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Design and Control of Electronic Motor Drives for Regenerative Robotics

Taylor Barto
Cleveland State University

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DESIGN AND CONTROL OF
ELECTRONIC MOTOR DRIVES FOR
REGENERATIVE ROBOTICS

Taylor Barto

Bachelor of Electrical Engineering
Cleveland State University
May 2016

submitted in partial fulfillment of requirements for the degree
MASTER OF SCIENCE IN ELECTRICAL ENGINEERING
at the
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ABSTRACT

Two regenerative motor drives, a voltage source converter and a bidirectional buck/boost converter, are studied for energy regeneration and joint trajectory tracking. The motor drives are applied to two different robotic systems—a PUMA560 robotic arm and a hip testing robot / prosthesis system. An artificial neural network controller is implemented with the two motor drives and provides joint trajectory tracking with an RMS error of 0.03 rad. The control signals produced by the artificial neural network contain a large amount of high frequency content which prevents practical implementation. A robust passivity-based motion controller is modified to include information about the motor drives to overcome the limitations of the artificial neural network controller. The modified robust passivity-based controller outperforms the artificial neural network controller by maintaining a 3 V RMS error between the voltage generated by the converter and the desired voltage while maintaining comparable trajectory tracking. The high frequency content of the robust passivity-based controller contains less high frequency content than the artificial neural network controller. The modified robust passivity-based controller is implemented inside the semiactive virtual control energy regeneration framework to demonstrate energy regeneration with one of the motor drives. The motor drive implemented with the energy regeneration framework shows that energy can be regenerated while using the bidirectional buck/boost converter.
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CHAPTER I
INTRODUCTION

Energy regeneration is a well-studied topic with respect to applications such as electric vehicles. In 1983, electric vehicles were claimed to operate at a range of 22 miles. With the advancement of energy regeneration, electric vehicles in 2012 had an operating range around 100 miles, an improvement of over 450% [4], [42]. With the success of energy regeneration in electric vehicles, other applications have also been studied.

Another example application of energy regeneration is electrically powered wheelchairs. One study [32] demonstrated the ability to harness the braking energy of an electric wheelchair. It was found that energy regeneration in electric wheelchairs have the ability to provide up to 50% efficiency, which was determined by how much mechanical energy could be stored in a capacitor bank relative to how much energy was provided mechanically and electrically for a desired braking pattern for the wheelchair.

Energy consumption of industrial robotics has been evaluated [5], [20], [24], [45]. Since the consumption of energy of industrial robots can be large depending on payload and trajectories, multiple methods have been evaluated to reduce energy consumption. For existing robots, optimal path planning has been studied, which focuses on control of robots without energy regeneration to reduce consumed en-
ergy \cite{20, 34, 45}. Motor drives that include the ability to regenerate energy have been introduced \cite{24}. These motor drives have been able to reduce the energy consumption of robots. The energy regenerated by the motor drive of one robot has been shared on a DC bus for industrial applications that contain more than a single robot \cite{24}. In one experimental case, the regenerative motor drives reduced energy consumption of industrial robots by 20\% \cite{24}.

The possibility of applying regenerative motor drives to consumer robots is evaluated in this work. In this work, two different regenerative motor drives are evaluated for use with an actively driven transfemoral prosthesis. The idea of using a shared bus between the ankle and knee motors is evaluated in addition to having a separate storage device for each joint. The main objective of this work is to provide a path towards improving the operating time of an actively driven prosthesis by evaluating and implementing different energy regeneration techniques.

1.1 Motivation

Advancements in lower limb prosthetics have the potential to affect a large number of people. One study shows that close to 68,000 people were discharged from a hospital after an amputation in 2009 \cite{11}. Of the 68,000 amputations, more than 75\% of the amputations were lower-limb \cite{18}. A traditional prosthesis only provides damping to assist the prosthesis during a braking motion.

Some commercially available prosthetic legs have improved the passive prosthesis by introducing controlled damping during gait. One example is the Rheo Knee, which uses a fluid that changes its damping properties based on a magnetic field \cite{22}. Another example is the C-leg which includes a microprocessor to control damping of the prosthesis \cite{23}. Although these prostheses have the ability to match sections of gait when braking motions occur, these prosthetic legs do not have the ability to
provide active actuation needed in other sections of gait such as during push-off. The lack of active actuation requires a prosthesis user to consume about 50% more energy than able-bodied individuals to provide knee flexion during stance phase and ankle plantarflexion during push-off [13].

Studies show that in addition to the higher energy consumption, amputees using a prosthesis are 25% more likely to develop osteoarthritis and 88% more likely to develop osteoporosis than an able-bodied individual [12], [37]. The additional health issues commonly faced by amputees can largely be associated with the unnatural hip movements required to maintain gait while using a passive prosthesis.

Further improvements were made to lower-limb prosthetics through the inclusion of motors. The Össur Power Knee is a commercially available prosthesis that includes a motor at the knee joint to actively provide power to the knee [21]. A research group at Vanderbilt University has also developed a transfemoral prosthesis that includes both knee and ankle motors [38]. Research groups at MIT and Georgia Tech have also developed a powered transtibial and transfemoral prosthesis with motors at each joint [16], [51].

The development of active prosthetic legs has enabled a prosthesis to closely match able-bodied gait. One of the main limitations of the actively powered prosthetics is the limited operating time. Due to size and weight constraints in a prosthesis, the power density of current batteries severely limits the operating time of a prosthesis. The Vanderbilt leg has an operating time of 1.8 hours of walking [38]. The MIT prosthesis operates for 3 hours of walking [51].

Instead of waiting for battery technology to improve the operating time of an actively driven prosthesis, the concept of energy regeneration is evaluated for use with prosthetic legs. The operating time of an actively driven prosthesis could potentially be improved by evaluating regenerative motor drives and different control methods, similar to the idea of industrial robots and electric vehicles.
1.2 Literature Review

Confidence in using regenerative motor drives to achieve energy regeneration in a transfemoral prosthesis is obtained by noting that the able-bodied knee produces more energy than it consumes in a typical gait cycle [48], meaning that the knee is producing energy during braking motions that is normally dissipated. In the same study, it was found that the ankle consumes more energy than it produces throughout a typical gait cycle. These results can be verified by looking at the flow of power at the knee and ankle joint during gait. As found in Figure 1, the able-bodied knee produces more energy (negative power flow) than it consumes (positive power flow) in a typical gait cycle. The opposite occurs with the power flow at the ankle, as seen in Figure 2 [44]. When integrating the power flow over time, it is found that the knee has a net production of 29.5 J and the ankle has a net consumption of 30.6 J. The net difference between the knee and ankle joint is 1.1 J. Regenerating energy at the knee and ankle could occur either separately (a distributed regeneration mode), or together on a single bus (a star regeneration mode).

![Figure 1](image_url):

Figure 1: Typical able-bodied power flow at the knee joint during one cycle of gait
During the 1980s, a research team from MIT designed a transfemoral prosthesis with energy regeneration [33], [39]. The motor drive was used to create an impedance that matched reference data. Due to the limitations of hardware at the time, the prosthesis was not able to reach its desired efficiency and had issues with impedance control when the capacitor voltage was low [39]. More recently an electrical prosthesis with energy regeneration was designed [41]. The prosthesis had a low efficiency of energy regeneration since the prosthesis was only able to charge a capacitor when the motor voltage exceeded the capacitor voltage, thereby limiting the amount of damping that the energy regeneration process can provide.

Some prosthesis research has attempted energy regeneration with non-electrical components. One example is a hydraulic prosthesis that uses accumulators which are pressurized before usage [29], [44], [50]. The braking motion of the prosthesis causes the fluid to move from one accumulator to another. A valve then releases the fluid when the prosthesis joint requires to be actively powered. The parallel with an electrical prosthesis can be observed as initial charge on a capacitor, or with a battery.

Similar to an industrial robot, creating a controller for a transfemoral prosthesis with respect to energy regeneration has been studied [17], [25], [28], [31]. In these studies, the control laws and regeneration relationships are designed with an
ideal electrical converter. If an evaluation of regenerative motor drives is completed, then a regenerative motor drive could be implemented with these studies to help improve the operating time of a transfemoral prosthesis that contains actively and semi-actively controlled joints.

1.3 Thesis Objectives and Organization

This thesis builds upon previous work in the area of regenerative robotics. Different regenerative motor drives and control methods are evaluated for use with a transfemoral prosthesis. The system parameters in each combination of motor drive and controller are selected with an evolutionary algorithm to meet various design goals. The different regenerative motor drives and controllers are compared to determine the preferred combination to provide the most energy regeneration without sacrificing joint trajectory tracking.

First, two different motor drives that can be used to regenerate energy are evaluated in Chapter II. Chapter III discusses two different robotic models, one of which is a PUMA560 robot, and the other of which is a system combining a hip testing robot and a prosthetic leg. In Chapter IV, different controllers for the motor drives are reviewed. The simulation results are given in Chapter V. A discussion is found in Chapter VI along with suggestions on how to continue the development of regenerative motor drives for prosthetic applications.
CHAPTER II

REGENERATIVE MOTOR DRIVE DESIGN

Energy regeneration in robots requires special motor drives that allow power to flow in all four quadrants of the velocity-torque plane shown in Figure 3. Regeneration occurs when the voltage and current have opposite signs (negative power flow) which occurs in quadrants II and IV. A motor drive must be able to modulate the direction of current flow to/from the motor. The standard method of accomplishing this task is through use of an H bridge as in Figure 4. Both motor drive designs will use this H bridge design. It can be noted that other domains can use similar circuits to provide four quadrant operation. For example, a four quadrant hydraulic converter is described in [43].

Before evaluating motor drives, motor current and voltage must be defined. In this work, motor current is defined to be positive when exiting the positive terminal of the DC motor as shown in Figure 4. For each motor drive, this standard will remain the same. Positive voltage is defined as an increase in potential when following the definition of positive current flow through the motor.

When switches s1 and s4 are closed, then current can flow into or out of the positive terminal of the motor. This condition is referred to as a positive mode of
operation. When switches s2 and s3 are closed, then current can flow into or out of the negative terminal of the motor. This condition is referred to as a negative mode of operation. Throughout this work, the motor is shown with its equivalent circuit inside a dashed box. $L_a$ corresponds to the inductance of the motor, $R_a$ corresponds to the resistance of the motor, and $\varepsilon$ is the back-emf of the motor.

When the net current flow is entering the motor, then the motor is said to be in motoring mode. The motoring mode corresponds to quadrants I or III in Figure 3. When the net current flow is exiting the motor, then the motor is said to be in generating mode. The generating mode corresponds quadrants II or IV in Figure 3. Table I summarizes the conditions required to make a motor drive operate in different modes and directions.

![Figure 3: The four quadrants of operation for a DC motor](image-url)
The two motor drives are presented in this chapter. The models obtained will allow for simulation of two robotic systems with the two different motor drives. A comparison of the capabilities of the motor drives will show that the bidirectional buck/boost converter is the better alternative for use in the remainder of this study.

2.1 Voltage Source Converter (VSC)

The first regenerative motor drive that is examined is the voltage source converter (VSC) in Figure 5. The VSC is the regenerative motor drive that was used
for the prosthesis that was designed in the 1980s [39]. The only change from the circuit of Figure 4 is the addition of a capacitor. The VSC was reviewed for prosthesis design in [3]. The results of [3] showed that the VSC can be implemented into a prosthesis with a supercapacitor by using an artificial neural network as a motor current controller; however, the motor current signal was unrealistic for implementation and that energy regeneration only occurred in only one of the multiple simulations. Even with these limitations, the VSC serves as a starting point to obtain a regenerative motor drive and control system that can provide realistic control signals and provide energy regeneration. In this study, a supercapacitor bank, $C$, replaces the standard capacitor that was originally introduced in [39].

A supercapacitor is selected due to the low equivalent series resistance (ESR) which is typically on the order of mΩ and its high capacity. Since supercapacitors have a small ESR, the time constant is also very small, which allows for fast charging and discharging, which is critical in energy regeneration. An ideal system would contain both supercapacitors for fast charging and discharging and batteries for high energy densities. In this study, only supercapacitors are considered. Future work can optimize the system with both technologies included. The material in this section is based on [2] and [3].
Figure 5: The voltage source converter schematic. This circuit is used as a regenerative motor drive

**Converter Model**

To evaluate the VSC, different circuits were drawn in order to obtain the set of equations that model each circuit depending on the operating conditions (mode and direction) of the robotic joint. The equations were obtained using Kirchoff’s laws and the physical relationships of the components in the motor drive. Diodes are treated as ideal switches in the state equations.

The switches for the VSC motor drive are operated to modulate the amount of current flowing through the motor. To achieve this, the VSC switches between two circuits for each of the four combinations of operating mode and operating direction. One circuit will connect the capacitor to the motor, and the other circuit will short the motor while leaving the capacitor disconnected. Due to this configuration, the VSC operates with respect to currents, not voltages.

The motor mode in the positive direction is shown in detail in Figure 6 and
Figure 7. The first circuit (Figure 6) in the positive motor mode is when the capacitor bank is connected, allowing power to flow from the capacitor bank to the motor. The following equations describe the circuits states when the capacitor bank is connected (switches $S_1$ and $S_4$ in Figure 5 are closed):

$$\dot{I}_m = \frac{\varepsilon - I_m(R_a + 2R_F) - V_C}{L_a}$$

$$\dot{V}_c = \frac{I_m}{C}$$

(2.1)

Figure 6: The VSC motor drive circuit while operating in the positive motor mode (quadrant I) with the capacitor bank connected. Switches $S_1$ and $S_4$ from Figure 5 are closed.

When the power flows from the capacitor to the motor, the current will decrease as described in Equation 2.1. The amount of power delivered to the motor is modulated by switching MOSFET $s1$ in order to disconnect the capacitor by using the circuit of Figure 7. In this second circuit, the power delivered to the motor is reduced as the current increases due the lack of the $V_c$ term in the following equations:

$$\dot{I}_m = \frac{\varepsilon - I_m(R_a + R_F)}{L_a}$$

$$\dot{V}_c = 0$$

$$I_m \leq 0$$

(2.2)
Figure 7: The VSC motor drive circuit while operating in the positive motor mode (quadrant I) with the capacitor bank disconnected. Switch $S_4$ from Figure 5 is closed.

Similar to the positive motor mode, the positive generator mode is shown in Figure 8 and Figure 9. The first circuit shown in Figure 8 connects the capacitor bank to the motor, allowing power to flow from the motor to the capacitor. When the power flows from the motor to the capacitor, the current will decrease as described in the following equation:

$$\dot{I}_m = \frac{\varepsilon - I_m(R_a + R_F) - V_C}{L_a}$$

$$\dot{V}_c = \frac{I_m}{C}$$

$$I_m \geq 0$$

(2.3)
The power delivered to the capacitor is modulated by switching MOSFET $S_3$ in order to disconnect the capacitor bank from the motor. When the second circuit of Figure 9 is active, the power from the motor is no longer directed to the capacitor and the current will start to increase as described in the following equation:

$$\dot{I}_m = \frac{\varepsilon - I_m (R_a + 2R_F)}{L_a}$$

$$\dot{V}_c = 0$$

The same process is repeated for the remaining two modes of operation. Table II summarizes the results from each circuit. For a more detailed look at these
equations, the schematic for each of the four modes can be found in Appendix A.

Table II: Voltage source converter equations

<table>
<thead>
<tr>
<th>Mode</th>
<th>Dir.</th>
<th>Quad.</th>
<th>Capacitor State</th>
<th>State Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motor</td>
<td>Positive</td>
<td>I</td>
<td>Connected</td>
<td>$\dot{I}_m = \frac{\varepsilon - I_m(R_a + 2R_F) - V_C}{L_a}$, $\dot{V}_c = \frac{I_m}{C}$</td>
</tr>
<tr>
<td>Motor</td>
<td>Positive</td>
<td>I</td>
<td>Disconnected</td>
<td>$\dot{I}_m = \frac{\varepsilon - I_m(R_a + R_F)}{L_a}$, $\dot{V}_c = 0$, $I_m \leq 0$</td>
</tr>
<tr>
<td>Motor</td>
<td>Negative</td>
<td>III</td>
<td>Connected</td>
<td>$\dot{I}_m = \frac{\varepsilon - I_m(R_a + 2R_F) + V_C}{L_a}$, $\dot{V}_c = \frac{I_m}{C}$</td>
</tr>
<tr>
<td>Motor</td>
<td>Negative</td>
<td>III</td>
<td>Disconnected</td>
<td>$\dot{I}_m = \frac{\varepsilon - I_m(R_a + R_F)}{L_a}$, $\dot{V}_c = 0$, $I_m \geq 0$</td>
</tr>
<tr>
<td>Generator</td>
<td>Positive</td>
<td>II</td>
<td>Connected</td>
<td>$\dot{I}_m = \frac{\varepsilon - I_m(R_a + R_F) - V_C}{L_a}$, $\dot{V}_c = \frac{I_m}{C}$, $I_M \geq 0$</td>
</tr>
<tr>
<td>Generator</td>
<td>Positive</td>
<td>II</td>
<td>Disconnected</td>
<td>$\dot{I}_m = \frac{\varepsilon - I_m(R_a + 2R_F)}{L_a}$, $\dot{V}_c = 0$</td>
</tr>
<tr>
<td>Generator</td>
<td>Negative</td>
<td>IV</td>
<td>Connected</td>
<td>$\dot{I}_m = \frac{\varepsilon - I_m(R_a + R_F) + V_C}{L_a}$, $\dot{V}_c = \frac{I_m}{C}$, $I_m \leq 0$</td>
</tr>
<tr>
<td>Generator</td>
<td>Negative</td>
<td>IV</td>
<td>Disconnected</td>
<td>$\dot{I}_m = \frac{\varepsilon - I_m(R_a + 2R_F)}{L_a}$, $\dot{V}_c = 0$</td>
</tr>
</tbody>
</table>
2.2 Bidirectional Buck/Boost Converter

The second regenerative motor drive that is examined is the bidirectional buck/boost converter in Figure 10. The bidirectional buck/boost converter has found applications in electric vehicles, but not yet in prosthetics [14]. This motor drive is used to address the limitations of the VSC. The bidirectional buck/boost converter allows for voltages to be bucked and boosted as power flows from the motor to the capacitor, or from the capacitor to the motor. This flexibility has the potential to improve both energy regeneration and the resulting control signal applied to the motor.

![Bidirectional Buck/Boost Converter Schematic](image)

Figure 10: The bidirectional buck/boost converter schematic. This circuit is used as a regenerative motor drive

The bidirectional buck/boost converter includes the H bridge configuration of Figure 4, but includes another H bridge circuit that contains an inductor, \( L_{cv} \). Similar to the speed-torque diagram of Figure 3, an electrical equivalent can be used to describe the H bridge with this inductance. Figure 12 shows a current-voltage plane. With the H bridge of Figure 11, when \( S_b \) is open, the body diode allows current to flow from the left side of the circuit (from \( C \)) to the right side of the circuit.
(to $C_M$). When $S_a$ is open, the body diode allows current to flow from the right side of the circuit (from $C_M$) to the left side of the circuit (to $C$). The flow of current can either be positive or negative with respect to the definition of positive current flow, $i_{CV}$, through the inductor, $L_{CV}$, giving both the positive and negative sides of the current axis in Figure 12. When this converter is combined with the H bridge circuit of Figure 4, positive and negative voltages can be used with the motor.

![diagram](image)

Figure 11: The H bridge for the inductor part of the bidirectional buck/boost converter shown in Figure 10
Converter Model

The process of obtaining the equations for the bidirectional buck/boost converter is the same as the process of obtaining the equations for the VSC. The switches for the bidirectional buck/boost converter motor drive are operated to either buck or boost the voltage between the capacitor bank and motor. Switches $S_a$, $S_b$, $S_c$, and $S_d$ control the amount of voltage bucking or boosting and the direction of current flow between the capacitor and motor. Switches $S_1$, $S_2$, $S_3$, and $S_4$ are used to direct current flow through the motor. The bidirectional buck/boost converter fundamentally operates on voltages, not currents.

The motor mode in the positive direction during voltage boosting is shown in Figure 12: The four quadrants of operation for the H bridge converter shown in Figure 11

![Diagram of four quadrants](image)
detail through Figure 13 and described by the following equations when $S_d$ is closed:

$$\dot{I}_m = \frac{\varepsilon - I_m(R_a + R_F) - V_{cm}}{L_a}$$

$$\dot{V}_c = \frac{I_{cv}}{C}$$

$$\dot{I}_{cv} = -\frac{V_c}{L_{cv}}$$

$$\dot{V}_{cm} = \frac{I_m}{C_m}$$

(2.5)

When $S_d$ is open, the following equations describe the circuit:

$$\dot{I}_m = \frac{\varepsilon - I_m(R_a + R_F) - V_{cm}}{L_a}$$

$$\dot{V}_c = \frac{I_{cv}}{C}$$

$$\dot{I}_{cv} = \frac{V_{cm} - V_c}{L_{cv}}$$

$$\dot{V}_{cm} = -\frac{I_{cv} - I_m}{C_m}$$

$$I_m \leq 0$$

(2.6)

Since MOSFET $S_b$ is always open in this mode, the body diode forces the motor drive to deliver power from the capacitor bank to the motor. MOSFET $S_a$ is always shorted during this mode similar to the topology of a standard boost converter. MOSFET $S_d$ is responsible for controlling the amount of voltage boosting from the capacitor to the motor. Switches $S1$ and $S2$ are used to obtain current flow in the negative direction through the motor.
Figure 13: The positive motor mode of the bidirectional buck/boost converter during a boosting operation

Similar to the boosting operation of the positive motor mode, the boosting operation of the positive generator mode is shown in Figure 13. When $S_c$ is open, the motor delivers power to the capacitor, and the system is described by the following equations:

$$
\dot{I}_m = \frac{\varepsilon - I_m (R_a + R_F) - V_{cm}}{L_a}
$$

$$
\dot{V}_c = \frac{I_{cv}}{C}
$$

$$
\dot{I}_{cv} = \frac{V_{cm} - V_c}{L_{cv}}
$$

$$
\dot{V}_{cm} = -\frac{I_{cv} - I_m}{C_m}
$$

$$
I_m \geq 0\tag{2.7}
$$

When $S_c$ is closed, the capacitor is disconnected from the motor and described by the
following equations:

\[
\dot{I}_m = \frac{\varepsilon - I_m (R_a + R_F) - V_{cm}}{L_a}
\]

\[
\dot{V}_c = 0
\]

\[
\dot{I}_{cv} = \frac{V_{cm}}{L_{cv}}
\]

\[
\dot{V}_{cm} = -\frac{(I_{cv} - I_m)}{C_m}
\]  \hspace{1cm} (2.8)

Since MOSFET \( S_a \) is always open in this mode of operation, current cannot flow from the capacitor to the motor since the body diode of MOSFET \( S_a \) is active. MOSFET \( S_b \) is always shorted during this mode similar to the topology of a standard boost converter. MOSFET \( S_c \) is responsible for controlling the amount of voltage boosting from the motor to the capacitor. Switches \( S_1 \) and \( S_4 \) are used to obtain current flow in the positive direction through the motor.

![Figure 14: The positive generator mode of the bidirectional buck/boost converter during a boosting operation](image)

The same process is repeated for the remaining six modes of operation. All of the possible circuit configurations can be found in Appendix B. The state equations
for the circuit configurations are also found in Appendix B.

When compared to the VSC, the bidirectional buck/boost converter operates naturally with voltages instead of currents. The different circuit configurations of the bidirectional buck/boost converter yield a standard buck converter or a standard boost converter. The output $V_o$ of the buck and boost circuits are related to the input voltage $V_i$ and the duty cycle $D$ of the modulated switches in Figure 11 through the equations

\begin{align*}
\text{Buck: } V_o &= V_i D \quad (2.9) \\
\text{Boost: } V_o &= \frac{V_i}{1 - D} \quad (2.10)
\end{align*}

2.3 Discussion

The design of two different motor drives that provide the ability to regenerate energy were reviewed in this chapter. Each design has advantages and disadvantages that are important to consider. A summary of these advantages and disadvantages follows.

The first motor drive design reviewed was the voltage source converter (VSC) model that was implemented in a prosthesis by Tabor [39]. This motor drive has the advantage of a low component count, verified operation, and contains only four operating modes. The VSC is designed in such a way that controlling current through the motor is easier than controlling the voltage across the motor.

The second motor drive design reviewed was the bidirectional buck/boost converter cascaded with an H bridge to control the direction of current flow through the motor. The bidirectional buck/boost converter has the disadvantage of having a larger component count than the VSC and having more modes of operation than the VSC, thus increasing the complexity of design/control and the potential cost of
building a motor drive. The bidirectional buck/boost motor drive has the advantage of having distinct bucking and boosting circuits that can modulate power flow between the motor and capacitor as commanded by a high-level torque controller. The ability to buck and boost voltages with power flow in both directions allows for more flexibility in the control signal generated by the high-level torque controller than is available with the VSC motor drive.
CHAPTER III

ROBOTIC APPLICATIONS

The applications of the developed models of the two regenerative motor drives are studied. The knowledge of the mechanical system allows for an understanding of how to implement control on the motor drives that were modeled in Chapter II. Two robotic systems are studied in this chapter. The first robotic system is a typical manipulator, and the second robotic system is a hip testing robot / transfemoral prosthesis system. By using two different robotic systems, the generality of the motor drives can be shown.

3.1 PUMA Robot

Due to the complexity of applying controllers to the prosthesis model, a test platform is first used to verify the feasibility of the proposed controllers. The testing platform is a PUMA560 robot that contains six degrees of freedom. A schematic diagram of the PUMA560 robot can be found in Figure 15. Although the PUMA560 robot has six degrees of freedom, only three are used when testing the feasibility of the converters, with only the third joint, $q_3$, using the converter model with a DC motor while the first two joints, $q_1$ and $q_2$, use a model of DC servomotors at each joint which are driven with current-mode PWM servo amplifiers that are not regenerative.
The three degrees of freedom correspond to the waist of the PUMA560 robot $q_1$, the shoulder of the PUMA560 robot $q_2$, and the elbow of the PUMA560 robot $q_3$ in Figure 15 where $q_1$ corresponds to the rotation of the waist, $q_2$ corresponds to the rotation of the shoulder, and $q_3$ corresponds to the rotation of the elbow. The coordinate system is defined per the Denavit-Hartenberg standard. Even though the PUMA560 robot is considered a test platform in this study, the PUMA560 robot demonstrates that the proposed converters in Chapter II are not specifically designed for a single application, but can be used in multiple robotic applications where energy regeneration is considered.

Figure 15: The schematic diagram of the PUMA560 robot which is used as a test platform for the converter circuits. Figure adapted from [8]
Model

The mathematical model for the PUMA robot is of the form

\[ u - T_e = M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) + R(\dot{q}) \]  \hspace{1cm} (3.1)

where \( q \) corresponds to the degrees of freedom in the robot, \( M \) is the mass matrix, \( C \) is the Coriolis matrix, \( g \) is a gravity vector, \( R \) is a dissipation matrix, \( T_e \) represents the external forces/torques, and \( u \) is the desired torque signal.

With the defined coordinate system conforming to the Denavit-Hartenberg standard, the matrices in Equation 3.1 are found by using the joint velocity Jacobians. The resulting robotics equation, while nonlinear with respect to the joint angles \( q \) and their derivatives, \( \dot{q} \) and \( \ddot{q} \), is a linear function of the mechanical parameters in the system. Due to linearity with respect to the mechanical parameters, Equation 3.1 can be written as

\[ u - T_e = Y(q, \dot{q}, \ddot{q})\Theta \]  \hspace{1cm} (3.2)

where \( Y \) is a regressor matrix that does not contain any mechanical parameters and \( \Theta \) is a vector of mechanical parameters.

Since three degrees of freedom are used with the PUMA560 robot model, \( q \in \mathbb{R}^3 \) so \( M \) and \( C \) contain three rows and three columns, \( g \) and \( R \) contain three rows and one column, and \( u \) and \( T_e \) contain three rows and a single column. In terms of Equation 3.2, \( Y \) is a \( 3 \times 10 \) matrix and \( \Theta \) is a 10 matrix. The matrix resulting from the multiplication of \( Y \) and \( \Theta \) is \( 3 \times 1 \), one element per degree of freedom. The mechanical properties of the PUMA560 robot are found in Table III. The \( M, C, g, R, Y, \) and \( \Theta \) matrices for the PUMA560 robot can be found in Appendix C. Simulation results will be presented in Chapter V.
Table III: The mechanical properties of the PUMA560 robot shown in Figure 15

<table>
<thead>
<tr>
<th>Mechanical Property</th>
<th>symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass of link 2</td>
<td>(m_2)</td>
<td>19.02 kg</td>
</tr>
<tr>
<td>mass of link 3</td>
<td>(m_3)</td>
<td>5.53 kg</td>
</tr>
<tr>
<td>mass attached to link 3</td>
<td>(M_3)</td>
<td>1.17 kg</td>
</tr>
<tr>
<td>length of link 1</td>
<td>(d_1)</td>
<td>0.67 m</td>
</tr>
<tr>
<td>distance between link 1 and 2</td>
<td>(d_2)</td>
<td>0.24 m</td>
</tr>
<tr>
<td>distance between link 2 and 3</td>
<td>(d_3)</td>
<td>0.09 m</td>
</tr>
<tr>
<td>length of link 2</td>
<td>(a_2)</td>
<td>0.43 m</td>
</tr>
<tr>
<td>center of mass link 2</td>
<td>(c_{2x})</td>
<td>-0.34 m</td>
</tr>
<tr>
<td>center of mass link 3</td>
<td>(c_{3x})</td>
<td>0.14 m</td>
</tr>
<tr>
<td>moment of inertia (y) for link 1</td>
<td>(I_{1y})</td>
<td>1.39 kg-m^2</td>
</tr>
<tr>
<td>moment of inertia (x) for link 2</td>
<td>(I_{2x})</td>
<td>0.13 kg-m^2</td>
</tr>
<tr>
<td>moment of inertia (y) for link 2</td>
<td>(I_{2y})</td>
<td>5.25 kg-m^2</td>
</tr>
<tr>
<td>moment of inertia (z) for link 2</td>
<td>(I_{2z})</td>
<td>0.54 kg-m^2</td>
</tr>
<tr>
<td>moment of inertia (x) for link 3</td>
<td>(I_{3x})</td>
<td>0.19 kg-m^2</td>
</tr>
<tr>
<td>moment of inertia (y) for link 3</td>
<td>(I_{3y})</td>
<td>0.12 kg-m^2</td>
</tr>
<tr>
<td>moment of inertia (z) for link 3</td>
<td>(I_{3z})</td>
<td>1.08 kg-m^2</td>
</tr>
<tr>
<td>acceleration of gravity</td>
<td>(g)</td>
<td>9.81 m/s^2</td>
</tr>
</tbody>
</table>

3.2 Transfemoral Prosthesis

After evaluation of techniques with the PUMA560 robot, the two circuit designs described in Chapter II will be evaluated on a transfemoral prosthesis model with the different control techniques from Chapter IV. The transfemoral prosthesis model contains four degrees of freedom as shown in Figure 16, where \(q_1\) represents the hip displacement, \(q_2\) represents the thigh rotation, \(q_3\) represents the knee rotation, and \(q_4\) represents the ankle rotation. In this model, the first two degrees of freedom come from a prosthesis testing (human hip emulation) robot developed by Cleveland State University [30] and are purely motion controlled. In actual implementation of the prosthesis, these two degrees of freedom would be controlled by the amputee. The hip robot provides the ability for repeatable experimental results for testing different prosthetic legs. The third and fourth degrees of freedom come from a prosthesis. Both of these joints will be controlled for trajectory tracking and impedance tracking.

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as defined in Section 3.3. The prosthesis model was adapted from previous studies at Cleveland State University [19], [46]. Similar to the PUMA560 robot model, the prosthesis model adheres to the Denavit-Hartenberg standard.

![Figure 16: The schematic diagram of the transfemoral prosthesis robot which is attached to a hip robot. Adapted from [10]](image)

**Model**

Following the same process as the PUMA560 robot, the mathematical model for the transfemoral prosthesis follows the form of Equation 3.1. The linearity property of the parameters in the prosthesis allows the model to be written in the form of Equation 3.2. The prosthesis model contains four degrees of freedom, \( q \in \mathbb{R}^4 \) so \( M \) and \( C \) contain four rows and four columns, \( g \) and \( R \) contain four rows and one column, and \( u \) and \( T_e \) contain four rows and a single column. In terms of Equation 3.2, \( Y \) is a 4 \( \times \) 15 matrix and \( \Theta \) is a 15 \( \times \) 1 matrix. The matrix resulting from the multiplication of \( Y \) and \( \Theta \) is 4 \( \times \) 1, one element per degree of freedom. The mechanical properties
of the hip testing robot / prosthesis system are found in Table IV. The \( M, C, g, R, Y, \) and \( \Theta \) matrices for the hip testing robot / prosthesis system can be found in Appendix D.

Table IV: The mechanical properties of the hip testing robot / prosthesis system shown in Figure 16

<table>
<thead>
<tr>
<th>Mechanical Property</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>equivalent inertial mass of link 1</td>
<td>( m_0 )</td>
<td>317.54 kg</td>
</tr>
<tr>
<td>mass of link 1</td>
<td>( m_1 )</td>
<td>40.60 kg</td>
</tr>
<tr>
<td>mass of link 2</td>
<td>( m_2 )</td>
<td>8.57 kg</td>
</tr>
<tr>
<td>mass of link 3</td>
<td>( m_3 )</td>
<td>2.99 kg</td>
</tr>
<tr>
<td>mass of link 4</td>
<td>( m_4 )</td>
<td>1.09 kg</td>
</tr>
<tr>
<td>sliding friction in link 1</td>
<td>( f )</td>
<td>83.33 N</td>
</tr>
<tr>
<td>rotary actuator damping</td>
<td>( b )</td>
<td>9.75 N-m-s</td>
</tr>
<tr>
<td>length of link 2</td>
<td>( l_2 )</td>
<td>0.43 m</td>
</tr>
<tr>
<td>center of mass of link 2</td>
<td>( c_2 )</td>
<td>0.09 m</td>
</tr>
<tr>
<td>length of link 3</td>
<td>( l_3 )</td>
<td>0.53 m</td>
</tr>
<tr>
<td>center of mass of link 3</td>
<td>( c_3 )</td>
<td>0.23 m</td>
</tr>
<tr>
<td>length of link 4</td>
<td>( l_4 )</td>
<td>0.10 m</td>
</tr>
<tr>
<td>center of mass of link 4</td>
<td>( c_4 )</td>
<td>0.05 m</td>
</tr>
<tr>
<td>rotary inertia of link 2</td>
<td>( I_{2z} )</td>
<td>0.44 kg-m²</td>
</tr>
<tr>
<td>rotary inertia of link 3</td>
<td>( I_{3z} )</td>
<td>0.18 kg-m²</td>
</tr>
<tr>
<td>rotary inertia of link 4</td>
<td>( I_{4z} )</td>
<td>0.02 kg-m²</td>
</tr>
<tr>
<td>acceleration of gravity</td>
<td>( g )</td>
<td>9.81 ( \text{m/s}^2 )</td>
</tr>
<tr>
<td>inertia of motor</td>
<td>( J_m )</td>
<td>( 1.82 \times 10^{-4} \text{kgm}^2 )</td>
</tr>
</tbody>
</table>
Ballscrew Transmission Model

The prosthesis model is expanded by adding a ballscrew actuator (Figure 17) at the knee joint [27]. The length of the ballscrew, $L$, is dependent on the knee velocity $\dot{q}_3$ through the relationship

$$\dot{L} = -\frac{bd_1 \cos(q + a)}{L}$$

(3.3)

where $b = \sqrt{d_2^2 + H^2}$, $d_2 = H \tan(a)$, and $L^2 = \sqrt{d_1^2 + b_2^2 - 2d_1b \sin(q + a)}$. The relationship between the angular velocity of the motor, $\omega$, and the linear velocity of the ballscrew is determined as $\dot{L} = l\omega$, where $l$ is the pitch of the screw (m/rad). Solving for the angular velocity

$$\omega = \left(\frac{-bd_1 \cos(q + a)}{lL}\right) \dot{\theta} \equiv n(q) \dot{\theta}$$

(3.4)
The resulting model for the combination of knee motor and ballscrew is described by

\[ \dot{\omega} + \frac{\alpha^2}{RJ_m} \omega = \left( \frac{\alpha}{RJ_m} \right) V - \left( \frac{\tau_f + \tau_L}{J_m} \right) \]  \hspace{1cm} (3.5) \]

where \( R \) is the armature resistance of the knee motor, \( \alpha \) is the torque constant of the knee motor, \( J_m \) is the inertia of the knee motor, \( \tau_f \) is the torque caused by friction, \( \tau_L \) is the load torque, and \( V \) is the voltage applied across the knee motor. When simplifying the model by neglecting the inertia and friction caused by the linearly moving mass, the linear force of the ballscrew is described by

\[ F_a = \frac{\tau_L}{l} - \frac{\tau_f}{l} - \frac{J_1}{l^2} \ddot{L} \]  \hspace{1cm} (3.6) \]

where \( J_1 \) is the inertia of the ballscrew and \( \tau_{f1} \) is the friction of the ballscrew. The moment produced by the actuator force is equivalent to \( T = F_a \ln(q) \). When combining this result with Equation 3.5, the load torque is calculated to be

\[ \tau_L = -J_m \dot{\omega} - \frac{\alpha^2}{R} \omega + \frac{\alpha}{R} V - \tau_f \]  \hspace{1cm} (3.7) \]

The knee torque is a function of the voltage applied across the motor, \( V \), knee position, \( q \), knee velocity \( \dot{q} \), and knee acceleration \( \ddot{q} \). To simplify notation, let \( n \equiv n(q) \), \( n' \equiv \frac{\partial n(q)}{\partial q} \), \( J_t \equiv J_m + J_1 \), and \( u \equiv n(q) \frac{\alpha}{R} V \). Then the torque is obtained by substituting \( \tau_L \) into Equation 3.6 and solving for \( \tau \) as a function of input voltage, knee position, knee velocity, and knee acceleration.

\[ \tau = -n^2 J_t \ddot{q} - nn' J_t \dot{q}^2 - \frac{(\alpha n)^2}{R} \dot{q} + u - n \tau_{f1} \]  \hspace{1cm} (3.8) \]

where \( n \tau_{f1} \) is the friction term and \( u \) is the control term. The knee torque relationship of Equation 3.8 is augmented to the standard robotic dynamics of Equation 3.1. When
the knee torque relationship is augmented to the robotic equation, the following modifications are applied to the robotic equation.

1. The term $n^2J_i$ is added to the third diagonal entry of the mass matrix.

2. The term $\left(\frac{an}{R}\right)^2\dot{q}$ is added to the third element of the dissipation matrix.

3. The term $n\tau_{ft}$ is combined with the third element of the dissipation matrix.

4. The term $u$ is used as the control term to produce knee angle displacement.

5. The term $nn'J_i\dot{q}$ is added to the third diagonal entry of the Coriolis matrix.

**Ground Reaction Force Model**

The prosthesis model is also expanded to include ground reaction forces, which comprises the external forces ($T_e$) acting on the robotic model in Equation 3.1. The ground reaction forces are observed when the heel or toe of the foot is in contact with the ground as shown on the prosthesis schematic in Figure 16. The ground reaction forces are composed of both a horizontal force acting on the heel, $F_{xh}$, and toe, $F_{xt}$, and a vertical force acting on the heel, $F_{zh}$, and on the toe, $F_{zt}$. The ground reaction forces were calculated [10], [19] as follows,

\[
\begin{align*}
z_t &= 0.24 \sin(q_2 + q_3 + q_4 + 0.79) + l_3 \sin(q_2 + q_3) \\
&\quad + l_2 \sin(q_2) + q_1 \tag{3.9} \\
zh &= -0.11 \sin(q_2 + q_3 + q_4 - 0.79) + l_3 \sin(q_2 + q_3) \\
&\quad + l_2 \sin(q_2) + q_1 \tag{3.10} \\
F_{zt} &= -k_b(z_t - sz) \left(1 + \text{sgn}(z_t - sz)\right) \tag{3.11} \\
F_{zh} &= -k_b(z_h - sz) \left(1 + \text{sgn}(z_h - sz)\right) \tag{3.12} \\
F_{xt} &= \beta F_{zt} \tag{3.13} \\
F_{xh} &= \beta F_{zh} \tag{3.14}
\end{align*}
\]
where $l_2$ and $l_3$ are the lengths of the thigh and shank of the prosthesis robot, $k_b$ is the stiffness of the belt (ground), $s_z$ is the standoff height of the treadmill, and $\beta$ is the coefficient of friction between the foot and the treadmill’s belt. The vertical position of the heel is denoted by $z_h$ and the vertical position of the toe is denoted by $z_t$. The ground reaction force model is simplified for ease of implementation in this study. A higher fidelity model [46] should be used in future studies. Simulation results will be presented in Chapter V.

### 3.3 Mechanical Control Structure

Since the PUMA560 robot and the prosthesis robot both follow the robotic dynamics of Equation 3.1, the same control technique can be used to determine the desired torques for each joint to cause the joint angles to follow a set of reference trajectories. Control of robotics that follow the form of Equation 3.1 has been the subject of many studies. With respect to a lower-limb prosthesis, control has been achieved with independent-joint sliding mode control and hybrid control [30], adaptive control methods [1], the functional approximation technique [9], and robust passivity-based control [19]. Each of these control techniques have also been studied with other approaches to control [47].

One way of expanding on well-known control methods is the introduction of impedance control. Impedance control was previously implemented with a robust passivity-based controller [19]. Impedance control allows for the management of the force and velocity relationship of robotic joints [6]. By mixing impedance control with robust passivity-based control, the prosthesis robot can include both joint trajectory tracking through passivity-based control and target impedance tracking through impedance control. The robotic leg works well with this control method since the hip robot includes thigh displacement and thigh rotation and is required to follow
reference trajectories while the prosthesis includes knee rotation and ankle rotation and is required to follow reference trajectories and also be flexible when interacting with the surrounding environment to reduce the effect of external forces on the joints. Since the prosthesis robot will be connected to the user, the flexible joints allow for desirable operation of the leg which emulates the human leg.

Due to the availability of the simulation files [19], a mixed tracking/impedance robust passivity-based controller is used when testing the motor drives described in Chapter II with the artificial neural network described in Section 4.1. Pure motion tracking is implemented with the hip and thigh joints, while mixed trajectory tracking / impedance tracking is implemented in the knee and ankle joints.

In [19], the passivity framework is used to define:

\[ r_{MC} = \dot{q}_{MC} - v_{MC} \]  \hspace{1cm} (3.15)
\[ v_{MC} = \dot{q}^d_{MC} - \Lambda_{MC} \tilde{q}_{MC} \]  \hspace{1cm} (3.16)
\[ a_{MC} = \dot{v}_{MC} \]  \hspace{1cm} (3.17)

where \( \tilde{q} = q - q_d \) is the joint trajectory tracking error for each motion-controlled joint, and \( \Lambda_{MC} \) is a diagonal positive definite matrix of controller gains for the two motion-controlled joints.

A desired impedance is selected for each of the impedance-controlled joints which follows the form of:

\[ I\ddot{q}_{IC} + b\dot{q}_{IC} + kq_{IC} = -T_{IC} \]  \hspace{1cm} (3.18)

where \( I, b, \) and \( k \) are the impedance parameters for inertia, damping, and stiffness, and \( T_{IC} \) is the effect of the external forces on the impedance-controlled joints. The subscript \( IC \) denotes the joints that contain a target impedance (the knee and an-
kle in this implementation). The desired impedance equation of Equation 3.18 is incorporated into a dynamic compensator with state $z$:

$$
\dot{z} = Az + K_p \ddot{q}_{IC} + K_d \dot{q}_{IC} + K_f T_{IC}
$$

(3.19)

where $A$ is Hurwitz and $K_p$, $K_d$, and $K_f$ are diagonal gain matrices to be determined.

The impedance-controlled joints follow the control law:

$$
\begin{align*}
\dot{r}_{IC} &= \dot{q}_{IC} - v_{IC} \\
v_{IC} &= \dot{q}_{IC}^d - \Lambda_{IC} \ddot{q}_{IC} - F_r z \\
a_{IC} &= \dot{v}_{IC}
\end{align*}
$$

(3.20)

(3.21)

(3.22)

where $F_r$ is a diagonal gain matrix to be determined.

If $r$ and $\dot{r}$ converge to zero, then Equation 3.20 yields:

$$
\begin{align*}
z &= -F_r^{-1}(\ddot{q}_{IC} + \Lambda_{IC} \ddot{q}_{IC}) \\
\dot{z} &= -F_r^{-1}(\dddot{q}_{IC} + \Lambda_{IC} \dot{\tilde{d}}e_{IC})
\end{align*}
$$

(3.23)

(3.24)

Equating the definition of $\dot{z}$ from Equation 3.24 with the definition of $\dot{z}$ from Equation 3.19 yields:

$$
-\dot{F}_r^{-1} \dddot{q}_{IC} + (F_r^{-1} \Lambda_{IC} - AF_r^{-1} + K_d) \ddot{q}_{IC} +
(AA_{IC} + K_p) \ddot{q}_{IC} = -K_f T_{IC}
$$

(3.25)

The gain matrices, $K_p$, $K_d$, and $F_r$ are determined from equating the desired impedances.
in the form of Equation 3.18 with Equation 3.25 as:

\[
\begin{align*}
F_r &= I^{-1} \\
K_p &= k + AL_{IC} \\
K_d &= b - IA_{IC} + AI \\
K_f &= I^{-1}
\end{align*}
\]

Since the desired impedances can be achieved with the assumption that \( r \) and \( \dot{r} \) converge to zero, the motion-controlled joints must provide trajectory convergence. The control law for the motion-controlled joints (Equation 3.15) is introduced into the robot dynamics of Equation 3.1 yielding:

\[
M\ddot{r} + Cr + Ma + Cv + g + R = \tau - T_{IC}
\]  

(3.30)

which can be written in the form of Equation 3.2 as:

\[
M\ddot{r} + Cr + Y\Theta = \tau - T_{IC}
\]  

(3.31)

A control law is selected of the form:

\[
\tau = Y\hat{\Theta} - Kr + T_{IC}
\]

(3.32)

where \( \hat{\Theta} \) is the estimate of the system parameters. If \( \hat{\Theta} \) is selected properly, then \( r \) can be shown to converge to zero. The estimate of the system parameters, \( \hat{\Theta} \), is defined as:

\[
\hat{\Theta} = \Theta_0 + \delta\Theta
\]

(3.33)

where \( \Theta_0 \) is the initial estimate of the system parameters and \( \delta\Theta \) is an adjustable
quantity selected through a Lyapunov function [19] to be:

\[
\delta \dot{\Theta} = \begin{cases} 
-\rho \frac{Y^T r}{||Y^T r||} & ||Y^T r|| \geq \varepsilon \\
\frac{\varepsilon}{\rho} Y^T r & ||Y^T r|| \leq \varepsilon
\end{cases}
\] (3.34)

which ensures that the system is uniformly ultimately bounded since a deadzone, \(\varepsilon\), is implemented. Here \(\rho\) is the bound of uncertainty in the system parameters.

The impedance parameters for the mixed trajectory / impedance tracking torque controller were selected in [19]. The parameters for the mixed trajectory / impedance tracking joints (the knee joint and the ankle joint) are found in Table V.

Table V: The target impedance values used with the mixed trajectory / impedance tracking torque controller. Values were obtained from [19].

<table>
<thead>
<tr>
<th>Impedance Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>target inertia of knee</td>
<td>(I_k)</td>
<td>0.9 kg-m(^2)</td>
</tr>
<tr>
<td>target damping of knee</td>
<td>(B_k)</td>
<td>1000 N-m-s</td>
</tr>
<tr>
<td>target stiffness of knee</td>
<td>(K_k)</td>
<td>3000 N-m</td>
</tr>
<tr>
<td>target inertia of ankle</td>
<td>(I_a)</td>
<td>0.9 kg-m(^2)</td>
</tr>
<tr>
<td>target damping of ankle</td>
<td>(B_a)</td>
<td>300 N-m-s</td>
</tr>
<tr>
<td>target stiffness of ankle</td>
<td>(K_a)</td>
<td>2000 N-m</td>
</tr>
</tbody>
</table>

The mixed trajectory/impedance robust passivity-based controller presented by [19] is used as a high-level torque controller with the artificial neural network (ANN) controller described in Section 4.1 to generate desired torques. The ANN is responsible for ensuring that the motor drives of Chapter II generates the torque determined by the mixed trajectory/impedance robust passivity-based controller. Due to the limitations of the ANN, as discussed Section 4.1, a robust voltage controller is introduced in Section 4.2, which expands on the mixed trajectory/impedance robust passivity-based controller by including information from the motor drives.
3.4 Discussion

Due to the properties of robotic systems, much of the modeling and control theory has been generalized for \( n \)-link systems. These controllers have been built under the assumption that an actuator is available for the joints. Following research in modeling and controlling robotic systems, it is desirable to model and control the motor drives actuate the robotic joints in a generalized fashion. As shown in Chapter II, the modeling of regenerative motor drives depends on the motor constant, which is a known quantity, and the joint velocity, which can be measured. The other motor drives require control signals to modulate the current through the joint motor as determined in Chapters III and IV. Further discussion of the generalization of controlling robotics with regenerative motor drives follows in Chapter IV. To demonstrate the ability of the motor drives, the motor drives are applied to the two robotic systems discussed in Chapter II. In addition to demonstrating the ability to be applied to different robotic systems, the PUMA560 robot allows for testing of the motor drives and control systems on a simple robot model before being tested on the prosthetic robot.
CHAPTER IV

REGENERATIVE MOTOR DRIVE
CONTROL AND OPTIMIZATION

To achieve joint trajectory tracking and joint impedance tracking, the high-level mechanical controller determines a desired torque for each joint. The desired torque is then converted to a desired voltage through input constants that capture motor torque constants, gear ratios, and amplifier gains. When ideal power supplies are used, the desired input voltage is connected across the motors for each respective joint. In practical implementation of certain robots, such as a prosthesis, ideal power supplies cannot be used, so motor drives (such as the ones described in Chapter II) are connected between an energy storage bank and the joint motors. The desired voltage determined by the high-level mechanical controller is commanded to the motor drive, which attempts to produce the desired voltage across the motor. Multiple methods of controlling power modulation between the supercapacitor and joint motor are studied in this chapter.

From previous studies [3], it was shown that controlling the VSC circuit with an artificial neural network did not provide realistic control signals. The goal of this chapter is to start with the VSC circuit and the neural network and to address the issue of high frequency content in the control signal as well as the issue of
energy regeneration. A robust voltage controller is implemented on the bidirectional buck/boost converter in order to address the high frequency content in the control signal. The bidirectional buck/boost converter with the robust voltage controller is then implemented in an energy regeneration framework to address energy regeneration.

Due to the construction of the voltage source converter (VSC), as described in Chapter II, the input constants that convert the desired torque to a desired voltage differ from the input constants for the bidirectional buck/boost converter since the VSC will operate with respect to motor current instead of motor voltage. To remain general to both the VSC and bidirectional buck/boost converter, the desired control signal will be discussed in terms of torque $\tau^d$, with the understanding that known constants will change the control signal to either a current (for the VSC) or a voltage (for the bidirectional buck/boost converter).

The remainder of this chapter will discuss the different controllers used with the motor drives on the robotic systems of Chapter III. The first controller is an artificial neural network, which has previous studies that can be used as a benchmark [2], [3]. The second controller is a robust voltage controller that was designed to reduce the high frequency content in the control signal as well as decrease the RMS error between the generated voltage and the desired voltage control signal. The final method is semiactive virtual control, which is a control framework used in applications of robotics with energy regeneration.

### 4.1 Artificial Neural Network

An artificial neural network (ANN) was implemented on a hydraulic prosthesis in [40]. Due to the success of that study, an ANN controller was evaluated with the VSC [3]. Although the control signal in that study was unrealistic, the controller provided desirable joint angle tracking and provided energy regeneration.
in simulation. The ANN in [3] is duplicated here with the bidirectional buck/boost converter to compare the performance of the bidirectional buck/boost converter with the performance of the VSC. The material in this section is based on [2] and [3].

**Voltage Source Converter**

An ANN is implemented to control the PWM duty cycle of the modulating switches in the VSC in order to provide the knee torque desired by the controller discussed in Chapter III. The error signal $e$ is defined as the difference between the reference knee torque $\tau^d$ and the output knee torque $\tau$. The error signal is used as one of the inputs to the ANN. To obtain a desired change in the PWM duty cycle, the output of the ANN is either added to or subtracted from the previous duty cycle. This operation depends on the directional operating condition of the prosthesis (Table I) and is determined later in this section.

The simulation uses the previous duty cycle by applying a single integration memory step to the generated duty cycle from the switch logic and uses the output of the ANN to modify the previous duty cycle. The power modulation ratio is then generated by saturating the PWM duty cycle between 0 and 1. The PWM signal controls the MOSFETs to switch between circuit configurations from Table II. A PWM signal of 0 connects the capacitor to the knee motor and a PWM signal of 1 disconnects the knee motor from the capacitor. A block diagram of this system is shown in Figure 18.
Figure 18: The ANN control system for the VSC that is used to track a reference joint torque $\tau^d$ determined by a high-level torque controller discussed in Section 3.3.

As previously mentioned, the desired change in PWM duty cycle is calculated by the ANN. The switch logic will either add or subtract the desired change in PWM duty cycle with the previous PWM duty cycle. The switch logic will determine whether increasing the PWM duty cycle will cause the knee torque to increase or decrease, based on the state equations from Table II for each directional operating mode. The switch logic will either decrease the PWM duty cycle (corresponding to the first row of Table II) or increase the PWM duty cycle (corresponding to the second row of Table II).

The switch logic is derived by considering each example circuit configuration. Consider an example when the VSC is operating in the positive direction and in the motoring mode (the first two rows of Table II) with the hypothetical torque tracking curve of Figure 19. Decreasing the PWM duty cycle causes a decreased current, and therefore a decreased knee torque through the motor and transmission. If the reference knee torque is larger than the output knee torque (right side of black line in Figure 19), then the error signal is positive. To reduce the error signal, the current through the knee motor must increase, so the PWM duty cycle must also increase. To increase the PWM duty cycle, the switch logic adds the positive error signal from
the previous duty cycle. If the reference knee torque is smaller than the output knee torque (left side of black line in Figure 19), then the error signal is negative. To reduce the magnitude of the error signal, the current through the motor must also decrease, so the PWM duty cycle must also decrease. To decrease the PWM duty cycle, the switch logic adds the negative error signal from the previous duty cycle.

Figure 19: A hypothetical torque tracking to demonstrate the VSC control system

**Artificial Neural Network**

The change in PWM signal is determined by the ANN. The proposed ANN of Figure 20 contains five input nodes, three hidden nodes, and a single output node. The five inputs to the ANN are: the torque error signal, the derivative of the torque error signal, and the three binary operating conditions. The three operating conditions are: the operating mode of the prosthesis (motoring or generating), the operating direction of the prosthesis (positive or negative), and the operating phase of the prosthesis (stance phase or swing phase). Each activation function (shown as circles in the hidden and output layers in Figure 20) is a sigmoid function which is scaled and vertically centered on the origin so that the minimum asymptote is -1 and the maximum asymptote is 1. The arrows connecting the different layers in Figure 20
represents a tunable weight.

The weights are selected with the biogeography-based optimization algorithm, which is described in Section 4.4.1. An error in the software for the ANN was present in [3]. This error caused the neural network weights between the hidden nodes (W matrix weights) and output nodes to be the same as the first four weights between the input nodes and the hidden nodes (V matrix weights). An improvement could be made by allowing the BBO algorithm to tune the W matrix weights independently of the V matrix weights. Since the BBO algorithm selects weights to minimize the knee angle tracking error and energy consumption, the drawbacks of the ANN as discussed in this section and in Section 5.1 would not have a large impact from this error; a large number of tuning parameters still require tuning. To maintain consistency with [3], the W matrix weights are selected to match the first four V matrix weights.

Similar studies have used other candidate controllers that require separate gains for each of the eight combinations of operating conditions [2]. The previously studied controllers contained over 100 gains and required a large amount of time to tune. This ANN contains only 22 gains to be tuned. Since the time it takes for the ANN to be tuned is lower with fewer parameters, its gains can be trained with multiple data sets to provide acceptable control results for multiple reference trajectories.
Bidirectional Buck/Boost Converter

The implementation of the ANN for the bidirectional buck/boost converter is similar to implementation for the VSC. Since the bidirectional buck/boost converter operates on voltages instead of currents, the switch logic is different. The bidirectional buck/boost converter contains an H bridge to direct the flow of current separately from the power modulation. Since the H bridge can force the voltage to be positive or negative depending on the directional operating condition, the desired change in PWM duty cycle calculated by the ANN is always added to the previous PWM duty cycle.

4.2 Robust Voltage Control

Since the ANN controller contained unrealistic control signals when applied to the VSC in [2] and [3], it is desirable to explore other control methods that can provide desirable tracking with a practical control signal. Since the robust passivity-based controller has already been shown to provide desirable joint angle
tracking [19], modifying the robust passivity-based controller to exploit information about the dynamics of the motor drives could provide desirable trajectory tracking and with reasonable control signals. The idea of adding motor drive information to a robot controller has been accomplished with adaptive controllers [15], which inspired the modification of the robust mixed impedance / trajectory tracking controller to use motor drive information [7]. Since the robot controller contains feedback information from the motor drive, better control can potentially be obtained. The material in this section is based on [7].

Adding motor drive information to the robotic controller of Section 3.3 starts with a dynamic equation that models motor dynamics:

$$L\ddot{I} + RI + K_b\dot{q} = u$$  \hspace{1cm} (4.1)$$

where $L$ is the inductance in series with the motor, $R$ is the resistance in series with the motor, $K_b$ is the back emf of the motor, $u$ is the control voltage, $\dot{q}$ is the velocity of the motor rotation, and $I$ is the current through the motor. The motor torque is calculated from the motor current through the equation

$$\tau = HI$$  \hspace{1cm} (4.2)$$

where $\tau$ is the motor torque and $H$ is an invertible electromechanical conversion matrix. The current error is defined as

$$e_I = I - I_d$$  \hspace{1cm} (4.3)$$

where $I_d$ is the desired current that will be determined from a current control law. The current error from Equation 4.3 is substituted into the motor dynamics of Equation 4.1
and rewritten as

\[ L(\dot{e}_I + \dot{I}_d) + RI + K_b \dot{q} = u \]  

(4.4)

An electrical regressor vector and electrical parameter matrix are selected as

\[ Y_e = \begin{bmatrix} I \\ \dot{q} \end{bmatrix}, \quad \theta_e = \begin{bmatrix} R & K_b \end{bmatrix} \]  

(4.5)

The electrical regressor vector and electrical parameter matrix are substituted into the electrical dynamics of Equation 4.4, yielding

\[ L\dot{e}_I = -\Theta_e Y_e - L\dot{I}_d + u \]  

(4.6)

The torque relationship from Equation 4.2 is substituted into the dynamics of Equation 3.30 to yield

\[ M\ddot{r} + Cr = -Y_m \Theta_m + H(e_I + I_d) - T_e \]  

(4.7)

A control law that provides stability is selected as follows [7]:

\[ I_d = H^{-1}[Y_m \hat{\Theta}_m + T_e - K_i r] \]  

(4.8)

where \( K_i \) is a diagonal gain matrix and \( \hat{\Theta}_m \) is the vector containing the estimates of the mechanical parameters. Substituting Equation 4.8 into the dynamics from Equation 4.7 gives

\[ M\ddot{r} + Cr = Y_m[\hat{\theta}_m - \Theta_m] + H e_I - k_i r \]  

(4.9)

The vector containing the estimates of the mechanical parameters \( \hat{\Theta}_m \) is perturbed
from its nominal values $\Theta_{m0}$ by $\delta \Theta_m$ determined by the switching law

$$\delta \hat{\Theta}_m = \begin{cases} 
-\rho_m \frac{Y_r^T}{||Y_r^T||} & ||Y_r^T|| \geq \varepsilon_m \\
\frac{-\rho_m}{\varepsilon_m} Y_r^T & ||Y_r^T|| \leq \varepsilon_m 
\end{cases} \quad (4.10)$$

where $\rho_m$ is the bound of the mechanical parameter estimation error and $\varepsilon_m$ is a deadzone parameter. This switching law provides stability for the reference trajectory tracking [7].

In addition to the reference trajectory tracking provided by the switching law of Equation 4.10, a current control law is designed to ensure that $I \rightarrow I_d$. The control law is

$$u = \hat{\Theta}_e Y_e - \rho_e \lambda_{max}(L)e_I - Hr \quad (4.11)$$

where $\lambda_{max}(L)$ is the maximum eigenvalue of the motor inductance matrix $L$ and $\hat{\Theta}_e$ is a vector containing the estimates of the electrical parameters. Substituting Equation 4.11 into the dynamics of Equation 4.9 yields

$$L\dot{e}_I = [-\rho_e \lambda_{max}(L)e_I - \dot{L}I_d] + \hat{\theta}_e - \Theta_e Y_e - Hr \quad (4.12)$$

Similar to the reference trajectory tracking, a switching law is selected as

$$\delta \hat{\Theta}_e = \begin{cases} 
-\rho_e \frac{e_I Y_e^T}{||e_I Y_e^T||} & ||e_I Y_e^T|| \geq \varepsilon_e \\
\frac{-\rho_e}{\varepsilon_e} e_I Y_e^T & ||e_I Y_e^T|| \leq \varepsilon_e 
\end{cases} \quad (4.13)$$

where $\rho_e$ is the bound of the electrical parameter estimation error and $\varepsilon_e$ is a deadzone parameter. The switching law of Equation 4.13 provides stability for motor current tracking [7].
4.3 Semiactive Virtual Control

Although the robust passivity-based controller with the robust voltage control law addresses the issue of oscillation in the motor drive control signal, the robust controller does not address energy regeneration. The controller for the motor drives should fit into a generalized framework for energy regeneration in robotics. Such a framework has been developed in [28]. This framework, known as semiactive virtual control, has successfully been applied to both the PUMA560 robot and the prosthesis robot with an underlying assumption of an ideal motor drive [17], [26], [28], [31]. Unlike the motor bidirectional buck/boost drive discussed in Chapter II, the ideal motor drives do not allow for power sources or energy storage banks to have a voltage lower than that demanded by a mechanical torque controller.

By itself, semiactive virtual control does not provide the capability for trajectory tracking. Semiactive virtual control requires a virtual controller for the mechanical dynamics of Equation 3.1 as discussed in Section 3.3. After a controller is designed to ensure that the joint trajectories track reference data, virtual control signal matching is achieved for joints that include energy regeneration.

Virtual control signal matching provides semiactive robotic joints (such as the one depicted in Figure 21) a means of controlling the joint indirectly. Since the control signal, \( u \), can not be directly controlled with motor drives such as those discussed in Chapter II, the control signal is a function of a modulation parameter, \( r \). The control signal follows the form

\[
u = \frac{\alpha r}{C R_a} Y \tag{4.14}
\]

where \( \alpha \) is the motor constant, \( C \) is the capacitance of the supercapacitor, \( R_a \) is the series resistance of the motor, and \( Y \) is the charge of the supercapacitor. The goal of virtual control matching is to make the desired control signal determined in
Section 3.3 fit into Equation 4.14 such that the control signal \( u \) becomes equivalent to the desired torque \( \tau^d \).

\[
C_{rr} R_a + \varepsilon - + V_m - + V_C -
\]

Figure 21: A supercapacitor connected to a semiactive robotic joint through a regenerative motor drive

The modulation that should be provided is given by solving Equation 4.14 for \( r \):

\[
r = \frac{\tau^d R_a}{\alpha V_C}
\]  

(4.15)

This modulation is then used to determine the duty cycle of the modulation switches that are described in Chapter II. Since modulation is defined in Figure 21 to be the ratio of the motor voltage, \( V_m \), to the supercapacitor voltage, \( V_C \), the modulation parameter, \( r \), is not always equivalent to the duty cycle, \( D \). In addition to the direction of power flow, the different operating configurations of the motor drives cause differences between the modulation parameter and duty cycle. Table VI summarizes the relationship between the modulation parameter and duty cycle for all four combinations of circuit configurations for the bidirectional buck/boost converter as determined by substituting the direction of power flow into the buck Equation 2.10 and the boost Equation 2.9.
Table VI: Relationships between the modulation parameter $r$ and the duty cycle $D$ for the different configurations of the bidirectional buck/boost converter

<table>
<thead>
<tr>
<th></th>
<th>Generator</th>
<th>Motor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buck</td>
<td>$D = \frac{1}{r}$</td>
<td>$D = r$</td>
</tr>
<tr>
<td>Boost</td>
<td>$D = 1 - r$</td>
<td>$D = 1 - \frac{1}{r}$</td>
</tr>
</tbody>
</table>

The semiactive virtual control framework has been tested with ideal converters that are unable to provide boosting, which requires that $|r| \leq 1$ [17]. By using a regenerative motor drive such as the bidirectional buck/boost converter, this requirement on $r$ can be relaxed. The described framework is implemented in this study with an ideal model of a DC motor that does not include inductance in Equation 4.14. To improve the implementation of semiactive virtual control, inductance and converter properties could be included. Semiactive virtual control does not take time delays of converter circuits into consideration. From previous studies with the motor drives proposed in Chapter II, the time delay in the system can cause the motor drive to not match the desired voltage if fast changes in the voltage signal are required [7]. The model which includes time delay could also be implemented to improve the performance. This study focuses on using semiactive virtual control in its idealized framework with the bidirectional buck/boost converter on the PUMA560 robot.

### 4.4 Optimization

The combined mechanical / electrical system contains many parameters that need to be optimized. All different combinations of robotic system, motor drive, and motor drive controllers can be optimized. The system configuration with the most tunable quantities (the ankle system with ANNs) contains more than 60 parameters.
The parameters should be selected to ensure that the joints track reference trajectories and to ensure that the system regenerates the largest amount of energy. Between the large number of system parameters and the multiple objectives that need to be minimized/maximized, manual tuning is not practical. An algorithmic method of selecting parameters is desired to reduce the amount of time required to tune the system to achieve high performance in multiple objectives.

4.4.1 Biogeography-Based Optimization

The various system configurations described in this study were optimized with biogeography-based optimization (BBO), an evolutionary algorithm based on the mathematical model of biogeography [35], [36]. The study of biogeography includes the migration and mutation of species. Migration and mutation is paralleled in BBO by viewing a candidate solution as an island and the system parameters as the individuals in the island. Each island contains a habitat suitability index (HSI), which quantifies the ability of the island to sustain species. A higher HSI corresponds to an island with many desirable species.

If an island has a large HSI, then many species live on the island and are able to migrate to other islands at a high rate $\mu$. Since the island contains many species, few additional species are able to migrate to the island due to lack of resources, so the rate of immigration $\lambda$ will be low. For this study, a linear relationship is assumed between the HSI of an island and its migration rates. Species will probabilistically decide whether to migrate to a new island. If selected to migrate, the individual will use a roulette wheel selection process to determine which island to migrate to. The roulette wheel selection process is normalized by the migration rates of each island.

When an individual is migrating to a new island, the possibility of mutation occurs with a low probability, which is set to 2%. If a migrating individual is selected to mutate, then the island that the individual migrated to might generate
new information about the parameter search space. Mutation helps the optimization routine by including a small amount of randomness to prevent islands from becoming stuck at a local minimum or maximum of the optimization problem. Mutation rates are typically set low enough so that BBO does not become a random selection of parameter values. After the mutation process, the optimization routine is repeated for a set number of generations until a desirable island (solution) is obtained that minimizes (or maximizes) a cost function.

To ensure that the optimization algorithm always provides equal, or improved, results after each generation, elitism is added. Elitism takes the best two individuals from generation $k - 1$ and uses them in generation $k$. In the worst case, the best island from generation $k$ will be equivalent to the best island from generation $k - 1$.

### 4.4.2 Multiple Objective Optimization

The BBO algorithm works for a single objective. In this study, energy regeneration is a secondary goal for multiple system configurations. Several methods exist for optimizing a cost function with multiple dimensions [36]. Ultimately, the non-dominated sorting algorithm (NSA) was selected here due to its processing requirements and ease of implementation [36].

The NSA works by taking an evaluated population and sorting the results into groups. The first group is not dominated by any other islands. After the first group is selected, its members are removed from the set of islands and a second group of non-dominated members is stored. The process repeats until no members remain in the population. After each island is assigned a group, BBO is used to share information between the islands. Members in the first groups will have a higher chance of sharing their characteristics than members of the later groups. During the sharing of information, the same mutation and elitism process is applied to the members in the
population. The process repeats for multiple generations until a desirable solution is obtained. Non-dominated sorting biogeography-based optimization (NSBBO) provides the capability to algorithmically determine the system parameters to achieve both trajectory tracking and energy regeneration.

The NSBBO algorithm requires the evaluation of a cost function. To quantify the joint trajectory tracking, the root mean square error (RMSE) of the tracking error is used:

$$\text{RMSE} = \sqrt{\frac{\sum (q - q^d)^2}{T}}$$  \hspace{1cm} (4.16)

where $T$ is the number of simulation steps, $q$ is the simulated joint trajectory, and $q^d$ is the desired reference trajectory. Energy regeneration is quantified by calculating the difference between the energy stored in the supercapacitor at the beginning of the simulation (from the initial charge of the supercapacitor) to the end of the simulation.

$$\Delta J = \frac{CV_f^2}{2} - \frac{CV_0^2}{2}$$  \hspace{1cm} (4.17)

where $C$ is the capacitance of the supercapacitor, $V_f$ is the voltage across the supercapacitor at the end of the simulation, $V_0$ is the initial voltage stored across the supercapacitor, and $\Delta J$ is the change in energy of the supercapacitor.

### 4.5 Discussion

Multiple methods to control the regenerative motor drive are available. Other methods also exist, such as active disturbance rejection control (ADRC). The methods listed in this chapter were selected based upon previous work on the VSC motor drive when applied to a prosthetic system that provides a baseline for continuing work on different controllers and on the bidirectional buck/boost converter [2], [3]. Previous studies have shown that the ANN provides desirable trajectory tracking with
undesirable control signals. Since robust voltage control uses information from the motor drives to determine desired torques and voltages, RVC appears to be a better option for controlling the motor drives.

In addition to trajectory tracking, this study is interested in energy regeneration capabilities of robotic joints. Since neither ANN, RVC, or other controllers such as ADRC provide this capability, a combination with SVC is necessary. The SVC framework has been applied previously with motor drives that are unable to boost the voltage of the supercapacitor. By applying SVC in addition to the robust passivity-based controller or RVC, the robotic joints should be able to regenerate energy while also providing joint trajectory tracking.
CHAPTER V

SIMULATION

The motor drives described in Chapter II can be implemented in the robotic systems of Chapter III when using the controllers described in Chapter IV. The reference trajectories for the prosthesis simulations were obtained from able-bodied human walking data that was obtained from the United States Department of Veterans Affairs (VA) [44]. The reference trajectories for the PUMA560 robot simulations were arbitrarily set to be sinusoidal signals. The input constants that convert the desired torques to voltages/currents were obtained from experimental testing of a hip testing / prosthesis robot and a PUMA560 robot. The main objective of this work is to evaluate the motor drives of Chapter II with different controllers. Five different system configurations are evaluated in this section. Trajectory tracking, control signals, and energy regeneration are discussed. For this chapter, a motor drive will be included at a single joint and the remaining joints will use ideal actuators.

5.1 Artificial Neural Network (ANN)

Since ANN results were available from previous studies with the VSC, the ANN is used as a benchmark for the bidirectional buck/boost converter [2], [3]. The ANN is applied only to the hip testing robot / prosthesis since the control signals
were not practical in the previous studies [2], [3]. The goal is to ensure that the bidirectional buck/boost converter can provide similar trajectory tracking to the VSC with the ANN.

The ANN system configurations were optimized with BBO. In addition to the parameters discussed in the previous chapters, additional resistance in series with the motor is included as an optimization parameter. The BBO algorithm used 25 generations with 24 members in each generation. The mutation rate was set to 2% and 2 elites were used to ensure that the cost function never increases. The cost function includes the root mean square error of the joint trajectory tracking and the difference between the energy stored in the supercapacitor at the end of the simulation and the energy stored in the supercapacitor at the beginning of the simulation. The sensitivity of the simulation output to changes in the optimized system parameters was not tested on the ANN system since the control signals were not desirable.

5.1.1 ANN for the prosthesis with the VSC

The hip testing robot / prosthesis was simulated with the VSC using the ANN. The system parameters can be found in Table VII and the ANN weights selected by BBO can be found in Table VIII (with the W matrix weights following the V matrix weights as described in Section 4.1). The system was optimized to start with an initial voltage of 89 V stored across the supercapacitor, which is high when considering the low-voltage rating of supercapacitors.
Table VII: The system parameters for the hip testing / prosthesis robot with the VSC and ANN

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Controller gain</td>
<td>$K_t$</td>
<td>diag(78 431 421 10) [-]</td>
</tr>
<tr>
<td>Controller gain</td>
<td>$\Lambda$</td>
<td>diag(432 358 320 10) [-]</td>
</tr>
<tr>
<td>Deadzone</td>
<td>$\varepsilon_m$</td>
<td>1 [-]</td>
</tr>
<tr>
<td>Mech. uncertainty bound</td>
<td>$\rho_m$</td>
<td>0.33 [-]</td>
</tr>
<tr>
<td>ANN Weights</td>
<td>$W$</td>
<td>see Table VIII [-]</td>
</tr>
<tr>
<td>$q_1$ torque/voltage gain</td>
<td>$k_1$</td>
<td>375 N/V</td>
</tr>
<tr>
<td>$q_2$ torque/voltage gain</td>
<td>$k_2$</td>
<td>15 Nm/V</td>
</tr>
<tr>
<td>$q_3$ torque/voltage gain</td>
<td>$k_3$</td>
<td>$n(q_3)\alpha$ Nm/A</td>
</tr>
<tr>
<td>$q_4$ torque/voltage gain</td>
<td>$k_4$</td>
<td>1 Nm/V</td>
</tr>
<tr>
<td>Series motor inductance</td>
<td>$L_{a2}$</td>
<td>505 µH</td>
</tr>
<tr>
<td>Storage bank capacitance</td>
<td>$C$</td>
<td>50 F</td>
</tr>
<tr>
<td>Total series motor resistance</td>
<td>$R_a+R_s$</td>
<td>0.10 Ω</td>
</tr>
<tr>
<td>Armature inductance</td>
<td>$L_a$</td>
<td>343 µH</td>
</tr>
</tbody>
</table>
Table VIII: The ANN weights for the hip testing / prosthesis robot with the VSC. See Figure 20 for matching weight locations

<table>
<thead>
<tr>
<th>Weight</th>
<th>Value</th>
<th>Weight</th>
<th>Value</th>
<th>Weight</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{1,1}$</td>
<td>3.17</td>
<td>$V_{1,2}$</td>
<td>8.50</td>
<td>$V_{1,3}$</td>
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<tr>
<td>$V_{2,1}$</td>
<td>1.00</td>
<td>$V_{2,2}$</td>
<td>1.00</td>
<td>$V_{2,3}$</td>
<td>1.00</td>
</tr>
<tr>
<td>$V_{3,1}$</td>
<td>-0.42</td>
<td>$V_{3,2}$</td>
<td>1.00</td>
<td>$V_{3,3}$</td>
<td>1.00</td>
</tr>
<tr>
<td>$V_{4,1}$</td>
<td>-2.22</td>
<td>$V_{4,2}$</td>
<td>-0.64</td>
<td>$V_{4,3}$</td>
<td>1.00</td>
</tr>
<tr>
<td>$V_{5,1}$</td>
<td>-1.68</td>
<td>$V_{5,2}$</td>
<td>-2.43</td>
<td>$V_{5,3}$</td>
<td>1.00</td>
</tr>
<tr>
<td>$V_{6,1}$</td>
<td>-9.75</td>
<td>$V_{6,2}$</td>
<td>1.00</td>
<td>$V_{6,3}$</td>
<td>8.22</td>
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<tr>
<td>$W_{1,1}$</td>
<td>3.17</td>
<td>$W_{2,1}$</td>
<td>1.00</td>
<td>$W_{3,1}$</td>
<td>1.00</td>
</tr>
<tr>
<td>$W_{4,1}$</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 22 shows the ability of the hip testing robot / prosthesis to track reference data. The hip and thigh joints are motion controlled while the knee and ankle joints are impedance controlled. The tracking of the hip and thigh joints is nearly perfect while the tracking of the knee and ankle joints demonstrates the tradeoff between motion tracking and impedance matching. The RMS tracking errors are: $2.32 \times 10^{-4}$ m for the hip displacement, $6.19 \times 10^{-4}$ rad for the thigh rotation, $2.1 \times 10^{-3}$ rad for the knee rotation, and 0.090 rad for the ankle rotation. The supercapacitor lost 68.38 J of energy during the simulation.
Figure 22: The trajectory tracking of all four prosthesis joints with the VSC and ANN

The control signals in Figure 23 are not practical. The hip, knee, and ankle all contain high frequency content as well as very large values. While the control signal is not a problem for simulation, actual implementation of this signal would not be acceptable. Since the VSC is unable to maintain the high frequency content in the desired knee torque determined by the robust passivity-based controller, the knee torque produced by the VSC only roughly follows the desired knee torque. The resulting RMS error between the torque produced by the VSC and the desired torque is 156 Nm. Since the robust passivity-based controller treats the VSC as a disturbance, the trajectory tracking in Figure 22 contains only a small RMS error. This result matches with the earlier study of [3].
Figure 23: The control signals for all four prosthesis joints with the VSC and ANN

The knee motor current and the combined (toe and heel) vertical ground reaction forces acting on the prosthesis are shown in Figure 24. The knee current is a scaled version of the knee torque, which contains high frequency content that would not be acceptable in a practical system. The peak ground reaction force is 536 Nm, which is 35% lower than the peak value of 820 N for a 78 kg able-bodied human during normal walking [49].
5.1.2 ANN for the prosthesis with the bidirectional buck/boost converter

To compare the bidirectional buck/boost converter with the VSC, the hip testing robot / prosthesis system was repeated, but using the bidirectional buck/boost converter instead of the VSC. The system parameters can be found in Table IX and the ANN weights selected by BBO can be found in Table X (with the W matrix weights following the V matrix weights as described in Section 4.1). The system was optimized to start with an initial voltage of 47 V stored across the supercapacitor, which is high when considering the low-voltage rating of supercapacitors.
Table IX: The system parameters for the hip testing / prosthesis robot with the bidirectional buck/boost converter and ANN

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Controller gain</td>
<td>$K_t$</td>
<td>diag(208 469 499 203) [-]</td>
</tr>
<tr>
<td>Controller gain</td>
<td>$\Lambda$</td>
<td>diag(400 395 263 190) [-]</td>
</tr>
<tr>
<td>Deadzone</td>
<td>$\varepsilon_m$</td>
<td>1 [-]</td>
</tr>
<tr>
<td>Mech. uncertainty bound</td>
<td>$\rho_m$</td>
<td>0.05 [-]</td>
</tr>
<tr>
<td>ANN Weights</td>
<td>$W$</td>
<td>see Table X [-]</td>
</tr>
<tr>
<td>$q_1$ torque/voltage gain</td>
<td>$k_1$</td>
<td>375 N/V</td>
</tr>
<tr>
<td>$q_2$ torque/voltage gain</td>
<td>$k_2$</td>
<td>15 Nm/V</td>
</tr>
<tr>
<td>$q_3$ torque/voltage gain</td>
<td>$k_3$</td>
<td>$n(q_3)\alpha$ Nm/A</td>
</tr>
<tr>
<td>$q_4$ torque/voltage gain</td>
<td>$k_4$</td>
<td>1 Nm/V</td>
</tr>
<tr>
<td>Converter inductance</td>
<td>$L_{CV}$</td>
<td>38 mH</td>
</tr>
<tr>
<td>Storage bank capacitance</td>
<td>$C$</td>
<td>44 F</td>
</tr>
<tr>
<td>Parallel motor capacitance</td>
<td>$C_m$</td>
<td>690 $\mu$F</td>
</tr>
<tr>
<td>Total series motor resistance</td>
<td>$R_a + R_s$</td>
<td>97.44 $\Omega$</td>
</tr>
<tr>
<td>Armature inductance</td>
<td>$L_a$</td>
<td>343 $\mu$H</td>
</tr>
</tbody>
</table>
Table X: The ANN weights for the hip testing / prosthesis robot with the bidirectional buck/boost converter. See Figure 20 for matching weight locations

<table>
<thead>
<tr>
<th>Weight</th>
<th>Value</th>
<th>Weight</th>
<th>Value</th>
<th>Weight</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{1,1}$</td>
<td>-0.87</td>
<td>$V_{1,2}$</td>
<td>5.85</td>
<td>$V_{1,3}$</td>
<td>4.85</td>
</tr>
<tr>
<td>$V_{2,1}$</td>
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<td>$V_{2,2}$</td>
<td>2.78</td>
<td>$V_{2,3}$</td>
<td>-1.37</td>
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<tr>
<td>$V_{3,1}$</td>
<td>4.26</td>
<td>$V_{3,2}$</td>
<td>3.56</td>
<td>$V_{3,3}$</td>
<td>5.50</td>
</tr>
<tr>
<td>$V_{4,1}$</td>
<td>-3.18</td>
<td>$V_{4,2}$</td>
<td>-8.04</td>
<td>$V_{4,3}$</td>
<td>1.49</td>
</tr>
<tr>
<td>$V_{5,1}$</td>
<td>5.44</td>
<td>$V_{5,2}$</td>
<td>-2.57</td>
<td>$V_{5,3}$</td>
<td>-2.73</td>
</tr>
<tr>
<td>$V_{6,1}$</td>
<td>6.57</td>
<td>$V_{6,2}$</td>
<td>9.19</td>
<td>$V_{6,3}$</td>
<td>-4.41</td>
</tr>
<tr>
<td>$W_{1,1}$</td>
<td>-0.87</td>
<td>$W_{2,1}$</td>
<td>5.85</td>
<td>$W_{3,1}$</td>
<td>4.85</td>
</tr>
<tr>
<td>$W_{4,1}$</td>
<td>-7.38</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 25 shows the ability of the hip testing robot / prosthesis to track reference data. The tracking of the hip and thigh joints is nearly perfect with the tracking of the knee and ankle joints demonstrates the tradeoff between motion tracking and impedance matching. The results for this system are not as desirable as the VSC, however; the supercapacitor lost 9.20 J of energy during the simulation, which is 59.18 J less energy than the VSC simulation to track the joints. The RMS tracking errors are: $4.47 \times 10^{-4}$ m for the hip displacement, $4.3 \times 10^{-3}$ rad for the thigh rotation, 0.03 rad for the knee rotation, and 0.14 rad for the ankle rotation. The supercapacitor lost 9.20 J of energy during the simulation.
Figure 25: The trajectory tracking of all four prosthesis joints with the bidirectional buck/boost converter and ANN

The control signals in Figure 26 are also not practical. High frequency content and large signals are still an issue. Outside of the large spikes around 0.6 seconds when the prosthesis transitions from stance phase to swing phase, the frequency content of the control signals is much lower than the system with the VSC. For the knee joint, the magnitude of the control signal is one order of magnitude smaller than the system with the VSC. Since the neural network was not optimized with respect to frequency content, modifying the gains of the controller could potentially improve the transition from stance to swing; however, due to previous studies and the results presented in this section, the ANN was no longer evaluated. The major disadvantages of the ANN include many gains to tune and lack of system information provided to the controller.
Similar to the VSC, the ground reaction force peaks at 557 N, which is less than a 10 N difference from the VSC and can be seen in Figure 27. The knee current, a scaled version of the knee torque, also contains high frequency content, but with much improvement from the knee current of the VSC.
Figure 27: The knee motor current and ground reaction force on the prosthesis due to contact with the treadmill with the bidirectional buck/boost converter and ANN

5.2 Robust Voltage Control (RVC)

Since the bidirectional buck/boost converter was able to provide desirable trajectory tracking with the ANN, further controller development was tested with the bidirectional buck/boost converter. Robust voltage control was developed to improve the control signal in the motor drive to the type of a signal that could be used for experiments. The goal is to ensure that the bidirectional buck/boost converter can provide similar trajectory tracking to the ANN while also achieving a practical voltage signal.

Since the RVC requires fewer parameters than the ANN, the control parameters were manually tuned to achieve desirable trajectory tracking while also maintaining a practical control signal. The system parameters were obtained from the optimized ANN values, with the exception of the additional series resistance, which
was decreased since additional series resistance provides for a larger amount of energy loss. The sensitivity of the simulation output to changes in the parameters was tested for the hip testing robot / prosthesis system since ground contact makes the hip testing robot / prosthesis system more complicated than the PUMA560 robot.

5.2.1 RVC for the PUMA560

The PUMA560 robot was simulated with the bidirectional buck/boost converter with the robust voltage controller (RVC). The system parameters can be found in Table XI. The system was simulated with an initial voltage of 0.91 V stored across the supercapacitor, which satisfies the typical low-voltage rating of supercapacitors and allows for verification of both the boost mode and the buck mode since the control signal will both be above and below 0.91 V during the simulation of the PUMA560 robot.
Table XI: The system parameters for the PUMA560 robot with the bidirectional buck/boost converter and RVC

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Controller gain</td>
<td>$K_t$</td>
<td>diag(5 4 6) [-]</td>
</tr>
<tr>
<td>Controller gain</td>
<td>$\Lambda$</td>
<td>diag(2 2 5) [-]</td>
</tr>
<tr>
<td>Deadzone</td>
<td>$\varepsilon_m, \varepsilon_e$</td>
<td>1 [-]</td>
</tr>
<tr>
<td>Mech. uncertainty bound</td>
<td>$\rho_m$</td>
<td>0.1 [-]</td>
</tr>
<tr>
<td>Elec. uncertainty bound</td>
<td>$\rho_e$</td>
<td>0.1 [-]</td>
</tr>
<tr>
<td>Uncertainty bound of $\dot{I_d}$</td>
<td>$\phi$</td>
<td>5 [-]</td>
</tr>
<tr>
<td>$q_1$ torque/voltage gain</td>
<td>$k_1$</td>
<td>18.4 Nm/V</td>
</tr>
<tr>
<td>$q_2$ torque/voltage gain</td>
<td>$k_2$</td>
<td>12.4 Nm/V</td>
</tr>
<tr>
<td>$q_3$ torque/voltage gain</td>
<td>$k_3$</td>
<td>9.3 Nm/V</td>
</tr>
<tr>
<td>Converter inductance</td>
<td>$L_{CV}$</td>
<td>38 mH</td>
</tr>
<tr>
<td>Storage bank capacitance</td>
<td>$C$</td>
<td>44 F</td>
</tr>
<tr>
<td>Parallel motor capacitance</td>
<td>$C_m$</td>
<td>690 $\mu$F</td>
</tr>
<tr>
<td>Total series motor resistance</td>
<td>$R_a + R_s$</td>
<td>15 $\Omega$</td>
</tr>
<tr>
<td>Armature inductance</td>
<td>$L_a$</td>
<td>343 $\mu$H</td>
</tr>
</tbody>
</table>

Figure 28 demonstrates the PUMA560 robot tracking reference data. Unlike the prosthesis, all joints are motion controlled since the robot does not interact with the environment. Since all three joints are motion controlled, the three joints have a similar ability to track the reference trajectories. The RMS errors are: 0.1327 rad for the first joint, 0.0723 rad for the second joint, and 0.0118 rad for the third joint.
Figure 28: The trajectory tracking of all three PUMA joints with the bidirectional buck/boost converter and RVC.

The control signals in Figure 29 do not exceed the practical limits of motors and power supplies for the PUMA560 robot. All of the joints contain smooth control signals with no high frequency content. The voltage generated by the bidirectional buck/boost converter follows the voltage desired by the robust voltage controller for the knee joint with an RMS error of 0.5 V. Outside of the initial transient, the difference between the voltage delivered by the converter circuit and the voltage desired by the robust controller is due to a switching between converter modes. A small amount of hysteresis was introduced to reduce the effect of the mode switching.
Figure 29: The control signals for all three PUMA joints with the bidirectional buck/boost converter and RVC

The motor current in Figure 30 does not exceed 0.163 A, which is within range of the PUMA560 robot. Some high frequency content is apparent in the signal, however; due to the magnitude of the current spikes, this would be unlikely to cause issues in implementation.
Figure 30: The motor current through the third joint on the PUMA

The direction control signal for the bidirectional buck/boost converter shown at the top of Figure 31 matches the desired motor voltage of the third joint in Figure 29. That is, when the desired voltage is positive, the direction control signal is 1, and the H bridge connects the motor to the voltage generated by the converter in the positive direction (closes switches \( S_1 \) and \( S_4 \)). When the desired voltage is negative, the direction control signal is 0 and the H bridge connects the motor to the voltage generated by the converter in the negative direction (closes switches \( S_2 \) and \( S_3 \)); see Figure 10.

The buck/boost control signal at the bottom of Figure 31 follows the desired motor voltage for the third joint in Figure 29. That is, when the magnitude of the desired voltage exceeds the voltage in the supercapacitor (initially 0.91 V and slightly decreasing with time) the converter operates in the boost mode (the buck/boost control signal is 1). When the magnitude of desired voltage is less than the voltage
in the supercapacitor, the converter operates in buck mode (the buck/boost control signal is 0).

The control signals in Figure 31 switch relatively quickly when operating close to the transition between states. The fast switching causes the generated voltage to vary from the desired voltage as well as reducing the trajectory tracking of the joint. Hysteresis is implemented to reduce the effect of switching. Additional studies may further reduce the effect of switching when operating near the transition of the converter states.

![Direction Control](image1)

![Buck/Boost Control](image2)

Figure 31: The direction and buck/boost control signals of the bidirectional buck/boost converter with RVC when implemented on the PUMA robot

### 5.2.2 RVC for the Prosthesis

The hip test robot / prosthesis system was simulated with the bidirectional buck/boost converter with the robust voltage controller (RVC). The system parameters can be found in Table XII. The system was simulated with an initial voltage of
11 V stored across the supercapacitor, which is less than half of the voltage rating of a typical 24 V motor. The low voltage also satisfies the typical low-voltage rating of supercapacitors. The results in this section are based on [7].

Table XII: The system parameters for the hip testing / prosthesis robot with the bidirectional buck/boost converter and RVC

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Controller gain</td>
<td>$K_t$</td>
<td>diag(250 500 10 5) [-]</td>
</tr>
<tr>
<td>Controller gain</td>
<td>$\Lambda$</td>
<td>diag(250 500 1 50) [-]</td>
</tr>
<tr>
<td>Deadzone</td>
<td>$\varepsilon_m, \varepsilon_e$</td>
<td>1 [-]</td>
</tr>
<tr>
<td>Mech. uncertainty bound</td>
<td>$\rho_m$</td>
<td>0.1 [-]</td>
</tr>
<tr>
<td>Elec. uncertainty bound</td>
<td>$\rho_e$</td>
<td>0.1 [-]</td>
</tr>
<tr>
<td>Uncertainty bound of $\dot{I}_d$</td>
<td>$\phi$</td>
<td>5 [-]</td>
</tr>
<tr>
<td>$q_1$ torque/voltage gain</td>
<td>$k_1$</td>
<td>375 N/V</td>
</tr>
<tr>
<td>$q_2$ torque/voltage gain</td>
<td>$k_2$</td>
<td>15 Nm/V</td>
</tr>
<tr>
<td>$q_3$ torque/voltage gain</td>
<td>$k_3$</td>
<td>$n(q_3)\frac{a}{R}$ Nm/V</td>
</tr>
<tr>
<td>$q_4$ torque/voltage gain</td>
<td>$k_4$</td>
<td>5 Nm/V</td>
</tr>
<tr>
<td>Converter inductance</td>
<td>$L_{CV}$</td>
<td>100 mH</td>
</tr>
<tr>
<td>Storage bank capacitance</td>
<td>$C$</td>
<td>44 F</td>
</tr>
<tr>
<td>Parallel motor capacitance</td>
<td>$C_m$</td>
<td>700 $\mu$F</td>
</tr>
<tr>
<td>Total series motor resistance</td>
<td>$R_a+R_s$</td>
<td>20 $\Omega$</td>
</tr>
<tr>
<td>Armature inductance</td>
<td>$L_a$</td>
<td>343 $\mu$H</td>
</tr>
</tbody>
</table>

Figure 32 demonstrates the hip testing / prosthesis robot tracking reference
data. The hip and thigh joints are motion controlled while the knee ankle ankle joints are impedance controlled. The tracking of the hip and thigh joints is nearly perfect while the tracking of the knee and ankle joints demonstrates the tradeoff between motion tracking and impedance matching. The RMS tracking errors are: 0.0002 m for the hip displacement, 0.0003 rad for the thigh joint, 0.005 rad for the knee joint, and 0.073 rad for the ankle joint.

Figure 32: The trajectory tracking of all four prosthesis joints with the bidirectional buck/boost converter and RVC

The control signals in Figure 33 do not exceed the practical limits of motors and power supplies for the hip testing / prosthesis robot. The voltage generated by the electric power converter follows the voltage desired by the robust controller for the knee joint with an RMS error of 3 V. The observed difference in the desired voltage for the knee joint and the voltage signal delivered by the converter circuit is due to the time delay of the converter circuit, which is not an issue with the directly controlled hip, thigh, and ankle joints.
Figure 33: The control signals for all four joints with the bidirectional buck/boost converter and RVC when implemented on the prosthesis robot

The knee motor current and the combined (toe and heel) vertical ground reaction forces acting on the prosthesis are shown in Figure 34. The maximum knee motor current is 1.07 A, which is a reasonable maximum for the prosthesis motor. The peak ground reaction force is 627 N, which is 23.5% lower than the peak value of 820 N for a 78 kg able-bodied human during normal walking [49].
The direction control signal for the bidirectional buck/boost converter shown in the top graph of Figure 35 matches the desired knee motor voltage of Figure 33. That is, when the desired voltage is positive, the direction control signal is 1, and the H bridge connects the motor to the voltage generated by the converter in the positive direction (closes switches $S_1$ and $S_4$). When the desired voltage is negative, the direction control signal is 0 and the H bridge connects the motor to the voltage generated by the converter in the negative direction (closes switches $S_2$ and $S_3$); see Figure 10.

The buck/boost control signal in the bottom graph of Figure 35 follows the desired knee motor voltage of Figure 33. That is, when the magnitude of the desired voltage exceeds the voltage in the supercapacitor (initially 11 V and slightly decreases with time) the converter operates in the boost mode (the buck/boost control signal is 1). When the magnitude of the desired voltage is less than the voltage in the supercapacitor, the converter operates in the buck mode (the buck/boost control signal is 0).
5.2.3 Parameter perturbation with RVC

The nominal physical parameters in Table XII were perturbed by ±10% while leaving the control parameters as defined in Table XII. Figure 36 shows that the controller was able to maintain joint trajectory tracking with the perturbed parameters, similar to the trajectory tracking of the unperturbed system in Figure 32. The control signals in Figure 37 remain within practical ranges when the system parameters are perturbed, similar to the control signals in the unperturbed system in Figure 33. The ground reaction forces and knee motor currents in Figure 38 remained within the same order of magnitude as the system with the nominal parameters as in Figure 34. The converter control signals in Figure 35 are also similar to the converter control signals in Figure 35. The simulation results from the perturbed systems are compared to the simulation results of the unperturbed system in Table XIII.
Table XIII: The results for RVC prosthesis simulation using the bidirectional buck/boost converter with the perturbed and unperturbed parameters

<table>
<thead>
<tr>
<th></th>
<th>Unperturbed</th>
<th>Perturbed −10%</th>
<th>Perturbed +10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_1$ RMSE m</td>
<td>$2.27 \times 10^{-4}$</td>
<td>$4.44 \times 10^{-4}$</td>
<td>$8.95 \times 10^{-4}$</td>
</tr>
<tr>
<td>$q_2$ RMSE rad</td>
<td>$2.84 \times 10^{-4}$</td>
<td>$3.12 \times 10^{-4}$</td>
<td>$2.73 \times 10^{-4}$</td>
</tr>
<tr>
<td>$q_3$ RMSE rad</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>$q_4$ RMSE rad</td>
<td>0.07</td>
<td>0.07</td>
<td>0.07</td>
</tr>
<tr>
<td>$u_1$ max V</td>
<td>4.72</td>
<td>4.52</td>
<td>4.92</td>
</tr>
<tr>
<td>$u_2$ max V</td>
<td>10.30</td>
<td>8.69</td>
<td>12.49</td>
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<tr>
<td>$u_3$ max V</td>
<td>24.30</td>
<td>25.27</td>
<td>23.27</td>
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<tr>
<td>$u_4$ max V</td>
<td>1.62</td>
<td>1.63</td>
<td>1.44</td>
</tr>
<tr>
<td>$I_m$ max A</td>
<td>1.20</td>
<td>1.39</td>
<td>1.05</td>
</tr>
</tbody>
</table>

Figure 36: The trajectory tracking of all four prosthesis joints with the bidirectional buck/boost converter and RVC with perturbed parameters
Figure 37: The control signals for all four joints with the bidirectional buck/boost converter and RVC when implemented on the prosthesis robot with perturbed parameters.

Figure 38: The knee motor current and ground reaction force on the prosthesis due to contact with the treadmill with the bidirectional buck/boost converter and RVC with perturbed parameters.
Figure 39: The direction and buck/boost control signals of the bidirectional buck/boost converter with RVC when implemented on the prosthesis robot with perturbed parameters

5.3 Semiactive Virtual Control (SVC)

Since RVC was able to maintain trajectory tracking and was able to generate a practical control signal, energy regeneration was considered with the bidirectional buck/boost converter to decrease the power required to operate robotic systems. The bidirectional buck/boost converter was implemented with the semiactive virtual control framework to achieve energy regeneration. The goal is to maintain the same trajectory tracking, while also regenerating some of the energy in the robotic system.

Since trajectory tracking and energy regeneration are of concern with the SVC implementation, BBO is used to optimize the system and control parameters. The BBO algorithm used 25 generations with 24 members in each generation. The mutation rate was set to 2% and 2 elites were used to ensure that the cost function never increases. The cost function includes the root mean square error of the joint trajectory tracking and the difference between the energy stored in the supercapacitor.
at the end of the simulation and the energy stored in the supercapacitor at the beginning of the simulation. The sensitivity of the simulation output to changes in the optimized system parameters was not tested on the SVC and is left as future work. Since the cost function did not penalize unrealistic control signals, the parameters were selected from the non-dominated population with the most reasonable control signals and a balance between trajectory tracking and energy regeneration.

The PUMA robot was simulated with the bidirectional buck/boost converter using robust voltage control inside the semiactive virtual control framework to obtain energy regeneration. The system parameters can be found in Table XIV. The system was simulated with an initial voltage of 1 V stored across the supercapacitor, which allows times when the supercapacitor voltage is lower than the desired voltage and times when the supercapacitor voltage is higher than the desired voltage. The low voltage also satisfies the typical low-voltage rating of supercapacitors.
Table XIV: The system parameters for the PUMA560 robot with the bidirectional buck/boost converter, RVC, and SVC

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Controller gain</td>
<td>$K_t$</td>
<td>diag(5 4 6) [-]</td>
</tr>
<tr>
<td>Controller gain</td>
<td>$\Lambda$</td>
<td>diag(10 10 5) [-]</td>
</tr>
<tr>
<td>Deadzone</td>
<td>$\varepsilon_m, \varepsilon_e$</td>
<td>1 [-]</td>
</tr>
<tr>
<td>Mech. uncertainty bound</td>
<td>$\rho_m$</td>
<td>0.1 [-]</td>
</tr>
<tr>
<td>Elec. uncertainty bound</td>
<td>$\rho_e$</td>
<td>0.1 [-]</td>
</tr>
<tr>
<td>Uncertainty bound of $\dot{I}_d$</td>
<td>$\phi$</td>
<td>5 [-]</td>
</tr>
<tr>
<td>$q_1$ torque/voltage gain</td>
<td>$k_1$</td>
<td>18.4 Nm/V</td>
</tr>
<tr>
<td>$q_2$ torque/voltage gain</td>
<td>$k_2$</td>
<td>12.4 Nm/V</td>
</tr>
<tr>
<td>$q_3$ torque/voltage gain</td>
<td>$k_3$</td>
<td>9.3 Nm/V</td>
</tr>
<tr>
<td>Converter inductance</td>
<td>$L_{CV}$</td>
<td>5 mH</td>
</tr>
<tr>
<td>Storage bank capacitance</td>
<td>$C$</td>
<td>50 F</td>
</tr>
<tr>
<td>Parallel motor capacitance</td>
<td>$C_m$</td>
<td>500 $\mu$F</td>
</tr>
<tr>
<td>Total series motor resistance</td>
<td>$R_a+R_s$</td>
<td>100 $\Omega$</td>
</tr>
<tr>
<td>Armature inductance</td>
<td>$L_a$</td>
<td>343 $\mu$H</td>
</tr>
</tbody>
</table>

Figure 40 demonstrates the PUMA560 robot tracking reference data. All three joints are motion controlled with RVC deciding a desired torque and SVC matching the torque to determine when the bidirectional buck/boost converter should regenerate energy from the third joint, or supply energy to the third joint. After some initial transients, the first two joints track the reference data. The third joint with the converter and SVC track the reference trajectory. The RMS tracking errors are:
0.094 rad for the first joint, 0.12 rad for the second joint, and 0.11 rad for the third joint.

Figure 40: The trajectory tracking of all three PUMA joints with the bidirectional buck/boost converter and SVC

The control signals in Figure 41 do not exceed the practical limits of motors and power supplies for the PUMA560 robot. The voltage generated by the electric power converter does not follow the voltage desired by the robust controller as well as the RVC without SVC. The RMS error value of the converter output with the desired voltage is 5.63 V. The difference between the converter output and the desired output is likely due to the large amount of mode switching in Figure 42. Since any change of operating condition, such as the converter mode, causes a jump in the system, changes between operating modes should be kept at a minimum, or should occur when the voltage signals are close to zero. Similar to the RVC PUMA560 robot simulation
without SVC, hysteresis was included to reduce the effects of the switching; however, future work should find a solution to reduce the amount of switching.

Figure 41: The control signals for all three PUMA joints with the bidirectional buck/boost converter and SVC
The motor current through the third joint is shown in Figure 43. The maximum knee motor current is 0.76 A, which is a reasonable maximum for the PUMA560 robot motor. Much like the RVC PUMA560 robot simulation without SVC, high frequency content is present in the current signal, which is undesirable. The high frequency content in the current signal is due to the large amount of mode switching.
Figure 43: The motor current through the third joint on the PUMA560 robot when implemented with the bidirectional buck/boost converter and SVC

The energy stored in the supercapacitor in Figure 44 shows that energy regeneration is possible with the bidirectional buck/boost converter using RVC inside of the SVC framework. The periods of energy regeneration are periodic due to the periodic nature of the reference trajectory signal. Overall, the capacitor lost 8.43 J of energy during the 10 second simulation. In this study sinusoidal trajectories are arbitrarily selected as the desired trajectories. This simulation provides a first step in including a more realistic converter model into the SVC framework with a simple trajectory. Since energy regeneration is a function of trajectory, in addition to system parameters, a detailed analysis of available energy and regenerated energy should be conducted when the bidirectional buck/boost converter is simulated with the hip testing robot / prosthesis system since energy regeneration is a major topic in that area of research. Improvements to using the bidirectional buck/boost converter could be implemented in the SVC framework, such as adding the converter model information into the framework instead of using the ideal system model of Figure 21. Adding model information could potentially help with the excessive mode
switching seen in Figure 42, which would likely improve the control signal tracking, the trajectory tracking, and the motor current.

![Graph showing voltage over time]

Figure 44: The energy stored in the supercapacitor during the PUMA560 robot simulation with the bidirectional buck/boost converter and SVC

5.4 Discussion

This chapter showed the results of different circuits and circuit controllers when applied to two different robotic systems. As discovered in previous studies, ANNs were difficult to optimize and resulted in undesirable control signals. To remedy this issue, a new controller was proposed that uses information from the circuit to improve the control system. The RVC was able to provide a practical control signal to provide trajectory tracking and impedance control.

To improve upon robotic motor drives, this chapter also demonstrated the ability of motor drives to apply energy regeneration techniques. The bidirectional buck/boost converter provides more flexibility than the VSC due to its operation that naturally works with voltages instead of currents. The bidirectional buck/boost
converter also provides more flexibility than previous studies with SVC due to the ability of the bidirectional buck/boost converter to boost and buck voltages in two directions.
CHAPTER VI

CONCLUSIONS AND FUTURE WORK

Two motor drive designs were evaluated for use with robotic joints. The motor drives were applied to two different robotic systems: a PUMA560 robotic arm and a hip testing robot/prosthesis system. Previously studied circuit controllers were found to be impractical. To remedy this issue, a robust voltage controller was applied to the bidirectional buck/boost motor drive since this motor drive provided more flexibility than the VSC. The bidirectional buck/boost converter with the RVC provided trajectory tracking with practical control signals. The bidirectional buck/boost converter was applied with an energy regeneration framework to show that the bidirectional buck/boost converter can regenerate energy when used on robotic joints.

The RVC success is largely due to the access of knowledge about the circuit, which is not used by the ANN. The RVC also contains fewer parameters than the ANN, leading to an easier implementation in an experimental prosthesis or PUMA560 robot. The RVC can be included with an energy regeneration framework to potentially improve the performance of robotic systems.
Future Work

Many paths could be taken to expand on the work presented in this thesis and are proposed in this section. The first idea is to apply the bidirectional buck/boost converter to multiple joints while using RVC and SVC. Different topologies such as a star configuration or distributed configuration can be evaluated to improve on systems that contain multiple joints, such as the prosthesis. Energy regenerated at the knee joint could be used to power the ankle joint.

Other mechanical controllers can be evaluated for use with the bidirectional buck/boost converter. For example, ADRC could be used to determine the desired voltages for each joint. The robust voltage controller could then be combined with ADRC. By evaluating other controllers, a comparison of the controllers could find a mechanical control method that provides the best tradeoff between energy regeneration and trajectory tracking while also maintaining practical control signals for an evaluated robotic system. It could be determined whether one general controller suits multiple robotic systems, or if each robotic system uniquely performs best with a different controller.

Supercapacitors contain very small series resistance, which allows for rapid charging/discharging. The high rate of charge/discharge is useful when regenerating energy; however, the introduction of a battery in addition to a supercapacitor could improve energy regeneration. The supercapacitor could be used to quickly manage the flow of energy, but a battery bank could be used to extend the operating time of mobile robotics, such as a prosthesis.

The bidirectional buck/boost converter should be constructed so that the circuit configuration models can be verified. The bidirectional buck/boost converter can be applied to a PUMA560 robot or a hip testing robot / prosthesis system to obtain experimental data. The experimental data would allow for enhancements to
be made to the model as well as extending the study of energy regeneration.
BIBLIOGRAPHY


[38] Frank Sup, Huseyn Varol, Jason Mitchell, Thomas Withrow, and Michael Goldfarb. Self-contained powered knee and ankle prosthesis: Initial evaluation on a


APPENDICES
APPENDIX A

Voltage Source Converter Schematics

The voltage source converter (VSC) discussed in Chapter II is expanded in this section of the appendix. The equivalent circuit and the corresponding state equations for each of the four possible combinations of operating conditions (mode and direction) are included.
Positive motor configuration for VSC

\[ \dot{I}_m = \frac{\varepsilon - I_m (R_a + 2R_F) - V_C}{L_a} \]

\[ \dot{V}_c = \frac{I_m}{C} \tag{A.1} \]

Figure 45: The VSC motor drive circuit while operating in the positive motor mode (quadrant I) with the capacitor bank connected

\[ \dot{I}_m = \frac{\varepsilon - I_m (R_a + R_F)}{L_a} \]

\[ \dot{V}_c = 0 \]

\[ I_m \leq 0 \tag{A.2} \]

Figure 46: The VSC motor drive circuit while operating in the positive motor mode (quadrant I) with the capacitor bank disconnected
Negative motor configuration for VSC

Figure 47: The VSC motor drive circuit while operating in the negative motor mode (quadrant III) with the capacitor bank connected

\[
\dot{I}_m = \frac{\varepsilon - I_m (R_a + 2R_F) + V_C}{L_a}
\]

\[
\dot{V}_c = \frac{I_m}{C}
\]  \hspace{1cm} (A.3)

Figure 48: The VSC motor drive circuit while operating in the negative motor mode (quadrant III) with the capacitor bank disconnected

\[
\dot{I}_m = \frac{\varepsilon - I_m (R_a + R_F)}{L_a}
\]

\[
\dot{V}_c = 0
\]

\[
I_m \geq 0
\]  \hspace{1cm} (A.4)
Positive generator configuration for VSC

\[ \dot{I}_m = \frac{\varepsilon - I_m (R_a + R_F)}{L_a} - V_C \]

\[ \dot{V}_c = \frac{I_m}{C} \]  

\[ I_m \geq 0 \]  

\[ \dot{V}_c = 0 \]  

Figure 49: The VSC motor drive circuit while operating in the positive generator mode (quadrant II) with the capacitor bank connected

\[ \dot{I}_m = \frac{\varepsilon - I_m (R_a + 2R_F)}{L_a} \]

Figure 50: The VSC motor drive circuit while operating in the positive generator mode (quadrant II) with the capacitor bank disconnected
Negative generator configuration for VSC

\[ \dot{I}_m = \frac{\varepsilon - I_m(R_a + R_F) + V_C}{L_a} \]

\[ \dot{V}_c = \frac{I_m}{C} \]

(A.7)

\[ I_m \leq 0 \]

Figure 51: The VSC motor drive circuit while operating in the negative generator mode (quadrant IV) with the capacitor bank connected

\[ \dot{I}_m = \frac{\varepsilon - I_m(R_a + 2R_F)}{L_a} \]

\[ \dot{V}_c = 0 \]

(A.8)

Figure 52: The VSC motor drive circuit while operating in the negative generator mode (quadrant IV) with the capacitor bank disconnected
APPENDIX B

Bidirectional Buck/Boost Converter

Schematics

The bidirectional buck/boost converter discussed in Chapter II is expanded in this section of the appendix. The equivalent circuit and the corresponding state equations for each of the eight possible combinations of operating conditions (mode, direction, buck/boost) are included.
Positive motor boost configuration

Figure 53: The positive motor mode of the bidirectional buck/boost converter during a boosting operation

The following equations describe the circuit of Figure 53 when $S_d$ is closed:

\[
\begin{align*}
\dot{I}_{cv} &= \frac{V_{cm} - V_c}{L_{cv}} \\
\dot{I}_m &= \frac{\varepsilon - R_a I_m - V_{cm}}{L_a} \\
\dot{V}_c &= \frac{I_{cv}}{C} \\
\dot{V}_{cm} &= \frac{-(I_{cv} - I_m)}{C_m} \\
I_{cv} &\leq 0
\end{align*}
\]
The following equations describe the circuit of Figure 53 when $S_d$ is open:

\[
\begin{align*}
\dot{I}_{cv} &= -\frac{V_c}{L_{cv}} \\
\dot{I}_m &= \frac{\varepsilon - R_a I_m - V_{cm}}{L_a} \\
\dot{V}_c &= \frac{I_{cv}}{C} \\
\dot{V}_{cm} &= \frac{I_m}{C_m} \\
\end{align*}
\]  
(B.2)
Negative motor boost configuration

Figure 54: The negative motor mode of the bidirectional buck/boost converter during a boosting operation

The following equations describe the circuit of Figure 54 when $S_d$ is closed:

$$
\dot{I}_{cv} = \frac{V_{cm} - V_c}{L_{cv}}
$$

$$
\dot{I}_{m} = \frac{\varepsilon - R_a I_m + V_{cm}}{L_a}
$$

$$
\dot{V}_c = \frac{I_{cv}}{C}
$$

$$
\dot{V}_{cm} = \frac{-(I_{cv} + I_m)}{C_m}
$$

$$
I_{cv} \leq 0
$$
The following equations describe the circuit of Figure 54 when $S_d$ is open:

\[
\begin{align*}
\dot{I}_{cv} &= -\frac{V_c}{L_{cv}} \\
\dot{I}_m &= \frac{\varepsilon - R_a I_m + V_{cm}}{L_a} \\
\dot{V}_c &= \frac{I_{cv}}{C} \\
\dot{V}_{cm} &= -\frac{I_m}{C_m}
\end{align*}
\] (B.4)
Positive motor buck configuration

![Diagram of the bidirectional buck/boost converter]

Figure 55: The positive motor mode of the bidirectional buck/boost converter during a bucking operation

The following equations describe the circuit of Figure 55 when $S_a$ is closed:

\[
\begin{align*}
\dot{I}_{cv} &= \frac{V_{cm}}{L_{cv}} \\
\dot{I}_m &= \frac{\varepsilon - R_a I_m - V_{cm}}{L_a} \\
\dot{V}_c &= 0 \\
\dot{V}_{cm} &= \frac{-(I_{cv} - I_m)}{C_m} \\
I_{cv} &\leq 0
\end{align*}
\]
The following equations describe the circuit of Figure 55 when $S_a$ is open:

\[
\begin{align*}
\dot{I}_{cv} &= \frac{V_{cm} - V_c}{L_{cv}} \\
\dot{I}_m &= \frac{\varepsilon - R_a I_m - V_{cm}}{L_a} \\
\dot{V}_c &= \frac{I_{cv}}{C} \\
\dot{V}_{cm} &= -\frac{(I_{cv} - I_m)}{C_m} \\
I_{cv} &\leq 0
\end{align*}
\]
Negative motor buck configuration

![Diagram of negative motor buck bucking operation]

**Figure 56:** The negative motor mode of the bidirectional buck/boost converter during a bucking operation

The following equations describe the circuit of Figure 56 when $S_a$ is closed:

\[
\begin{align*}
\dot{I}_{cv} &= \frac{V_{cm}}{L_{cv}} \\
\dot{I}_m &= \frac{\varepsilon - R_a I_m + V_{cm}}{L_a} \\
\dot{V}_c &= 0 \\
V_{cm} &= -\frac{(I_{cv} + I_m)}{C_m} \\
I_{cv} &\leq 0
\end{align*}
\]
The following equations describe the circuit of Figure 56 when $S_a$ is open:

\[\begin{align*}
\dot{I}_{cv} &= \frac{V_{cm} - V_c}{L_{cv}} \\
\dot{I}_m &= \frac{\varepsilon - R_a I_m + V_{cm}}{L_a} \\
\dot{V}_c &= \frac{I_{cv}}{C} \\
\dot{V}_{cm} &= \frac{-(I_{cv} - I_a)}{C_m} \\
I_{cv} &\leq 0
\end{align*}\]
Positive generator boost configuration

![Circuit Diagram]

Figure 57: The positive generator mode of the bidirectional buck/boost converter during a boosting operation

The following equations describe the circuit of Figure 57 when $S_c$ is closed:

\[
\begin{align*}
\dot{I}_{cv} &= \frac{V_{cm} - V_c}{L_{cv}} \\
\dot{I}_m &= \frac{\varepsilon - R_a I_m - V_{cm}}{L_a} \\
\dot{V}_c &= \frac{I_{cv}}{C} \\
\dot{V}_{cm} &= \frac{- (I_{cv} - I_m)}{C_m} \\
I_{cv} &\geq 0
\end{align*}
\]
The following equations describe the circuit of Figure 57 when $S_c$ is open:

$$\dot{I}_{cv} = \frac{V_{cm}}{L_{cv}}$$

$$\dot{I}_m = \frac{\varepsilon - R_a I_m - V_{cm}}{L_a}$$

$$\dot{V}_c = 0$$

$$\dot{V}_{cm} = -\frac{(I_{cv} - I_m)}{C_m}$$

(B.10)
Negative generator boost configuration

\[ \dot{I}_{cv} = \frac{V_{cm} - V_c}{L_{cv}} \]

\[ \dot{I}_m = \frac{\varepsilon - R_a I_m + V_{cm}}{L_a} \]

\[ \dot{V}_c = \frac{I_{cv}}{C} \]

\[ \dot{V}_{cm} = -\frac{(I_m + I_{cv})}{C_m} \]

\[ I_{cv} \geq 0 \] (B.11)

Figure 58: The negative generator mode of the bidirectional buck/boost converter during a boosting operation

The following equations describe the circuit of Figure 58 when \( S_c \) is closed:
The following equations describe the circuit of Figure 58 when $S_c$ is open:

\[
\begin{align*}
\dot{I}_{cv} &= \frac{V_{cm}}{L_{cv}} \\
\dot{I}_m &= \frac{\varepsilon - R_a I_m + V_{cm}}{L_a} \\
\dot{V}_c &= 0 \\
\dot{V}_{cm} &= -\frac{(I_m + I_{cv})}{C_m}
\end{align*}
\]
Positive generator buck configuration

Figure 59: The positive generator mode of the bidirectional buck/boost converter during a bucking operation

The following equations describe the circuit of Figure 59 when \( S_b \) is closed:

\[
\begin{align*}
\dot{I}_{cv} &= -\frac{V_c}{L_{cv}} \\
\dot{I}_m &= \frac{\varepsilon - R_a I_m - V_{cm}}{L_a} \\
\dot{V}_c &= \frac{I_{cv}}{C} \\
\dot{V}_{cm} &= \frac{I_m}{C_m} \\
I_{cv} &\geq 0
\end{align*}
\]
The following equations describe the circuit of Figure 59 when $S_b$ is open:

\[
\begin{align*}
\dot{I}_{cv} &= \frac{V_{cm} - V_c}{L_{cv}} \\
\dot{I}_m &= \frac{\varepsilon - R_a I_m - V_{cm}}{L_a} \\
\dot{V}_c &= \frac{I_{cv}}{C} \\
\dot{V}_{cm} &= -\frac{(I_{cv} - I_m)}{C_m} \\
I_{cv} &\geq 0
\end{align*}
\] (B.14)
Negative generator buck configuration

Figure 60: The negative generator mode of the bidirectional buck/boost converter during a bucking operation

The following equations describe the circuit of Figure 60 when $S_b$ is closed:

\[
\begin{align*}
\dot{I}_{cv} &= \frac{V_c}{L_{cv}} \\
\dot{I}_m &= \frac{\varepsilon - R_a I_m + V_{cm}}{L_a} \\
\dot{V}_c &= \frac{I_{cv}}{C} \\
\dot{V}_{cm} &= \frac{-I_m}{C_m} \\
I_{cv} &\geq 0 
\end{align*}
\]
The following equations describe the circuit of Figure 60 when $S_b$ is open:

\[
\begin{align*}
\dot{I}_{cv} &= \frac{V_{cm} - V_c}{L_{cv}} \\
\dot{I}_m &= \frac{\varepsilon - R_a I_m + V_{cm}}{L_a} \\
\dot{V}_c &= \frac{I_{cv}}{C} \\
\dot{V}_{cm} &= \frac{- (I_m + I_{cv})}{C_m} \\
I_{cv} &\geq 0
\end{align*}
\] (B.16)
APPENDIX C

PUMA560 Robot Matrices of the Dynamic Robot Equation

The matrices from the dynamic equations for the PUMA560 robot described in Section 3.1 are provided where \( M \) is the mass matrix, \( C \) is the Coriolis matrix, and \( G \) is the gravity matrix. For ease of notation and separation of parameters, the mass, coriolis, gravity, and dissipation matrices are written in terms of the system parameters, \( \Theta \).
\[ M_{11} = \Theta_2 \cos(q_2 + q_3)^2 + 2\Theta_3 \cos(q_2 + q_3) \cos(q_2) + \Theta_1 \cos(q_2)^2 + \Theta_0 \]
\[ M_{12} = \Theta_4 \sin(q_2 + q_3) + \Theta_7 \sin(q_2) \]
\[ M_{13} = \Theta_4 \sin(q_2 + q_3) \]
\[ M_{21} = M_{12} \]
\[ M_{22} = \Theta_5 + 2\Theta_3 \cos(q_3) \]
\[ M_{23} = \Theta_6 + \Theta_3 \cos(q_3) \]
\[ M_{31} = M_{13} \]
\[ M_{32} = M_{23} \]
\[ M_{33} = \Theta_6 \]
\[
C_{11} = -\dot{q}_2 (\Theta_2 \cos(q_2 + q_3) \sin(q_2 + q_3) + \Theta_3 \cos(q_2 + q_3) \sin(q_2) + \Theta_3 \sin(q_2 + q_3) \cos(q_2) + \Theta_1 \cos(q_2) \sin(q_2)) \\
- \dot{q}_3 (\Theta_2 \cos(q_2 + q_3) \sin(q_2 + q_3) + \Theta_3 \sin(q_2 + q_3) \cos(q_2))
\]

\[
C_{12} = \dot{q}_2 (\Theta_4 \cos(q_2 + q_3) + \Theta_7 \cos(q_2)) \\
- \dot{q}_1 (\Theta_2 \cos(q_2 + q_3) \sin(q_2 + q_3) + \Theta_3 \cos(q_2 + q_3) \sin(q_2) + \Theta_3 \sin(q_2 + q_3) \cos(q_2)) + \Theta_1 \cos(q_2) \sin(q_2)) + \Theta_4 \dot{q}_3 \cos(q_2 + q_3)
\]

\[
C_{13} = \Theta_4 \dot{q}_2 \cos(q_2 + q_3) - \dot{q}_1 (\Theta_2 \cos(q_2 + q_3) \sin(q_2 + q_3) + \Theta_3 \sin(q_2 + q_3) \cos(q_2)) + \Theta_4 \dot{q}_3 \cos(q_2 + q_3)
\]

\[
C_{21} = \dot{q}_1 (\Theta_2 \cos(q_2 + q_3) \sin(q_2 + q_3) + \Theta_3 \cos(q_2 + q_3) \sin(q_2) + \Theta_3 \sin(q_2 + q_3) \cos(q_2)) + \Theta_1 \cos(q_2) \sin(q_2)
\]

\[
C_{22} = -\Theta_3 \dot{q}_3 \sin(q_3)
\]

\[
C_{23} = -\Theta_3 \dot{q}_2 \sin(q_3) - \Theta_3 \dot{q}_3 \sin(q_3)
\]

\[
C_{31} = \dot{q}_1 (\Theta_2 \cos(q_2 + q_3) \sin(q_2 + q_3) + \Theta_3 \sin(q_2 + q_3) \cos(q_2))
\]

\[
C_{32} = \Theta_3 \dot{q}_2 \sin(q_3)
\]

\[
C_{33} = 0
\]

\[
G_1 = 0
\]

\[
G_2 = -\Theta_8 \cos(q_2 + q_3) - \Theta_9 \cos(q_2)
\]

\[
G_3 = -\Theta_8 \cos(q_2 + q_3)
\]
\[ \Theta_0 = (m_2 + m_3)d_2^2 - 2m_3d_2d_3 + m_3d_3^2 + I_{1y} + I_{2x} + I_{3x} \]
\[ \Theta_1 = m_2c_{2x}^2 + 2m_2c_{2x}a_2 + (m_2 + m_3)a_2^2 - I_{2x} + I_{2y} \]
\[ \Theta_2 = m_3c_{3x}^2 - I_{3x} + I_{3y} \]
\[ \Theta_3 = c_{3x}a_2m_3 \]
\[ \Theta_4 = c_{3x}m_3(d_3 - d_2) \]
\[ \Theta_5 = m_2c_{2x}^2 + 2m_2c_{2x}a_2 + m_3c_{3x}^2 + (m_2 + m_3)a_2^2 + I_{2z} + I_{3z} \]
\[ \Theta_6 = I_{3z} + M_3c_{3x}^2 \]
\[ \Theta_7 = -d_2m_2(c_{2x} + a_2) + a_2m_3(d_3 - d_2) \]
\[ \Theta_8 = c_{3x}gm_3 \]
\[ \Theta_9 = gm_2(c_{2x} + a_2) + a_2gm_3 \]
APPENDIX D

Hip Robot and Prosthesis Matrices of the Dynamic Robot Equation

The matrices from the dynamic equations for the hip robot / prosthesis system described in Section 3.2 are provided where $M$ is the mass matrix, $C$ is the Coriolis matrix, $G$ is the gravity matrix, and $R$ is the dissipation matrix. For ease of notation and separation of parameters, the mass, Coriolis, gravity, and dissipation matrices are written in terms of the system parameters, $\Theta$. The ball screw mechanism is included at the knee joint.
\[M_{11} = \Theta_1 + \Theta_0\]
\[M_{12} = \Theta_2 \cos(q_2) + \Theta_3 \cos(q_2 + q_3 + q_4) + \Theta_4 \cos(q_2 + q_3)\]
\[M_{13} = \Theta_3 \cos(q_2 + q_3 + q_4) + \Theta_4 \cos(q_2 + q_3)\]
\[M_{14} = \Theta_3 \cos(q_2 + q_3 + q_4)\]
\[M_{21} = M_{12}\]
\[M_{22} = \Theta_5 + 2\Theta_6 \cos(q_3 + q_4) + 2\Theta_7 \cos(q_3) + 2\Theta_8 \cos(q_4)\]
\[M_{23} = \Theta_9 + \Theta_7 \cos(q_3) + 2\Theta_8 \cos(q_4) + \Theta_6 \cos(q_3 + q_4)\]
\[M_{24} = \Theta_{10} + \Theta_6 \cos(q_3 + q_4) + \Theta_8 \cos(q_4)\]
\[M_{31} = M_{13}\]
\[M_{32} = M_{23}\]
\[M_{33} = \Theta_9 + 2\Theta_8 \cos(q_4) + J_i n_q^2\]
\[M_{34} = \Theta_{10} + \Theta_8 \cos(q_4)\]
\[M_{41} = M_{14}\]
\[M_{42} = M_{24}\]
\[M_{43} = M_{34}\]
\[M_{44} = \Theta_{10}\]
\[ C_{11} = 0 \]
\[ C_{12} = -\dot{q}_3 (\Theta_3 \sin(q_2 + q_3 + q_4) + \Theta_4 \sin(q_2 + q_3)) \]
\[ - \dot{q}_2 (\Theta_2 \sin(q_2) + \Theta_3 \sin(q_2 + q_3 + q_4) + \Theta_4 \sin(q_2 + q_3)) - \Theta_3 \dot{q}_4 \sin(q_2 + q_3 + q_4) \]
\[ C_{13} = -\dot{q}_2 (\Theta_3 \sin(q_2 + q_3 + q_4) + \Theta_4 \sin(q_2 + q_3)) \]
\[ - \dot{q}_3 (\Theta_3 \sin(q_2 + q_3 + q_4) + \Theta_4 \sin(q_2 + q_3)) - \Theta_3 \dot{q}_4 \sin(q_2 + q_3 + q_4) \]
\[ C_{14} = -\Theta_3 \dot{q}_2 \sin(q_2 + q_3 + q_4) - \Theta_3 \dot{q}_3 \sin(q_2 + q_3 + q_4) - \Theta_3 \dot{q}_4 \sin(q_2 + q_3 + q_4) \]
\[ C_{21} = 0 \]
\[ C_{22} = -\dot{q}_3 (\Theta_6 \sin(q_3 + q_4) + \Theta_7 \sin(q_3)) - \dot{q}_1 (\Theta_6 \sin(q_3 + q_4) + \Theta_8 \sin(q_4)) \]
\[ C_{23} = -\dot{q}_2 (\Theta_6 \sin(q_3 + q_4) + \Theta_7 \sin(q_3)) - \dot{q}_3 (\Theta_6 \sin(q_3 + q_4) + \Theta_7 \sin(q_3)) \]
\[ - \dot{q}_4 (\Theta_6 \sin(q_3 + q_4) + \Theta_8 \sin(q_4)) \]
\[ C_{24} = -\dot{q}_2 (\Theta_6 \sin(q_3 + q_4) + \Theta_8 \sin(q_3)) - \dot{q}_3 (\Theta_6 \sin(q_3 + q_4) + \Theta_8 \sin(q_3)) \]
\[ - \dot{q}_4 (\Theta_6 \sin(q_3 + q_4) + \Theta_8 \sin(q_4)) \]
\[ C_{31} = 0 \]
\[ C_{32} = \dot{q}_2 (\Theta_6 \sin(q_3 + q_4) + \Theta_7 \sin(q_3)) - \Theta_8 \dot{q}_4 \sin(q_4) \]
\[ C_{33} = -\Theta_8 \dot{q}_4 \sin(q_4) + J_i n_\gamma n_\gamma' \dot{q}_3 \]
\[ C_{34} = -\Theta_8 \dot{q}_2 \sin(q_4) - \Theta_8 \dot{q}_3 \sin(q_4) - \Theta_8 \dot{q}_4 \sin(q_4) \]
\[ C_{41} = 0 \]
\[ C_{42} = \dot{q}_2 (\Theta_6 \sin(q_3 + q_4) + \Theta_8 \sin(q_4)) + \Theta_8 \dot{q}_3 \sin(q_4) \]
\[ C_{43} = \Theta_8 \dot{q}_2 \sin(q_4) + \Theta_8 \dot{q}_3 \sin(q_4) \]
\[ C_{44} = 0 \]
\[ G_1 = -g \Theta_1 \]
\[ G_2 = -g(\Theta_2 \cos q_2 + \Theta_3 \cos(q_2 + q_3 + q_4) + \Theta_4 \cos(q_2 + q_3)) \]
\[ G_3 = -g \Theta_4 \cos(q_2 + q_3) - g \Theta_3 \cos(q_2 + q_3 + q_4) \]
\[ G_4 = -g \Theta_3 \cos(q_2 + q_3 + q_4) \]

\[ R_1 = \Theta_{12} \text{sign}(\dot{q}_1) \]
\[ R_2 = \Theta_{11} \dot{q}_2 \]
\[ R_3 = \frac{(n_q \alpha)^2}{R \dot{q}_3 + (f_f l n_q)} \]
\[ R_4 = 0 \]
\[ \Theta_0 = m_0 \]
\[ \Theta_1 = m_1 + m_2 + m_3 + m_4 \]
\[ \Theta_2 = c_2 m_2 + l_2 m_3 + l_2 m_4 \]
\[ \Theta_3 = c_4 m_4 \]
\[ \Theta_4 = c_3 m_3 + l_3 m_4 \]
\[ \Theta_5 = I_{2z} + I_{3z} + I_{4z} + c_2^2 m_2 + c_3^2 m_3 + c_4^2 m_4 + l_2^2 m_4 + l_3^2 m_4 + J_m r^2 \]
\[ \Theta_6 = c_4 l_2 m_4 \]
\[ \Theta_7 = c_3 l_2 m_3 + l_2 l_3 m_r \]
\[ \Theta_8 = c_4 l_3 m_4 \]
\[ \Theta_9 = m_3 c_3^2 + m_4 c_4^2 + m_4 l_3^2 + I_{3z} + I_{4z} \]
\[ \Theta_{10} = I_{4z} + c_4^2 m_4 \]
\[ \Theta_{11} = b \]
\[ \Theta_{12} = f \]