 and 41 CFU/100ml. That is, there is at most 41 CFU/100ml and at the very
least 0 (or no) CFU/100ml. The model output could also simply be interpreted that there is a 95% probability that the groundwater will not get contaminated or that there is a 5% probability that contamination occurs.

By nature of the model as a risk analysis, the interpretations of the results are subjective. It may be acceptable for one community that there is a 5% probability that the bacterial levels might exceed 40 CFU/100ml. Another community might consider this to be contamination and not find the result acceptable. In the example provided, the result would be interpreted by the author as no contamination of groundwater. Other circumstances may occur in which if contamination occurs, there is an adverse effect as a result. For example, depending on whether or not the groundwater on the site is used by people for drinking or recreational purposes, the risk of contamination may not be an issue. If no one drinks the contaminated water, no one will be infected.

To illustrate the use of the second output screen, consider the same scenario described above with the same conditions except that instead of the groundwater table 1 foot below the trench bottom, there is a confining layer of bedrock 10 inches below the bottom of the trench. In addition to the confining layer, there is also a groundwater well located 20 ft from the system. As mentioned previously, when water (and contaminated water) is flowing vertically through the subsurface encounters a confining layer, it will begin to flow horizontally. This concept introduces one of the two additional input parameters for the second output screen of the model that is, the “Horizontal Distance of Concern”. The other input parameter is the vertical distance to the confining layer or “Confining Layer Depth”. The second output screen is used to determine the probability
of contamination occurring as a result of horizontal flow of contaminated water due to the interaction with a confining layer (See Figure 6-3 below).

**Figure 6-3: Graphic of second program output screen, “Program Output (Confining Layer Only).**

The use of the second output screen requires same inputs as in the previous scenario with the two additional inputs mentioned above. It should be noted that for a “Horizontal Distance of Concern” greater than 3 feet, the model cannot calculate the range of values and probability of contamination due to the limitations of some of the equations used in the model with respect to the time parameter. As time increases, variation in the model also increases which causes large, unrealistic confidence intervals in the model. In addition, as mentioned previously, as bacterial contact time increases, inactivation also increases. Therefore, the large confidence intervals are not practical or realistic.
CHAPTER VII

CONCLUSION & FUTURE DIRECTIONS

7.1 Conclusions

A stochastic model has been developed that provides the probability of a contaminant concentration reaching a user specified point below the trench bottom of a subsurface absorption system. The model has been calibrated and validated using existing field data. However, limitations with the model exist that would benefit greatly from future work. Although calibrated using data taken from literature, concentrations produced by the model are slightly lower than that reported in field data for the vadose zone. As a result, this section of the model should be recalibrated to produce a more conservative output concentration. In addition, there is an absence of inactivation rate constant data in the literature for the infiltration zone. This fact lead to the use of an inactivation rate constant that was a product of model calibration and not one that was reported in literature. As a result, the value for the inactivation rate constant for the infiltration zone is only theoretical. In general, the model would benefit from improved data regarding inactivation rate constants. Furthermore, as mentioned previously, the model produces unrealistic confidence intervals in situations with horizontal separation.
distances larger than approximately three feet. This is attributable to the lack information in the literature with regards to inactivation rate constant variation over time.

7.2 Future Work

7.2-1: Improved Model Calibration: From the calibration, validation, and regression analysis in Section 5, it can be seen that there is variation between the model output concentration data and real-world data. The field data and the data produced by the calibrated model (See Table 5-2) correlate only with respect to the concentrations at the upper end of the vadose zone. It appears as though die-off, and therefore the inactivation rate constant, is described accurately for the infiltration zone region of the model program. Although die-off in the vadose zone was calibrated through the use of inactivation rate constants corresponding to different soil types taken from existing literature, it appears that die-off in the vadose zone occurs much faster in the model program than in the reality described by the field data. As a result, this portion of the model should also be recalibrated against field data, similar to the calibration of the infiltration zone inactivation rate constant, in order to produce bacterial die-off that is more representative of the real world.

In order to understand this variation more clearly, a sensitivity analysis was performed on the inactivation rate constant for the infiltration zone and the vadose zone as well as on the hydraulic conductivity in the vadose zone. A sensitivity analysis is a study of how the variation in the output of mathematical model can be apportioned, qualitatively or quantitatively, to different sources of variation in the input of the model.
(Saltelli et al., 2008). Put more simply, it is a way to understand how the model responds to the effects of changes in input parameters.

7.2-2: Inactivation Rate Constant for the Infiltration Zone: The sensitivity analysis for the infiltration zone was performed by varying the inactivation rate constant for the infiltration zone at three depths (2, 4, and 6 inches) and recording the output for that specific layer at that specific depth (not the output of the entire model). In addition, the change in concentration per incremental change in inactivation rate constant was also determined at the same three depths. The results are presented in Tables 7-1 and 7-2 and in figures 7-1 and 7-2

Table 7-1: Incremental changes in the inactivation rate and the corresponding outputs from that layer at 3 different depths in the infiltration zone (i.e., biological layer).

<table>
<thead>
<tr>
<th>Biological Layer $k_{bi}$</th>
<th>Output Concentration $C_e$ @ depth = 2&quot;</th>
<th>Output Concentration $C_e$ @ depth = 4&quot;</th>
<th>Output Concentration $C_e$ @ depth = 6&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>(day$^{-1}$)</td>
<td>(CFU/100 ml)</td>
<td>(CFU/100 ml)</td>
<td>(CFU/100 ml)</td>
</tr>
<tr>
<td>0.000</td>
<td>8011466.7</td>
<td>8011466</td>
<td>8011466</td>
</tr>
<tr>
<td>0.050</td>
<td>5865956.3</td>
<td>4295024.2</td>
<td>3144795</td>
</tr>
<tr>
<td>0.100</td>
<td>4295024.2</td>
<td>2302603</td>
<td>1234477</td>
</tr>
<tr>
<td>0.150</td>
<td>3144795.5</td>
<td>1234477</td>
<td>483466</td>
</tr>
<tr>
<td>0.200</td>
<td>2302603.7</td>
<td>661799</td>
<td>190210</td>
</tr>
<tr>
<td>0.250</td>
<td>1685955</td>
<td>354797</td>
<td>74664</td>
</tr>
<tr>
<td>0.300</td>
<td>1234477.9</td>
<td>190210</td>
<td>29308</td>
</tr>
<tr>
<td>0.350</td>
<td>903856</td>
<td>101973</td>
<td>11504</td>
</tr>
<tr>
<td>0.400</td>
<td>61799</td>
<td>54668</td>
<td>4516</td>
</tr>
</tbody>
</table>
Table 7-2: The change in output concentration for incremental changes in the inactivation rate constant at 3 different depths in the infiltration zone (i.e., biological layer).

<table>
<thead>
<tr>
<th>Biological Layer $k_{st}$ (day$^{-1}$)</th>
<th>$\Delta C_t @ 2''$</th>
<th>$\Delta C_t @ 4''$</th>
<th>$\Delta C_t @ 6''$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>2145510.4</td>
<td>3716441.8</td>
<td>4866671</td>
</tr>
<tr>
<td>0.100</td>
<td>1570932.1</td>
<td>1992421.2</td>
<td>1910348</td>
</tr>
<tr>
<td>0.150</td>
<td>1150228.7</td>
<td>1068156</td>
<td>749851</td>
</tr>
<tr>
<td>0.200</td>
<td>842191.8</td>
<td>572848</td>
<td>294356</td>
</tr>
<tr>
<td>0.250</td>
<td>615648.7</td>
<td>307312</td>
<td>115546</td>
</tr>
<tr>
<td>0.300</td>
<td>451507.1</td>
<td>184597</td>
<td>45356</td>
</tr>
<tr>
<td>0.350</td>
<td>330591.9</td>
<td>88237</td>
<td>17604</td>
</tr>
<tr>
<td>0.400</td>
<td>242057</td>
<td>47305</td>
<td>6983</td>
</tr>
</tbody>
</table>

Figure 7-1: Concentration as a function of inactivation rate constant for the infiltration zone (biological layer).
Figure 7-2: Change in concentration as a function of inactivation rate constant for the infiltration zone (biological layer).

It can be seen from the tables and graphs that both the output concentration for the infiltration zone (Table and Figure 7-1) and the change in output concentration for the infiltration zone (Table and Figure 7-2) both decrease with increasing inactivation rate constant, as would be expected. In addition, as the depth of the biological layer increases, the output concentration also decreases.

The results of this sensitivity analysis highlight the importance and necessity of using accurate inactivation rate constant data in the model. At present, accurate data is lacking, if existent at all. As mentioned previously in chapter 5, prior to calibration, there was a large discrepancy between field data and model data when comparing bacterial concentrations in the infiltration zone and just beneath this zone in the first few inches of the vadose zone. Through calibration it was determined that this was largely due to the lack of inactivation rate constant data for the infiltration zone and its incorporation into the model resulting in no inactivation for the infiltration zone in the model. Once calibrated, it was shown in Figure 5-1 and Table 5-1 that, although the concentrations...
representing the model data are slightly lower than those of the field data, overall they follow a similar pattern.

**7.2-3: Inactivation Rate Constant for the Vadose Zone:** The same procedure outlined in section 7.2-2 was used to perform a sensitivity analysis on the inactivation rate constant for the vadose zone. The vadose zone inactivation rate constant was varied incrementally at 4 depths (6, 18, 30, 42 inches) and the model output concentration was recorded in order to determine the effect of vadose zone inactivation rate constant on the output of the model. The results are presented below in Table 7-3 and Table 7-4 as well as illustrated graphically in Figure 7-3 and Figure 7-4.

**Table 7-3:** Incremental changes in the inactivation rate and the corresponding outputs for the model at 4 different depths in the vadose zone.

<table>
<thead>
<tr>
<th>Vadose Zone k_{SOIL} (day^{-1})</th>
<th>Output Concentration C_t @ VSD = 6&quot;</th>
<th>Output Concentration C_t @ VSD = 18&quot;</th>
<th>Output Concentration C_t @ VSD = 30&quot;</th>
<th>Output Concentration C_t @ VSD = 42&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>937312</td>
<td>937312</td>
<td>937312</td>
<td>937312</td>
</tr>
<tr>
<td>0.050</td>
<td>587309</td>
<td>230585</td>
<td>905311</td>
<td>35543</td>
</tr>
<tr>
<td>0.100</td>
<td>358001</td>
<td>56725</td>
<td>8744</td>
<td>1347</td>
</tr>
<tr>
<td>0.150</td>
<td>230585</td>
<td>13954</td>
<td>844</td>
<td>51</td>
</tr>
<tr>
<td>0.200</td>
<td>144492</td>
<td>3433</td>
<td>61</td>
<td>2</td>
</tr>
<tr>
<td>0.250</td>
<td>905311</td>
<td>844</td>
<td>8</td>
<td>0.1</td>
</tr>
<tr>
<td>0.300</td>
<td>56725</td>
<td>207</td>
<td>1</td>
<td>0.01</td>
</tr>
<tr>
<td>0.350</td>
<td>35543</td>
<td>51</td>
<td>0.1</td>
<td>0.001</td>
</tr>
<tr>
<td>0.400</td>
<td>22271</td>
<td>12</td>
<td>0.01</td>
<td>0.0001</td>
</tr>
</tbody>
</table>
Table 7-4: The change in output concentration for incremental changes in the inactivation rate constant at 4 different depths in the vadose zone.

<table>
<thead>
<tr>
<th>Vadose Zone $\kappa_{\text{ser.}}$ (day$^{-1}$)</th>
<th>$\Delta C_r @ 6^\circ$</th>
<th>$\Delta C_r @ 18^\circ$</th>
<th>$\Delta C_r @ 30^\circ$</th>
<th>$\Delta C_r @ 42^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>350003</td>
<td>706727</td>
<td>846781</td>
<td>901769</td>
</tr>
<tr>
<td>0.050</td>
<td>219308</td>
<td>173860</td>
<td>81787</td>
<td>34196</td>
</tr>
<tr>
<td>0.150</td>
<td>137416</td>
<td>42771</td>
<td>7900</td>
<td>1296</td>
</tr>
<tr>
<td>0.200</td>
<td>86163</td>
<td>10521</td>
<td>763</td>
<td>49</td>
</tr>
<tr>
<td>0.250</td>
<td>53581</td>
<td>2589</td>
<td>73</td>
<td>1.9</td>
</tr>
<tr>
<td>0.300</td>
<td>33806</td>
<td>637</td>
<td>7</td>
<td>0.09</td>
</tr>
<tr>
<td>0.350</td>
<td>21182</td>
<td>156</td>
<td>0.9</td>
<td>0.009</td>
</tr>
<tr>
<td>0.400</td>
<td>13272</td>
<td>39</td>
<td>0.09</td>
<td>0.0009</td>
</tr>
</tbody>
</table>

Figure 7-3: Concentration as a function of inactivation rate constant for the vadose zone at 4 different depths.
Results similar to the sensitivity analysis for the infiltration zone can be seen when analyzing the information in Tables 7-3 and 7-4 as well as, Figures 7-3 and 7-4. That is, the output concentration of the model decreases with increasing vadose zone inactivation rate constant as well as decreases with increasing depth. Again, these results highlight the importance and necessity of using accurate inactivation rate constant data in the model. However, as highlighted in section 7.2-1, although calibrated through the use of inactivation rate constants taken from existing literature, die-off in the vadose zone occurs much faster in the model program than in the reality described by the field data. This result can be largely attributed to the natural variation that occurs in the die-off equation, or more specifically in the inactivation rate constant. Values for the inactivation rate constant reported in the literature will only provide rough estimates for use in the model.
7.2-4: Hydraulic Conductivity in the Vadose Zone: Again, the same procedure used above was used to perform a sensitivity analysis on hydraulic conductivity. The hydraulic conductivity was varied incrementally from 0 to 180 gpd/ft² and the resulting average concentration examined at 4 different depths (6, 18, 30, 42 inches). In addition, the change in concentration for each incremental change in hydraulic conductivity was determined. The results are presented below in Table 7-5 and Table 7-6. A plot was made of the concentration as a function of hydraulic conductivity as well as, a plot of the change in concentration as a function hydraulic conductivity. The graphs are presented in Figure 7-5 and Figure 7-6 below.

Table 7-5: Incremental change in hydraulic conductivity and the corresponding output concentrations at 4 different depths.

<table>
<thead>
<tr>
<th>$K_{sat}$ (hydraulic conductivity)</th>
<th>Output Concentration $C_r \oplus VSD = 6^\circ$</th>
<th>Output Concentration $C_r \oplus VSD = 18^\circ$</th>
<th>Output Concentration $C_r \oplus VSD = 30^\circ$</th>
<th>Output Concentration $C_r \oplus VSD = 42^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(gpd/ft²)</td>
<td>(CFU/100ml)</td>
<td>(CFU/100ml)</td>
<td>(CFU/100ml)</td>
<td>(CFU/100ml)</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>20</td>
<td>47.2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>40</td>
<td>19445</td>
<td>0.11</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>60</td>
<td>144700</td>
<td>47.2</td>
<td>0.02</td>
<td>0</td>
</tr>
<tr>
<td>80</td>
<td>354712</td>
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Table 7-6: Change in concentration as a result of incremental change in hydraulic conductivity at 4 different depths.

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Figure 7-5: Concentration as a function hydraulic conductivity at 4 different depths.

Figure 7-6: Change in concentration as a function of hydraulic conductivity at 4 different depths.

It can be seen from Figure 7-5 and Figure 7-6 that below a vadose zone depth of 6 inches (i.e., vadose zone depth of 18, 30, and 42 inches) there is very little change in model output concentration as a result of hydraulic conductivity. Due to the fact that output concentration also decreases with increasing depth, this result could be attributed to the
fact that bacterial inactivation which, directly effects output concentration, increases with increasing depth. In addition, the results also show that within the first six inches below the trench bottom, increasing hydraulic conductivity results in an increase in model output concentration. Simply put, the faster effluent moves through the first 6 inches below the trench bottom the less time is available for inactivation to occur. At increasing depths beyond 6 inches, although there is still less time for inactivation, overall inactivation is greater than at more shallow depths. Comparing all three sensitivity analyses, it can be seen that the inactivation rate constant (both infiltration and vadose constants) has a much greater effect on model output concentration than hydraulic conductivity. This can be seen when comparing Figures 7-2, 7-4, and 7-6. The magnitude of the change in concentration for both Figures 7-2 and 7-4 is much greater than that for Figure 7-6. In Figure 7-6, the magnitude of change in concentration is minimal for depths below 6 inches beneath the trench bottom.

7.2-5 Summary: The fact that values reported in the literature for vadose zone and infiltration zone inactivation rate constants could only provide rough estimates for use in models highlights the importance of a stochastic, or risk-based, approach for examining bacterial concentrations within the vadose zone as a result of on-site wastewater treatment systems. In addition, as depth increases within the vadose zone the output concentration becomes more variable, that is the confidence interval increases exponentially as the model output concentration decreases. This is directly attributed to the fact that the equation used to calculate the variance (equation 4-2) contains time as a variable and uncertainty by nature increases with time. For example, it is easier to predict the whether a few hours from now than it is to predict it a few days from now.
However, as mentioned previously, bacterial inactivation increases with time resulting in a discrepancy within the model with regards bacterial inactivation over time.

### 7.3 Future Directions

Based on the conclusions above, there are two major areas, which would benefit from further research. These areas are:

- Inactivation rate constant for the infiltration zone (biological layer)
- Inactivation rate constant for the vadose zone

As mentioned previously, due to the natural variation involved in the inactivation rate constant, there is a need for a better understanding of how parameters mentioned in chapter 2 (e.g., soil temperature, pH, moisture content…etc) effect bacterial inactivation in the vadose zone. In addition, as shown in the sensitivity analysis regarding the infiltration zone, this layer is also very important with regards to bacterial inactivation within on-site wastewater treatment systems. Further research into how this layer functions would also benefit this model as well the understanding of how on-site treatment systems function in general. Furthermore, once research was conducted in these areas, the model should be re-calibrated and re-validated using the new data in order to produce a more accurate model.
BIBLIOGRAPHY


# Appendix A

## Determination of Inactivation Rate Constant

### Table 1: Inactivation Rate Constants for Different Soil Types

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### Graph

**Coarse Sand**

- **y = -161x + 6.6305**
- **y = -69.923x + 2.8796**

- **LN**
- **LOG**

**Time (days)**

**Concentration (MPN/gram)**

- **Linear (LN)**
- **Linear (LOG)**

69
## Appendix B
### Linear Regression

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