Design and Control of a Powered Rowing Machine with Programmable Impedance

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DESIGN AND CONTROL OF A
POWERED ROWING MACHINE WITH
PROGRAMMABLE IMPEDANCE

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Bachelor of Science in Mechatronics Engineering
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ABSTRACT

Due to the rise of obesity, diabetes and cardiovascular disease, research in human performance and physical activity has received increased attention. Rowing machines are used for performance improvements through concentric exercises, however a combination of concentric and eccentric actions is known to improve the effectiveness of training. In this work, a conventional rowing machine was modified to include an electric motor and a robust impedance control system, enabling programmable impedance with concentric and eccentric capabilities. Eccentric exercises are known to contribute significantly to the efficacy of training and to diminish the detrimental effects of humans operating in microgravity for long periods. The powered machine was developed by replacing the conventional elements of the rowing machine such as the flywheel, fan and the one-way clutch with a torque-controlled motor and a belt transmission. Components were selected on the basis of forces and velocities encountered in the rowing exercise. The powered machine is capable of producing controlled forces during the return stroke, allowing eccentric exercise. Machine parameters such as the flywheel inertia, linear and quadratic damping and spring constants were estimated from a set of data obtained from real-time tests. Subsequently a hybrid robust impedance controller was incorporated into the model. The controller incorporates two discrete states corresponding to the pull and return phases. Discrete states transitions are determined from a law driven by force sensing and the state of a virtual flywheel. The prototype and control system were tested in real-time, replicating the operation of the conventional rowing machine. Moreover, arbitrary impedances were programmed and a 1:1 eccentric/concentric power ratio was demonstrated.
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Lately the human-machine interaction has become a more relevant research field than a few years ago due to the large number of opportunities in their applications. Some of them are related to the rehabilitation and some others to the high-performance physical conditioning. The efficiency of an exercise regime is based on several aspects such as the human dynamics (postures and coordination of body segments) and on the versatility that a machine can provide. For this research, a Concept 2 Model E rowing machine [5] [6] was used and adapted. The conventional rowing machines operate in such a way that they store part of the energy used by the athlete in a flywheel and dissipate the rest in resistive elements such as adjustable air dampers. The limitations present in the current models are the fixed inertia, limited damping adjustments and especially the unloaded return. This lack of a significant load during the return stroke impedes the generation of an important modality known as eccentric exercise or exercise through muscle lengthening (see Figure 1). The eccentric exercises are very important -principally in microgravity- to make exercise sessions shorter and more effective.

Muscles are able to provide positive work (produced by shortening contraction) and negative work (produced by lengthening contraction). On one hand, shortening contraction is related to the delivering of work to a load by the muscle, for example
when lifting a weight. On the other hand, lengthening contraction is related to the load returning energy to the muscles, for example when lowering a weight in a slow and controlled motion. A complete and successful workout incorporates a combination of exercises with concentric and eccentric actions [26] [2].

### 1.1 Motivation

One of the greatest challenges to humans operating in space for long periods of time is that they have to deal with a considerable alteration of the musculoskeletal loading and the muscle toning. Some of the alterations produced are motion sickness, muscle degeneration, bone demineralization and change in body fluids. These variations have repercussions responding by losing mass proportional to the time of exposition to the microgravity (an average of 1% BMD per month) becoming weaker in a similar way to how osteoporosis affects the human anatomy [28] [15] [41] (see Figure 2).
The eccentric exercises are infrequent in most workouts and most especially during spaceflights. Since this kind of contraction is related to the counteract of the muscle while it is lengthening against the gravity force, the eccentric exercises are practically null in microgravity [40]. This research is very important because through the use of powered machines capable of producing eccentric exercises, the detrimental effects of the astronauts could be significantly diminished and thus would make possible more durable expeditions without compromising the health and well-being of the crew.

![Figure 2: Normal bone compared to an osteoporotic bone. Adapted from [28]](image)

The powered exercise machines have been developing for a long time and for different purposes. Some are being used for training (see the HealthRider model HRTL-59215 in Figure 3) while others for rehabilitation purposes. For instance, nowadays we have centers of aggressive physical rehabilitation which used powered machines. These programs have emerged and seem to be very efficient restoring motor skills even years after injury [25]. Besides, there are other powered machines focused on rehabilitation-research such as the exoskeleton for gait rehabilitation of motor-impaired patients [1], the robot-assisted therapy for stroke victims [4] or the gait rehabilitation based on programmable footplates [38].

The main characteristic of all these powered designs is the versatility that they can provide to the training and rehabilitation. However, the design developed
in this research afford not only the great adaptability for different kind of training, what makes it different and special is its ability to make training more complete and efficient.

![Figure 3: Powered machines performance-oriented. Adapted from [9]](image)

In this research, a conventional ergometer was modified by removing and replacing the flywheel, fan and one-way clutch by a torque-controlled motor and a belt transmission which were selected on the basis of the forces and velocities encountered.

![Figure 4: Rehabilitation-oriented powered machines. Adapted from [25].](image)
in the rowing training. The investigation incorporates the required experiments for the evaluation of all variants of the system, from its conventional design, for generating an extensive study of the dynamics of a conventional machine and the biomechanics of the rowing exercise, to the one that includes the final robust impedance control. The approach to this research deals with improving human performance, with a focus on subjects with normal biomechanical function and athletes.

1.2 Literature Review

Powered exercise designs and eccentric contraction research have been reported in different publications but independently. Powered machines oriented towards rehabilitation and physical training remotes for many years. Different variants have been presented over time ranging from simple powered treadmills to sophisticated robust trackers and impedance control [27] robots for controlling a lower-limb prosthesis with energy regeneration [42].

This research presents a powered ergometer machine capable of giving a solution to the problem of the lack of eccentric exercise training. This same research has been used for the paper [24] for the Dynamic Systems and Control Conference 2017 (to be published).

As early as the 1980s, NASA began with the idea of taking advantage of the eccentric exercises in order to reduce the damaging effects on astronauts exposed to microgravity and thereby make long-term space travel more viable [15]. To this day, the related research is a highly relevant area evaluating the concentric and eccentric contractions of every astronaut before and after having left the planet [32].
1.2.1 Powered Designs

Motor-driven ergometers have been generally used for the purpose of assisting and rehabilitating of people with weakness or muscle fatigue. Some others, in lesser proportion, have been also developed for physical training.

This is the case, for example, of a motor-driven ergometer which works together with an automatic functional electrical stimulation controller for treating patients with paraplegia [21]. In addition is the case of this research focused on the functional electrical stimulation of people with spinal cord injury participating on an equal basis with professional athletes in rowing competitions [16]. Or the case of this which incorporates fuzzy logic for a better control of the functional electrical stimulation on patients with paraplegia assisted in a powered ergometer [10].

Some other investigations related to powered ergometers have been focused on the physical training of people with normal motor skills. For instance, a research on admittance control was performed on a reduced scale of an electromechanical rowing machine for imitation and learning about the machine [31]. And others have inquired into impedance control with energy regeneration in machines for advanced training [35].

1.2.2 Rowing Exercises

The indoor rowing machines, often known as ergometers, were initially developed to simulate the action of the watercraft rowers. Although the ergometer has been established as a different sport to the rowing, the biomechanics of their exercises [2] are very similar.

During a full rowing exercise training cycle, four components of a basic stroke can be identified: the catch, the drive, the finish and the recovery (a, b, c and d pictures respectively in Figure 5 shows each of them). The catch begins when
the user is pulling the handle, this person applies force to add momentum to the flywheel, overcoming air resistance and the restoring force of the spring. With the one-way clutch coupled and the chain, sprocket and flywheel moving as a unit, the drive begins. Then the finish is performed when the end of the stroke is reached. Finally, when the user reverses motion and the force on the chain is reduced, equaling only the force due to the spring, the recovery begins. The clutch becomes decoupled and the flywheel decelerates due to air resistance, while the chain and sprocket rotate in the opposite direction. At the end of the return phase, the user reverses motion again, causing the clutch to re-engage and initiate a new catch. The mechanisms of the rowing machine will be later detailed in the section of modeling.

1.2.3 Modeling of Rowing Exercise

The rowing exercise can be studied better by separately considering the human side and the machine side. This research focuses mainly on the machine which is conceived as follows:

A conventional rowing ergometer has one degree of freedom represented as its
sprocket rotary displacement. This machine has a flywheel joined to a one-way clutch connected to a sprocket through a chain and a spring. The system operates between two discrete states defined by its transition laws. These discrete states are related to the coupling and decoupling relationship between the flywheel and sprocket. The clutch is modeled as an ideal element with an instantaneous transition between its coupling modes. The mathematical modeling will be explained in details in Chapter II.

The modeling of the user depends on many factors, however the following research very accurately details very important aspects of it [44]. It is known that an ergometer has some mechanical properties which affect the performance of the user (movements and loads on its tissues) such as the air resistance from the flywheel where most of the energy is dissipated. The effects of these mechanical parameters of the machine on the human performance and the effects of the exercise on the arm motion and tissue loads could be identified by means of a computational model of an arm based on a human musculoskeletal simulation and optimal control. To make it possible, a scaled ergometer and a biceps muscle were modeled. With a system identification, a Concept2 ergometer parameters were identified and the model was tested. Subsequently, the machine parameters were changed to examine the effects of the variations.

1.3 Thesis Contributions and Organization

The proposed control system attempts to solve the problems of the low versatility of the conventional machines. Since its operation does not depend on gravitational components but of a motor, the solution tries to provide the additional benefit of operation in a microgravity environment to diminish the detrimental effects of the long spaceflights on astronauts. Due to the great adaptability of the system, in
addition to the performance of eccentric exercises, the resulting machine is capable of producing an endless number of training modalities ranging from a simple spring to real simulations with progressive resistance.

The thesis is organized as follows: Chapter II - Hybrid Mathematical Model contains information about the dynamic behavior of the conventional rowing machine required for the modeling. This includes a brief outline about Bond Graph used for the modeling, the state equations in each of the modes of operation (coupled and decoupled), the discrete transition laws and a basic simulation. Chapter III - Reference Data describes the first stage of experimentation. Since it is performed with the conventional rowing machine, the results allow estimating the model and the sizing of the equipment to be used in the powered ergometer. Chapter IV - Servomotor and Accessories Selection explains the process of the motor, gears and belt selection based on the critical results that produced the highest power. Furthermore, the frame adaptation is presented and briefly discuss the reasons for its design. Chapter V - Hybrid Model Simulations describes the second stage of experimentation. In this chapter, the hybrid model is evaluated using the real force recorded in the previous test. The modeling and the results are explained. The conclusions drawn from this experiment make possible to evaluate the accuracy of the model and of the estimated parameters of the conventional machine. Chapter VI - Controller System Design consists of the development and test of the basic impedance control and the robust impedance control. In this section, the third stage of experimentation is explained. This last stage of experimentation includes the system parameter identification of the new system, the basic impedance control, the robust impedance robust based on sliding modes and the analysis of the final results. Chapter VII - Conclusions and Future Work analyzes the shortcomings and provides the conclusions. Improvements and future work are also discussed in this section.
2.1 Outline of Bond Graph Modeling Method

The state equations of the rowing machine were derived using the bond graph method. Details on the calculations can be seen in Appendix A.

The bond graph is a modeling method that is based on the conservation of power. The graphical bond representation of any dynamic system allows to convert it into a state-space representation. It consists in the use of power bonds denoting the direction of power transmission and causality to relate all elements of a system. The causality assignment classifies the variables as either inputs or outputs. A complete bond graph provides enough information regarding the number of states and algebraic loops present in the system. The bond graph method accepts multi-energy domains. Each power bond contains information about effort and flow present in the system (see Table I for some examples) [43].

The multiplication of any pair of effort and flow returns the power of the bond. Furthermore, it provides more information such as the power direction and the causality which indicates the input between the two options of effort and flow. By means of the diagram obtained, it is possible to obtain the state equations [22].

In bond graphs, there are the following four groups of basic symbols:
Table I: Effort and flow variables in some domains [43].

<table>
<thead>
<tr>
<th>System</th>
<th>Effort</th>
<th>Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mechanical</td>
<td>Force (F)</td>
<td>Velocity (v)</td>
</tr>
<tr>
<td>Mechanical</td>
<td>Torque (τ)</td>
<td>Angular velocity (ω)</td>
</tr>
<tr>
<td>Electrical</td>
<td>Voltage (V)</td>
<td>Current (i)</td>
</tr>
<tr>
<td>Hydraulic</td>
<td>Pressure (P)</td>
<td>Volume flow rate (dQ/dt)</td>
</tr>
<tr>
<td>Thermal</td>
<td>Temperature (T)</td>
<td>Entropy change rate (ds/dt)</td>
</tr>
<tr>
<td>Thermal</td>
<td>Pressure (P)</td>
<td>Volume change rate (dV/dt)</td>
</tr>
<tr>
<td>Chemical</td>
<td>Chemical potential (µ)</td>
<td>Mole flow rate (dN/dt)</td>
</tr>
<tr>
<td>Magnetic</td>
<td>Magneto-motive force (e_m)</td>
<td>Magnetic flux (φ)</td>
</tr>
</tbody>
</table>

- Three basic one-port passive elements: it is formed by the R-element, C-element and I-element. The constitutive relationship between effort \( e \) and flow \( f \) is given just as with the resistors, capacitors and inductors elements:
  \[
  \text{R-element:} \quad e = Rf \\
  \text{C-element:} \quad e = C^{-1} \int f(t)dt \\
  \text{I-element:} \quad e = L(df(t))
  \]

- Two basic active elements: A basic active element can be effort or flow source. It gives the source of reaction. It has a half arrow pointing away from the source as its graphical representation. The difference between the two of them can be seen with their causality at the end for the effort source and at the beginning for the flow source (see Figure 6).

- Two basic two port elements: A basic two port element can be a transformer or a gyrator. A transformer relates flow-to-flow and effort-to-effort while a gyrator relates effort to flow and flow to effort (see Figure 7). The constitutive relationship is derived as follows:
  \[
  \text{Transformer:} \quad f_j = r f_i \quad \text{and} \quad e_j = r^{-1} e_i \\
  \text{Gyrator:} \quad e_j = \mu f_i \quad \text{and} \quad e_i = \mu f_j
  \]

- Two basic junctions: A basic junction can be the 1 or the 0 junction. They
Figure 6: The two basic active elements: effort and flow source.

Figure 7: The two basic two port elements: transformer and gyrator. Adapted from [43].

conserve power and are reversible. In the 1 junction, all flows are equal and the sum of efforts with the same orientation is equal to zero. In the 0 junction, all efforts are equal and the sum of flows with the same orientation is equal to zero.

The number of state equations can be determined by the C and I elements causalities. When the C-element has flow causality (the line at the beginning of the arrow) means one state as well as when the I-element has effort causality (the line at the end of the arrow).

2.2 Modeling

The first objective of the project was to mimic the behavior of a conventional rowing machine. In order to achieve this objective, initial tests were performed using the
conventional machine. It was necessary to know the dynamics of the conventional rowing session with the idea of being able to size and estimate a model. Details of the experiment and results used as reference data are explained in Chapter III.

The rowing machine has the sprocket rotary displacement as its degree of freedom. Figure 8 shows the schematic of the rowing machine used for the research. The machine is modeled as a flywheel joined to a one-way clutch connected to a sprocket through a chain and a spring. Most of them present a linear behavior with the exception of the air resistance in the flywheel which seems to be quadratic [36]. For this reason, the total friction ($\phi$), including the linear friction due to the bearings, was considered as a linear-quadratic damper. Figure 9 shows the physical representation of the model developed.

The dynamic model of the rowing machine has two discrete states which are related to the coupling of the flywheel (coupled or decoupled from the sprocket). The equations of motion depend directly on these discrete states.

The rowing machine has a free-wheeling clutch (FWC) to make possible the coupling and decoupling. This device is a one-way mechanical coupling which is used to transmit motion in one direction and to inhibit it in the opposite direction. It is usually used in the rear wheel of bicycles. The modeling of the FWC can be done in
two main ways, considering it as an ideal or non-ideal element.

In the FWC modeled as a non-ideal element, the viscous friction depends on the sign of the relative speed between input and output. In the ideal model, there is no slip during the coupled dynamics nor there is either transmitted torque in the decoupled dynamics. However in the non-ideal model, a little slip and some transmitted torque are considered for the coupled and decoupled dynamics respectively. A flow-effort relationship can be seen in Figure 10 [34].

The use of an infinite value for RH (from the flow-effort relationship in Figure 10) could be problematic introducing simulation artifacts and causing numerical solver. For that reason, the discrete transition laws for the rowing machine were performed considering the free-wheeling clutch as an ideal element (infinite contact stiffness and no slip). This model implies that the coupled state does not involve energy storage or dissipation, the decoupled state is not affected by friction and there is an instantaneous transition between these states [34].

Figure 9: Physical model of the conventional rowing machine.
2.2.1 Coupled Dynamics

The physical representation of the model when the flywheel is coupled from the sprocket can be represented in Bond graph as follows:

The discrete state \( q \) on the coupled mode has a value of zero \( (q = 0) \). There are two state equations (one C-element with flow causality and one I-element...
with effort causality). They are derived as follows:

\[
x_1' = \frac{x_2}{M_h} \tag{2.1}
\]

\[
x_2' = \left( Fr_s - x_1 r_s K_s - \phi \left( \frac{x_2}{M_h r_s} \right) \right) \frac{M_h r_s}{M_h r_s^2 + J_F} \tag{2.2}
\]

Where \( x^T = [x_1, x_2] \) is the state vector, \( x_1 \) (Eqn. 2.1) is the linear position of the handle and \( x_2 \) (Eqn. 2.2) is its momentum. The function \( \phi \) (Eqn. 2.3) represents the total friction which is modeled as a linear and a quadratic damper with the following representation:

\[
\phi(\omega) = C_F \omega^2 + b_F \omega \tag{2.3}
\]

In this coupled mode, the angular velocity of the flywheel is equal to the angular velocity of the sprocket.

### 2.2.2 Decoupled Dynamics

The physical representation of the model when the flywheel is decoupled from the sprocket can be represented in Bond graph as follows:

![Bond graph representation for decoupled dynamics in the rowing machine.](image)

Figure 12: Bond graph representation for decoupled dynamics in the rowing machine.
The discrete state \((q)\) on the decoupled mode has a value of one \((q=1)\). There are three state equations (one C-element with flow causality and two I-element with effort causality). They are derived as follows:

\[
\dot{x}_1 = \frac{x_2}{M_h} \tag{2.4}
\]

\[
\dot{x}_2 = Fr_s - X_1K_s \tag{2.5}
\]

\[
\dot{x}_3 = -\phi \left( \frac{x_3}{J_F} \right) \tag{2.6}
\]

Where \(x^T = [x_1, x_2, x_3]\) is the state vector, \(x_1\) (Eqn. 2.4) is the linear position of the handle, \(x_2\) (Eqn. 2.5) is the momentum of the handle, \(x_3\) (Eqn. 2.6) is the angular momentum on the flywheel and \(\phi\) (Eqn. 2.3) is the same friction function used for the coupled mode. While the flywheel is remaining decoupled from the sprocket, the angular velocity of the flywheel and the sprocket are independent to each other. Furthermore, the velocity of the flywheel decreases due to the linear and quadratic damper (friction in bearings and the air resistance produced by the flywheel) while the sprocket velocity is governs by the trajectory followed by the user.

### 2.2.3 Discrete Transition Laws

The discrete transition law is implemented on the basis of the force alongside the chain and the velocity feedback. Suppose the system starts in the coupled mode (with a value of zero in its discrete state). Once the transmitted force (Eqn. 2.8) reaches the small threshold value \((F_{TH})\), the discrete state changes to the value of one (to the decoupled mode). The transition from coupled to decoupled occurs at the end of the pull phase when the user is about to start the reversing motion back to the beginning point releasing the flywheel for a free spin. The flywheel decoupled from the sprocket starts to decelerate because of the air friction (linear and quadratic
dampers presented in the model). The user then reaches the end of the return stroke and reverses motion. When the relative speed (the difference between the flywheel angular velocity and the sprocket angular velocity) reaches a small threshold value ($\omega_{TH}$), the clutch is reattached and the cycle repeats. The transition laws (See Eqn. 2.7) can be summarized as follows:

$$q = \begin{cases} 
0 & \text{if } t = 0 \\
0 \rightarrow 1 & \text{if } t > 0 \text{ and } F_t < F_{TH} \\
1 \rightarrow 0 & \text{if } t > 0 \text{ and } (\omega_{flywheel} - \omega_{sprocket}) < \omega_{TH}
\end{cases}$$

(2.7)

where $F_t$ (Eqn. 2.8), $\omega_{flywheel}$ (Eqn. 2.9) and $\omega_{sprocket}$ (Eqn. 2.10) are the produced force due to the torque transmitted by the clutch, the flywheel angular velocity and the sprocket angular velocity respectively. The discrete transition laws use these variables as follows:

$$F_t = \frac{(F - x_1 K_s) J_F}{M_h r_s^2 + J_F} - \phi \left( \frac{x_2}{M_h} \right) \frac{M_h r_s}{M_h r_s^2 + J_F}$$

(2.8)

$$\omega_{flywheel} = \frac{x_3}{J_F}$$

(2.9)

$$\omega_{sprocket} = \frac{x_2}{M_h r_s}$$

(2.10)

The model diagram in Figure 13 represents the hybrid dynamics of the system.

### 2.3 First Simulations

A basic simulation was performed using arbitrary model parameters in order to test the model. This basic simulation shows the system response for an applied rectangular force.
Figure 13: Hybrid dynamic model for the conventional rowing machine.

Figure 14 shows the generated force applied to the system and the transmitted to the flywheel. The transmitted force is less than the force produced because part of it is used to overcome the inertia of the chain and the spring connected.

Figure 15 shows the velocities of the sprocket and the flywheel. As it is expected, both velocities are the same while the system is coupled. When the return stroke begins, the sprocket velocity starts to decrease due to the flywheel loses its momentum. In that moment the flywheel starts its free spin however due to the present friction (linear and quadratic), its speed does not remain constant. Once the sprocket velocity reaches the flywheel velocity, the system is coupled again and the cycle is repeated.
Figure 14: Force applied to the system together with the transmitted force to the flywheel.
Figure 15: Simulation of velocities generated by the model from the applied rectangular force.
CHAPTER III

REFERENCE DATA

There are many studies on rowing machines and their interaction with the user [2] [26], however none of them has done a complete study considering the machine as a set of pieces and not as only one. Due to this lack of information related to the conventional rowing machine operation and the human dynamics during rowing sessions, the first stage of experimentation was developed taking samples of each part of the rowing machine. The main objective of the test was to record the required data for being later used as our reference. The data obtained was used in the modeling and sizing of the powered ergometer. The test was developed using the conventional ergometer on the Parker Hannifin Human Motion and Control Laboratory of Cleveland State University. Figure 16 shows a subject performing the exercise during a trial. The experiment consisted of 10 minutes of warm-up followed by a total of 9 workout trials of 2 minutes each (Figure 17 shows the mechanical dataset performed). Each workout comprised a combination between three different cadences (quantified in strokes per minute) and three different intensity levels set by opening and closing the machine vents (the resistance setting was from 1 to 10 corresponding to the highest intensity). The data repository and report are available in [8].
3.1 Systems and Devices for Data Acquisition

Multiple systems were required to record the data (related to the human and machine sides). The conducted experiment for the human side included motion capture, measurement of metabolism and oxygen consumption and electromyography for muscle activation. On the other hand, for the mechanical side, included the handle force, flywheel velocity and sprocket velocity. The systems and devices used for the data recording are explained in detail below:

**dSpace MicroLabBox**

The MicroLabBox [12] is a system class which provides a high-performance control, test and measurements. It is a simple and fast system at a compact size. It has a Freescale QorlQ P5020 dual-core 2 GHz processor with 1 GB DRAM and 128 MB flash memory. It admits until 32 analog-inputs, 16 analog-outputs with ±10 V and ±8mA and 48 bidirectional digital channels with functionality for:
Figure 17: Description of each of the 9 workout trials.

- 6 x Encoder sensor input
- 2 x Hall sensor input
- 2 x EnDat interface
- 2 x SSI interface
- Synchronous multi-channel PWM
- Block commutational
- PWM

Figure 18: dSpace MicroLabBox.

The MicroLabBox (see Figure 18) was used to record the data from the encoders and the load cell.
**Encoders**

An encoder is an electromechanical device which allows measuring the rotation of an object connected to it. The measurement is achieved by converting the rotation into digital pulses that are later interpreted by a controller. Two 256-line optical encoders (see Figure 19) were used. One of them was used to record the angular speed of the sprocket and the other one to record the angular speed of the flywheel.

![256-line Bourns optical encoder](image19)

**Load Cell**

A load cell is a transducer which converts from force to an electrical signal. The load cell can be for tension, compression or both measurements. The load cell used was a compression/tension 500 lbf one (see Figure 20). It was used to measure the strength.

![500 lbf load cell](image20)
at all times of training. It was set up along the chain which connects the handle to the sprocket.

**Polar Beat Monitor**

The Polar Beat monitor (see Figure 21) is a heart rate sensor [33].

![Polar Beat Monitor](image)

**Figure 21: Polar Beat - Heart rate monitor.**

The heart rate measurements can be performed in real-time or recording for later study.

**Cosmed K4b2**

The Cosmed is a corporation specialized in metabolic and cardio pulmonary diagnostics. The Cosmed K4b2 (see Figure 22) is the device used in the experiment. It is an oxygen consumption meter [7]. The results allow performing a metabolic diagnostic. The results recorded gave an estimate on the oxygen consumption during training with the rowing machine. The objective of this assessment is subsequently compare the results with the powered rowing machine. Furthermore, comparing both results based on the oxygen consumption, the benefits of the eccentric exercises during training will be evaluated.

**Cortex System**

The Cortex system (see Figure 23) is a group of devices capable of recording and processing the movement of objects or people.
A total of 26 reflective markers were strategically located at each point of interest for the human motion capture. The human dynamics (range of motion flexion and extension) were measured by the Cortex System composed by 10 infrared cameras.

**Wireless Delsys System**

The Delsys Trigno Wireless EMG system (see Figure 24) is a high-performance device capable of making electromyography (EMG) signal detection easy and reliable [11]. A total of 16 EMG were used for the test. They were strategically located
at each muscle of the right side of the body responsible for the activity (see Figure 25). The activation of the muscles was successfully measured and recorded.

Figure 25: Subject before test in the at the Parker-Hannifin Motion and Control Lab, Cleveland State University.

3.2 Data Synchronization

The data processing was performed with the following systems: the dSpace Micro-LabBox, the Cortex system, the Wireless Delsys system, the Polar Beat monitor and
the Cosmed K4b2. The three first had the same rate of 1 kHz and the last two did not require synchronization since their data do not suffer immediate variation as in the first cases. Since the Cortex and the Delsys systems were connected to the same computer, they had the same time bases however they were independent of the DSpace's. In order to synchronize both datasets, one of the EMGs, in addition to be measured by the Delsys system, it was also measured by the dSpace. Both set recorded having the first EMG as a common data were run in an autocorrelation algorithm. The program was able to find the relative delay between them and so then, a synchronization between them was successfully performed.

3.3 Results

The following results show one of the datasets for each of the resistance levels.

The high-intensity level of resistance was achieved by adjusting the air damper to the highest (level ten). The cadence performed for these results was 40 strokes per minute.

The medium-intensity level of resistance was achieved by adjusting the air damper to the medium (level five). The cadence performed for these results was 40 strokes per minute.

The low-intensity level of resistance was achieved by adjusting the air damper to the lowest (level one). The cadence performed for these results was 40 strokes per minute.

3.4 Discussion

A total of 9 separate datasets were recorded for being used as a basis of this work. They were performed by a professional rower with a good physical condition. The first three datasets correspond to the highest resistance level and they were performed
with three different cadences. Although each of the datasets was extremely necessary for the fulfillment of the objectives, the third trial can be considered as the most important. Since the third trial was performed with the highest resistance level and the fastest cadence, it was the one that got the highest values of force and power. For this reason it was used for the sizing of the motor. The second and the third three sets correspond to the medium and the lowest resistance level respectively. They were also performed with different cadences. This reference data was subsequently used for the model and parameter estimation required for the accurate imitation of the conventional ergometer.

Figure 26: Data recorded with high-intensity.
Figure 27: Power during a workout on conventional rowing machine with high-intensity.

Figure 28: Data recorded with medium-intensity.
Figure 29: Power during a workout on conventional rowing machine with medium-intensity.

Figure 30: Data recorded with low-intensity.
Figure 31: Power during a workout on conventional rowing machine with low-intensity.
CHAPTER IV
SERVOMOTOR AND ACCESSORIES
SELECTION

In this section the motor, gears and belt will be selected in order to replace the flywheel and the one-way clutch. The handle, chain and spring are the original elements that will remain in the powered machine.

Having available the data recorded from the rowing machine experiments, a DC motor model was developed. Since the torque-mode servo drive makes an AC motor to operate as a DC motor, the model developed is suitable for the motor sizing of both DC and servomotor.

For the motor model, the electric circuit of the armature and the free-body diagram of the rotor were considered. The input of the system is the voltage applied to the motor’s armature and the output is the angular velocity. The following conditions were considered: rotor and shaft rigid, viscous friction and friction torque proportional to angular velocity [14].

Once the model was developed in symbolic form, a gear arrangement and a belt were added to the model. Subsequently, with the model ready, several tests were performed replacing the symbolic variables for real values extracted from the datasheet of different kind of motors. Finally, the model was subjected to the critical
values that showed the highest peaks of torque and speed in the dataset and the voltage, current, torque, speed and power were estimated. Based on these results, a motor was selected. Details in the next sections.

4.1 Motor and Transmission Model

The conventional model for the DC motor (see Figure 32) was considered for the evaluation. The complete model includes the motor and the set of gears synchronized by a timing belt (see Figure 33).

Since the model had to be tested with different parameters related to different kind of motors, the calculations were performed symbolically (see Table II).

![Figure 32: DC motor model.](image)

1. According to the Kirchhoff’s voltage law, the voltage is defined by:

$$ V = e_{mf} + (iR_a) + \left( \frac{d}{dt} (i) L_a \right) $$

(4.1)

Where: $e_{mf} = \alpha \dot{\theta}_m$

2. According to Newton’s 2nd law, the electro-mechanical transmission is defined by:

$$ \tau_m = J_m\omega_m + b_m\omega_m - \alpha i $$

(4.2)
Table II: Variables considered in the DC motor model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variable Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius of the sprocket</td>
<td>$r_s$</td>
</tr>
<tr>
<td>Radius of the gear from the motor</td>
<td>$r_m$</td>
</tr>
<tr>
<td>Radius of the gear connected to the sprocket</td>
<td>$r_g$</td>
</tr>
<tr>
<td>Voltage supplied to the motor</td>
<td>$V$</td>
</tr>
<tr>
<td>Current on the armature circuit</td>
<td>$i$</td>
</tr>
<tr>
<td>Armature resistance of the motor</td>
<td>$R_a$</td>
</tr>
<tr>
<td>Armature inductance of the motor</td>
<td>$L_a$</td>
</tr>
<tr>
<td>Electromotive force constant of the motor</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>Combined moment of inertia of the rotor and the driver gear</td>
<td>$J_m$</td>
</tr>
<tr>
<td>Motor viscous friction constant</td>
<td>$b_m$</td>
</tr>
<tr>
<td>Moment of inertia of the driven gear</td>
<td>$J_g$</td>
</tr>
<tr>
<td>Motor viscous friction constant of the driven gear</td>
<td>$b_g$</td>
</tr>
<tr>
<td>Torque on the motor</td>
<td>$\tau_m$</td>
</tr>
<tr>
<td>Torque on the sprocket</td>
<td>$\tau_s$</td>
</tr>
<tr>
<td>Angular position of the motor</td>
<td>$\theta_m$</td>
</tr>
<tr>
<td>Angular velocity of the motor</td>
<td>$\omega_m$</td>
</tr>
<tr>
<td>Angular position of the sprocket</td>
<td>$\theta_s$</td>
</tr>
<tr>
<td>Angular velocity of the sprocket</td>
<td>$\omega_s$</td>
</tr>
</tbody>
</table>

3. The following equations relate the motor torque and velocity with those of the sprocket:

   - The torque:
     \[ \tau_m + J_g\omega_s + b_g\omega_s = \tau_s \]  \hspace{1cm} (4.3)

   - The angular velocity:
     \[ r_g > r_m \Rightarrow \frac{r_g}{r_m} = n \Rightarrow \omega_m = n\omega_s \]  \hspace{1cm} (4.4)

4. Replacing Equation 4.1 in Equation 4.2:

   \[ \tau_m = b_m\omega_m + J_m\dot{\omega}_m - \left( \frac{\alpha V - \alpha^2\omega_m}{R_a} \right) \]  \hspace{1cm} (4.5)

5. Replacing Equation 4.5 in Equation 4.3:

   \[ n\left( \frac{\alpha^2\omega_m - \alpha V}{R_a} + J_m\dot{\omega}_m + b_m\omega_m \right) + J_g\dot{\omega}_g + b_g\omega_g = \tau_s \]  \hspace{1cm} (4.6)
6. Replacing Equation 4.4 in Equation 4.6:

\[
\left( \frac{\alpha^2 n^2 \omega_s - \alpha n V}{R_a} \right) + J_m n^2 + J_g \dot{\omega}_s + b_m n^2 + b_g \omega_s = \tau_s
\]  \hspace{1cm} (4.7)

7. Performing variable change:

- Reflected inertia: \( J_M n^2 + J_G \rightarrow J_T \)
- Reflected damping: \( b_M n^2 + b_G \rightarrow b_T \)
- Electro-mechanical damping: \( \alpha n \rightarrow a \)

8. Replacing the new variables in Equation 4.7:

\[
\tau_s + \left( \frac{a V}{R_a} \right) = J_T \dot{\omega}_s + \left( b_T + \frac{a^2}{R_a} \right) \omega_s
\]  \hspace{1cm} (4.8)

9. And from Equation 4.8, the voltage is defined by:

\[
V = \left( \frac{J_T R_a}{a} \right) \dot{\omega}_s + \left( \frac{b_T R_a + a^2}{a} \right) \omega_s - \left( \frac{\tau_s R_a}{a} \right)
\]  \hspace{1cm} (4.9)
Table III: DC motor parameters.

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_s$</td>
<td>13.50</td>
<td>mm</td>
</tr>
<tr>
<td>$r_m$</td>
<td>35.00</td>
<td>mm</td>
</tr>
<tr>
<td>$r_g$</td>
<td>89.10</td>
<td>mm</td>
</tr>
<tr>
<td>$R_a$</td>
<td>30.00</td>
<td>ohms</td>
</tr>
<tr>
<td>$L_a$</td>
<td>0.80</td>
<td>H</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.37</td>
<td>V.s</td>
</tr>
<tr>
<td>$J_m$</td>
<td>0.18</td>
<td>N.mm.s$^2$</td>
</tr>
<tr>
<td>$b_m$</td>
<td>1.03</td>
<td>N.mm.s</td>
</tr>
<tr>
<td>$J_g$</td>
<td>0.59</td>
<td>N.mm.s$^2$</td>
</tr>
<tr>
<td>$b_g$</td>
<td>1.03</td>
<td>N.mm.s</td>
</tr>
</tbody>
</table>

The gears and belt selection was carried out following the steps from the technical manual for timing belts from the brand Optibelt (see [30]). The details on the selection of each of the accessories can be seen in Appendix B.

4.2 Model Subjected to the Critical Values

The results of the motor and accessories selection can be seen below. After testing in simulation many kinds of motors with different capacities, using the dataset of the most critical values, the motor sizing was successfully performed by choosing a servomotor of 1kW of power. Below are the parameters of the selected servomotor and approximate values for the parameters of the gears based on the required ratio (see Table III) followed by the performance of the system subjected to the data from the highest intensity human test (see Figure 34):

As it can be seen in Figure 34, the system is expected to operate correctly within the permissible and safe ranges of the motor. Note that operation is not continuous, so the short-duration running range has been considered.
4.3 Frame Adaptation

The design of the frame had the following main objectives:

- Provide safety and protection to the user.
- It had to be attachable to the main frame as an only one piece.
- The required amount of material had to be the least possible.
- It had to be easily machined.

It was decided to design a two-piece support to attach the motor to the frame of the rowing machine. An intermediate support with elongated holes was required due to the necessity of having to tension the belt without placing adjustable holes in the rowing machine frame either in the motor support. Since the dimensions of the
motor were known, a first support was developed (see Figure 35) to be joint to the motor. Afterward, a second support was developed in order to work as an intermediate adapter between the first and the rowing machine frame. With this combination, once the motor is fixed to the first support, through the sliding between the first and the second support, the belt can be tensioned (see Figure 36). Subsequently, since the driven gear had to be located in the sprocket and the length of the belt was known (defined by the calculations in Appendix B), the driver gear only could be located in any place of the circumference that makes the belt be tight. Since the most resistant part of the rowing machine was under the main frame, the support of the motor was located there (see Figure 37). Finally, in order to meet the proposed objectives, a protective cover was designed with the aim of preventing on the one hand that
someone accidentally inserts part of the body or any garment into the running motor or on the other hand that any part of the system could get out of there causing an accident (see Figure 38).

The overall purpose of the motor and accessories selection was achieved. The assembly with the rowing machine was also achieved by meeting each of the specific objectives related to the user safety and the design efficiency. The necessary machining processes for the adapters were done, the protection part was made in 3d printing and all parts were assembled (see Figure 39, 40 and 41).
Figure 38: 3D printed protective case.

Figure 39: Powered rowing machine.
Figure 40: Two-piece support attached to the powered rowing machine frame.

Figure 41: Powered rowing machine with 3D printed protective case.
CHAPTER V

HYBRID MODEL SIMULATIONS

Prior to conducting real-time experiments with the powered ergometer, some simulations were performed to validate the model against the real data recorded from experiments.

5.1 Simulation of the Virtual Flywheel and Clutch

The discrete transition law is implemented on the basis of the force and the velocity feedback. The system starts in the coupled mode (with a value of zero in its discrete state). When the transition from coupling to decoupling occurs, a real-time simulation of the flywheel is started using as initial condition its velocity at the time of transition. This is accomplished with a reset integrator triggered by the transitions to the decoupled mode. Simultaneously, since the target impedance of the coupling and decoupling mode are different, the parameters are switched to the set corresponding to the return phase. Due to the decrease in inertia, damping and the spring whose action is permanent, the user is able to return to the initial position. Meanwhile, the virtual flywheel decelerates under the action of linear (bearings) and quadratic damping (air). Later the user reverses motion, accelerates and finally gets in contact with the virtual flywheel. A transition to the coupled mode is triggered and the cycle
Table IV: Real parameters on the conventional rowing machine [36].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sprocket Radius ($r_s$)</td>
<td>13.5</td>
<td>mm</td>
</tr>
<tr>
<td>Handle Mass ($M_h$)</td>
<td>1</td>
<td>kg</td>
</tr>
<tr>
<td>Spring Stiffness ($K_s$)</td>
<td>14.85</td>
<td>N/m</td>
</tr>
<tr>
<td>Flywheel Inertia ($J_F$)</td>
<td>885</td>
<td>kg.cm$^2$</td>
</tr>
<tr>
<td>Flywheel Linear Damper ($b_F$)</td>
<td>9.1e$^{-4}$</td>
<td>N.m.s</td>
</tr>
<tr>
<td>Flywheel Low Quadratic Damper ($C_F$)</td>
<td>9e$^{-5}$</td>
<td>N.s$^2$.m</td>
</tr>
<tr>
<td>Flywheel Medium Quadratic Damper ($C_F$)</td>
<td>12.75e$^{-5}$</td>
<td>N.s$^2$.m</td>
</tr>
<tr>
<td>Flywheel High Quadratic Damper ($C_F$)</td>
<td>22.2e$^{-5}$</td>
<td>N.s$^2$.m</td>
</tr>
</tbody>
</table>

is repeated.

5.2 Model Validation of the Conventional Ergometer

In order to validate the model, the real parameters of the rowing machine were required. Some of them were measured directly such as the sprocket radius, mass of the handle and the spring constant while some others were fitted using simulations together with the actual rowing data (from the first test performed with the conventional machine - Chapter III). These estimations were first performed on Parker Hannifin Human Motion and Control Laboratory [36]. Subsequently, the parameters were re-estimated on Control, Robotics and Mechatronics Laboratory by using other methods but achieving the same results. These parameters can be seen in Table IV. The values of the estimated parameters were introduced into the mathematical model. Afterward, the simulation was performed using the same force of the dataset used in the parameter estimation.
5.3 Results

A simulation for each of the nine trials was performed. The following picture (see Figure 42) shows an example result of the nine simulations performed. It can be appreciate the accuracy of the model and of the estimated parameters. The model was able to predict by simulation the angular velocity of the sprocket and the flywheel only using the measured force of the load cell as the input. The results show that the model can capture the essential dynamics of the rowing machine, including the coupled and decoupled behavior and the overall scale of the force and velocity. The accuracy of these results is sufficient enough for the model-based control development. The next picture (see Figure 43) shows the velocities of the sprocket and the flywheel together making it possible to see how the coupling between them occurs. In this last
picture can be seen how the flywheel and sprocket velocities are the same while they are coupled and how they become independent when they are decoupled. The flywheel velocity decreases due to the linear-quadratic damping and the sprocket velocity is governed by the trajectory made by the user. The oscillations showed in the sprocket velocity are due to the sudden movement made by the user before making the change of direction to retake the catch phase.

It was possible to verify the accuracy of the model and the estimated parameters comparing the velocities of the sprocket and flywheel with the recorded from the data. However, there are other ways to do it such as an impedance verification or using the sliding function (the last one just for the robust impedance controller) in greater detail in the next chapter, section 6.4.
The designing of the control system is composed of 3 stages. The preliminary step is a system and parameters identification, an intermediate is a development of an impedance control and the last one is a robust impedance control based on sliding modes. From this point, the rowing machine becomes powered by the motor. The system and parameters change since there is a motor and there is not the physical flywheel either the physical clutch anymore. However, the motor is able to replace these physical parts by virtual ones in order to the system behave as the conventional rowing machine or as a more general exercise machine.

6.1 System and Parameters Identification

Due to the changes performed, a dynamic model for the powered system must be obtained and the parameters must be identified. The system and parameters identification made possible the estimation of these parameters and the dynamic model. It is given below in linear coordinates by:

\[ \tau = M\ddot{x} + C\dot{x} - F \]  

(6.1)
Where $\tau$ is the control torque applied by the motor:

$$\tau = \left( \frac{K_m}{n} \right) u \quad (6.2)$$

$K_m$ is the motor constant, $n$ is the effective moment arm, $u$ is the analog input to the servo amplifier and it is proportional to the torque applied to the motor, $x$ is the linear displacement of the handle (tangential to the sprocket in the direction of motion), $M$ is:

$$M = \left( \frac{J_T}{n^2} \right) \quad (6.3)$$

$J_T$ is the total inertia, $C$ is:

$$C = \left( \frac{b_T}{n^2} \right) \quad (6.4)$$

$b_T$ is the total damping and $F$ is the tension force on the chain. For this stage of identification, an analysis of the architecture of the powered ergometer is required (see Figure 44). From the architecture, the effective moment arm is derived:

$$n = \frac{r_s}{Z_r} = 0.0053 \text{ m} \quad (6.5)$$

Where $r_s$ is the radius of the internal sprocket of measurement 13.5 mm and $Z_r$ is the ratio of the gears of 112 and 44 respectively. In order to estimate the remaining parameters, the dynamic model (Eqn. 6.1) is transferred to joint coordinates:

$$\left( \frac{J_T}{n} \right) \ddot{\omega} + \left( \frac{b_T}{n} \right) \dot{\omega} = \left( \frac{K_m}{n} \right) u + F \quad (6.6)$$

Then, (Eqn. 6.7 is transformed to Laplace:

$$\left( \frac{J_T}{n} s + \frac{b_T}{n} \right) \omega(s) = \left( \frac{K_m}{n} \right) U(s) + F(s) \quad (6.7)$$
The tests performed for computing the values are explained in the following subsections.

### 6.1.1 Random Velocity at Zero Force

The first test consisted of applying a random voltage to the servo amplifier ($u$) without any resistance force on the motor. In order to perform this experiment, the belt was removed and the motor was allowed to turn freely while the random voltage is applied.

$$\omega(s) = U(s) \left( \frac{K_m}{s+b_T/J_T} \right)$$  \hspace{1cm} (6.8)

The angular velocity measured by the encoder of the motor ($\omega$) was recorded to be used in the System Identification Toolbox of Matlab to predict the transfer function values and afterward to obtain the values of $K_m/J_T$ and $b_T/J_T$. In the System Identification Toolbox, the signals were analyzed in time-domain, the input and out-
put of the transfer function were selected from the Eqn. 6.8, the sample time was 1 ms (see Figure 45) and the following preprocessing of data were performed (see Figure 46):

1. Remove the means.

2. Filter in a range from 0.01 to 5 Hz.

3. Estimation of the Transfer Function.

The transfer function estimated was the following:

\[ U(s)\left( \frac{400}{s + 1.253} \right) = W(s) \rightarrow K_m/J_T = 400 \quad ; \quad b_T/J_T = 1.253 \]  \hspace{1cm} (6.9)

And from Eqn. 6.9, the following grouping variables were derived:

\[ K_m/J_T = 400 \]  \hspace{1cm} (6.10)
Figure 46: Preprocess of data for the system identification toolbox.

\[ b_T/J_T = 1.253 \]  

(6.11)

### 6.1.2 Static Force Test

The second configuration consisted of finding the relationship between the input voltage and the force when the system is in a static position. It was developed applying constant input voltages \( u \) and measuring the force necessary to keep the system at rest.

\[ u \left( -\frac{K_m}{n} \right) = F \]  

(6.12)

After 14 measurements and a linear regression (see Figure 47), the relationship between the input voltage and the force was estimated.

\[ u(-253.5) = F \]  

(6.13)
And by the equality between Eqn. 6.13 and Eqn. 6.12 and using the Eqn. 6.5, the motor constant is derived:

\[ K_m = 1.3446 \text{ Nm/V} \]  \hspace{1cm} (6.14)

### 6.1.3 Final Identification

The final identification is given using the values of Eqn. 6.10, 6.11 and 6.14:

\[ J_T = 0.0034 \text{ Nms}^2 \quad , \quad b_T = 0.0042 \text{ Nms} \]
6.2 Impedance Control

6.2.1 Outline

The impedance control is an approach which makes possible to control the dynamic interaction between the machine and its environment. That is possible through the regulation of the relationship between the force and the position, velocity and acceleration [17] [18] [19] [20].

To use this approach in this research was very appropriate because it makes possible to provide an arbitrary type of resistance making the training more varied and at the same time more complete. The target impedance parameters are switched constantly during the test according to the coupling state.

6.2.2 Development of the Basic Impedance Controller

The impedance control is developed in order to target the following impedance:

\[ M_d \ddot{x} + B_d \dot{x} + C_d \dot{x} |\dot{x}| + K_d x = F \]  \hspace{1cm} (6.15)

Where \( M_d \), \( B_d \), \( C_d \) and \( K_d \) are the desired inertia, linear damping, quadratic damping and stiffness respectively. By computing \( \ddot{x} \) from Eqn. (6.15), the linear acceleration in function of the desired parameters is described as:

\[ \ddot{x} = \frac{1}{M_d} \left( F - K_d x - B_d \dot{x} - C_d \dot{x} |\dot{x}| \right) \]  \hspace{1cm} (6.16)

And using the dynamic model Eqns. (6.2), (6.3), (6.4) and computing \( \ddot{x} \) from Eqn. (6.1), the linear acceleration in function of the estimated parameters is described as:

\[ \ddot{x} = \frac{F n^2}{J_T} + \frac{K_m n}{J_T} u - \frac{b_T}{J_T} \dot{x} \]  \hspace{1cm} (6.17)
And by equality between the Eqn. (6.16) and Eqn. (6.17) the torque control of the impedance model is derived as:

\[
\tau = \left( \frac{b_T}{n^2} - \Gamma B_d \right) \ddot{x} - C_d |\dot{x}| - K_d \Gamma x + F(\Gamma - 1) \tag{6.18}
\]

Where \( \Gamma \) is the inertia ratio which replaces the following relationship:

\[
\frac{J_T}{M_d n^2} = \Gamma
\]

### 6.3 Robust Impedance Control

#### 6.3.1 Outline

A basic impedance controller is a means of controlling the relationship between the force and velocity without ensuring that the impedance obtained is as the expected. A robust impedance controller is an approach which makes possible to control the relationship between the force and velocity in spite of disturbances or the inaccurate estimates of the parameters. This controller is perfect for this research because it is extremely important to have full control of the programmed impedances. The fact of having the control over the programmed impedances makes possible to focus accurately the training and with this to obtain better results. A robust controller works by compensating for those uncertainties and guarantees stability. It also provides an adequate and proper operation [3] [13]. The impedance obtained with the previous controller was not expected to be exactly as desired due to possible errors or lack of precision during:

- The conventional rowing parameters estimation.
- The system parameter estimation.
• The mathematical model.

• Disturbances present in the experiments.

To design a controller capable of providing a desired performance in the presence of disturbances or uncertainties in the model is a challenge. This has led to the development of robust control methods in order to solve this problem. One of the existing approaches which is the one used in this research is the sliding mode control technique [39]. This sliding mode robust impedance controller was developed to replace the previous control system in order to achieve the performance required. A sliding mode is a nonlinear control method. It works altering the dynamics of the system by applying a control signal. This control signal forces the system to compensate the errors by ”sliding” their values to zero. The performance of the controller can be easily estimated by measuring the slide (or switching) variable. For this thesis, a previously-published sliding mode impedance control was suitably modified for the powered machine. The modifications performed were the integration of a nonlinear parameter and the hybrid aspect. The hybrid condition made possible to switch the controller according to the phase of motion and the target impedances but keeping a single sliding variable. The robust impedance control implemented is mainly based on the work developed by Chan [3] with some adaptations for working with a nonlinear damper.

The control torque required to be applied to the system in order to achieve the desired impedance is specified as:

$$\tau_R = \hat{M}\ddot{x}_{eq} + \hat{C}\dot{x}_{eq} - T_d - Ds - \epsilon \text{sgn}(s) - F$$  \hspace{1cm} (6.19)$$

where $\hat{M}$ is the estimated M term of the Eqn. (6.3), $\ddot{x}_{eq}$ is the equivalent linear
acceleration derived as:

\[ \ddot{x}_{eq} = -F_1 \dot{x} - F_2 \dot{z} \quad (6.20) \]

\( F_1 \) and \( F_2 \) are any value different to zero, \( A \) is any negative or zero value, \( \dot{z} \) is the dynamic compensator derived as:

\[ \dot{z} = Az + K_{pz}x + K_{vz} \dot{x} + K_{qz} |\dot{x}| + K_{fz} F \quad (6.21) \]

The switching function derived as:

\[ s(x, \dot{x}, z) = \dot{x} + F_1 x + F_2 z \quad (6.22) \]

\( \hat{C} \) is the estimated \( C \) term of the Eqn. (6.4), \( \dot{x}_{eq} \) is the equivalent linear velocity derived as:

\[ \dot{x}_{eq} = -F_1 \dot{x} - F_2 \dot{z} \quad (6.23) \]

\( T_d \) is

\[ T_d = \left( \delta M |\ddot{x}_{eq}| + \delta C |\dot{x}_{eq}| \right) sgn(s) \quad (6.24) \]

\( \delta M \) and \( \delta C \) are the modeling error parameters, \( sgn(s) \) is the sign function replaced by the sigmoidal:

\[ \frac{2}{1 + e^{-u/\Delta}} - 1 \quad (6.25) \]

\( \Delta \) is any small value, \( D \) is any positive value, \( \epsilon \) is any value and \( F \) is the same tension force on the chain used before.

**6.3.2 Coefficient Selection for Target Impedance Matching**

Solving for \( z \) from Eqn. (6.22) when the switching function reaches the value of zero:

\[ z = -F_2^{-1}(\dot{x} + F_1 x) \quad (6.26) \]
Obtaining $\dot{z}$ from Eqn. (6.26):

$$\dot{z} = -F_2^{-1}(\ddot{x} + F_1\dot{x}) \quad (6.27)$$

Solving $\ddot{x}$ from Eqn. (6.27):

$$\ddot{x} = \dot{x}(A - F_2K_{pz} - F_1) + x(AF_1 - F_2K_{pz}) + \dot{x}|\dot{x}|(-F_2K_{qz}) - F_2K_{fz}e_f \quad (6.28)$$

And considering the following target impedance:

$$M_m\ddot{x} + B_m\dot{x} + C_m|\dot{x}| + K_m e = -K_f F \quad (6.29)$$

Solving $\ddot{x}$ from Eqn (6.29):

$$\ddot{x} = \frac{-B_m}{M_m} \dot{x} - \frac{C_m}{M_m} |\dot{x}| - \frac{K_m}{M_m} x - \frac{K_f F}{M_m} \quad (6.30)$$

And by the equality Eqn (6.28) and Eqn (6.30), the constants can be related as follows:

$$K_{pz} = \frac{(K_d/M_d + AF_1)}{F_2} \quad (6.31)$$

$$K_{vz} = \frac{(B_d/M_d - F_1 + A)}{F_2} \quad (6.32)$$

$$K_{qz} = \frac{C_d/M_d}{F_2} \quad (6.33)$$

$$K_{fz} = \frac{K_f}{F_2M_d} \quad (6.34)$$

Where $K_f$ is any non-dimensional value different to zero.
6.4 Target Impedance Verification

A robust impedance control verification was used in order to validate the accuracy of the controller. The process was executed as follows:

A model scheme can be seen in Figure 48. For the target impedance verification by simulation, the rowing machine model compute the acceleration by receiving the force from the experiment. Subsequently by integration, the position and velocity are obtained. The velocity output from the rowing machine model is used in order to be compared with the output of the target impedance verification block. This block also receives the force from the experiment and using the velocity and position from the integration of its own output, the block computes the acceleration. The target impedance verification in real-time follows the same procedure for the target impedance verification block, however the velocity obtained is then compared with the velocity measured from the encoders. An error lower than 1% is expected for an accurate performance.

Figure 48: Model scheme for the target impedance verification.

In order to validate the method for the impedance verification, it was initially performed in simulation. Both the control and the verification block received a square signal as input and they derived the velocity. Figure 49 shows the results of this simulation. Since it was not a real experiment, the error expected was zero.
Once the method was verified, it was applied in real-time in the experiments. Figure 50 shows an example test in which the powered rowing machine was set to work as the conventional machine. The velocity obtained and the desired were practically the same and the error between them was much smaller than the 1% expected. Figure 51 shows an example test in which the powered rowing machine was set to work producing a ratio of 1-1 in concentric and eccentric contractions. The velocity obtained and the desired were also practically the same. Although the error was less than the 1% expected, was bigger than the previous experiment. This result was due to the high eccentric loading that demanded a great effort from the user. This great amount of effort provoked shaking that caused perturbations to the system.

There are other ways to verify the impedance obtained, for instance in the robust impedance controller, the sliding function can be used to verify the accuracy of the controller by checking its value (see Figure 52). It depends on the error between the desired and the obtained impedance. The control system aims to reduce its value by achieving the required performance. In the example provided can be seen that the value converges to zero demonstrating the accuracy of the controller.
Table V: Target impedance parameters for imitate the conventional rowing machine.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Discrete State</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spring constant</td>
<td>Coupled</td>
<td>40</td>
</tr>
<tr>
<td>Spring constant</td>
<td>Decoupled</td>
<td>50</td>
</tr>
<tr>
<td>Inertia flywheel</td>
<td>Coupled</td>
<td>200</td>
</tr>
<tr>
<td>Inertia flywheel</td>
<td>Decoupled</td>
<td>6</td>
</tr>
<tr>
<td>Linear damping</td>
<td>Coupled</td>
<td>100</td>
</tr>
<tr>
<td>Linear damping</td>
<td>Decoupled</td>
<td>0.5</td>
</tr>
<tr>
<td>Quadratic damping</td>
<td>Coupled and Decoupled</td>
<td>50</td>
</tr>
</tbody>
</table>

Table VI: Target impedance parameters for a ratio of 1-1 in concentric to eccentric power.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Discrete State</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spring constant</td>
<td>Coupled</td>
<td>180</td>
</tr>
<tr>
<td>Spring constant</td>
<td>Decoupled</td>
<td>40</td>
</tr>
<tr>
<td>Inertia flywheel</td>
<td>Coupled</td>
<td>100</td>
</tr>
<tr>
<td>Inertia flywheel</td>
<td>Decoupled</td>
<td>15</td>
</tr>
<tr>
<td>Linear damping</td>
<td>Coupled</td>
<td>100</td>
</tr>
<tr>
<td>Linear damping</td>
<td>Decoupled</td>
<td>0.5</td>
</tr>
<tr>
<td>Quadratic damping</td>
<td>Coupled and Decoupled</td>
<td>50</td>
</tr>
</tbody>
</table>

6.5 Discussion

The robust impedance controller implemented on the powered rowing machine was effective at producing the desired target impedances for the pull and return phases. It was possible to imitate the operation of the conventional rowing machine (see Figure 50). The target impedance parameters for achieving the desired performance can be seen in Table VI. Furthermore, the return phase of the rowing exercise could be achieved under load by introducing eccentric training in rowing. It was shown that the ratio of concentric to eccentric power can be approached to one (see Figure 50) using the target impedance parameters shown in Table??.
Figure 50: Robust impedance verification with force and instantaneous power at low eccentric loading in the powered machine with robust impedance control.
Figure 51: Robust impedance verification with force and instantaneous power at high eccentric loading in the powered machine with robust impedance control.
Figure 52: Example of sliding function during an arbitrary simulation.
In conclusion, the goals have been met in a successful way providing security at all times to both the user and the system operator. Furthermore, the results throughout the document have demonstrated the precision of the model and the versatility of the design.

In addition to making possible the imitation of the conventional rowing machine, new training patterns have been shown to be possible. These new patterns are those which provide the versatility to the powered machine. For instance, the return phase of the rowing exercise could be achieved under load by introducing eccentric training.

The training was improved by making it more complete through the use of both the concentric and eccentric contractions. It was shown that the ratio of concentric to eccentric power can be easily programmed for instance approaching to one. It has not been shown previously a machine capable of producing training of equal magnitude for both concentric and eccentric contractions which makes it the first able to perform it. Besides, since the eccentric exercise (lengthening contraction) is extremely important principally in microgravity for being related to the training against the gravity force, this powered machine has much potential for being used both on earth and in space.
Since a minimum error between the target parameters and the obtained parameters is required for an effective control of the powered machine, different methods were successfully performed to the control system. The results obtained have shown that the robust impedance controller implemented on the powered rowing machine was effective at producing the desired impedances for the pull and return phases. The controller implemented successfully showed its ability to reproduce the operation and “feel” (as judged by an expert rower) of the conventional machine. Since the control system has the ability to virtually change the parameters (such as the flywheel inertia, spring stiffness, linear and quadratic damping) that are fixed in the conventional rowing machine, an unprecedented versatility was demonstrated. Besides, the design and controller allow the hardware to be used in a wider range of applications, including exercise in microgravity and rehabilitation. While the target impedance used in this project corresponds to a classical mass-spring-damper system, other synthetic impedances can be programmed without changes to the hardware. In particular, position-dependent impedance coefficients can be introduced to tailor the needs of individuals with muscle weaknesses or injuries.

As a soon future work, it is planning to perform a complete test similar to the first one including human motion capture, electromyography, metabolic consumption, etc. only replacing the conventional rowing machine by the powered one. Furthermore, as another future work, it is expecting to develop an autonomous intelligent system capable of automatically change the powered rowing machine parameters in order to, regardless of the rhythm that the user has, the training level is maintained in the desired. Similarly, the autonomous intelligent system has to be capable of providing a particular training based on the special necessities of the user in order to be able to use it as a rehabilitation machine. Moreover, more development in advanced exercise machines will be performed. For instance, the next one which is been adapting is a Nautilus exercise machine which will be also performed for advanced
training for the upper part of the body. It is expecting to use the background of this research to adapt them, to improve them and to extend them for new features.
BIBLIOGRAPHY


APPENDIX A

Bond Graph Dynamic Equation

Derivation

Since the dynamic behavior of the rowing machine depends on its discrete state, the equation derivation was performed independently for each of the coupling modes.

Coupled Dynamics

Figure 53: Bond graph representation of the rowing machine - Coupled dynamics.

The bond graph (see Figure 53) represents the model when the sprocket and
the flywheel are coupled. In the present discrete state the system has 2 states. The
state equations derived from the bond graph representation are the following:

1st State-Equation: \( \dot{q}_2 = \frac{P_4}{M_h} \)

2nd State-Equation: \( \dot{P}_4 = (F - q_2.K_s - \frac{\phi(P_h)}{r_s}) \cdot \frac{M_h.r_s^2}{M_h.r_s^2 + J_f} \)

Where \( q_2 \) (analogously in Eqn. 2.1) is the linear position of the handle, 
\( P_4 \) (analogously in Eqn. 2.2) is the momentum of the handle and the function \( \phi \) 
(analogously in Eqn. 2.3) represents the total friction which is modeled as a linear 
and a quadratic damper with the following representation:

\[
\phi(f) = C_1.f^2 + b.f \quad \rightarrow \quad \phi\left(\frac{P_h}{J_f}\right) = C_1.\left(\frac{P_h}{J_f}\right)^2 + b.\left(\frac{P_h}{J_f}\right)
\]

From this same representations was possible to calculate the force transmitted to the 
flywheel (analogously to the Eqn. 2.8) which is determined as follows:

\[
E_3 = \frac{(F - q_2.K_s).J_f}{M_h.r_s^2 + J_f} + \left(\frac{C_1}{r_s}\left(\frac{P_4}{M_h.r_s}\right)^2 + \frac{b}{r_s}\left(\frac{P_4}{M_h.r_s}\right)\right) \cdot \frac{M_h.r_s^2}{M_h.r_s^2 + J_f}
\]

**Decoupled Dynamics**

The bond graph (see Figure 54) represents the model when the sprocket and the 
flywheel are decoupled. In the present discrete state the system has 3 states. The 
angular velocity of the flywheel is independent to the angular velocity of the sprocket 
and its magnitude decreases proportionally to the coefficients of the linear and the 
quadratic damper. The state equations derived from the bond graph representation 
are the following:

1st State-Equation: \( \dot{q}_2 = \frac{P_4}{M_h} \)
2\textsuperscript{nd} State-Equation: \[ \dot{P}_4 = (F - q_2.K_s) \]

3\textsuperscript{rd} State-Equation: \[ \dot{P}_6 = -\phi\left(\frac{P_6}{J_f}\right) = -\left(C_1\cdot\left(\frac{P_6}{J_f}\right)^2 + b\cdot\left(\frac{P_6}{J_f}\right)\right) \]

Where \( q_2 \) (analogously in Eqn. 2.4) is the linear position of the handle, \( P_4 \) (analogously in Eqn. 2.5) is the momentum of the handle, \( P_6 \) (analogously in Eqn. 2.6) is the angular momentum on the flywheel and \( \phi \) is the same friction function used for the coupled mode.
APPENDIX B

Gears and Belt Selection

1. Total service factor: Medium duty drives, intermittent operation with low to medium shock loading, less than 16 hours daily operating period.

\[ C_0 = 1.8 \quad ; \quad C_3 = 0 \quad ; \quad C_6 = 0 \quad \rightarrow \quad C_2 = C_0 + C_3 + C_6 \quad \rightarrow \quad C_2 = 1.8 \]

2. Design power: Based on the maximum power of the selected motor (1 KW).

\[ P_B = P(C_2) = 1(1.8) = 1.8 \text{ KW} \]

3. Selection of timing belt section and construction: Based on the maximum power and speed, the section is obtained from the diagram for OMEGA HP (see Figure 55).

\[ (1.8 \text{ KW}; 3000 \text{ RPM}) \quad \rightarrow \quad \text{Optibelt OMEGA 5M - HP} \]

4. Speed ratio: based on the ratio used in the simulation.

\[ i = 2.54 \]
5. Number of teeth of the pulleys: based on the dimensions used in the simulation.

\[ Z_1 = 44 \rightarrow d_{w1} = 70.03 \text{ mm} \]

\[ Z_2 = 112 \rightarrow d_{w2} = 178.25 \text{ mm}; \]

6. Check driven speed.

\[ \frac{112}{44} = 2.54 \rightarrow 1100(2.54) = 2794 \text{ RPM} \rightarrow 2794 < 3000 \text{ CHECKED.} \]

7. Recommended centre distance.

\[
((70.03 + 178.25)0.5 + 15) < a < (2(70.03 + 178.25))
\]

\[ 139.14 < a < 496.56 \]

\[ a_{\text{temp}} = 375 \]

8. Pitch length of the timing belt.

\[
L_{wth} = 2(375) + \left( \frac{248.28\pi}{2} \right) + \left( \frac{108.22^2}{4(375)} \right)
\]

\[ L_{wth} = 1147.8 \text{ mm} \rightarrow \text{Standard} \rightarrow L_{wst} = 1135 \text{ mm} \rightarrow 1135 \text{ 5M HP - 227 teeth} \]

9. Centre distance from the length of the timing belt.

\[
k = \left( \frac{1135}{4} \right) - \left( \frac{248.28\pi}{8} \right) = 186.25
\]

\[ a_{\text{nom}} = k + \sqrt{k^2 - \frac{108.22^2}{8}} = 368.53 \text{ mm} \]
10. Minimum adjustment of centre distance for tensioning.

\[ x > 0.004(368.53) \rightarrow x > 1.47 \text{ mm} \]

11. Minimum adjustment for fitting belts.

Flange on both timing pulleys \( y = 19 \text{ mm} \)

12. Number of teeth in mesh on the small pulley.

\[ Z_e = \frac{44}{6} \left( 3 - \frac{108.22}{368.53} \right) = 19.85 \rightarrow 20 \]

13. Belt length factor: based on the standard belt length (1135 mm).

\[ C_\gamma = 1 \]

14. Teeth in mesh factor.

\[ C_1 = 1 \]

15. Belt width above nominal power rating.

\[ P_U > P_B \ ; \ P_U = 4.7(1)(1)(0.61) = 2.87 \text{ KW} \rightarrow 2.87 \text{ KW} > 1.80 \text{ KW} \]

According to the results, the timing belt and pulleys selected are:

- 1 Optibelt OMEGA HP Timing Belt 1135 5M HP 6
- 1 Optibelt ZRS Timing Belt Pulley (Z=44) 5M HP 6
- 1 Optibelt ZRS Timing Belt Pulley (Z=112) 5M HP 6

Or their equivalents (or better) in other brands.
Figure 55: Diagram - Omega HP.