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Vibration Analysis of Anisotropic Plates, Special Case: Violin

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VIBRATION ANALYSIS OF ANISOTROPIC PLATES,

SPECIAL CASE: VIOLIN

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VIBRATION ANALYSIS OF ANISOTROPIC PLATES, SPECIAL CASE: VIOLIN

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ABSTRACT

This thesis presents vibration analysis of the top plate of Stradivari violin by creating a 3D model using SolidWorks and finding mode shapes and natural frequencies using SolidWorks simulation. The top plate was affixed to the bottom plate via a side wall following the contour of the violin plates. Assuming the input excitation is sinusoidal, it was applied at the location of bridge where the strings rest on it.. The static component of the force was calculated to be 83.17 N. The first five natural frequencies of the violin top plate are in the range of 150 to 450 Hertz. The fact that frequencies associated with initial pitches of sound lie in the same region validates the analysis conducted since sound is generated in form of pressure waves at the resonant frequencies.

Initial step was to validate the computational code used in the finite element software. This was achieved with 0.7% error as compared to the theoretical values of a thin flat steel plate clamped at all four ends. In the next step violin sound box (top plate, rib and bottom plate) were modelled using Stradivari violin specifications.

Designing aspects of SolidWorks were successfully explored. Conventional vibration analysis (modal analysis) has an experimental approach using carefully devised instruments and sensors. The downside of this approach is that it does not look at the situation where the violin is actually played by a human. This simulation study focuses on the aforementioned situation. Vibration simulation saves on experimentation cost and enables design engineers to study machine components that undergo deformations due to vibration to avoid catastrophic failure.

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CHAPTER I

INTRODUCTION

1.1 General Problem

Vibration is a mechanical phenomenon in which a component or a structure undergoes oscillations that occur about an equilibrium point. Vibration and study of sound are closely related topics. In most cases creation of sound or ‘pressure wave’ is due to vibration of structures. Sound is created by pressurized wave transmitted through a medium such as air or water consisting of frequencies that lie within the spectrum of hearing. Violin is a musical instrument that is capable of creating such a pressure wave. The distinct sound created by a violin is due to the interaction between its various components.

Vibration Analysis is the study of frequencies and mode shape of a mechanical system due to a force input or an initial disturbance. A free vibration is the one in which energy is imparted to the system which subsequently sets the system into an oscillatory motion. This will cause the system to vibrate freely at one or more of its natural frequency. A forced vibration occurs when an external force is applied to the system. Vibration inducing input in a violin is applied through the strings as complex periodic input. Strings vibrate at different frequencies which in turn produces ‘pitches’ of sound. In this study, vibration analysis of the top plate of a violin has been undertaken.

This study was consists of two major tasks. Initial task was the creation of a 3D model of a Stradivari violin. The next task was harmonic analysis of the model generated in task 1. The uneven thickness of plates in a Stradivari violin made it a challenging job to create a working model. The anisotropic nature of the physical properties of wood is another source of complication in performing a vibration analysis of a typical violin.

Anisotropy is the property of being directionally dependent as opposed to isotropy that implies identical properties in all directions. Naturally anisotropic (orthotropic) material wood tend to split easier along the grain than perpendicular to the grain. Wood’s strength and hardness is different when measured in different orientations.

1.2 Stradivari Violin History

Violin is a bowed string instrument. String instruments have been around since 2500 to 3000 BC. Lyres (Figure 1.1- Lyre) were the first string instruments with a wooden body that used to be held against the body. String instruments can be classified in the way instruments are played namely plucking, bowing and striking. Guitar, harp and sitar are the instruments that are played by plucking the strings using finger or plastic plectra. Piano is an instrument that uses striking the string method to create vibrations and eventually sound. Violin and Cello are among the instruments in which a bow made of stretched hair is used to cause vibration by a 'stick-slip phenomenon'.

Figure 1.1- Lyre

Stick-slip is a phenomenon in which two surfaces alternate between sticking to each other and sliding over each other. Sticking occurs when the applied force is less than the static friction. When the applied force surpasses that static resistive friction force, kinetic friction is applied instead of static and the two surfaces start to slip relative to each other.

Stradivari violin was first made by Antonio Stradivari in 1716 currently located in the Ashmolean Museum of Oxford. Antonio Stradivari was an Italian crafter of string instruments such as violins, cellos, guitars and harps. Stradivarius violins are well known for their design, construction and sound quality. The wood used for top plate was ‘spruce’ and ‘maple’ for the back plate, ribs and neck.

1.3 Background and Literature Review

1.3.1 Finite Element Method

According to the research done to date, the conventional method of performing vibration analysis using mathematical equations such as differential equations with numerical solutions was being used for simple structures. As the complexity of structures increases, it became imperative to come up with a method that would simplify complex structures. The Finite Element method was invented in the 1960s in which the shape of a structure is approximated with a finite number of smaller geometrical segments for which analytical equations were formulated and solved. A set of these segments consists of ‘elements’. Each element consists of several ‘nodes’ with different DOFs.

A shape function that is a polynomial function is used to interpolate and calculate the mass and stiffness matrices of the element. An equation of motion can be written as follows.

Equation 1-1- Equation of Motion

$$[\boldsymbol{M}][\ddot{\boldsymbol{X}}] + [\boldsymbol{C}][\dot{\boldsymbol{X}}] + [\boldsymbol{K}][\boldsymbol{X}] = [\boldsymbol{F}]$$

Where M is the mass matrix, C is the damping matrix, K is the stiffness matrix and F is the external force. Every element has a mass, damping and stiffness matrices. After assembling them into Equation 1-1, the natural frequencies and displacements at each node can be calculated as a quadratic eigenvalue problem. This is a mathematical overview of the procedure that Finite Element Analysis computer programs go through. ‘Comparison of Finite Element Analysis and Modal Analysis of Violin Top Plate’ by Ye Lu discusses the detailed formulation of Finite Element Method and compares it to an Experimental Method.

1.3.2 Experimental Methods

Unlike Finite Element Modal Analysis that uses a program to simulate vibration behavior, modal analysis in a generic sense means an experimental analysis using physical equipment and taking data readings. In the vibration study of a violin by Ye Lu [1], the central idea is to get Frequency Response Function (FRF) of the structure. For this experiment a measurable excitation is applied to a structure at a specific point and the FRF is measured at several other points on the structure. An accelerometer was used for measuring the acceleration of the excited structure.

Chladni method is another experimental method invented by Ernst Chladni that studies mode shapes of plates under vibration. In this method, a rigid plate is covered with sand and undergoing vibrations at resonant frequency. The sand arranges itself in a nodal pattern depending on the frequency. Figure 1.2 Chladni Method shows the sand patterns associated with varied frequencies.

Figure 1.2 Chladni Method

In this study, a scenario where Stradivari violin is being played at different sound pitches is being considered. Also, the type of fixtures on the sound box of a violin and the remote forces acting due to the fingerboard are taken into consideration. Under this scenario, the vibration behavior of the violin plates was studied using SolidWorks simulation based on finite element analysis.

1.3.3 Reason for choosing this problem

SolidWorks is an excellent tool for creating complicated structures. Variable thickness in both horizontal and vertical axis makes violin a challenging structure to design as a 3D CAD model. The purpose of choosing SolidWorks was to explore the numerous features that are currently used in the industry to design and manufacture components. Based on the previous work done, it seemed unique to create a CAD model of a Stradivari violin and perform a virtual analysis on it. SolidWorks simulation uses finite element method to calculate parameters such as displacement and stress. The aim of this study was to compare the frequencies and mode shapes of the experimental studies performed previously and analyze the capabilities and accuracy of SolidWorks simulation as a tool. Many industries use this program to design optimum products with low cost and high sustainability.

Secondarily, violin is made of wood which is an example of anisotropic material. It has unique and independent mechanical properties in the directions of three mutually perpendicular axes: longitudinal, radial and tangential. The longitudinal axis is parallel to the fiber whereas radial axis is normal to the growth rings and tangential axis is perpendicular to the grain but tangent to the growth rings [2].

Vibration properties that are of particular interest when it comes to wood are speed of sound and damping capacity. The speed of sound across the grain is much less (about $1/5^{th}$) than the speed of sound along the grain [2]. This is because its transverse modulus of elasticity is much less than the longitudinal. Violin makers choose wood as the material because of its lightness, strength and flexibility.

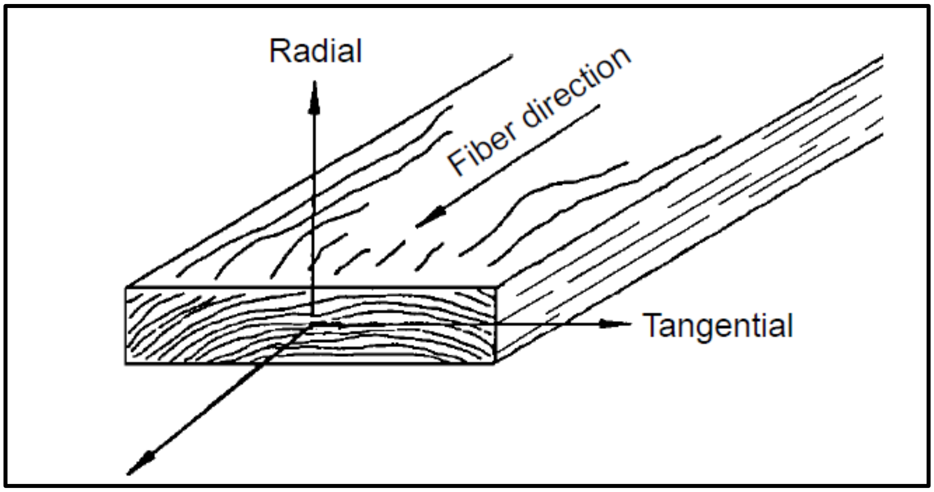


Figure 1.3 - Axis orientation of Wood

CHAPTER II

PROBLEM FORMULATION

2.1 Problem falls into Vibrations of plates and shells

Vibration Analysis of Violin plates can be performed experimentally or via simulation. Before recent years, the analytical approach was fundamental to finding frequencies and mode shapes. Currently there has been ample amount of research on vibration analysis of plates and shells using finite element analysis.

Plates have been one of the most vital structures in the field of engineering be it civil, hydraulic, aerospace, ships or machine equipment. When in service, plates undergo dynamic loading that could lead to critical conditions. This study uses SolidWorks Simulation to study the natural frequencies and mode shapes of the top plate of a violin.

In order to perform Vibration analysis, a Stradivari Violin was modeled in SolidWorks. The Geometry of Stradivari violin was constructed as follows.

2.2 Development of Geometry

Drawing

Geometry of the Stradivari Violin consists of three primary parts. The top plate, rib and bottom plate (Figure 1.1). Secondarily, the bass bar and the sound post are two parts of violin that contribute towards making violin an unsymmetrical instrument despite of its symmetric configuration from a geometric point of view. The four strings attached at both ends of the violin are rested on the bridge through which vibrations are transmitted to the top plate at different frequencies. The top plate is made of soft wood whereas the rib and bottom plate is made of relatively hard maple.

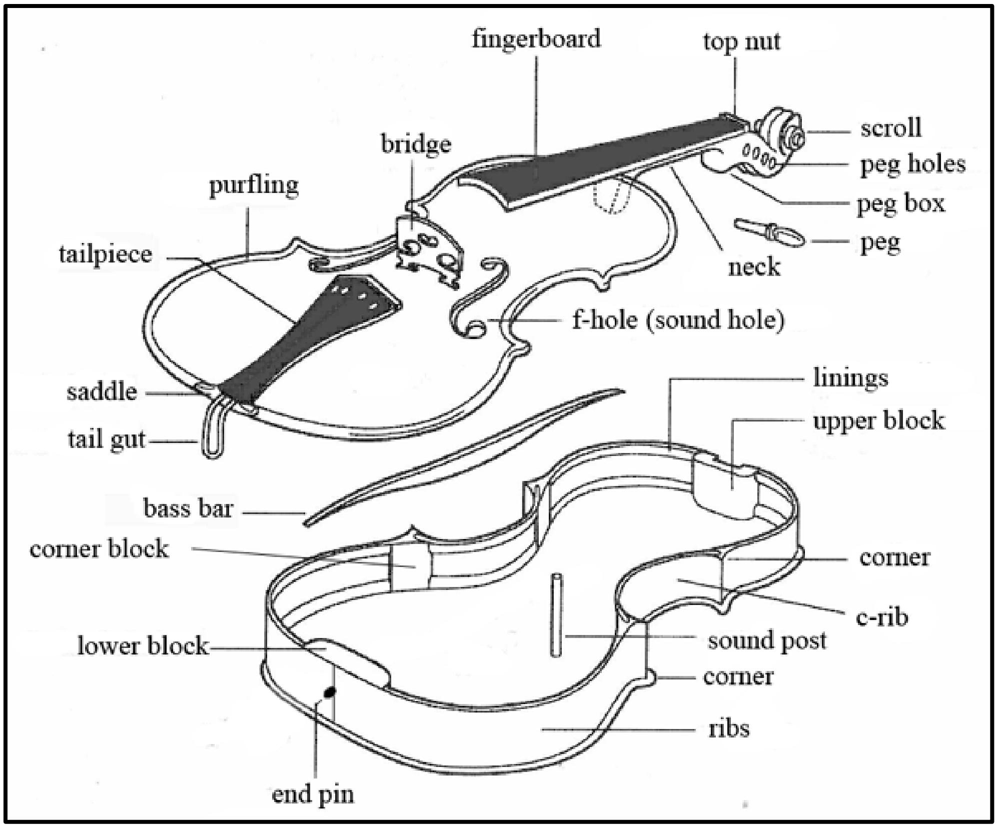


Figure 1.1- Violin Assembly (Exploded View)

The sound of a violin relies on its shape, the type of wood that is used and the thickness profile of top and bottom plates. For this study, the thickness profile of Stradivari Violin was taken into consideration. The following figure 2.2shows the drawing of a Stradivari bottom and top plate on left and right side of the central axis. Each section is marked with a specific thickness which is maximum at the center and gradually decreases away from center of both horizontal and vertical axis. The most challenging aspect of the geometry to be created in the SolidWorks was the aforementioned variable thickness along both x and y axis.

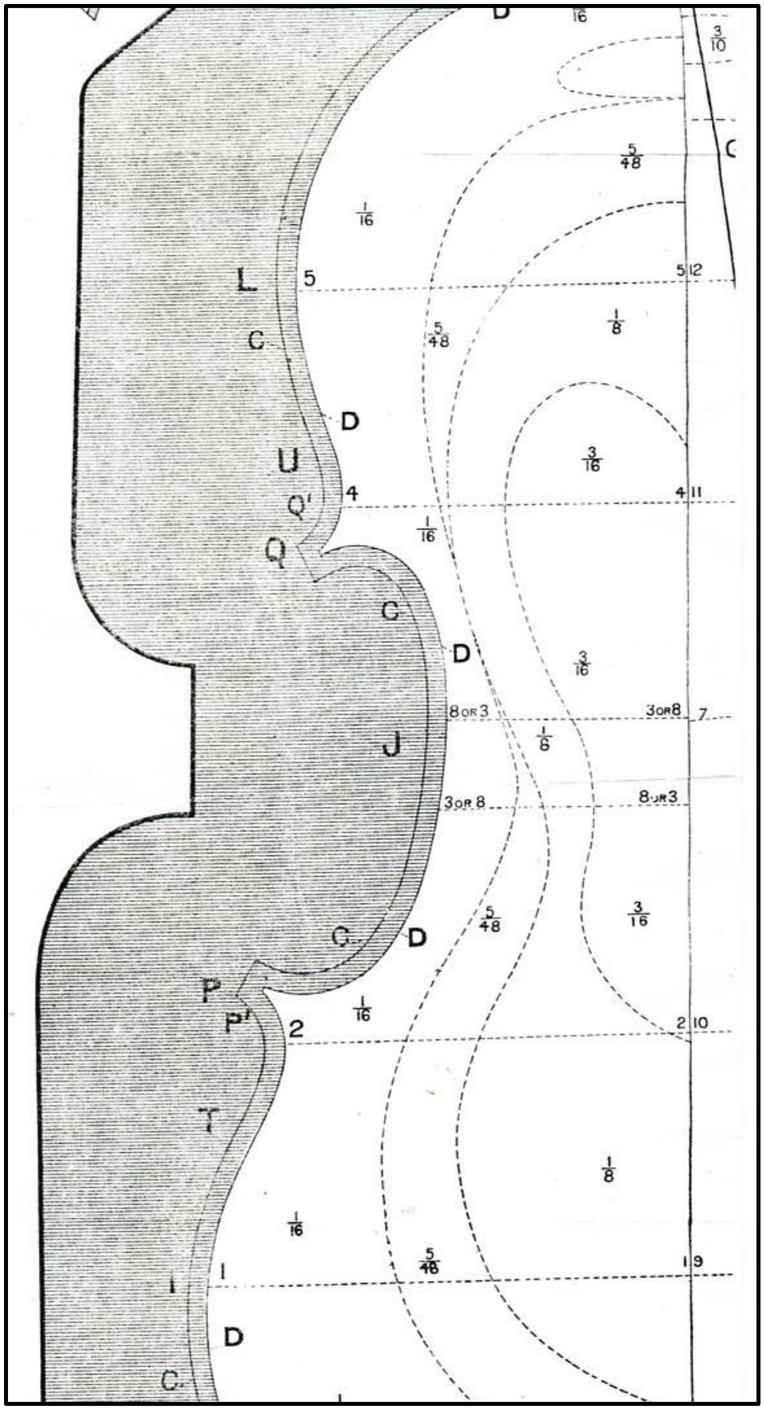


Figure 2.2 - Bottom Plate Drawing with thickness profile

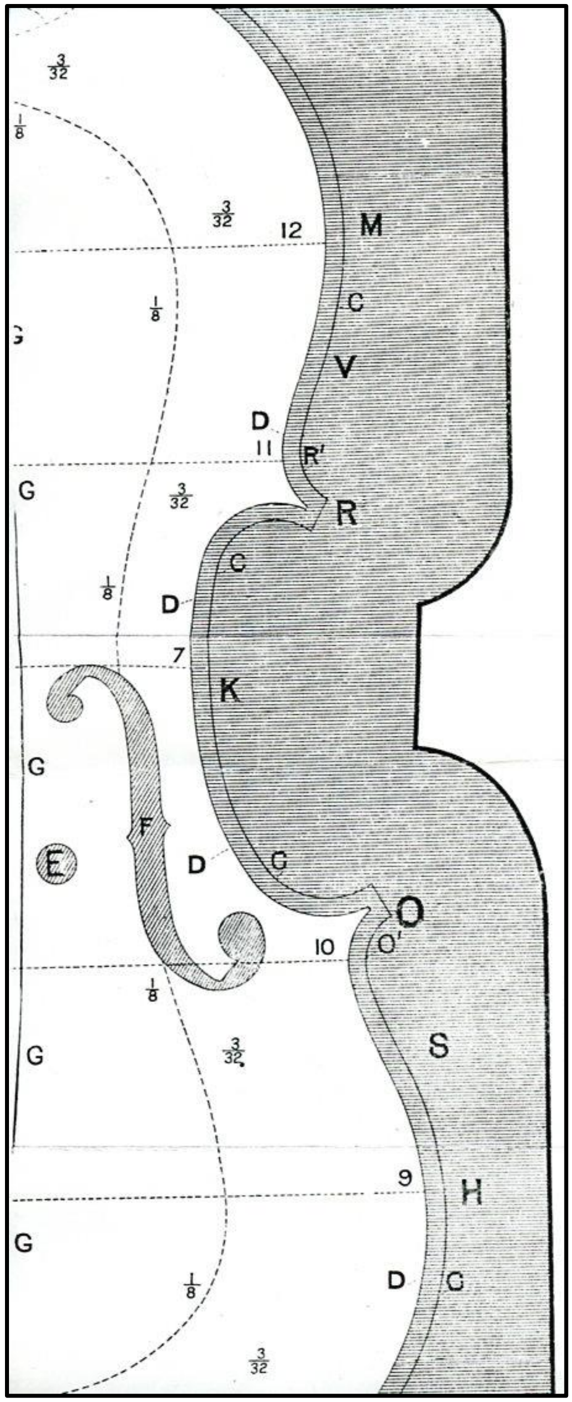


Figure 2.3 - Top plate drawing with thickness profiles

For simplicity of solid modeling, each top and bottom plate was divided into three distinct parts, ‘Middle Section’, ‘Big C’ and ‘Small C’.

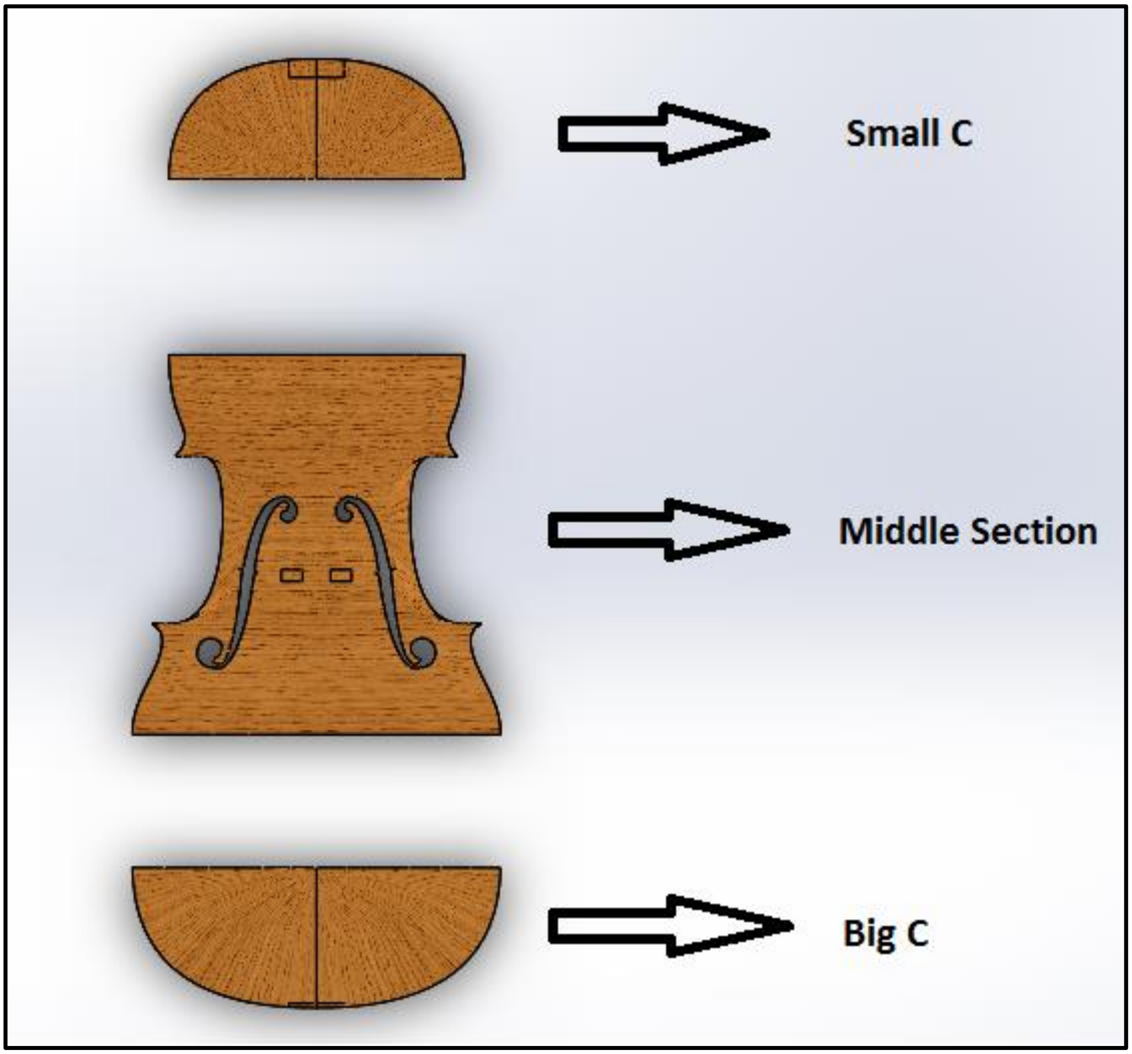


Figure 2.4 - Illustration of Violin parts

Geometry of Bottom Plate

Middle Section

The Middle part of bottom plate was modeled using the ‘loft’ feature in SolidWorks. As visible from Figure 2.5, blue sketches were created on planes parallel to X-Y plane whereas purple sketches were created on planes parallel to Y-Z plane. Both profiles and guided curves were created using 'splines' in SolidWorks. When using the loft feature, an extrusion is created through profiles (blue sketches) along the guided curves (purple sketches). The Loft feature enables us to create complicated asymmetrical geometries.

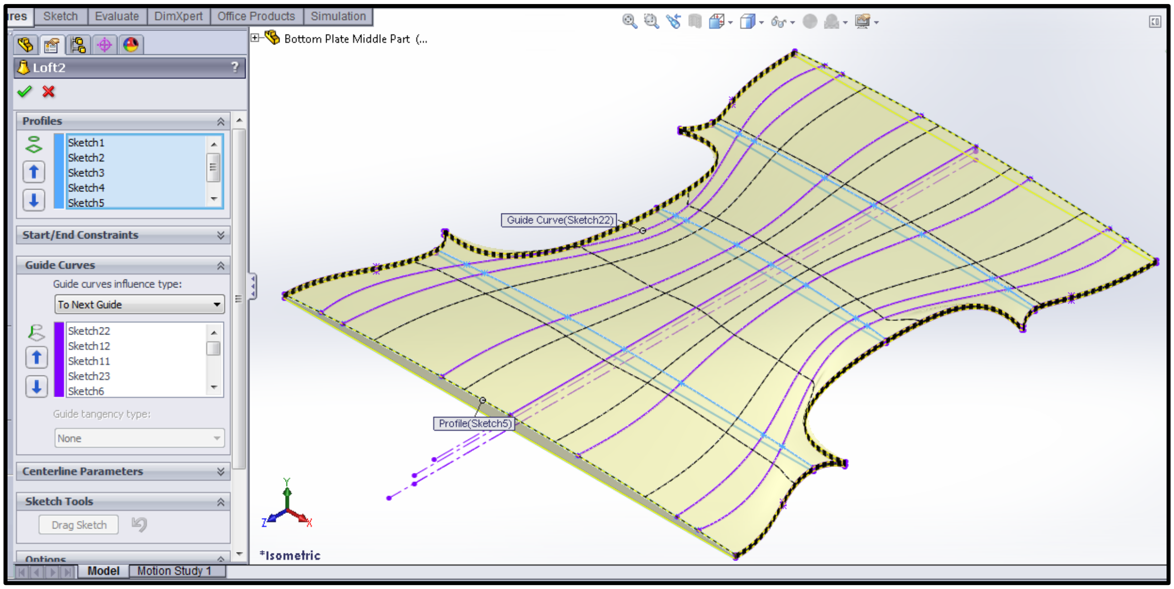


Figure 2.5: Bottom Plate Middle Part (Loft Feature)

Big C

Similarly, using splines and loft the remaining parts of the Bottom plate namely 'Big C' and 'Small C' were modeled. Figure 2.6 shows a half portion of Bottom plate Big C that was created using the 'loft'. The entire part was then created by mirror imaging (Figure 2.7) along the central axis.

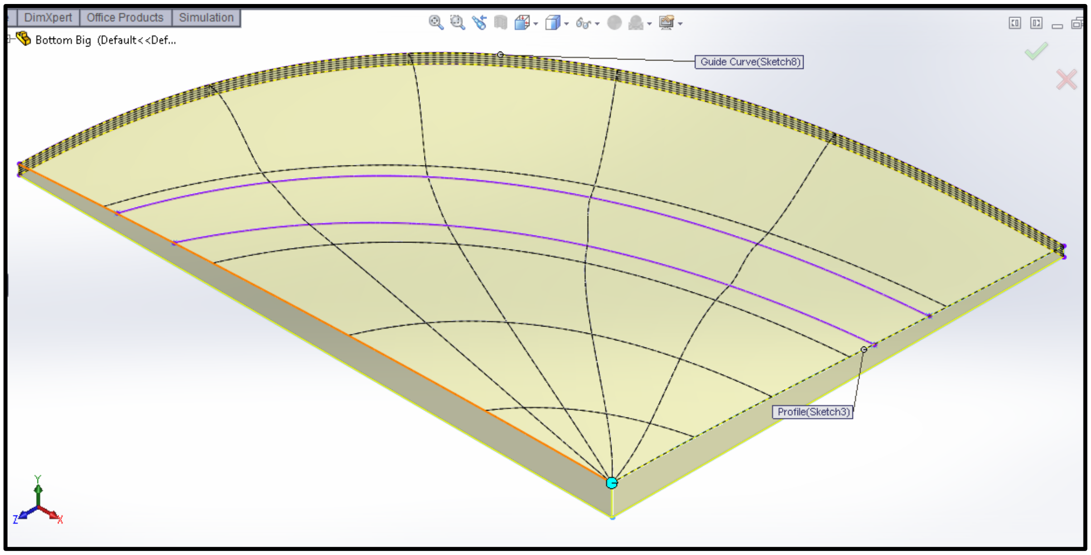


Figure 2.6 - Bottom Plate Big C (Loft Feature)

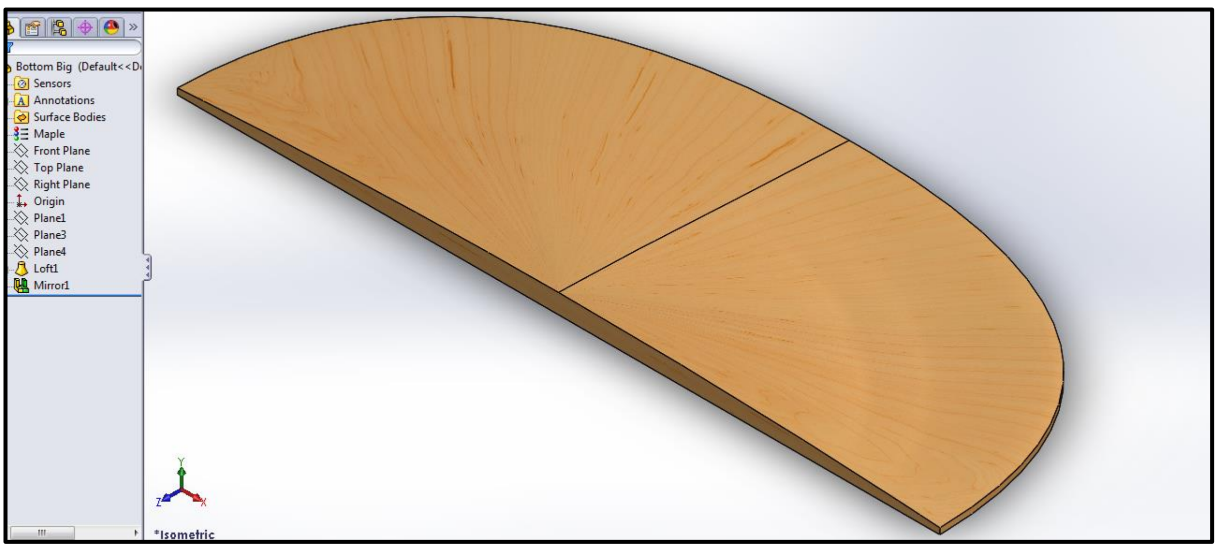


Figure 2.7 - Bottom Plate Big C (mirror imaging)

Small C

The third and final section of the bottom plate was similarly modeled using loft feature and mirror imaging.

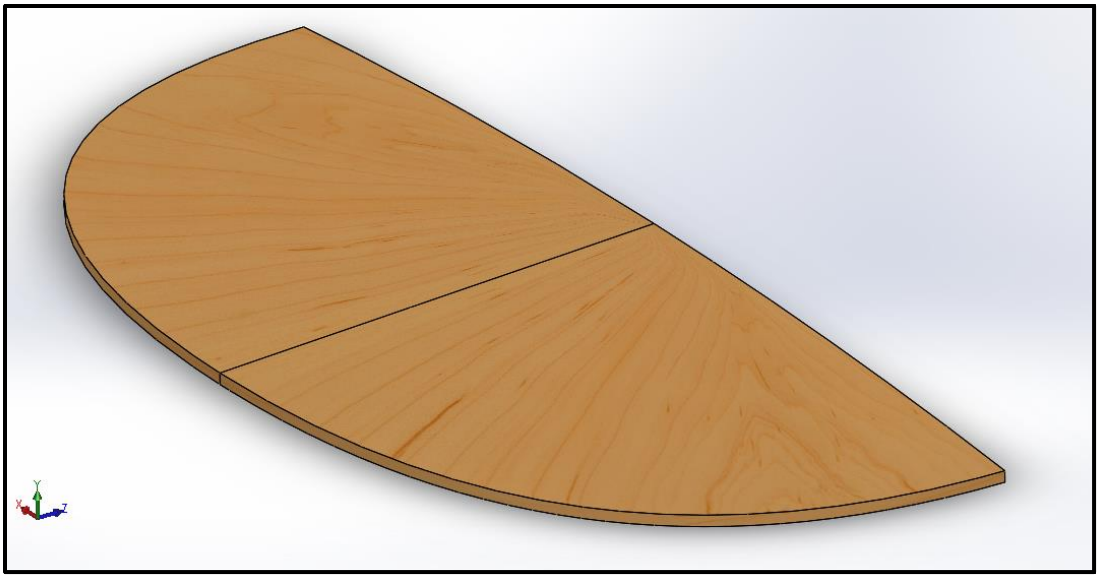


Figure 2.8 - Bottom Plate Small C

Using Assembly mates in SolidWorks, the three parts were merged to create a single component 'Bottom plate' of the violin.

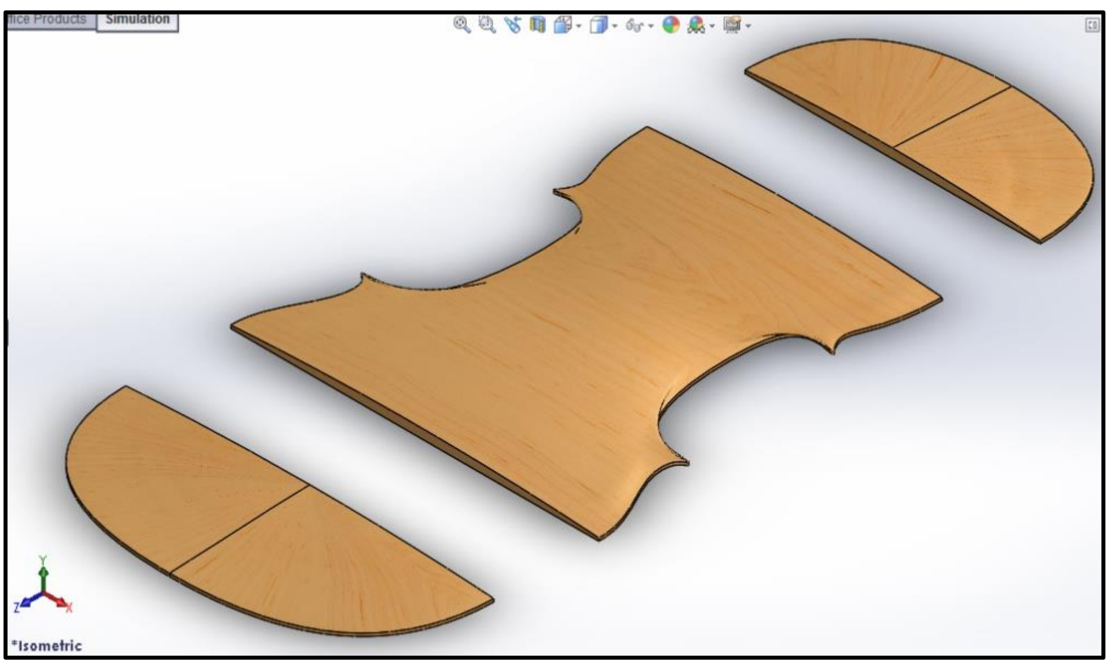


Figure 2.9 - Bottom Plate Assembly

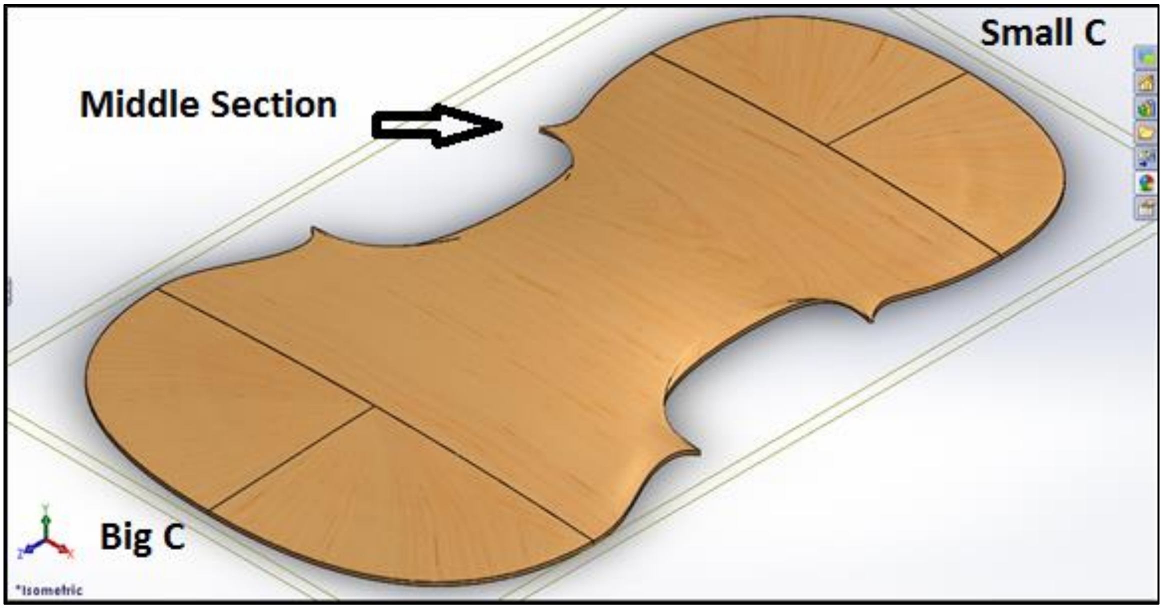


Figure 2.10 - Bottom Plate Assembly (Dimetric View)

Geometry of the Top Plate

Using the same methodology for modeling ('Loft' feature and 'mirror imaging' the sketch), Top plate was created in parts namely middle section, big C and small C. Thickness profiles shown in figure 2.3 were incorporated in the loft feature as the guided curves. The sketches perpendicular to the curved sections that are parallel to each other were incorporated as the profiles in the loft feature. The resulting Top plate is visible below in Figure 2.12.

The sound hole plays a crucial role in reproduction of sound in a violin. This was integrated in the top plate by creating a sketch parallel to the plate and applying an extrusion through the plate as seen from the Figure 2.11 below.

The sound post and bass bar were added to the top assembly that is seen in the Figure 2.13.

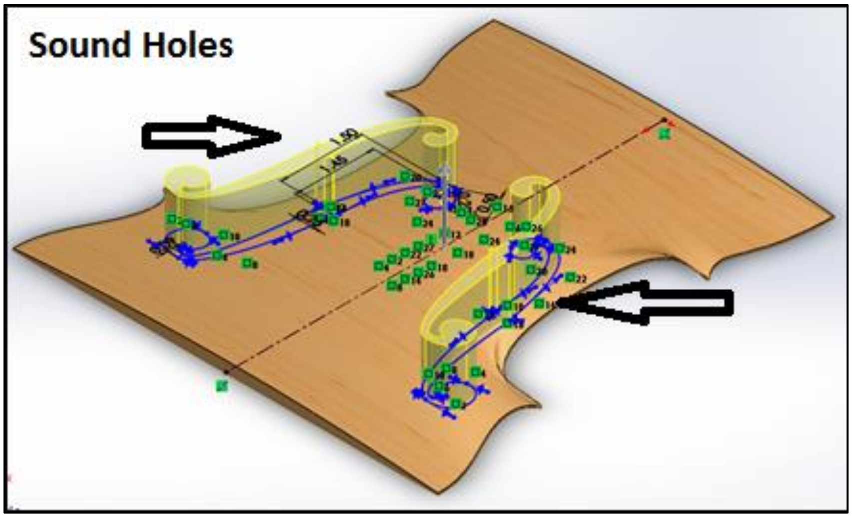


Figure 2.11 - Sound Holes on Top Plate

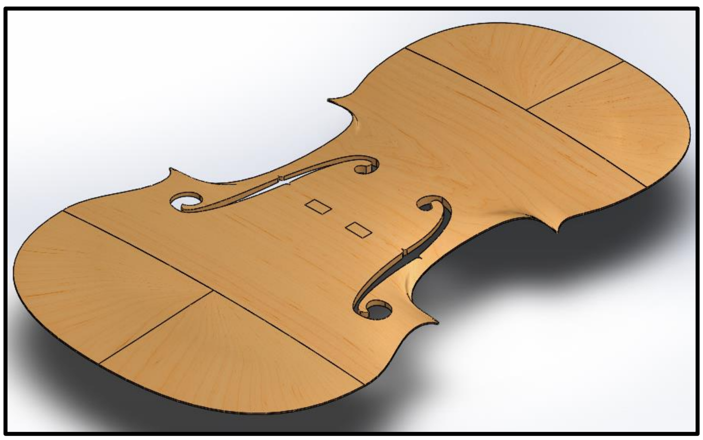


Figure 2.12 - Top Plate (Dimetric view)

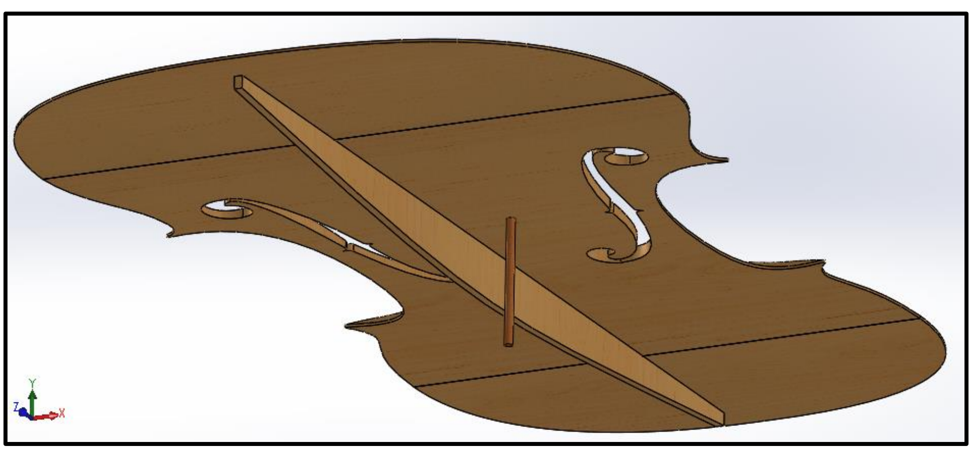


Figure 2.13 - Top Plate (Bass bar and Sound post)

Geometry of the 'Rib'

The Rib was created using 'Extruded cut' visible in Figure 2.14.

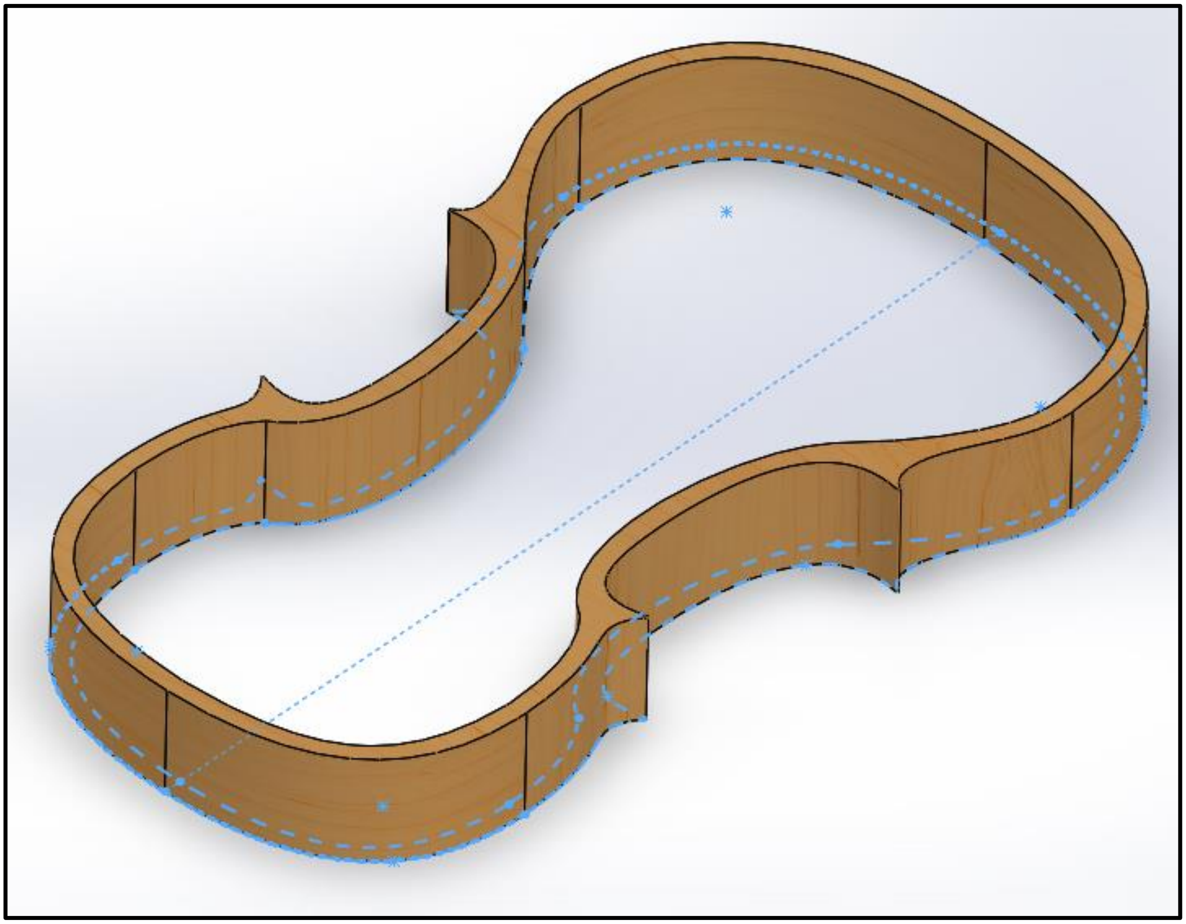


Figure 2.14 – Rib

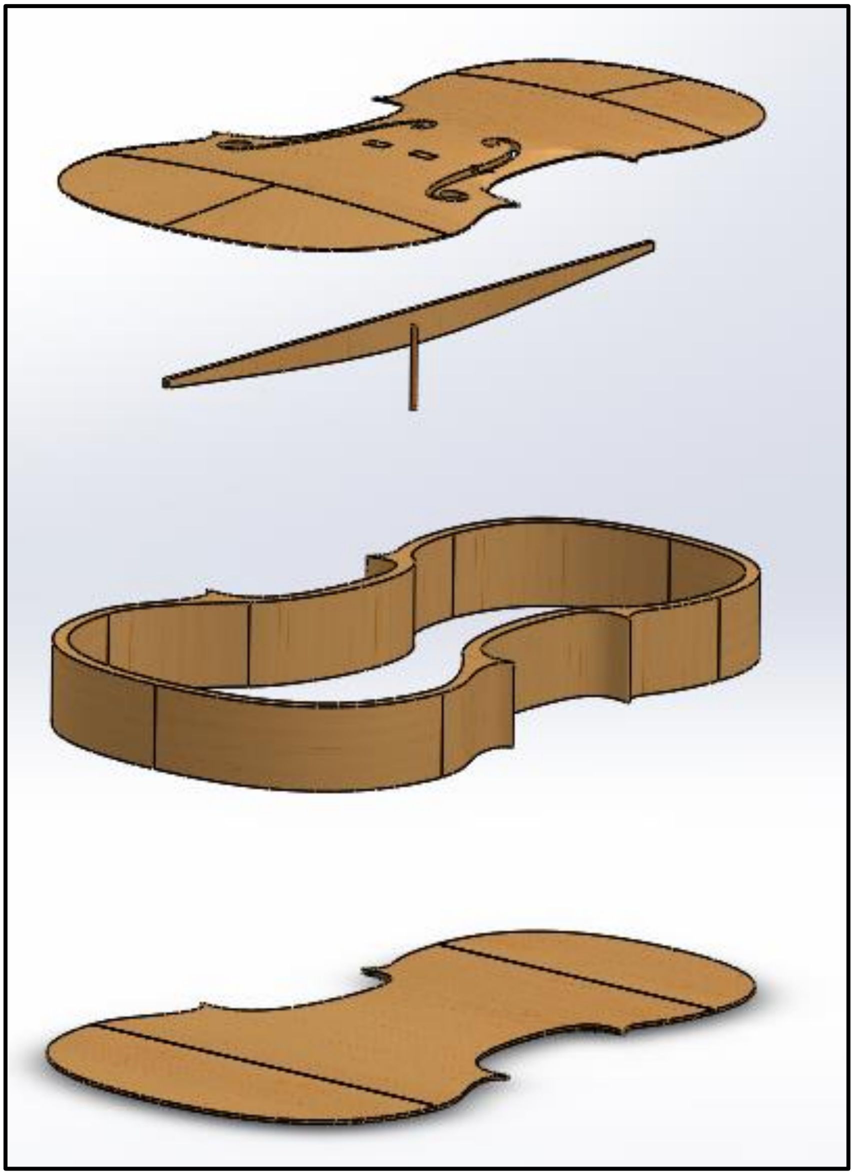


Figure 2.15 - Violin (Exploded View)

SolidWorks component set allows creating a group of components and analyzing the type of contact between every component. Applying component contact creates a bond at the interface of selected component.

CHAPTER III

CODE VALIDATION

Analogical to checking grammar in a sentence before progressing on to write an entire paragraph, it is of vital importance to apply planned methodology to a simplified subject. In this study, a 30cm by 30cm squared simple plate with 2.5mm thickness was considered as the subject for code validation. The aim of this study was to use an isotropic material such as steel and compare the results (mode shapes) with known analytical results.

3.1 Flat Steel Plate

A simple frequency analysis was conducted on a flat steel plate with all four boundaries fixed Figure 3.1

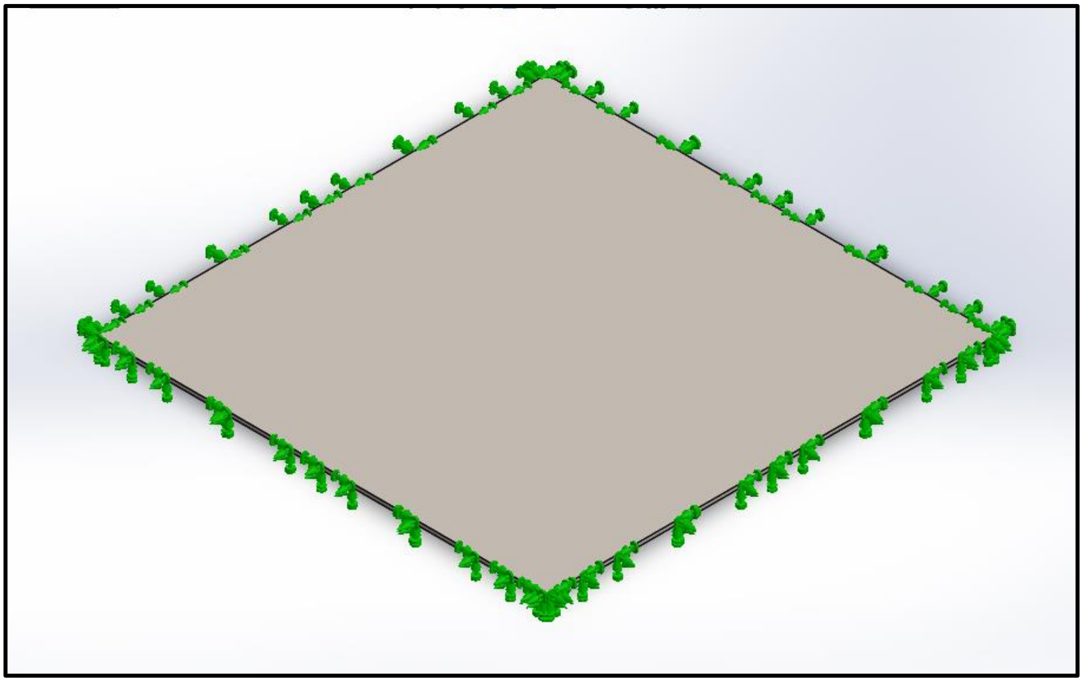


Figure 3.1 - 30cm by 30cm flat steel plate

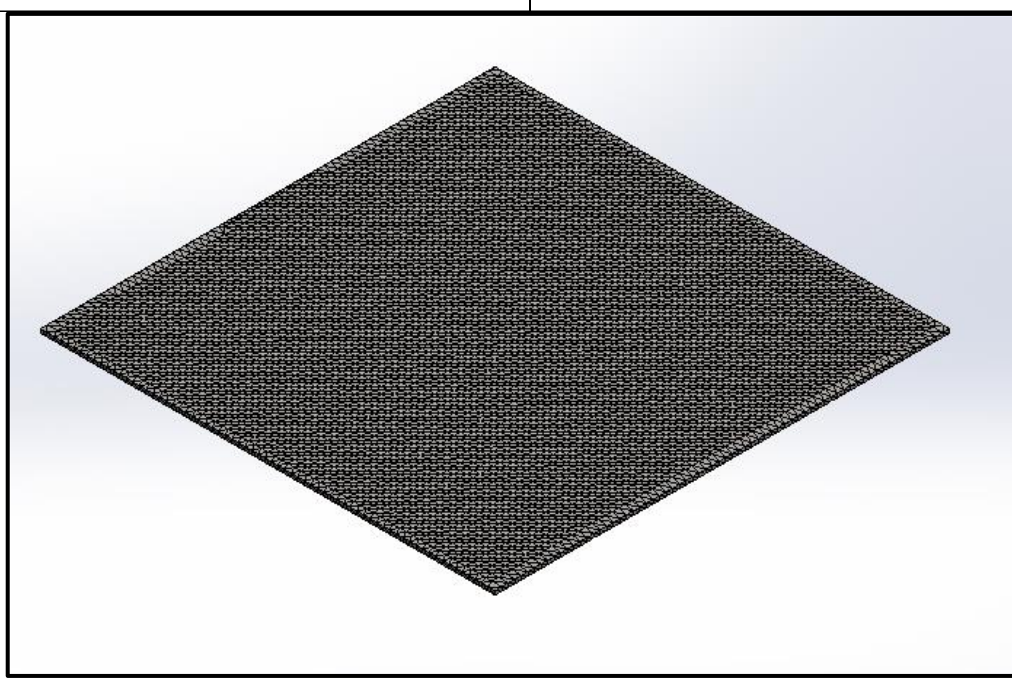
Flat plate was meshed into smaller elements. SolidWorks meshing specifications are listed in the Table 3-1.

Table 3-1- Mesh type and dimensions

Mesh type	Solid Mesh
Mesher Used:	Standard mesh
Automatic Transition:	Off
Include Mesh Auto Loops:	Off
Jacobian points	4 Points
Element Size	0.172511 in
Tolerance	0.00862554 in
Mesh Quality	High

Table 3-2 - Mesh Details

Total Nodes	60030
Total Elements	30561
Maximum Aspect Ratio	12.475
% of elements with Aspect Ratio < 3	90
% of elements with Aspect Ratio > 10	0.0196
% of distorted elements(Jacobian)	0
Time to complete mesh(hh;mm;ss):	00:00:16



The resulting mode shapes from Frequency Analysis (Simulation) are listed

Table 3-3 - Frequency from Simulation

Study name: Frequency Study			
Mode No.	**Frequency(Rad/sec)**	Frequency(Hertz)	Period(Seconds)
1	**1597.8**	254.3	0.0039324
2	**3257.3**	518.41	0.001929
3	**4804**	764.58	0.0013079
4	**5837.6**	929.09	0.0010763
5	**5865.4**	933.51	0.0010712

For code validation, natural frequencies were found using the formula in equation for thin flat plates of uniform thickness

Equation 3-1 - Natural frequency formula for thin flat plates of uniform thickness

$$\omega_n = B\sqrt{\frac{Et^2}{\rho a^4(1-\vartheta^2)}}\, rad/sec$$

Where,

E = Young's Modulus of Alloy Steel

t = thickness of plate in meter

ρ = mass density in kg/m^3

a = length of the square plate in meter

ϑ = Poisson's ratio

Using equation 3-1, the natural frequencies of the thin steel plates were found using values of B corresponding mode [6].

Table 3-4 - Frequencies calculated using equation 3-1

Mode	Value of B	Natural Frequency (rad/second)
1	10.4	1585.23307
2	21.21	3232.960904
3	31.29	4769.417572
4	38.04	5798.294804
5	38.22	5825.731531

On comparing the theoretical results to the simulation results Table 3-5 shows the accuracy of the simulation. Average error is 0.7% when compared to theoretical results. This confirms and validates the computational code used by the simulation software (SolidWorks).

Table 3-5 - Percentage error of natural frequency (theoretical and finite element simulation)

Mode	Theoretical Natural Frequency (rad/second)	Frequency(Rad/sec) using Simulation	**% Error**
1	1585.23307	1597.8	**0.793**
2	3232.960904	3257.3	**0.753**
3	4769.417572	4804	**0.725**
4	5798.294804	5837.6	**0.678**
5	5825.731531	5865.4	**0.681**

Figure 3.2 - Flat plate mode shape 1

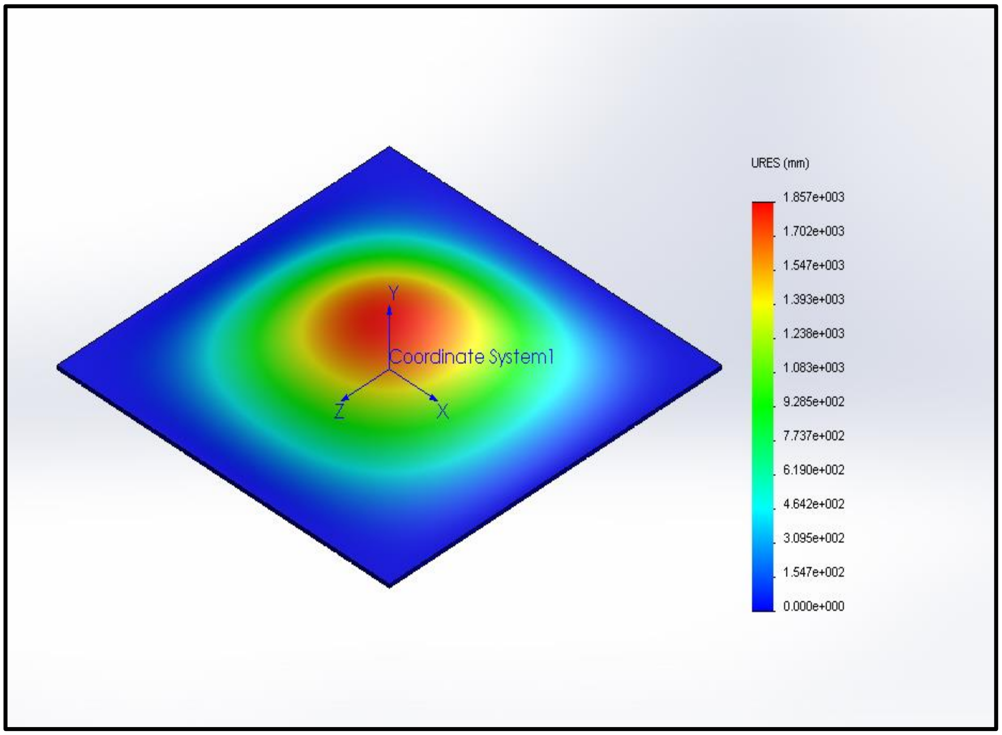


Figure 3.3 - Flat plate mode shape 2

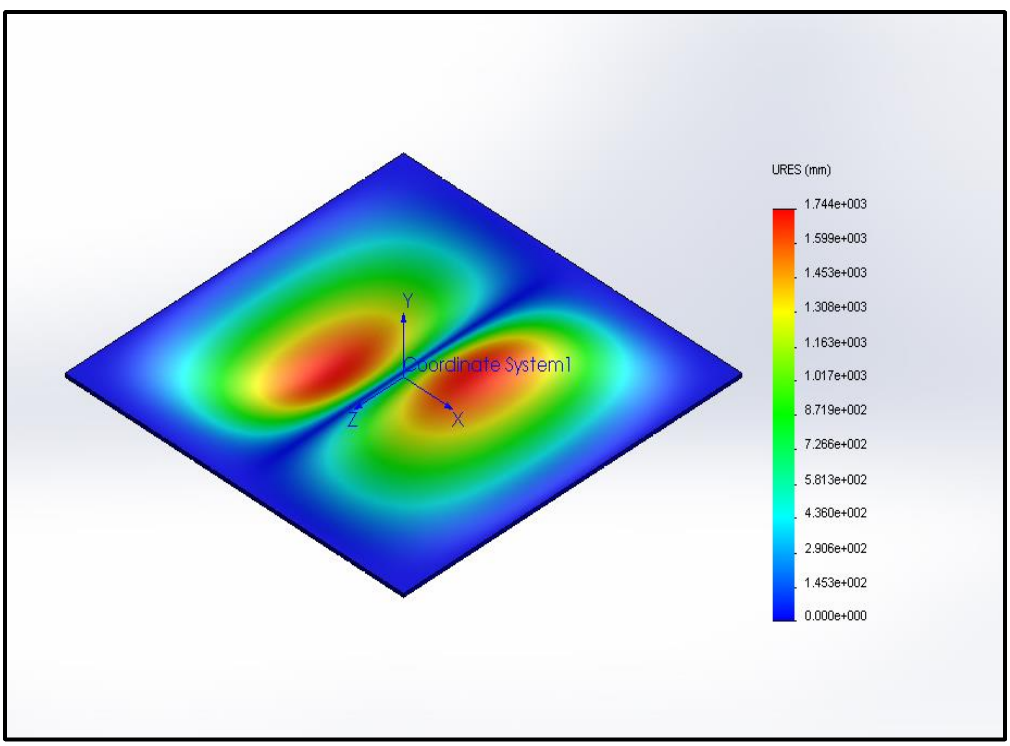


Figure 3.4 - Flat plate mode shape 3

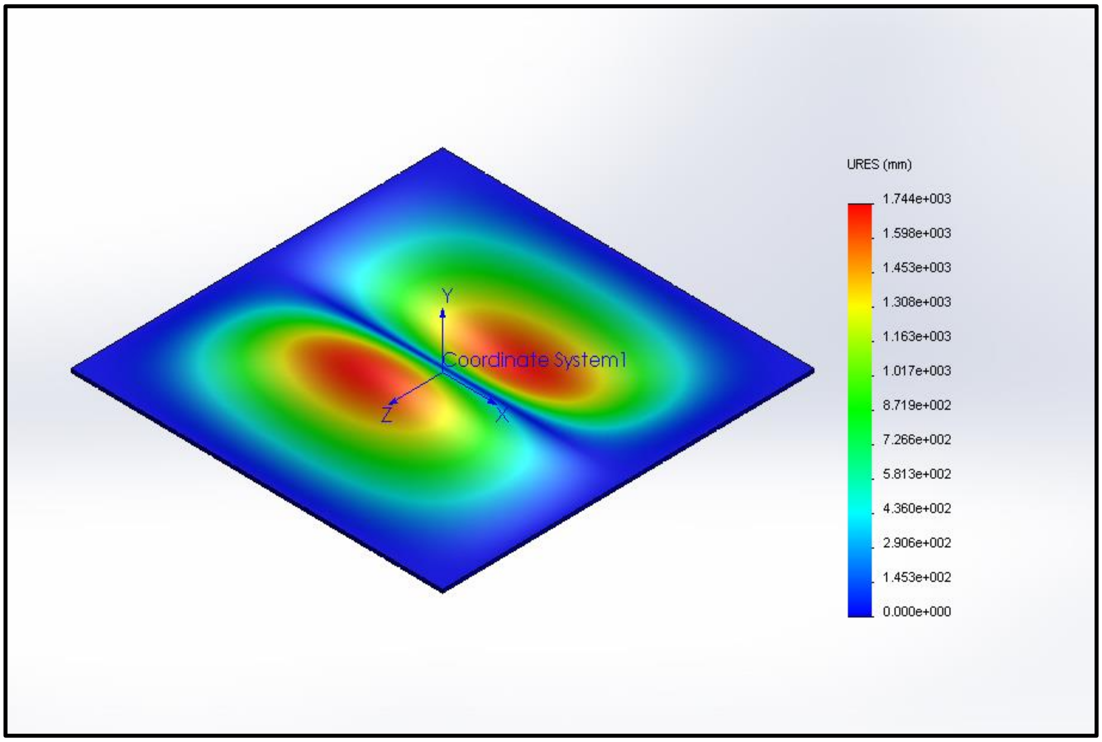
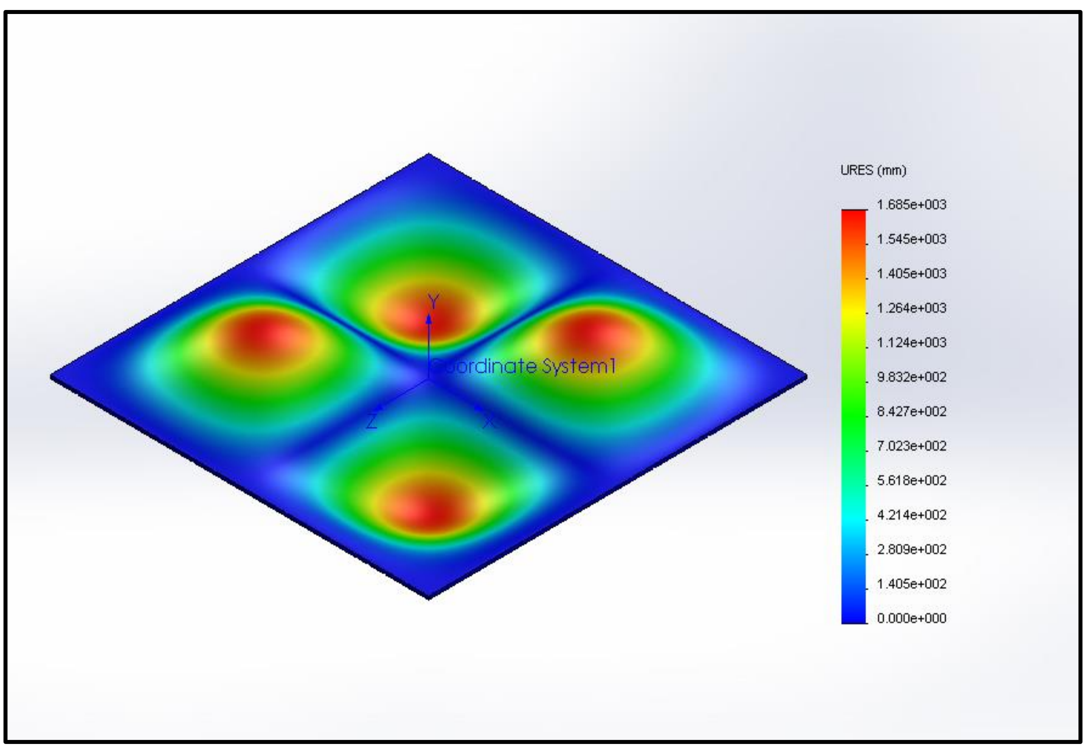


Figure 3.5 - Flat plate mode shape 4



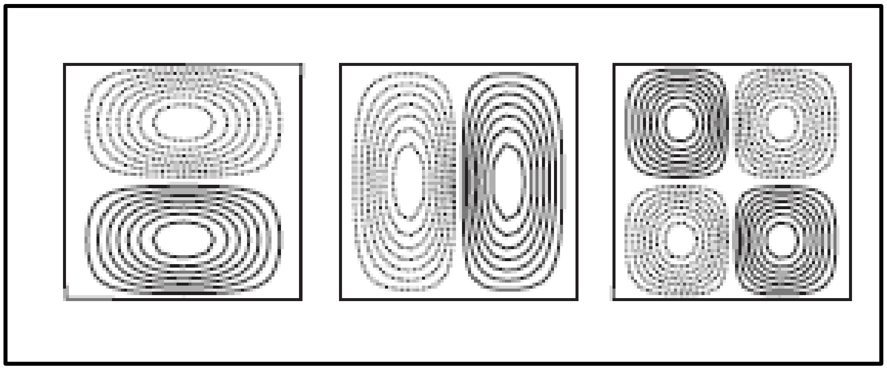


Figure 3.6 - Flat pate mode shapes 2, 3 and 4 (Analytical results)

'Free vibration analysis of plates by using a four-node finite element formulated with summed natural transverse shear strain' by S.J. Lee illustrates the patterns that resemble mode shapes of a square plate fixed at the boundaries [3]. Comparing the analytical results to the acquired results from SolidWorks simulation, it was confirmed that the computational code used by the finite element software is valid. Looking at the known mode shape patterns, the mode shape results from SolidWorks comply with each other.

3.2 Flat Wooden Plate

Having validated the computational code using steel as the material, the next step was using wood as the material for the flat plate with identical dimensions (30cm by 30cm).

Violins are made of two types of wood. The top plate is made from softwood and the rib and bottom plate is made from maple wood. For code validation, maple was applied as the material of the simple plate. SolidWorks has the ability to add a custom material to the material list and enter specific values for the mechanical properties such as modulus of elasticity and poison's ratio. Using 'Mechanical properties of wood' by David Green

[1] as the guide, material properties of maple wood were entered into the custom material list of SolidWorks. The material properties of maple wood are listed in table 3-6

Table 3-6- Material properties of maple

Model type:	**Linear Elastic Orthotropic**
Default failure criterion:	**Unknown**
Mass density:	**490 kg/m^3**
Elastic modulus in x:	**1.12e+010 N/m^2**
Elastic modulus in y:	**1.4916e+009 N/m^2**
Elastic modulus in z:	**7.345e+008 N/m^2**
Poisson's ratio in xy:	**0.424**
Poisson's ratio in yz:	**0.774**
Poisson's ratio in xz:	**0.476**
Thermal expansion coef in x:	**4e-006 /Kelvin**
Shear modulus in xy:	**1.25e+010 N/m^2**

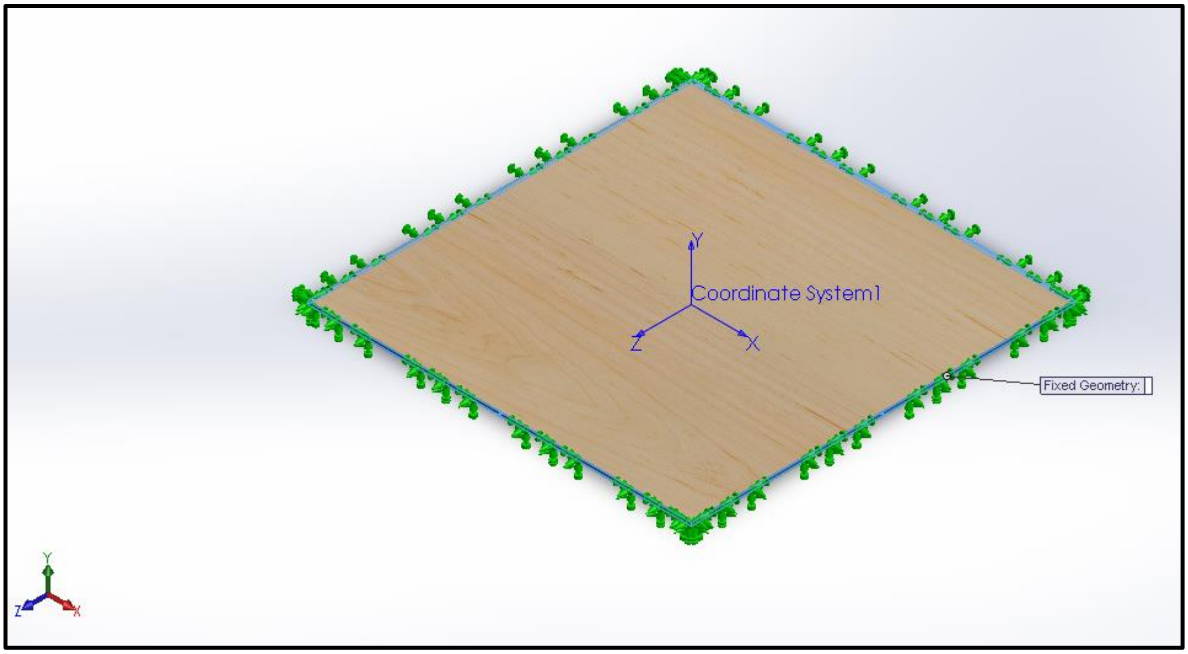


Figure 3.7 - Simple maple plate with end fixtures

The meshing details are listed in table 3-7

Table 3-7- Mesh Details flat maple plate

Mesh type	Solid Mesh
Mesher Used:	Standard mesh
Jacobian points	4 Points
Element Size	0.336882 in
Tolerance	0.0168441 in
Mesh Quality	High

Table 3-8 - Meshing Specifications

Total Nodes	16542
Total Elements	8299
Maximum Aspect Ratio	15.206
% of elements with Aspect Ratio < 3	4.16
% of elements with Aspect Ratio > 10	0.265
% of distorted elements(Jacobian)	0
Time to complete mesh(hh;mm;ss):	00:00:05

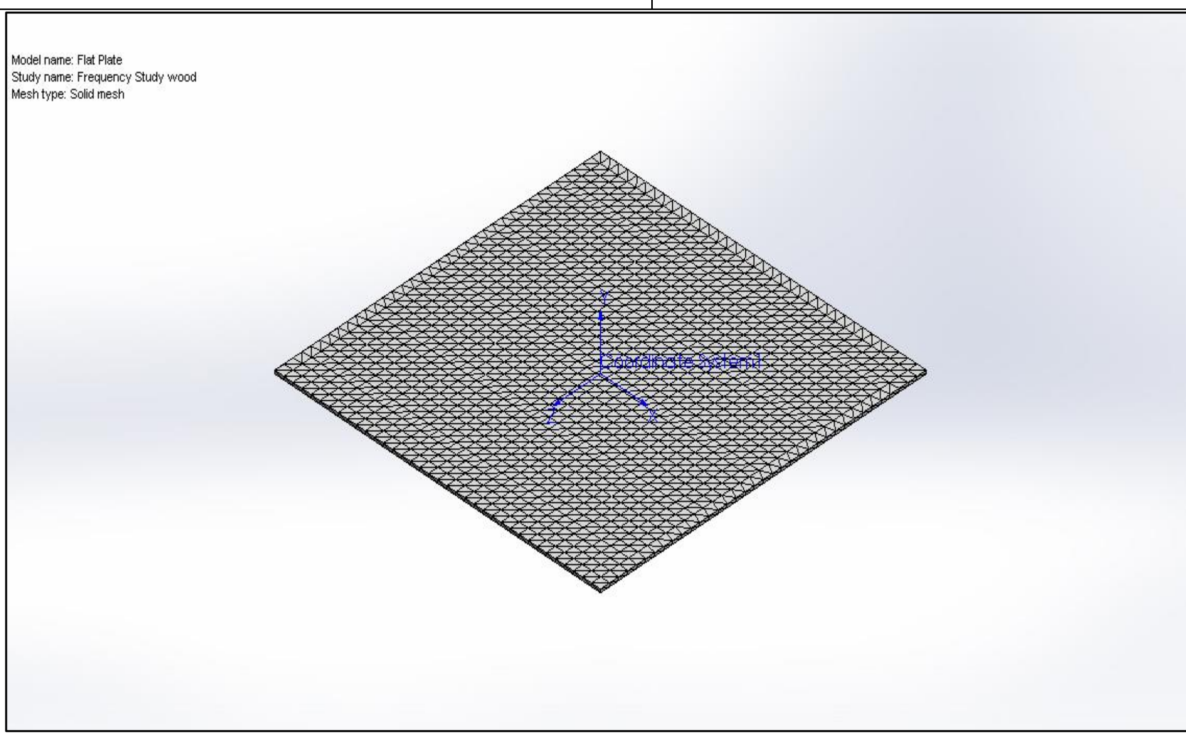
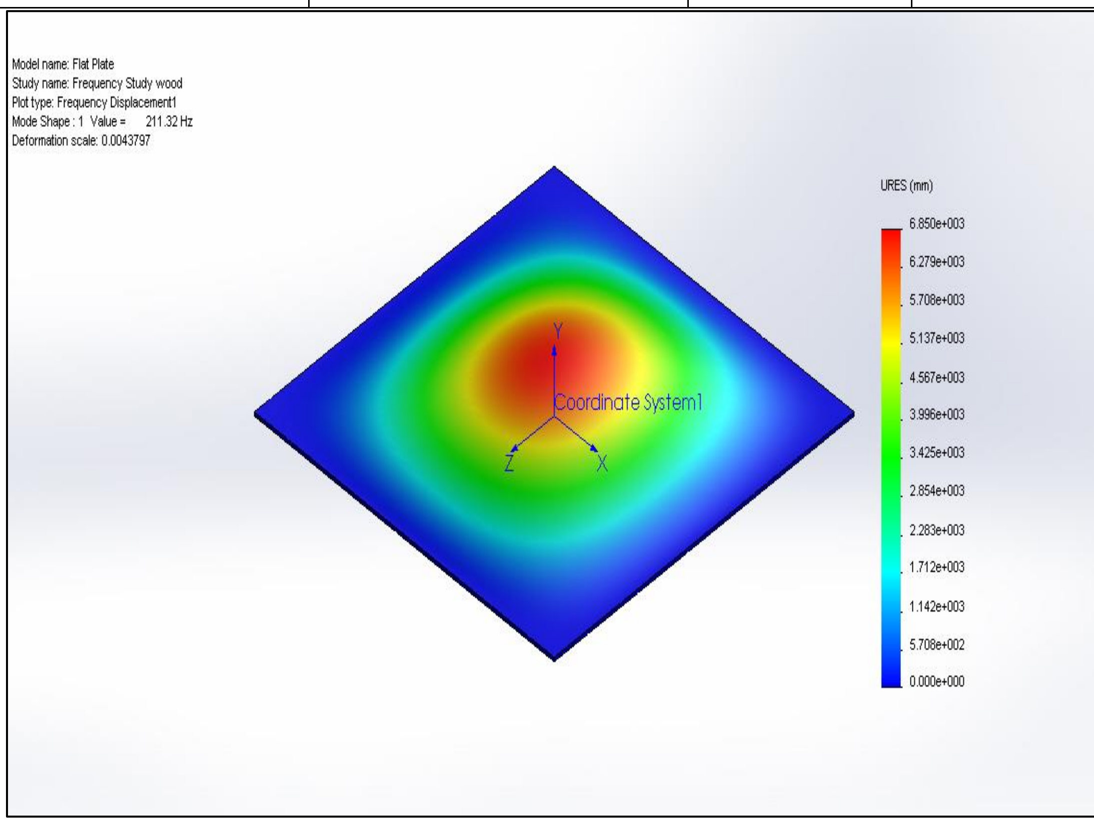


Figure 3.8 - Flate plate maple mode shape 1

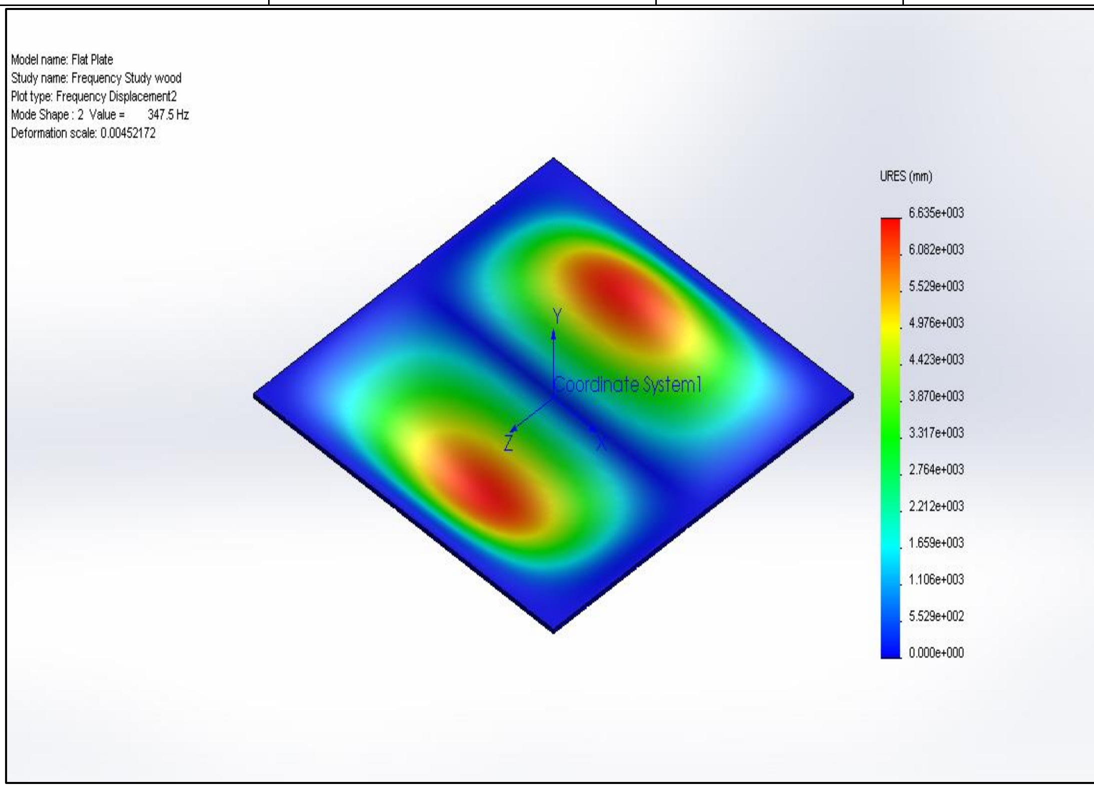
Name	Type	Min	Max
Displacement1	URES: Resultant Displacement Plot for Mode Shape: 1(Value = 211.324 Hz)	0 mm Node: 1	6849.78 mm Node: 14955



Flat Plate-Frequency Study wood-Displacement-Displacement1

Figure 3.9 - Fplat plate maple mode shape 2

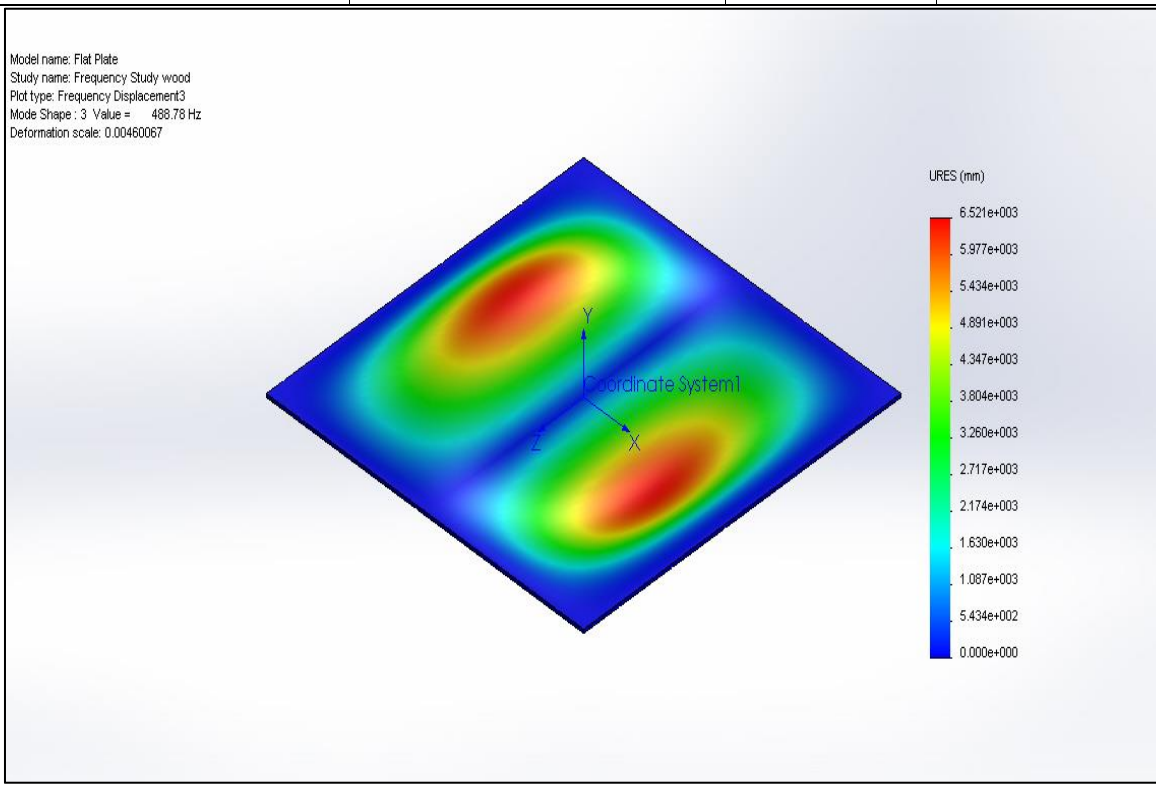
Name	Type	Min	Max
Displacement2	URES: Resultant Displacement Plot for Mode Shape: 2(Value = 347.503 Hz)	0 mm Node: 1	6634.63 mm Node: 14876



Flat Plate-Frequency Study wood-Displacement-Displacement2

Figure 3.10 - Flat plate maple mode shape 3

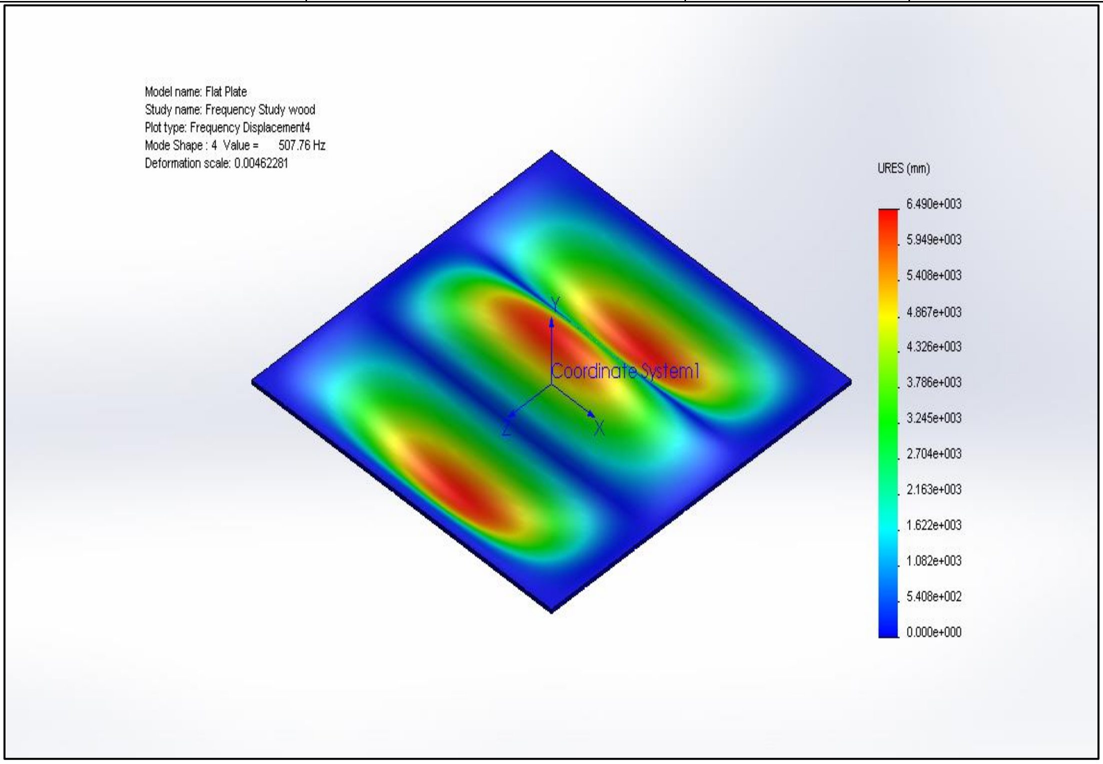
Name	Type	Min	Max
Displacement3	URES: Resultant Displacement Plot for Mode Shape: 3(Value = 488.776 Hz)	0 mm Node: 1	6520.78 mm Node: 12976



Flat Plate-Frequency Study wood-Displacement-Displacement3

Figure 3.11- Flat plate maple mode shape 4

Name	Type	Min	Max
Displacement4	URES: Resultant Displacement Plot for Mode Shape: 4(Value = 507.758 Hz)	0 mm Node: 1	6489.54 mm Node: 15055



Flat Plate-Frequency Study wood-Displacement-Displacement4

After comparing mode shapes of the isotropic material (steel) to the anisotropic material (wood) it can be concluded that even though the first three mode shapes are the same, the fourth mode shape pattern differs. This is because the patterns are same with respect to the equilibrium point in case of steel as the material is uniform in both X and Y axis. On the contrary, mode shape patterns are mirror images of each other along the axis parallel or perpendicular to the orientation of fibers in wood. This validated the data processing code and provided the basis for further using the program on a complicated structure such as violin.

CHAPTER IV

PARAMETRIC STUDY AND LOADING

In this chapter an analysis was conducted to examine the force distribution due to strings on the main body of the violin. The distinctive sound created by violin is a byproduct of the vibrational input through the strings and the interaction of various parts of violin with each other. Typically, four strings are rested on the bridge that transmits vibrations to the plates and are tied at both ends. A vibrating string produces sound at a constant frequency. The Free body diagram below shows the force distribution.

The excitation force is the one acting on the bridge when vibrations are transmitted through the violin strings using the violin bow.

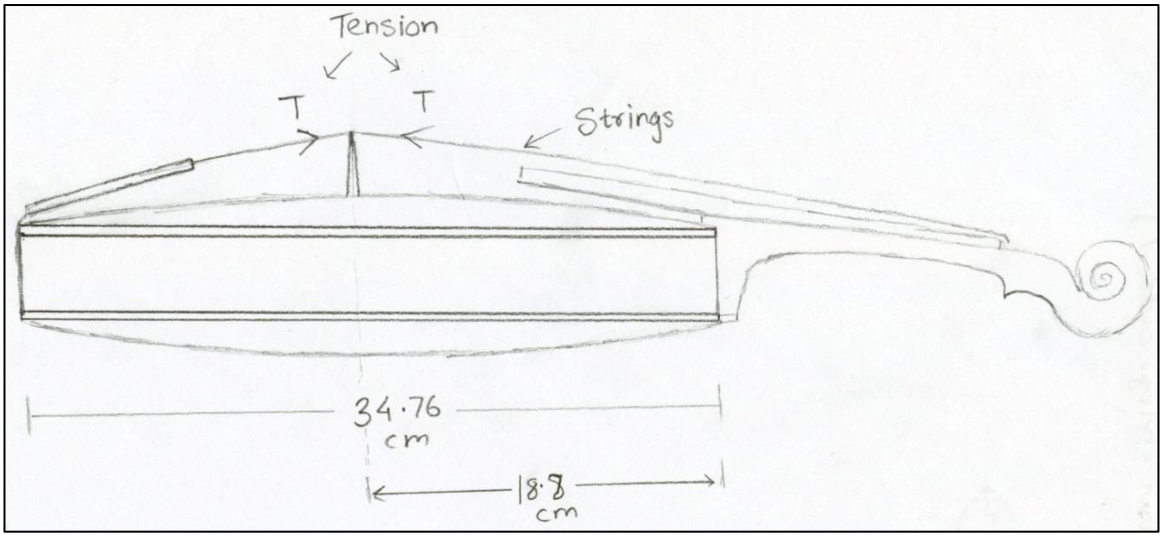


Figure 4.1 - FBD of Violin Strings

The force input is assumed to be in the form of a simple sine wave as expressed in the Equation 4-1

Equation 4-1 - Force Equation:

$$P = P^{*} + \bar{P}\sin(wt)$$

Where,

P = Total excitation force on the violin through the strings

P^{*} = Force on the violin plate due to the tension in strings

$\bar{P}$ = The amplitude of the dependent component of the force

Equation 4-2 - Remote Load Equation:

$$P^* = Tsin\alpha + Tsin\beta$$

Where,

Remote Load is the static component of the force on top plate

α and β are the angles between the strings and the top plate

$\bar{P}\sin(wt)$ = Sinusoidal Force input using vibration

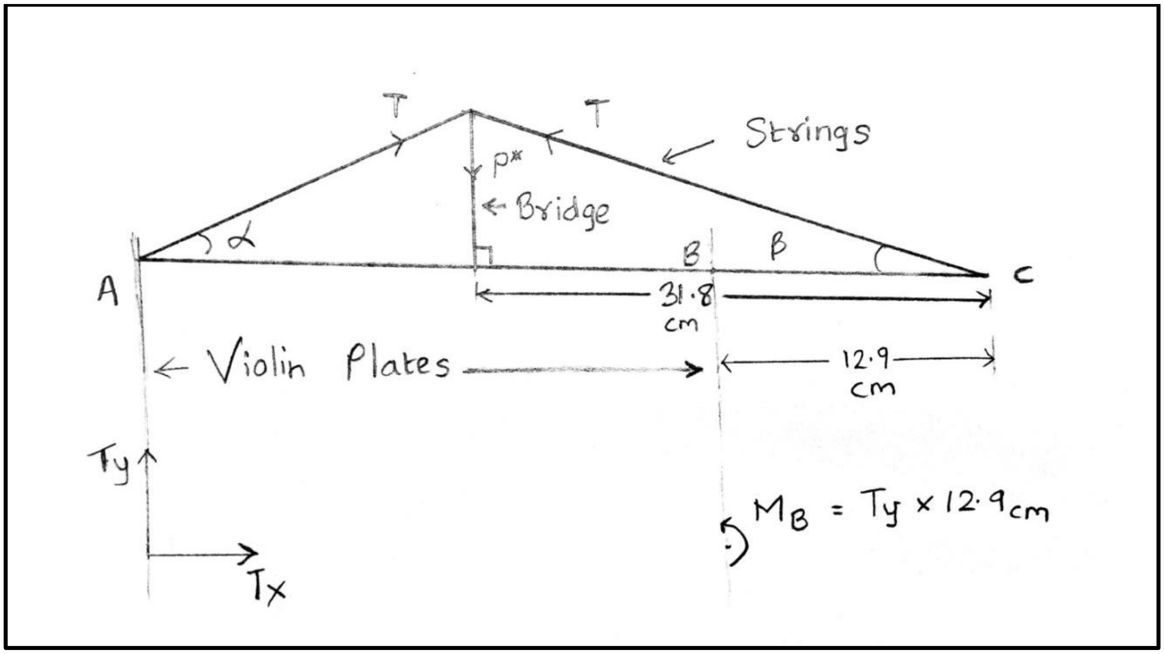


Figure 4.2 - Free Body Diagram (Remote Loads)

4.1 Calculating static component of the force on top plate

The remote load is the part of excitation force due to the tension in the violin strings. It was calculated using the free body diagram in Figure 4.2.

From the 'Violin String Tension Chart' [4] it can be seen that the average tension on all the four strings added up in about 50 pounds. Using the tension and string angles, the static component of the force can be calculated using Equation 4-3.

Using Stradivari Violin specifications [5] the string angles were calculated (Appendix A) to be

$$\alpha = 11.76° \text{ and } \beta = 9.8°$$

Equation 4-3 - Static component of the force:

$$P^{*} = 50 sin 9.8 + 50 sin 11.76 = 18.7\ lbs = 83.18\ N$$

Figure 4.2 illustrates the remote loads acting on the violin plate namely

$$T_{Y} = T sin\alpha = 50\ sin 11.76 = 10.19\ lbs$$

$$T_{X} = T cos\alpha = 50\ cos 11.76 = 48.95\ lbs$$

The Geometry of violin is such that four strings are attached to the fingerboard that in turn is attached to the main body of the violin through the neck (See Figure 1.1). This causes the tension in the string to create a moment at point B as illustrated in figure 2.2.

$$M_{B} = T_{Yc} \times 12.9cm = T_{Yc} \times 0.129m = 22.68kg \times 0.129m = 2.925\ N.m$$

4.2 Frequency Study Formulation

When excited by an initial disturbance (Initial velocity and/or displacement) each structure has a tendency to vibrate at a certain frequency that is called natural frequency. Each frequency is associated with a certain deformed shape of the structure after the initial disturbance that is called mode shape. SolidWorks Simulation enables us to apply boundary conditions and simulate the structure in order to find the natural frequencies. In this study, the natural frequencies of the top plate of violin were desired.

The top plate is bonded to the rib (purple) and fixed geometry is applied at the ends of the plate (green) where fingerboard is attached with strings. Pink lines denote static component of the force due to tension in strings.

The middle section has a complicated geometry due to the curved sound holes. For this reason, it was necessary to apply a mesh control on order to get more accurate results. Mesh control incorporates finer mesh that in turn results in more nodes and elements. Regular mesh element size is 0.349 inches whereas the mesh control applied created finer mesh with element size 0.177 inches. Material applied was spruce (table 4-1)

Table 4-1 - Material properties of spruce

Model type:	**Linear Elastic Orthotropic**
Default failure criterion:	**Max von Mises Stress**
Mass density:	**350 kg/m^3**
Elastic modulus in x:	**5.25e+008 N/m^2**
Elastic modulus in y:	**1.103e+009 N/m^2**
Elastic modulus in z:	**8.9e+009 N/m^2**
Poisson's ratio in xy:	**0.435**
Poisson's ratio in yz:	**0.372**
Poisson's ratio in xz:	**0.467**
Shear modulus in xy:	**8.9e+007 N/m^2**
Shear modulus in yz:	**1.103e+009 N/m^2**
Shear modulus in xz:	**1.068e+009 N/m^2**

Figure 4.3 - Boundary conditions and static load

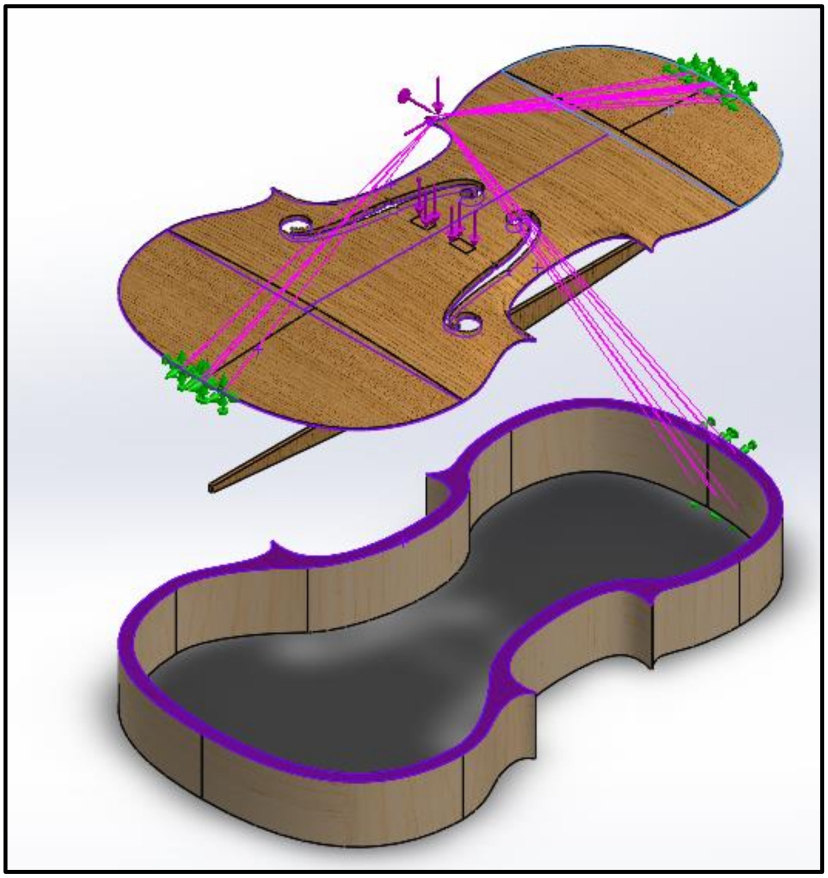


Figure 4.4 - Excitation Force (83.18N)

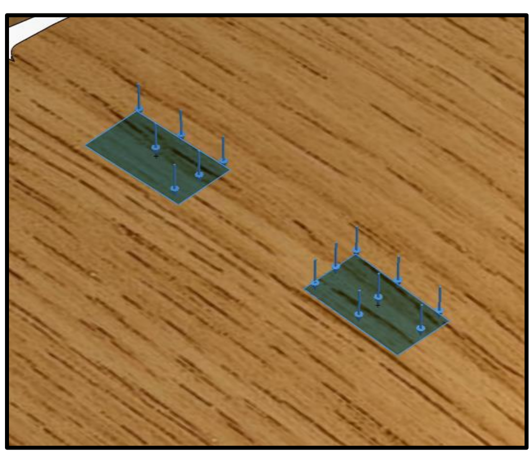
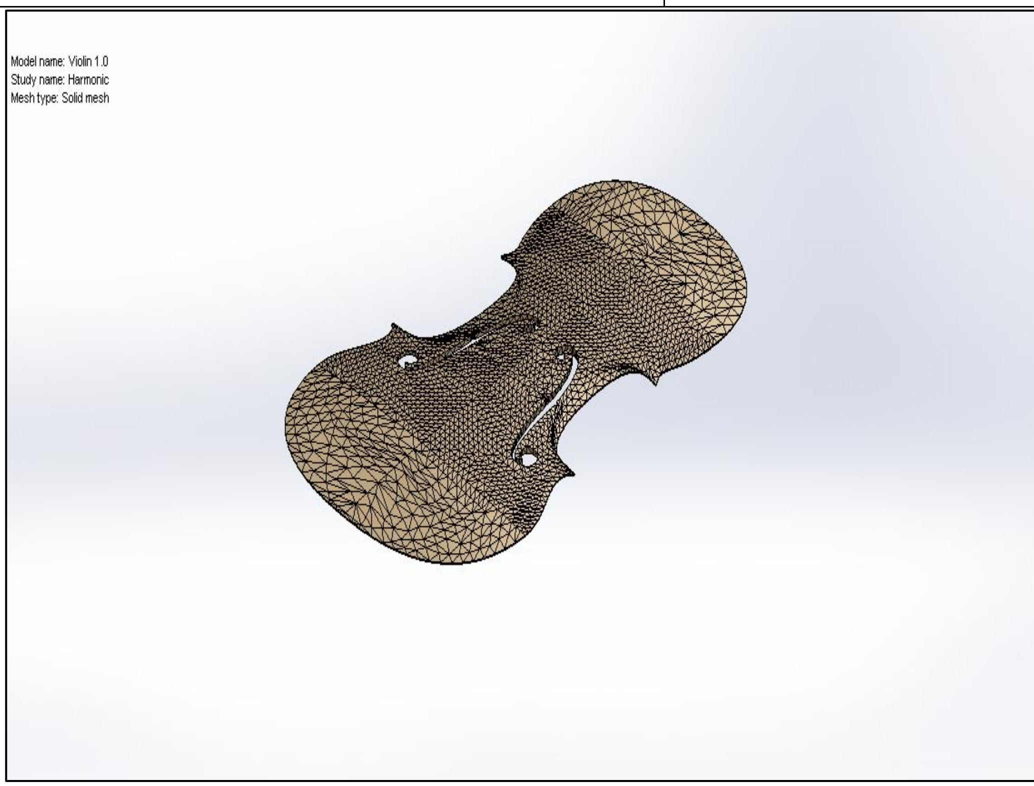


Table 4-2 - Mesh type and dimensions

Mesh type	Solid Mesh
Mesher Used:	Standard mesh
Automatic Transition:	Off
Include Mesh Auto Loops:	Off
Jacobian points	4 Points
Element Size	0.349549 in
Tolerance	0.0174775 in
Mesh Quality	High

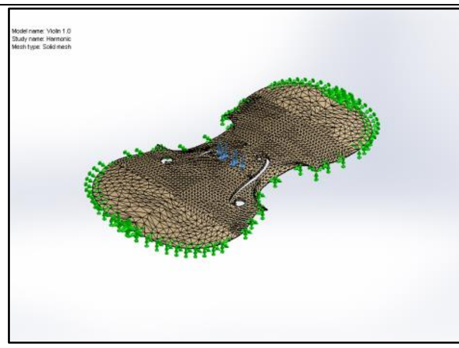
Table 4-3 - Mesh details and specifications

Total Nodes	27853
Total Elements	14247
Maximum Aspect Ratio	56.517
% of elements with Aspect Ratio < 3	76.7
% of elements with Aspect Ratio > 10	1.91
% of distorted elements(Jacobian)	0
Time to complete mesh(hh;mm;ss):	00:00:10



Meshing

Table 4-4 - Mesh Control

Mesh Control Name	Mesh Control Image	Mesh Control Details
Control-1		**Entities:** 1 **Units:** in **Size:** 0.177112 **Ratio:** 1.5

CHAPTER V

RESULTS AND DISCUSSION

5.1 Top Plate Mode Shapes

Figure 5.1 - Mode Shape 1 (158 Hz)

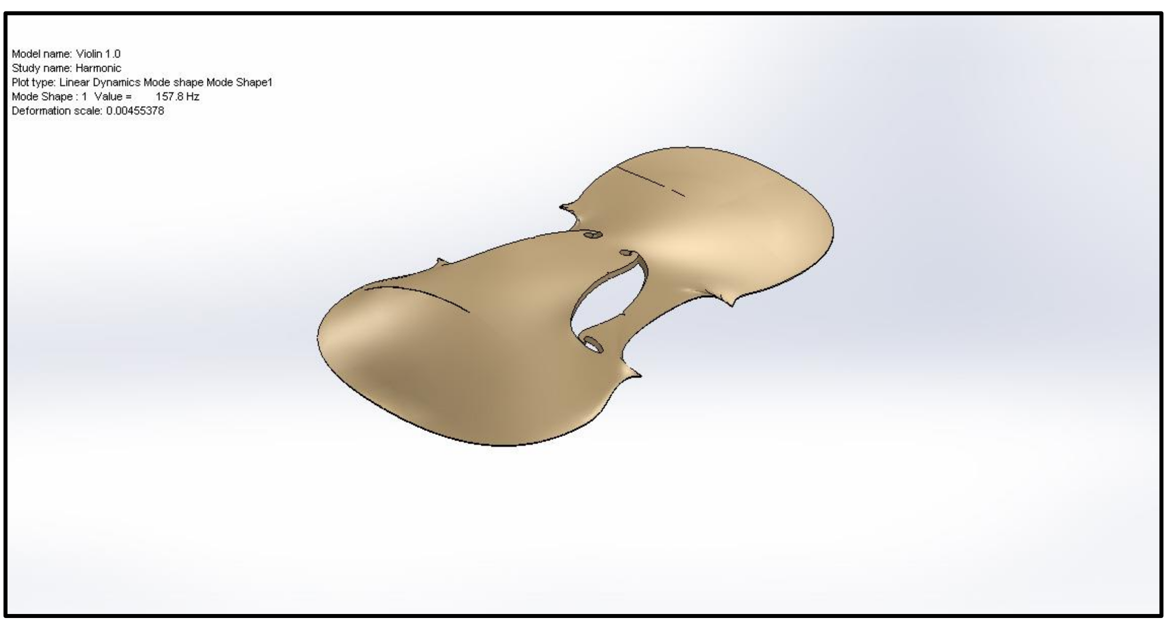


Figure 5.2 - Mode Shape 2 (269.461 Hz)

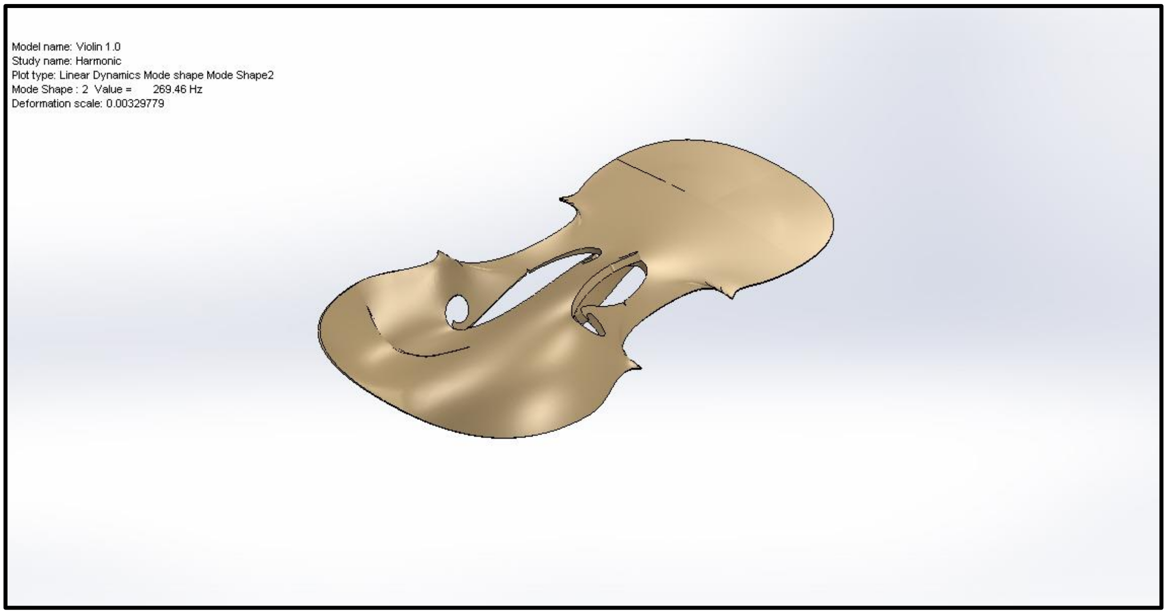


Figure 5.3 - Mode Shape 3 (287 Hz)

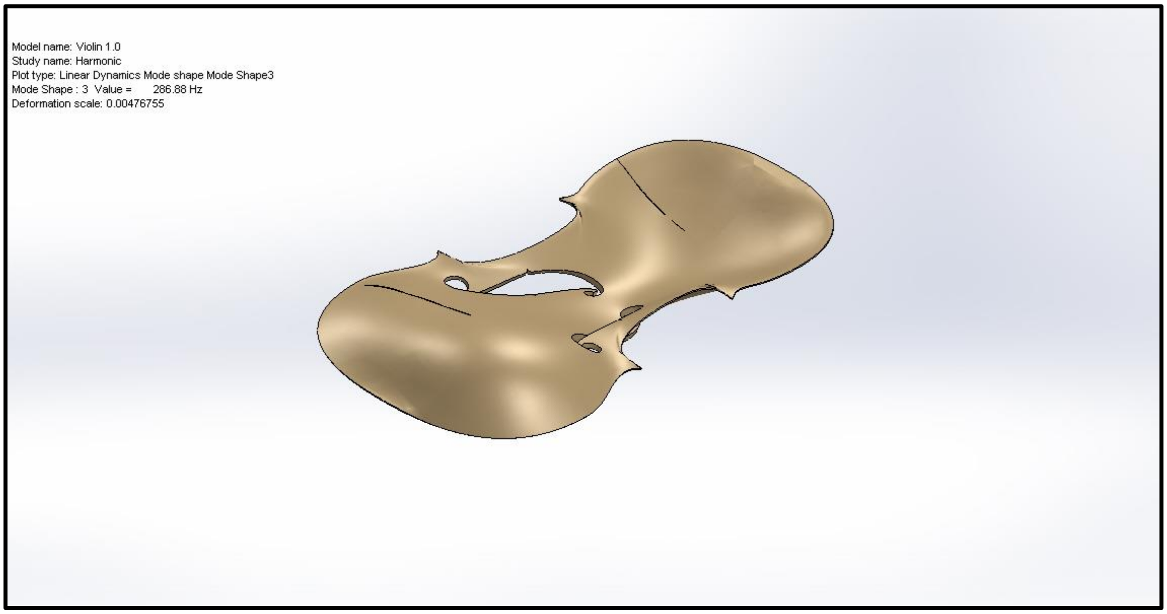


Figure 5.4 - Mode Shape 4 (380 Hz)

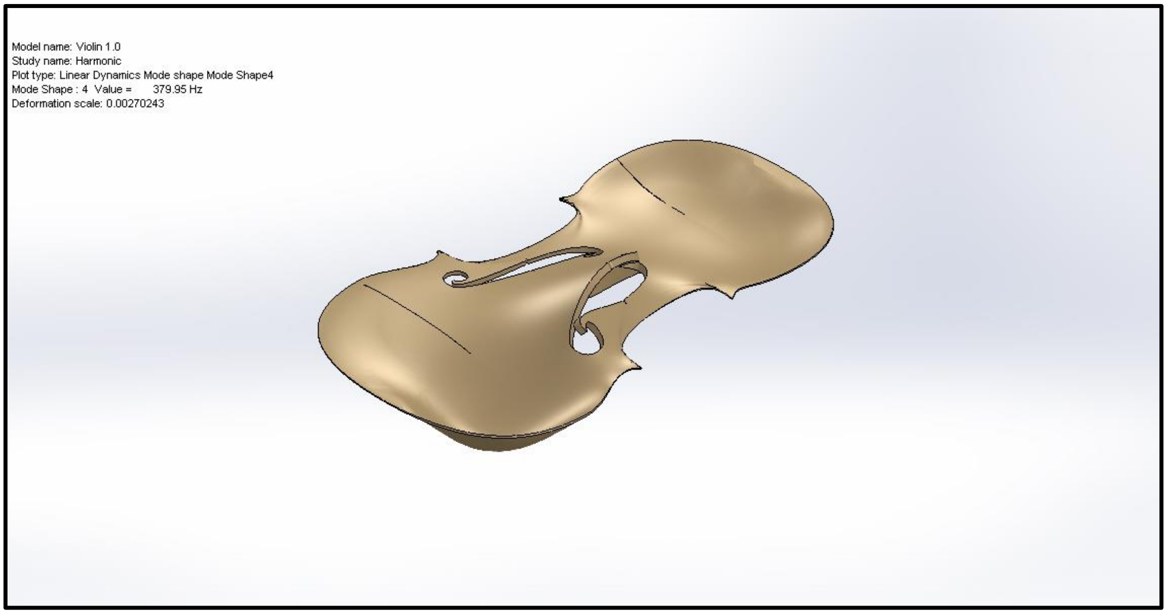


Figure 5.5 - Mode Shape 5 (430 Hz)

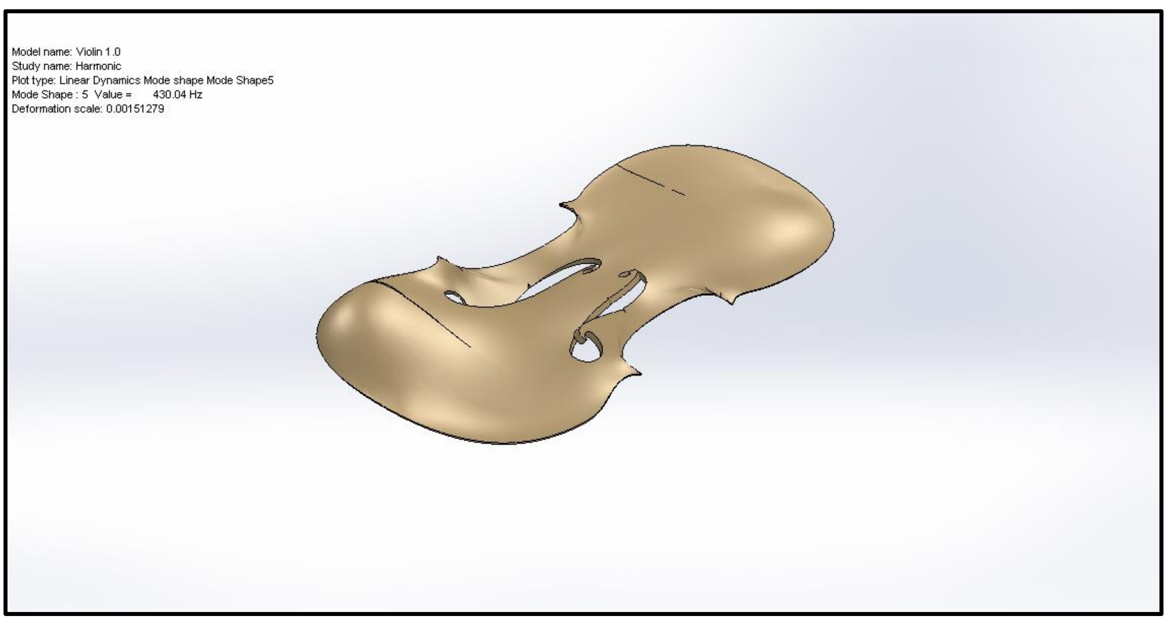


Figure 5.6 - Natural Frequencies (Top Plate)

Mode Number	Frequency(Hertz)
1	**157.8**
2	**269.46**
3	**286.88**
4	**379.95**
5	**430.04**

Figures 5.1 to 5.5 show the mode shapes obtained from finite element simulation. Motion described by the normal modes resembles resonance. In the figures 5.1 through 5.5, motion of the violin is associated with respective shapes such that at the resonant frequency sound is generated. These normal modes are called harmonics. At every natural frequency, a pitch of sound (do, re, mi, Etc.) is created. The pitch increases as the frequency goes higher. Also as the frequency increases, the amplitude decreases. This can be seen in Figures 5.1 to 5.5 where the deformation is more at mode shape 1 and less at mode shape 5.

5.2 Top Plate Displacements

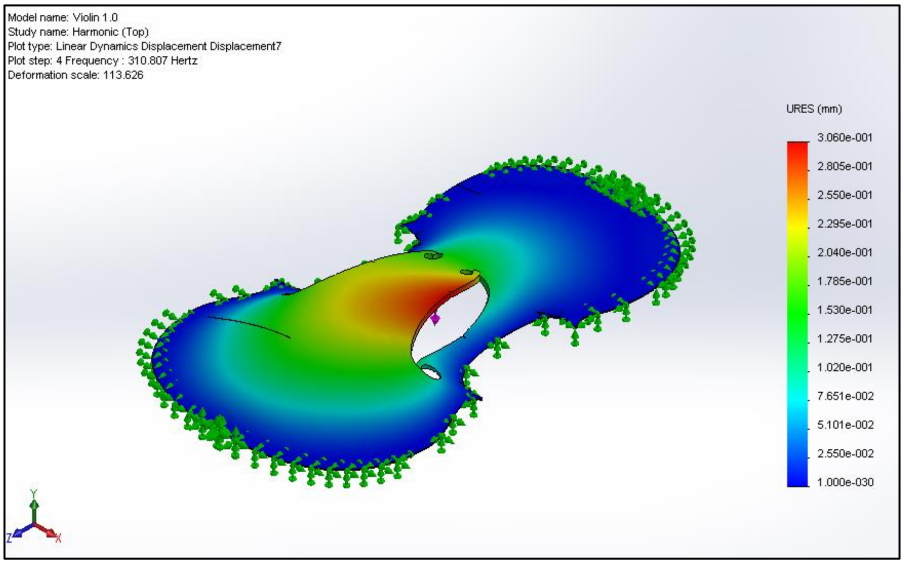


Figure 5.7 – Displacement at 310 Hertz – 0.3mm

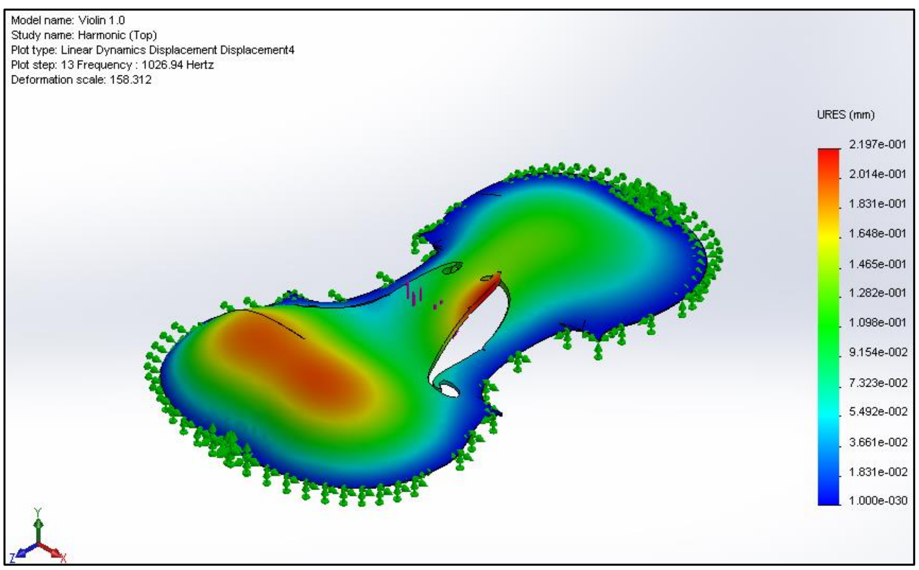


Figure 5.8 - Displacement at 1027 Hertz - 0.2mm

The maximum displacement at 310 hertz is 0.3mm and 0.2mm at 1027 hertz. The laws of vibration is satisfied from the fact that higher the frequency, lower is the amplitude.

CHAPTER VI

CONCLUSION AND FUTURE WORK

6.1 Limitations

Although this study concentrates on the material wood in specific, the dimensions of violin geometry were considered in a precise manner. Since it was a simulation study, it was difficult to include every part of the violin (Fingerboard, strings and tail piece) other than the sound box (Top plate, rib and bottom plate) due to lack of accurate dimensions of those parts. Inclusion of those parts would have increased the accuracy of the results.

6.2 Conclusion

The aim of this study was to create a 3D model of a complicated structure such as violin. Computer aided design in SolidWorks is capable of creating such complex geometries. Its varied thickness profiles made it challenging to create a near identical replica of violin plates. Exploration of modelling features in SolidWorks was the first major positive aspect of this study. The loft feature enabled to create profiles with specified thickness that could be extruded along guided curves. Vibration analysis by finite element analysis as opposed to experimental analysis was successfully achieved. SolidWorks simulation not only made applying anisotropic material possible but also the fixtures that come into play when the violin is actually played were integrated as well. Majority of the previous modal analysis have been experimental. This study let us virtually look at the behavior of violin top plate in a working condition. Vibration engineers can analyze machine components that undergo high excitation forces and alter the design to avoid catastrophic failures caused by large deformations.

6.3 Future Work

Including the bottom plate in vibration study is a future task in this thesis. Generation of pressure waves (sound) in a violin is a result of orientation of all parts including the bottom plate. The next step in this study will be to study frequencies and mode shapes of the bottom plate and relate them to the top plate results. It will be interesting to look at the mode shapes of both top and bottom plate at a natural frequency. Also inclusion of the remaining parts along with their respective materials is a major step ahead in this study. This can improve the distribution of static component of the force on the two plates and improve results.

WORKS CITED

[1] Ye Lu, 2013, "Comparison of Finite Element Method and Modal Analysis of Violin Top Plate," Department of Music Research, McGill University, .

[2] ‘David W. Green’, ‘Jerrold E. Winandy’, and ‘E. Kretschmann’, 2002, "Wood Handbook,"Algrove Publishing, .

[3] 'S.J. Lee', 2004, "Free Vibration Analysis of Plates using a Four-Node Finite Element Formulated with Assumed Natural Transverse Shear Strain," Journal of Sound and Vibration, **278**(3) pp. 657-684.

[4] Shawn Boucké, "Http://www.Violinstringreview.com/tension-Chart.Html," .

[5]"Http://www.Oosterhofonline.net/pdf/proportionele%20maten%20Strad_Specs.Pdf," Stradivarius Violin Specifications, .

[6] 'Cyril M. Harris', 1961, "Shock and Vibration Handbook," McGraw-Hill Book Company, New York, .

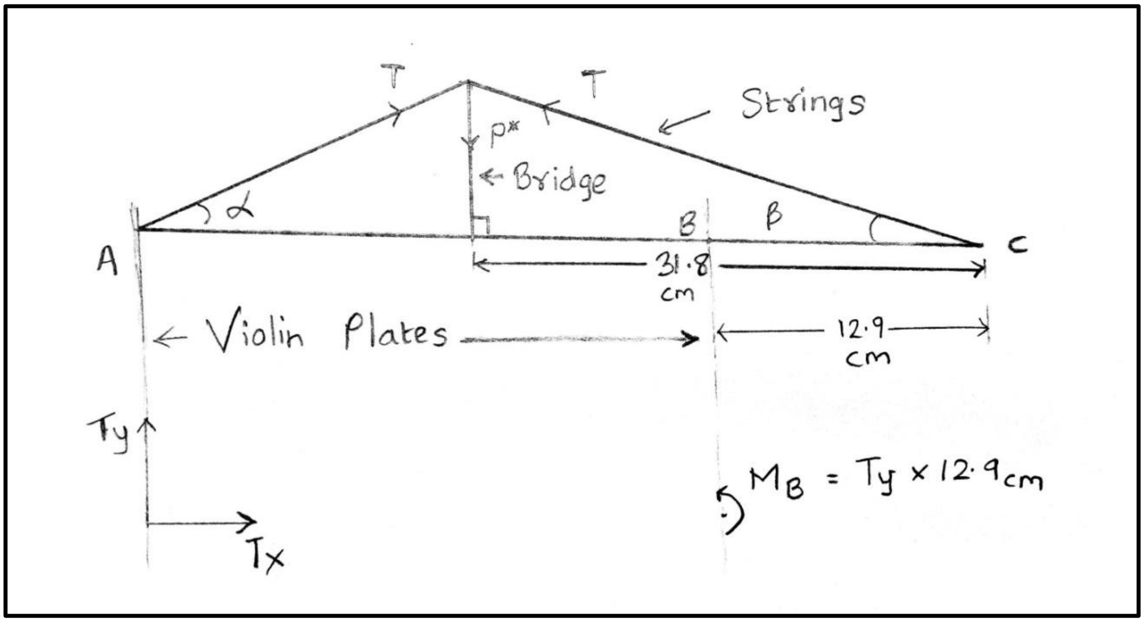
APPENDICES

Appendix A

Stradivarius Violin Specifications
in Centimeters

Size	Entire Length	Corpus	From Bridge to Nut Edges	Width	Width	Fingerboard	F holes	From Fingerboard to Bridge, tailpiece	Tailpiece	Neck	Bridge width lower, under, high	String Division	Bassbar	Arching back,top	Bouts, Upper, Lower
Violin	A	B	C 1,2,3	D 1,2,3	E 1,2	F 1,2,3	G 1,2,3	H 1,2	J 1,2	K 1,2,3	L 1,2,3	M 1,2	N 1,2,3	O 1,2	P 1,2
4/4	59	35.8	32.5 19.5 16.3	17.0 11.4 20.8	16.2 18.8	27 4.2 2.3	4.2 13.5 7.6	6 5	11.5 4.2	24.6 13.7 11.3	4.2 4.8 3.3	3.6 1.7	28.0 1.2 .6	1.5 1.6	2.9 3.1
7/8	57	34.6	31.4 18.8 15.7	16.4 11.0 20	15.7 18.2	26 4.1 2.2	4.1 13 7.3	5.8 4.8	11 4.1	23.8 13.2 11	4.1 4.6 3.2	3.5 1.6	27 1.15 .57	1.44 1.54	2.8 3
3/4	55	33.4	30.3 18.2 15.2	15.8 10.6 19.4	15.1 17.5	25.2 3.9 2.1	3.9 12.6 7.1	5.6 4.7	10.7 3.9	23 12.8 10.5	3.9 4.5 3.1	3.4 1.6	26 1.11 .55	1.39 1.49	2.7 2.9
1/2	52	31.6	28.6 17.2 14.4	15.0 10.0 18.3	14.3 16.6	23.8 3.7 2	3.7 12 6.7	5.3 4.4	10 3.7	21.7 12 10	3.7 4.2 2.9	3.2 1.5	24.7 1.05 .52	1.32 1.41	2.6 2.7

From the Stradivarius Violin Specifications [5], violin dimensions with size 7/8 were used.



$$\alpha = \sin^{-1} 3.2/15.7 = 11.76°$$

$$\beta = \sin^{-1} 3.2/18.8 = 9.8°$$