Summer 1-1-2020

Cost Optimization of Concentric Loaded Rectangular Combined Footings Using Different Matlab Solvers

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COST OPTIMIZATION OF CONCENTRIC LOADED RECTANGULAR COMBINED FOOTINGS USING DIFFERENT MATLAB SOLVERS

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Bachelor of Engineering in Civil Engineering

Palestine Polytechnic University

January 2016

In Partial Fulfillment of the Requirements for the Degree

MASTER OF SCIENCE IN CIVIL ENGINEERING

at the

CLEVELAND STATE UNIVERSITY

AUGUST 2020

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DEDICATION

To my Mother

For her unconditional endless love, her kindness, and her faith.

To my Father

My first teacher and mentor, for his inspiration and encouragement.

And To my Siblings

For always being there for me.

ACKNOWLEDGEMENTS

By the grace of almighty GOD, who indeed created us in the best form and taught us how to read, whose blessings are far beyond counting, I was able to complete my MSc thesis successfully. I am thankful to everyone who has helped me in any possible way in my education journey. I am thankful to my teachers, family members, and friends.

I am deeply indebted to my supervisor Dr. Josiah Owusu-Danquah, for his support, guidance, and mentorship. His advice enlightened me and gave me new insights. His wise words will continue to inspire me throughout my life.

I am also thankful to my thesis committee members, Dr. Lutful Khan and Dr. Stephen Duffy for their mentorship, continuous support, and guidance. I am also grateful to all of the faculty of the civil and environmental engineering department at Cleveland State University for sharing their knowledge and experience. I am also thankful to Ms. Diane Tupa for being helpful in every way.

Lastly, I am obliged to my friends Abdallah, Mohammed, and Tariq, whose support and motivation kept me going during the hard times.

My gratitude to you all for all you have done, which I never will forget.

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ABSTRACT

Conventional design methods for combined footings comprise a series of iterations. Generally, this involves an initial guess for the dimensions which are evaluated as guided by the existing design code. This is then followed by several iterations to reduce the cost without any detriment to structural safety. In most cases, the result from the final iteration does not reflect the minimum cost design. This necessitates optimization models capable of establishing efficient and accurate designs within a short period, especially under several design variables.

For this purpose, an optimization model for concentric loaded rectangular combined footings was developed in this research. The model was built in a general form and can perform optimization with different soil and material properties. The model encompasses an accurate objective function, subjected to the structural, geotechnical, and logical constraints to satisfy the requirements of the strength and serviceability limit states in accordance with ACI 318-11M specifications. The model works to find the minimal construction cost of the structure, adequate dimensions, and steel areas in different sections that correspond to that minimal cost.

The model was developed using five solvers available within the MATLAB Global Optimization toolbox. Model capabilities were investigated by optimizing a case of concentric loaded rectangular combined footing with a known solution. The model capabilities were also assessed by testing the effect of using different material properties

and varying site conditions on the resulting objective function. The optimization results showed identical results compared to the conventional design methodology. The results also showed the cost tends to decrease with the use of higher steel grades for all load variations. Moreover, there was no major effect for the concrete compressive strength in the range of 20 to 35 MPa on the value of the objective function. However, for higher concrete strengths >35MPa, the objective function value increased significantly.

The influence of changing the foundation depth was significant in terms of cost reduction for the depth ranges between 0.5 to 2.0m, then the cost remained almost constant with the depth increase. Finally, the results showed no significant impact of the column shape on the total cost.

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CHAPTER I

INTRODUCTION

1.1 PROBLEM OVERVIEW

Reinforced concrete foundations are structural members used to support other load-bearing members such as walls and columns by transmitting and distributing their loads to the soil. Like any other structural member, foundations should be designed to transfer the load safely. Foundations are generally divided into two groups, (i) shallow foundations and (ii) deep foundations. Common shallow foundations include spread or isolated footings, strip footings, combined footings, wall footings, and mat or raft slab. Common deep foundations include piles, piers, and caissons [1]. The traditional objective of structural analysis and design is to come up with a design that can safely maintain the applied loads under the defined boundary conditions without failure or excessive deformation that may affect the serviceability of the structure for the intended lifetime of the project. Advances in engineering technology have led to much more complex structures than ever. Designing a structure solely on the bases of safety is no longer satisfying; rather, several other considerations became as much important as safety nowadays. Foundation design must consider the following: (1) safety against collapse or failure of the soil; (2) settlements and movements are controlled; (3) factors related to the environment are considered (including frost action, shrinking, swelling of the soil, underground waters and adjacent structures or excavations), and (4) economically rational in regards to its function and the overall cost [2].

A good engineering solution or design is the one that finds the right balance between safety, time, and cost. However, finding such a balance could be very costly and time-consuming when using traditional or conventional design methods. Conventional design methods involve a series of iterations. Generally, an initial guess will be made and evaluated, then several iterations are made in a trial to minimize the cost as much as possible. The number of iterations performed depends on how much time is allocated for the design job and also on the quality of the initial guess, which relies solely on the engineering intuition, assumptions, and experience of the designer. This iteration process usually will be terminated after a few attempts to save time.

The idea of the structural optimization process is to identify the optimal values of the design variables that give the best value of the objective function while meeting the imposed bounds and constraints. The objective function is a function of the design variables; this could be cost, weight, stiffness, or any function that could be written in terms of the design variables. The design code requirements and workability or availability limitations are introduced through sets of explicit and implicit constraints that govern the design to ensure the safety and applicability of the design. In other words, conventional design methods evaluate the economics of a design after satisfying all the constraints. On the other hand, formulating design jobs as optimization problems from the beginning will ensure the results will be safe and the cost will be minimum. Thus, it saves both money and time.

The optimization process is based on performing series of iterations, which, with each iteration, the value of the objective function will move in the desirable direction (i.e., minimization or maximization). The number of iterations performed to reach the optimal values could be very high. Therefore, there is a need for a specific procedure and algorithm that can be written in terms of a computer program to solve the problem in a timely manner.

In the search for the most economical, safe solution for concentric loaded rectangular combined footings, this research introduces a cost optimization model that can find the optimum values for the design variables and the corresponding value of the cost function easily and effectively. This optimization scheme considers a comprehensive cost analysis of the footing and constraints that ensure the solution will adhere to the geotechnical and structural requirements of the structure. The main challenge in implementing the optimization model is that establishment of global optimum for non-linear, non-convex functions with tens of nonlinear inequalities could be a very lengthy process and the results do not necessarily reflect the global optimum in many cases. However, this problem could be overcome by using several algorithms and solvers with adequate constraint and optimality tolerances that increase the chance of finding the global optimum.

The present research was motivated by the fact that the construction industry uses the most material by weight among all other industries which can lead to a shortage of domestic resource supplies and severe environmental impact [3]. Achieving the optimum design could significantly reduce the material used and thus the total cost and environmental impact. To this end, design optimization is performed using five different existing solvers from MATLAB Global Optimization Toolbox. Global Optimization Toolbox provides functions that search for global solutions to problems that contain multiple maxima or minima. Toolbox solvers include surrogate, pattern search, genetic algorithm, particle swarm, simulated annealing, multistart, and global search [4]. That being said, the optimization was conducted using all the applicable solvers available for this type of problem in the Toolbox. Different options and search functions of each solver were tuned and adjusted to improve the effectiveness of the solver, and the results were compared.

1.2 OBJECTIVES

The main objective of this study was to come up with an optimization model that can find the optimum values for the following design variables (also defined in Figures 4 and 5). These are: (a) the length from the left edge to the centre of the left column, i.e., *x(1)* in Figure 4, (b) the length from the right edge to the centre of the right column, *x(2)*, the width of the foundation ,i.e., *x(3)* in Figure 5, the effective depth, *x(4)*, and the area of steel reinforcement in different sections denoted as *x(5)* to *x(10)* in Figure 5. It is anticipated that once the equations are set rightly, the corresponding objective function value can be effectively and easily evaluated, while accounting for geotechnical, structural, and logical constraints. The specific objectives of the study were:

1. To develop a cost optimization model for a concentrically loaded rectangular combined footing using Matlab optimization solvers.
2. To calibrate the Matlab model dimensional parameters with a known conventional design solution.

3. To compare the computational efficiency of the various available optimization methods for cases involving high number of variables and non-linear constraints.
4. To investigate the effect of material properties on the total cost of design.
5. To investigate the effect of changing the foundation depth on the total cost of design.
6. To investigate the effect of the column shape on the total cost of the footing.

1.3 SCOPE OF WORK

The scope of this research is to develop optimization model for concentrically loaded concrete rectangular combined footings using existing MATLAB solvers. A total of five solvers will be used and tested. The developed model should account for the geotechnical limitations of bearing capacity and primary consolidation settlement, all the applicable structural constraints based on the ACI 318-11M code, and any workability limitations.

Once the optimization model is developed, the model will be tested against a design example, the example which shown on appendix G is a design example of a concentrically loaded rectangular combined footing with a limited space on one of the sides (neighbor column),the chosen length must ensure uniform pressure distribution along the footing length, also the chosen area must satisfy the maximum allowable settlement of 2 inches and also a bearing capacity factor of safety equals to 3. The results from the five solvers will be compared to determine the most effective solver for optimization cases with non-linear, non-convex functions with several nonlinear inequalities. The chosen solver should yield the global optimum solution in the least time possible. This selected solver will then be used for several optimization scenarios. This includes cases that investigate the effect of changing reinforcement steel yield strength, concrete compressive strength under varying load percentages, and the foundation depth on the total cost. Another optimization scenario will be conducted to investigate the effect of changing the columns shape on the overall objective function.

The outline of the remaining part of the thesis is as follows. First, a background review of combined footings and the presently available optimization methods is given in chapter 2. The optimization model development is discussed in chapter 3. In chapter 4, the optimization model capabilities are investigated by optimizing a concentric loaded rectangular combined footing example. In chapter 5, the model capabilities are assessed by testing the effect of using different material properties, varying the depth, and using different column shapes on the optimization process results. The results and findings are summarized in the final chapter 6.

CHAPTER II

BACKGROUND

2.1 COMBINED FOOTINGS

A combined footing is a long footing supporting two or more columns in one row [5]. Usually, combined footings are used when two columns are close to each other or where soil bearing capacity is low and causing overlap of adjacent isolated footings. In other cases, the proximity of a property line or existing building or sewer, adjacent to a building column might necessitate combined footings [6]. Combined footings can take many shapes depending on the site conditions and the loads coming from the substructure. For instance, when one of the columns lies adjacent to the property line the combined foundations will be trimmed on the property line and extended on the other side. Moreover, when one of the column loads is much larger than the other column, the common practice is to use a trapezoidal shape for the footing, which makes it makes more economical. Different shapes for the combined footings are shown in figures 1 and 2.

Figure 1. Rectangular combined footings [7].

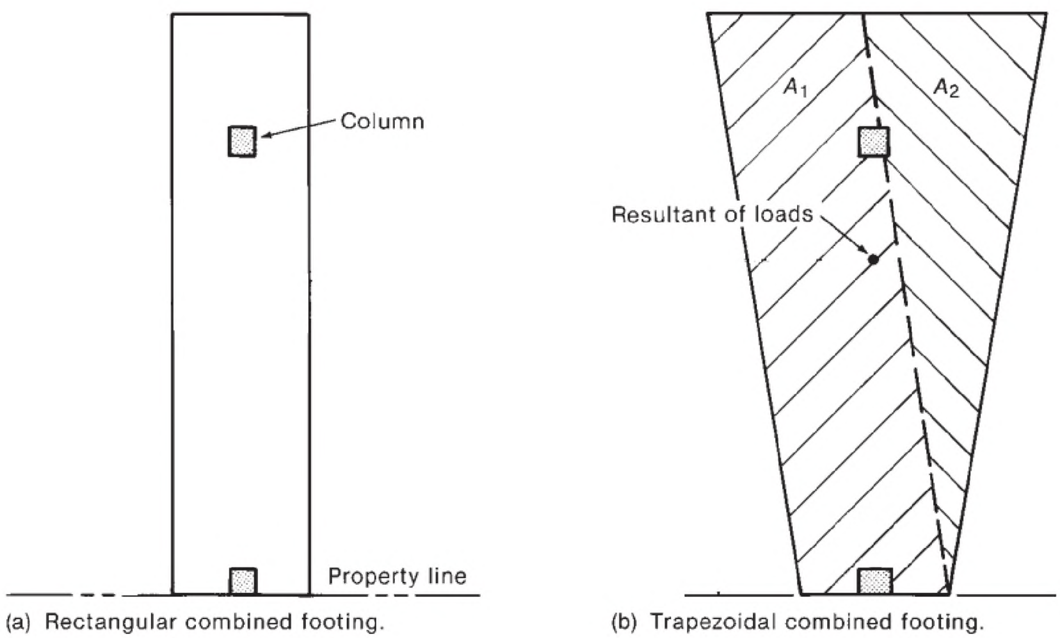

Figure 2. Rectangular property line and trapezoidal combined footings [8].

Usually combined and mat foundations are assumed rigid for design purposes. The rigid design method assumes that the footing or mat is infinitely rigid, and therefore, the deflection of the footing or mat does not influence the pressure distribution. Moreover, the soil pressure is linearly distributed and the pressure distribution will be uniform if the geometric centroid of the footing coincides with the location of the resultant of the applied loads acting on the foundation [6, 9, 10]. Also, almost exclusively dimensions of footings are designed based on the allowable stress design method where the dimensions will be based on the allowable stresses acting on the soil at service loads [8].

The reinforced concrete design is based on the Strength Design Method. The strength design method requires the conditions of static equilibrium and strain compatibility across the depth of the RC section to be satisfied. These strains in reinforcement and concrete are directly proportional to the distance from the neutral axis. This implies that the variation of strains across the section is linear. Concrete sections are considered to have reached their flexural capacities when they develop a strain of 0.003 in the extreme compression fiber. The stress in the reinforcement varies linearly with strain up to the specified yield strength. Hence, the strain hardening of steel is ignored. Also, the tensile strength of concrete is neglected, and the compressive stress distribution in concrete may be simplified by a rectangular stress distribution.

Comprehensive design of footings follows these general steps: (1) determining the required area of the footing and selecting the dimensions so that the centroids coincide; (2) drawing the shear diagram along the length of the footing using the factored loads; (3) determining the depth required for one-way shear and checking its adequacy for two-way shear; (4) designing the reinforcing bars in the longer direction; (5) designing the reinforcing steel in the short direction; (6) Checking and selecting the minimum amount of steel for temperature and shrinkage for parts of the footing required [6]. It is satisfactory to assume that each column load is spread over a width equal to the column width plus d/2 on each side [44-46].

2.2 OPTIMIZATION IN STRUCTURAL ENGINEERING

Structural optimization has been studied by many researchers, Stolyarov (1974) presented a method for minimization of the volume of foundations. However, it was realized that the least volume design did not necessarily correlate to the best design in terms of cost. This is because the total cost of the foundation is a function of several other variables such as the weight of reinforcing steel, area of formwork, excavation volume, insulation, and blinding concrete volume [11]. Another model for optimization for isolated footings has been presented by Bhavikatti and Hegde (1979) and the results showed that there is about 8–10% cost reduction when their method is used. However, the proposed model was based on linear optimization which affects the accuracy of the results [12]. In (1982) Naaman presented an optimized design for a prestressed concrete tensile member. The optimization was to minimize the cost of materials which includes the concrete and the prestressed steel [13]. Desai et.al (1984) presented an optimization design of an isolated sloped square footing resting on dry granular medium. The results showed significant savings in cost compared to the conventional design approach [14].

Namiq and Al-Ani (1985) presented cost optimization of spread footings subjected to eccentricity in both axes by using graphical and Rosenbrock's s method. The main findings were that the optimum ratio of footing length to width *(L/B)* is directly proportional to the ratio of the difference between the eccentricities in both directions to the eccentricity in the short direction *(eL-eB/eB),* and the ratio of the steel to concrete price does not affect the optimum *L/B* ratio [15]. Basudhar (2006), used nonlinear programming optimization techniques successfully to determine the optimum cost analysis of the rigid raft foundation and found that the variation in the cost is due to variation in area ratios [16]. Madan Mohan (2006) developed an optimization program for settlement controlled shallow isolated footings based on allowable differential settlements, the results showed (10% to 40%) savings in cost [17]. Wang and Kulhawy (2008) presented a design approach that explicitly considers construction economics in the design of isolated footings with the goal is to minimize construction costs [18].

Although there have been substantial efforts exerted towards the optimization of geotechnical structures, the studies on the combined footings are limited. Eman M. Farhan Al-Douri(2007) presented a study of optimizing the cost of trapezoidal combined footings based on the Hooke and Jeeves model, the structural constraints used in their model did not represent all of the structural requirements and the steel area was not treated as a design variable [19]. Muhammed Rizwan (2013) presented combined footing optimization using a modified complex method. The model was limited to a property line combined footing and the reinforcement area calculation and the cost function was not comprehensive [20]. Chavarría and Sandra (2017) presented an optimization model for corner combined footings considering real soil pressure. In their study, they considered real soil pressure with eccentrically loaded columns. However, the model was based on optimizing the contact area with the soil and does not necessarily reflect a cost optimization [21].

Francisco Velázquez-Santillán et al. (2018) presented numerical experimentation on optimization of eccentrically loaded rectangular footings based on real soil pressures and under varying loading conditions. The model was limited in capacity to property line foundation type where one of the columns lies on the property line. The constraints did not take into account the spacing limitations for reinforcement steel, and the cost function was limited to concrete and reinforcement steel costs [22].

CHAPTER III

PROBLEM FORMULATION AND METHODOLOGY

3.1 PROBLEM FORMULATION

A local minimum of a function is a point where the function value is smaller than at nearby points but possibly greater than at a distant point. On the other hand, a global minimum is a point where the function value is smaller than at all other feasible points. The local minimum could be also the global minimum, but if not, it has no significant meaning.

In general, optimization problems should take the form of:

$$Min\, f(x), where \begin{cases} c(x) \leq 0 \\ ceq = 0 \\ ib \leq x \leq ub \end{cases} \tag{G1}$$

where *c(x)* and *ceq(x)* are functions that return vectors, and *f(x)* is a function that returns a scalar. Here, *f(x), c(x),* and *ceq(x)* can be nonlinear functions. Also, *x, lb,* and *ub* can be passed as vectors or matrices [23].

3.2 METHODOLOGY

The search for the global optimum for non-linear, non-convex with tens of nonlinear inequalities could be a very lengthy process and the results do not necessarily reflect the global optimum in many cases. However, using several algorithms and solvers with adequate constraints and optimality tolerances could increase the chance of finding the global optimum. The optimization was performed using five different existing solvers from MATLAB Global Optimization Toolbox. This global optimization toolbox provides functions that search for global solutions to problems that contain multiple maxima or minima. Toolbox solvers include surrogate, pattern search, genetic algorithm, particle swarm, simulated annealing, multistart, and global search [23].

The optimization was conducted using all the applicable solvers available for this type of problem on the Toolbox. Different options and search functions of each solver were tuned and adjusted to improve the effectiveness of the solver.

3.2.1 Optimization solvers

In our research, five different solvers have been utilized in trying to find the global minimum efficiently, each algorithm has its advantages and disadvantages. An overview and comparison of the different algorithms used are stated below.

3.2.1.1 Derivative based optimization: GlobalSearch and MultiStart

GlobalSearch and MultiStart are both algorithms that start a local solver from different points to find the global optimum. The algorithms use multiple starting points to sample multiple basins of attraction.

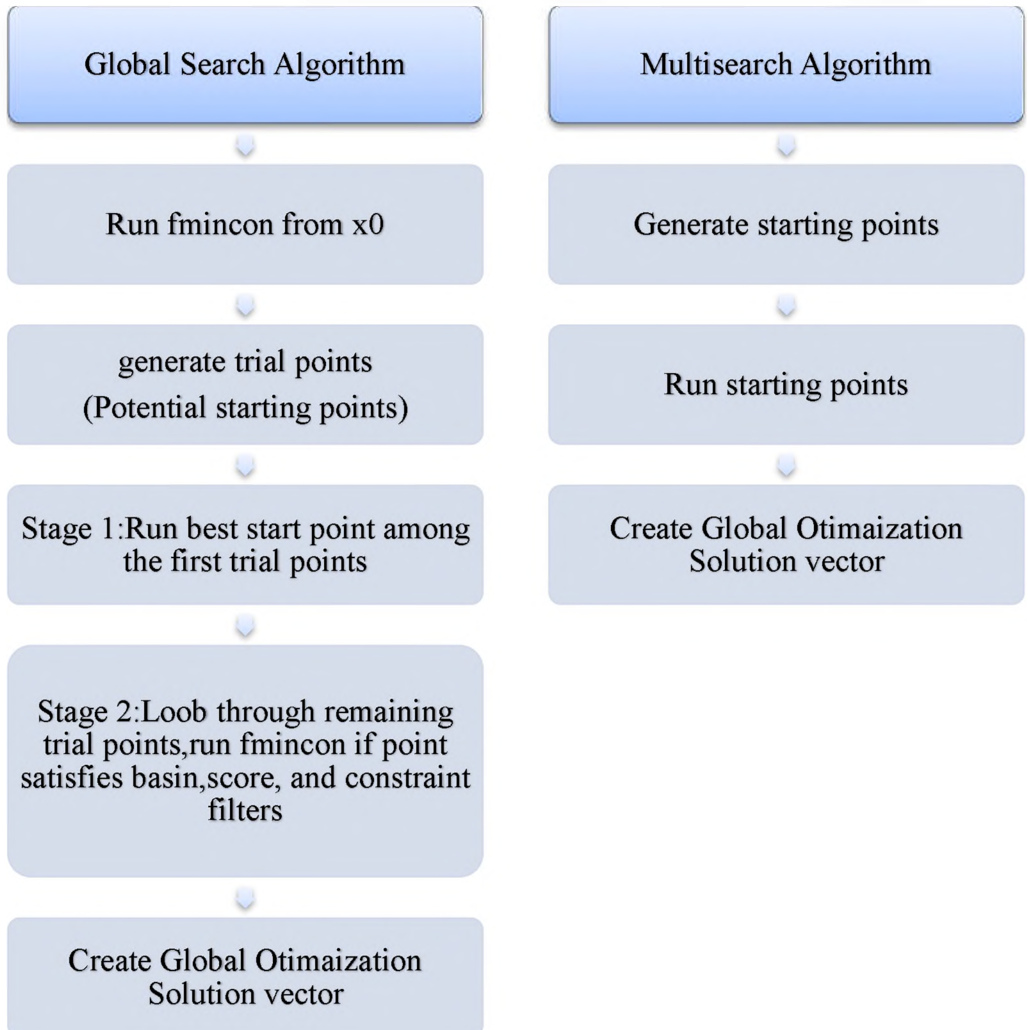

Figure 3.Sketch of the GlobalSearch and MultiStart algorithms [24].

> "GlobalSearch uses a scatter-search mechanism for generating start points then analyzes these points and rejects those points that are unlikely to improve the best local minimum found so far. MultiStart uses uniformly distributed start points within bounds, or user-supplied start points and then runs all these points" [24].

There are multiple available local solvers. However, globalsearch can only be used with Fmincon. So Fmincon will be the choice for both algorithms.

Fmincon is a nonlinear programming solver provided in MATLAB's optimization Toolbox. Fmincon performs nonlinear constrained optimization and supports linear and nonlinear constraints. Solver options including algorithms, convergence criteria, maximum iterations, and the method to calculate the gradients can be specified. Several algorithms can be used with fmincon, for constrained nonlinear optimization problems the options are interior-point and sequential quadratic programming (SQP). The interior-point algorithm handles both large, sparse problems and small dense problems. 'SQP' is a medium-scale algorithm, but both algorithms satisfy bounds at all iterations [25].

Large scale optimization algorithms use linear algebra that does not need to store, nor operate on full matrices. This is done by storing sparse matrices, and sparse linear algebra for computations whenever possible. In contrast, medium-scale methods use full matrices and dense linear algebra. For large problems, full matrices take up a significant amount of memory, and the dense linear algebra may require a long time to run [25-29].

3.2.1.2 Derivative-free optimization

1. Genetic algorithm

The genetic algorithm is based on natural selection and is used for both constrained and unconstrained optimization problems, it is built similarly to the process that drives biological evolution. It repeatedly modifies a population of individual solutions. The genetic algorithm works to select random individual points as parents at each step from the current population and uses them to produce the children for the next generation. With the repeated process, the population advance toward the optimal solution.

A major difference between genetic and derivative-based algorithms is that the genetic algorithms generate a population of points at each iteration and the best point of the population approaches the optimal solution. While the later generates a single point at each iteration and the sequence of the points approaches an optimal solution.

Another difference between them is that the genetic algorithm selects the next population by computation using random numbers. While the classic algorithms select the next point by deterministic computation. [30-32].

2. *Pattern search algorithm*

The pattern search algorithm uses the initial starting point to search for a set of points such as the value of the objective function in the new point is lower than the initial point. And by computing a sequence of points the solution approach the optimal point. With every step the algorithm searches different points around the current point, these points are called a mesh. “The mesh is formed by adding the current point to a scalar multiple of a set of vectors called a pattern. If the pattern search algorithm finds a point in the mesh that improves the objective function at the current point, the new point becomes the current point at the next step of the algorithm” [33].

There are several used pattern search algorithms i.e. the generalized pattern search (GPS) algorithm, the generating set search (GSS) algorithm, and the mesh adaptive search (MADS) algorithm. Both GPS and GSS algorithms use fixed direction vectors. For optimization with nonlinear constraints GSS and GPS algorithms are identical. The MADS algorithm uses a random selection of vectors to define the mesh [34][35].

3.2.2 Optimization model

Following the above solvers’ formulations, the optimization model was developed. To put the model in great use, it was developed in general form, such that the optimization can be performed for any concentric rectangular combined footing. Thus, it applies to a foundation with or without a property line and for different soil properties, different vertical loads, different material properties, and rates. The model comprises of an accurate objective function that reflects all of the material fabrication and working hands costs, subjected to the structural, geotechnical, and logical constraints to satisfy the requirements of the strength and serviceability limit states in accordance with ACI 318-11 specifications.

MATLAB code was developed with an excel sheet for data inputs/outputs. The optimization model consists of input data (constants for every optimization process), design variables (that changes along the process), and cost objective function (see Table 1), which is subjected to several sets of implicit and explicit constraints.

All the symbols used in the optimization model are illustrated in Table 1 and Figures 4 and 5.

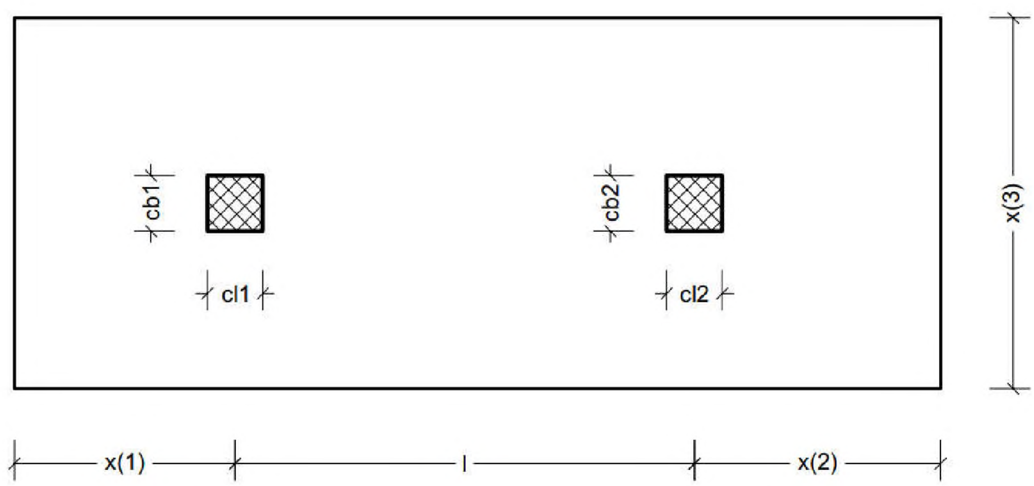

Figure 4. Plan view of the combined footing.

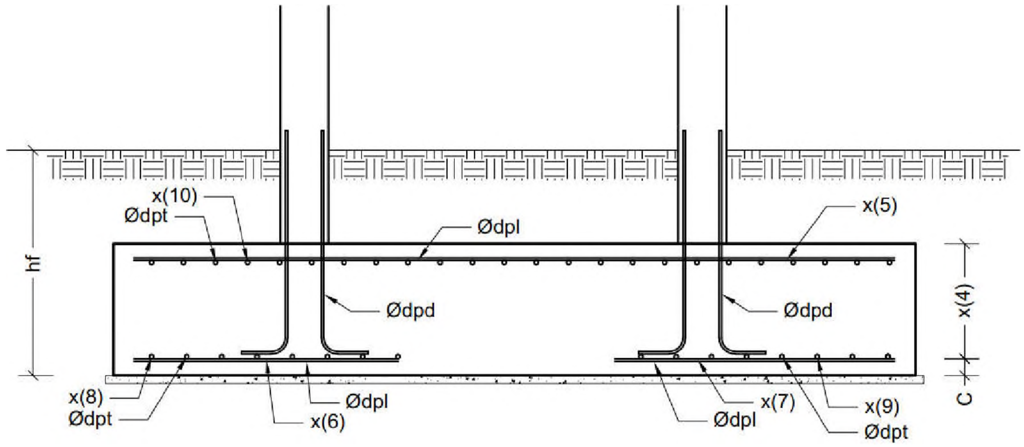

Figure 5.Section of the combined footing.

1. *Inputs*

Summary of the design parameters and analysis results essential for the design of the combined footing are listed in the table below. Moreover, one excel sheet was used to feed the bearing capacity factors Nc, Nq *and* $N\gamma$ to the model. These factors vary as a function of the friction angle and are calculated using Terzaghi equations [42].

2. *Design variables*

Design variables are represented as vector $x(i)$ with 10 elements as shown in Figures 4 and 5 and described as follows: $x(1)$ is the length from the left edge to the centre of the left column, $x(2)$ is the length from the right edge to the centre of the right column, $x(3)$ is the width of the foundation, $x(4)$ is the effective depth, $x(5)$ to $x(10)$ is the area of steel reinforcement for the *-ve* moment at mid-span, *+ve* moment under column 1, *+ve* moment under column 2, transverse beam under column 1, transverse beam under column 2 and temperature and shrinkage reinforcement respectively.

Table I. Optimization examples input data.

Description	Unit	Symbol
Clear cover	m	C
Depth of footing	m	hf
Excavation Margin	m	E
Soil initial void ratio		e0
Dead load on column 1	KN	PD1
LIVE load on column 1	KN	PL1
Dead load on column 2	KN	PD2
LIVE load on column 2	KN	PL2
Column 1 length	m	cl1
Column 1 width	m	cb1
Column 2 length	m	cl2
Column 2 length	m	cb2
Center to center column spacing	m	l
Diameter for longitudinal reinforcement	m	dpl
Diameter for transverse reinforcement	m	dpt
Diameter for dowels	m	dpd
Moist nit weight of soil	KN/m^3	Ws
Unit weight of concrete	KN/m^3	Wc
Surcharge	KN/m^2	qanet
Factor of safety for bearing capacity		FS
Allowable settlement	m	Sca
Depth of water table from surface	m	Dw
Depth of soil layer	m	D
Cohesion of soil	KN/m^2	C
Soil friction angle	degree	ø
Compression index		Cc
Recompression index		Cs

3. Objective function

A feasible solution is any solution that satisfies all the system constraints. However, not all feasible solutions have significant meaning. An objective function is necessary to compare all the feasible solutions to determine the optimum solution or design which has the best objective function value. Typically, an objective function will be a function of the variables that define the design [36].

In structural engineering, some typical objective statements, and their associated objective functions are selecting the least cost member (minimizing cost) and selecting the least weight member (minimizing weight) [37]. A cost objective function should calculate the total cost of the footing by calculating the volume and areas of the subcomponents of the foundation and multiplying them with the corresponding rate.

Table II. Cost calculation.

Item	**Calculation**	
Concrete	*(x(1)+x(2)+l)*(x(4)+C+(dpl/2))*x(3)*	Volume(m3)
Blinding Concrete	*(x(1)+x(2)+l+0.2)*(x(3)+0.2)*0.1*	Volume(m3)
Excavation	*(x(1)+x(2)+l+2*E)*(x(3)+2*E)*hf*	Volume(m3)
Reinforcement steel	*((x(5)*(x(1)+x(2)+l))+(x(6)*(x(4)+cl1))+(x(7)*(x(4)+cl2))+(x(8)*x(3))+(x(9)*x(3))+(x(10)*x(3)))*7.85*	Weight (metric ton)
Insulation	*((x(1)+x(2)+l+x(3))*(x(4)+C+(dpl/2))*2)+((x(1)+x(2)+l)*x(3))-(cl1*cb1)-(cl2*cb2)*	Area(m2)
Form Work	*(x(1)+x(2)+l+x(3))*(x(4)+C+(dpl/2))*2*	Area(m2)

$$Objective\ Function = \sum items\ cost \qquad (G2)$$

$$Objective\ Function = \sum amount * rate \qquad (G3)$$

$$\begin{aligned} Objective\ Function \\ &= (Cocrete\ volume * Rcon) \\ &+ (Blinding\ concrete\ volume * Rb) \\ &+ (Excavation\ volume * Rexc) \\ &+ (Reinforcment\ steel\ weight * Rst) \\ &+ (Insulation\ Area * Rins) + (Form\ work\ Area * Rfw) \end{aligned} \qquad (G4)$$

$$Objective\ Function =((((x(1)+x(2)+l+2*E)*(x(3)+2*E)*hf)*Rexc)+(((x(1)+x(2)+l+0.2)*(x(3)+0.2)*0.1)*Rb)+(((x(1)+x(2)+l+x(3))*(x(4)+C+(dpl/2))*2)*Rfw)+(((x(1)+x(2)+l)*(x(4)+C+(dpl/2))*x(3))*Rcon)+((((x(1)+x(2)+l+x(3))*(x(4)+C+(dpl/2))*2)+((x(1)+x(2)+l)*x(3))-(cl1*cb1)-(cl2*cb2))*Rins)+(((x(5)*(x(1)+x(2)+l))+(x(6)*(x(4)+cl1))+(x(7)*(x(4)+cl2))+(x(8)*x(3))+(x(9)*x(3))+(x(10)*x(3)))*7.85*Rst)) \qquad (G5)$$

4. Constraints

In structural design, design constraints are frequently referred to as limit states. The limit states are the potential failure conditions, where failure is any state that makes the design infeasible (will not work for its intended purpose). Two categories of Limit states are generally considered in structural engineering, i.e., strength, and serviceability [38].

Strength limit states are potential modes of structural failure. For reinforced concrete combined footings, the failure may be by yielding of the reinforcement steel (permanent deformation), one-way shear and punching shear. These limit states are represented by the general form: Required Strength < Nominal Strength. The required strength is the internal force obtained from the structural analysis of the structure or system. nominal strength is the capacity of the member [39-41]. The serviceability limit states are conditions that are not strength-based but may affect the intended use of the structure. deflection, vibration, and slenderness are common serviceability limit states. Serviceability limit states may be written in the general form: actual behaviour < allowable behaviour.

Constraints also are divided into two groups of explicit and implicit as described below:

a. Explicit constraints

Explicit constraints or bounds (lower and upper bounds) are rules that restrict each xi to take on values only from a given set. These constraints are imposed by either code requirements or applicability considerations. Each design variable *x(i)* can have lower and upper bounds. The bounds are passed as two arrays *lb* and *ub,* where each array contains 10 elements corresponding to each design variable.

Since *x(1)* and *x(2)* are measured from column centres, these values could not be less than half the column length. Hence, the lower bounds for *x(1)* and *x(2)* are *cl1/2* and *cl2/2*, respectively. Although there is no mandatory upper bound for these variables, in most of the common applications there will be a limit for these values based on the site circumstances and it can be passed to *ub* array. *x(3)* cannot be less than the largest of the column's width *cb1* and *cb2* and the upper limit can be also specified. The lower limits of *x (4)* are imposed by a minimum thickness required for the design of foundations as prescribed in Section 15.7 of the ACI 318-11, and the fact that enough compression development length has to be provided for the column dowels as stated in Section 12.3, ACI 318-11.

$x(4) \geq 150$ Section 15.7, ACI 318M-11

$x(4) \geq \left(0.24 * fy * \frac{dpd}{\lambda * \sqrt{f\acute{c}}} \right) + dpd + dpt + dpl/2$ Section 12.3, ACI 318M-11

$x(4) \geq (0.043 * fy * dpd) + dpd + dpt + dpl/2$ Section 12.3, ACI 318M-11

$x(4) \geq 0.2 + dpd + dpt + dpl/2$ Section 12.3, ACI 318M-11

$x(4) \leq hf$ Logical bound

X(5) to *x(10)* has no bounds since the minimum reinforcement requirements by the ACI Code and the spacing requirements are both functions of the design variables, instead, these limitations are introduced as nonlinear constraints.

The complete lower and upper bounds arrays are as follows:

$$lb = [cl1/2, \quad cl2/2, \quad cb1 \geq cb2, x(4)min, 0,0,0,0,0,0]$$
$$ub = [r1, \quad r2, \quad r3, hf, inf, inf\ , inf, inf, inf, inf]$$

b. Implicit constraints

Implicit constraints are the rules that determine which of the variable's values in the solution space satisfy the criterion function and describe how the *xis* or variables must relate to each other. These constraints are imposed by the applicable codes and practices to ensure the safety of the design, both in terms of geotechnical and structural constraints are described below with the reference for each one.

a) The first constraint is equality constraint to ensure a uniform pressure distribution between the soil and the foundation base. This constraint is based on moment equilibrium around column 1 and its set to limit the total length of the footing to be equal to twice the distance from the left edge to the center of force.

$$\sum M_{c1} = 0 \tag{G1}$$

$$\sum M_{c1} = (PD1 + PL2) * l - (PD1 + PL1 + PD2 + PL2) * y = 0 \tag{G2}$$

$$y = \frac{(PD1 + PL2) * l}{(PD1 + PL1 + PD2 + PL2)} \tag{G3}$$

$$x(1) + x(2) + l = 2 * \left(\left(\frac{(PD1 + PL2) * l}{(PD1 + PL1 + PD2 + PL2)}\right) + x(1)\right) \tag{G4}$$

$$x(1) + x(2) + l - \left(2 * \left(\left(\frac{(PD1 + PL2) * l}{(PD1 + PL1 + PD2 + PL2)}\right) + x(1)\right)\right) = 0 \tag{C1}$$

b) The second set of constraints is imposed by the geotechnical requirements. It guarantees that the pressure does not to exceed the allowable bearing capacity of the soil, and the settlements calculated does not to exceed the specified allowable settlement.

The bearing capacity for continuous or strip foundations according to Terzaqhi method can be calculated as per the following equation [42]:

$$qu = C * Nc + \gamma * hf * Nq + 0.5 * \gamma * B * N\gamma \tag{G5}$$

where

$Nc, Nq\ and\ N\gamma$: Bearing capacity factors according to Terzaghi.

An approximate procedure to adjust the bearing capacity to take into account the presence of the water table is by multiplying the established qu by a factor Cw [43]:

$$C_w = 0.5 + 0.5(\frac{Dw}{hf + B}) \tag{G6}$$

$$q_{anet} = \frac{q_u - q_0}{FS} \tag{G7}$$

$$q_{anet} = \frac{\left(0.5 + 0.5\left(\frac{Dw}{hf + B}\right)\right) * (CNc + \gamma hfNq + 0.5\gamma BNq) - (\gamma hf) - Surchrge}{FS} \tag{G8}$$

$$\frac{Total\ service\ load}{qanet} \leq Area \tag{G9}$$

$$((PD1 + PL1 + PD2 + PL2) * FS)/((0.5 + 0.5(Dw/(hf + x(3)))) * (C * Nc + \gamma * hf * Nq + 0.5 * \gamma * x(3) * Nq) - (\gamma * hf) - Surchrge) \leq ((x(1) + x(2) + l) * x(3)\ )) \tag{G10}$$

$$((PD1 + PL1 + PD2 + PL2) * FS)/((0.5 + 0.5(Dw/(hf + x(3)))) * (C * Nc + \gamma * hf * Nq + 0.5 * \gamma * x(3) * Nq) - (\gamma * hf) - Surchrge) - ((x(1) + x(2) + x(3)) * x(3))) \leq 0 \tag{C2}$$

The primary consolidation settlement can be calculated using the following equations

If *Dw=0 :*

$$\sigma_0' = (\gamma_m - \gamma_w) * \frac{D + hf}{2} \tag{G11}$$

If *0<Dw<hf:*

$$\sigma_0' = (\gamma_m * Dw) + \big((\gamma_m - \gamma_w) * (hf - Dw)\big) + \left((\gamma_m - \gamma_w) * \frac{D - hf}{2}\right) \tag{G12}$$

If *hf<Dw<(D+hf)/2 :*

$$\sigma_0' = (\gamma_m * Dw) + \left((\gamma_m - \gamma_w) * \left(\frac{D}{2} + \frac{hf}{2} - Dw\right)\right) \tag{G13}$$

Else:

$$\sigma_0' = (\gamma_m) * \left(\frac{D}{2} + \frac{hf}{2}\right) \tag{G14}$$

$$\Delta\sigma_{avg}' = \frac{1}{6} * (\Delta\sigma_T' + 4\Delta\sigma_M' + \Delta\sigma_B') \tag{G15}$$

$$\Delta\sigma_T' = \frac{PD1 + PL1 + PD2 + PL2}{(X(1) + X(2) + l) * X(3)} \tag{G16}$$

$$\Delta\sigma_M' = \frac{PD1 + PL1 + PD2 + PL2}{\left(X(1) + X(2) + l + \frac{D - hf}{2}\right) * \left(x(3) + \frac{D - hf}{2}\right)} \tag{G17}$$

$$\Delta\sigma'_B = \frac{PD1 + PL1 + PD2 + PL2}{\left(X(1) + X(2) + l + (D - hf)\right) * \left(x(3) + (D - hf)\right)} \quad \text{(G18)}$$

For normally consolidated clay $\sigma'_0 = \sigma'_c$:

$$Sc = \frac{C_c * H_c}{1 + e_0} * \log\left(\frac{\sigma'_0 + \Delta\sigma'_{avg}}{\sigma'_0}\right) \quad \text{(G19)}$$

For over consolidated clay where $\sigma'_0 + \Delta\sigma' \leq \sigma'_c$:

$$Sc = \frac{C_s * H_c}{1 + e_0} * \log\left(\frac{\sigma'_0 + \Delta\sigma'_{avg}}{\sigma'_0}\right) \quad \text{(G20)}$$

For over consolidated clay where $\sigma'_0 \leq \sigma'_c \leq \sigma'_0 + \Delta\sigma'$:

$$Sc = \frac{C_s * H_c}{1 + e_0} * \log\left(\frac{\sigma'_c}{\sigma'_0}\right) + \frac{C_c * H_c}{1 + e_0} * \log\left(\frac{\sigma'_0 + \Delta\sigma'_{avg}}{\sigma'_0}\right) \quad \text{(G21)}$$

$$S_c - S_{call} \leq 0 \quad \text{(C3)}$$

c) The third set of constraints is imposed by the ACI code and it takes care of the failure caused by flexural shear. According to Section 11.1.3.1 of ACI 318-M11, the critical section for flexure shear is at a distance d "*x(4)*" from the face of the support.

$$\emptyset Vn \geq Vu \quad \text{ACI 318M-11}$$

$$Vn = Vc + Vs \quad \text{ACI 318M-11}$$

It is a common practice not to provide shear reinforcement in foundations, therefore in this model, it is assumed there is no provided shear reinforcement and the nominal strength of the section is solely provided by the concrete.

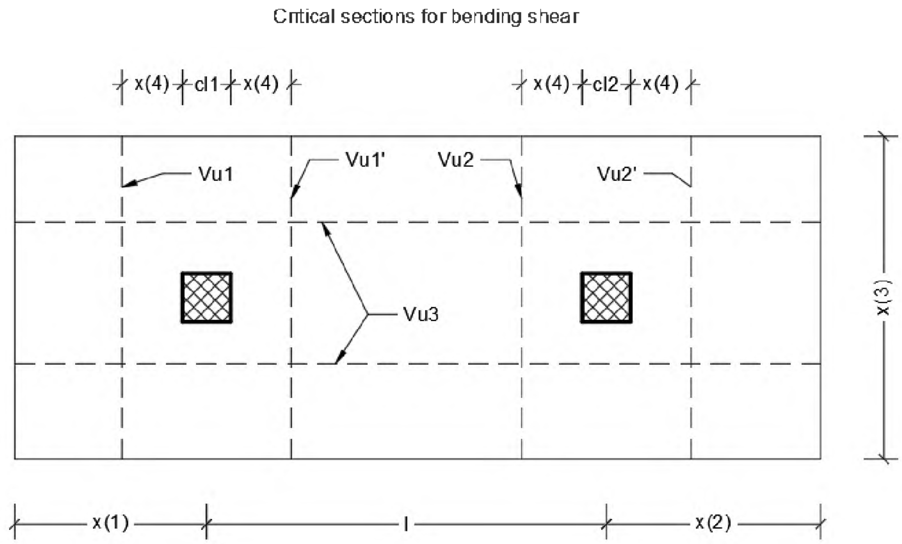

Figure 6. Critical area for flexural shear.

$$Vn = Vc = 0.17\lambda\sqrt{fc'} * bw * d$$ ACI 318M-11

$$Vn = Vc = 0.17\lambda\sqrt{fc'} * x(3) * x(4)$$ ACI 318M-11

And the factored shear force at the section can be calculated as follows:

$$Vu1 = \left(\frac{PU1 + PU2}{x(1) + x(2) + l}\right) * \left(x(1) - \left(\frac{cl1}{2}\right) - x(4)\right) \quad \text{(G22)}$$

$$Vu1' = PU1 - (\left(\frac{PU1 + PU2}{x(1) + x(2) + l}\right) * \left(x(1) + \left(\frac{cl1}{2}\right) + x(4)\right)) \quad \text{(G23)}$$

$$Vu2 = \left(\left(\frac{PU1 + PU2}{x(1) + x(2) + l}\right) * \left(x(1) + l - \left(\frac{cl2}{2}\right) - x(4)\right)\right) - PU1 \quad \text{(G24)}$$

$$Vu2' = \left(\frac{PU1 + PU2}{x(1) + x(2) + l}\right) * \left(x(2) - \left(\frac{cl2}{2}\right) - x(4)\right) \quad \text{(G25)}$$

$$Vu3 = \left(\frac{PU1 + PU2}{x(1) + x(2) + l}\right) * \left(x(3) - \left(\frac{cb2}{2}\right) - x(4)\right) * (x(1) + x(2) + l) \quad \text{(G26)}$$

$$\left(\frac{PU1 + PU2}{x(1) + x(2) + l}\right) * \left(x(1) - \left(\frac{cl1}{2}\right) - x(4)\right) - \phi * 0.17\lambda\sqrt{fc'} * x(3) * x(4) \leq 0 \quad \text{(C4)}$$

$$PU1 - \left(\left(\frac{PU1 + PU2}{x(1) + x(2) + l}\right) * \left(x(1) + \left(\frac{cl1}{2}\right) + x(4)\right)\right) - \phi * 0.17\lambda\sqrt{fc'} * x(3) * x(4) \leq 0 \quad \text{(C5)}$$

$$\left(\left(\frac{PU1 + PU2}{x(1) + x(2) + l}\right) * \left(x(1) + l - \left(\frac{cl2}{2}\right) - x(4)\right)\right) - PU1 - \phi * 0.17\lambda\sqrt{fc'} * x(3) * x(4) \leq 0 \quad \text{(C6)}$$

$$\left(\frac{PU1 + PU2}{x(1) + x(2) + l}\right) * \left(x(2) - \left(\frac{cl2}{2}\right) - x(4)\right) - \phi * 0.17\lambda\sqrt{fc'} * x(3) * x(4) \leq 0 \quad \text{(C7)}$$

$$\left(\frac{PU1 + PU2}{x(1) + x(2) + l}\right) * \left(x(3) - \left(\frac{cb2}{2}\right) - x(4)\right) * (x(1) + x(2) + l) - \phi * 0.17\lambda\sqrt{fc'} * (x(1) + x(2) + l) * x(4) \leq 0 \quad \text{(C8)}$$

where

PU1: Total factored load for column 1.
PU2: Total factored load for column 2.

According to Section 11.1.3.1, ACI 318M-11 the total factored loads for each column is the greatest of:

$$PU = 1.4PD \tag{G27}$$

$$PU = 1.2PD + 1.6PL \tag{G28}$$

d) The fourth set of constraints is also imposed by the ACI Code and its concerned about punching shear failure:

$$\emptyset Vn \geq Vu$$ ACI 318M-11

$$Vn = Vc + Vs$$ ACI 318M-11

It is also a common practice not to provide punching shear reinforcement in foundations, therefore the nominal strength of the section will be solely provided by the concrete.

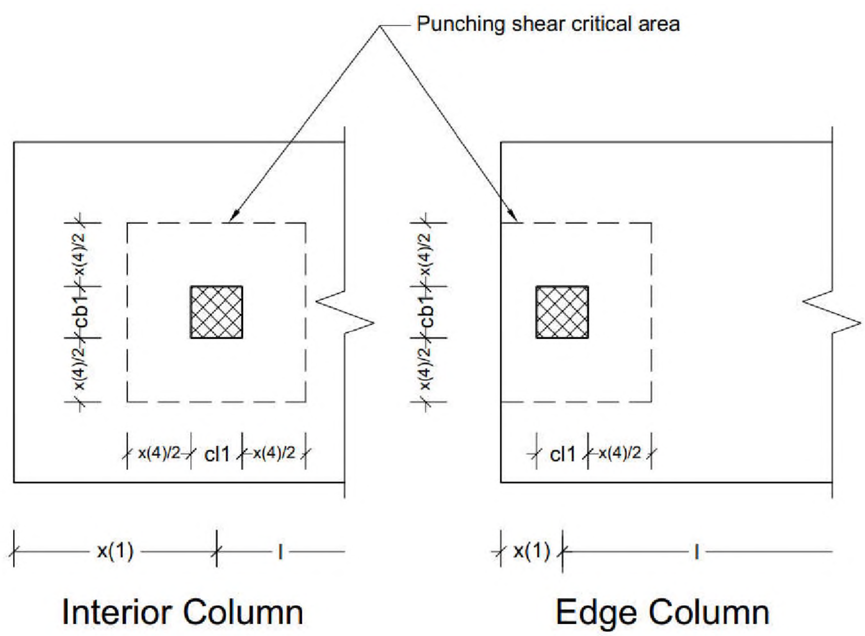

Figure 7. Critical area for Punching shear.

Section 11.11.2 allows a shear strength Vc in footings without shear reinforcement for two-way shear action, the smallest of

$$Vc \leq \left(\frac{1}{6}\right) * \left(1 + \left(\frac{2}{\beta}\right)\right) * \lambda * \sqrt{fc`} * b_0 * d$$ ACI 318M-11

$$Vc \leq \left(\frac{1}{12}\right) * \left(\left(\frac{\alpha_s * d}{b_0}\right) + 2\right) * \lambda * \sqrt{fc`} * b_0 * d$$ ACI 318M-11

$$Vc \leq \left(\frac{1}{3}\right) * \lambda * \sqrt{fc`} * b_0 * d$$ ACI 318M-11

where

β : ratio of long side to short side of the rectangular column.
b_0: perimeter of the critical section taken at from the loaded area.
d: effective depth of footing x(4).
λ : for normal-weight concrete 1.
α_s is assumed to be 40 for interior columns, 30 for edge columns, and 20for corner columns.

For column 1 when its interior column;

$$b_0 = (cl1 + x(4) + cb1 + x(4)) * 2 \tag{G29}$$

For column 1 when its edge column;

$$b_0 = (cb1 + x(4)) + \left(x(1) + \frac{cl1}{2} + \frac{x(4)}{2}\right) * 2 \tag{G30}$$

For column 2 when its interior column;

$$b_0 = \left(cl2 + x(4) + cb2 + x(4)\right) * 2 \tag{G31}$$

For column 2 when its edge column;

$$b_0 = (cb2 + x(4)) + \left(x(2) + \frac{cl2}{2} + \frac{x(4)}{2}\right) * 2 \tag{G32}$$

$$PU1 - \left(\frac{PU1 + PU2}{x(1) + x(2) + l}\right) * \left(cl1 + x(4)\right) * \left(cb1 + x(4)\right) - \phi Vc \leq 0 \tag{C9}$$

$$PU1 - \left(\frac{PU1 + PU2}{x(1) + x(2) + l}\right) * \left(x(1) + \frac{cl1}{2} + \frac{x(4)}{2}\right) * \left(cb1 + x(4)\right) - \phi Vc \leq 0 \quad \text{(C10)}$$

$$PU2 - \left(\frac{PU1 + PU2}{x(1) + x(2) + l}\right) * \left(cl2 + x(4)\right) * \left(cb2 + x(4)\right) - \phi Vc \leq 0 \quad \text{(C11)}$$

$$PU2 - \left(\frac{PU1 + PU2}{x(1) + x(2) + l}\right) * \left(x(2) + \frac{cl2}{2} + \frac{x(4)}{2}\right) * \left(cb2 + x(4)\right) - \phi Vc \leq 0 \quad \text{(C12)}$$

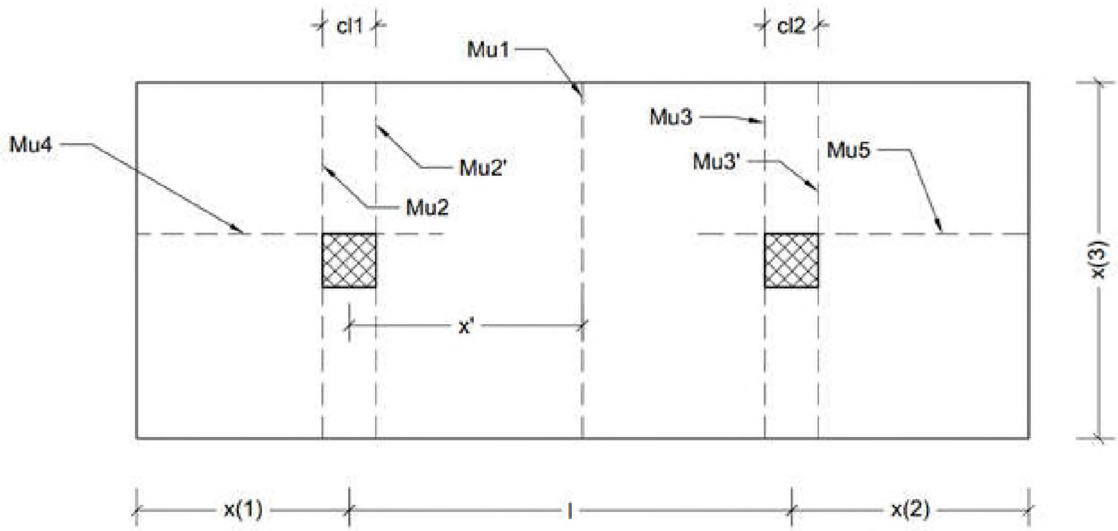

Figure 8. Critical section for flexural moment.

e) Another set of constraints are also imposed by the ACI Code and its concerned about flexural failure due to bending moments:

$$qu = \frac{PU1 + PU2}{(x(1) + x(2) + l) * x(3)} \quad \text{(G33)}$$

$$x' = \frac{\left(PU1 - \left(qu * B * x(1)\right)\right) * l}{qu * B * l} \quad \text{(G34)}$$

The factored moment at each section can be calculated as follows:

$$Mu1 = PU1 * x' - 0.5 * qu * x(3) * \left(x(1) + x'\right)^2 \tag{G35}$$

$$Mu2 = 0.5qu * x(3) * (x(1) - \frac{cl1}{2})^2 \tag{G36}$$

$$Mu2' = \left(qu * x(3) * \frac{\left(x(1) + \frac{cl1}{2}\right)^2}{2}\right) - \frac{PU1 * cl1}{2} \tag{G37}$$

$$Mu3 = -PU1 * \left(l - \frac{cl2}{2}\right) + \left(qu * x(3) * \frac{\left(x(1) + l - \frac{cl2}{2}\right)^2}{2}\right) \tag{G38}$$

$$Mu3' = -PU1 * \left(l + \frac{cl2}{2}\right) - \left(PU2 * \frac{CL2}{2}\right) + \left(qu * x(3) * \frac{\left(x(1) + l + \frac{cl2}{2}\right)^2}{2}\right) \tag{G39}$$

$$Mu4 = \frac{PU1}{2X(3)} * (\frac{x(3)}{2} - \frac{cb1}{2})^2 \tag{G40}$$

$$Mu5 = \frac{PU2}{2X(3)} * (\frac{x(3)}{2} - \frac{cb2}{2})^2 \tag{G41}$$

where

Mu1-5: the factored moment at each section

Based on section 10.3.1, ACI 318M-11. Design of cross-sections subject to flexure or axial loads, or to combined flexure and axial loads, shall be based on stress and strain compatibility using assumptions in section 10.2.

$$T = C \tag{G42}$$

$$As * fy = 0.85fc' * a * b \tag{G43}$$

$$\rho = \frac{As}{b * d} \tag{G44}$$

$$\rho * b * d * fy = 0.85fc' * a * b \tag{G45}$$

$$a = \frac{\rho * b * d * fy}{0.85 * fc' * b} \tag{G46}$$

$$Mn = T(d - \frac{a}{2}) \tag{G47}$$

$$Mn = \rho * b * d * fy * \left(d - \frac{\rho * fy * d}{2 * 0.85 * fc'}\right) \tag{G48}$$

$$m = \frac{fy}{0.85 * fc'} \tag{G49}$$

$$Rn = \frac{Mn}{b * d^2} = \rho f_y \left(1 - \rho * \frac{m}{2}\right) \tag{G50}$$

$$\rho \geq \frac{1}{m} * \left(1 - \sqrt{1 - \left(\frac{2mRn}{f_y}\right)}\right) \tag{G51}$$

$$\frac{As}{b * d} \geq \frac{1}{m} * \left(1 - \sqrt{1 - \left(\frac{2mRn}{f_y}\right)}\right) \tag{G52}$$

$$\frac{As}{b * d} \geq \frac{0.85 * fc'}{f_y} * \left(1 - \sqrt{1 - \left(\frac{2mMn}{f_y * b * d^2}\right)}\right) \tag{G53}$$

$$\frac{As}{b*d} \geq \frac{0.85*fc'}{f_y}*\left(1-\sqrt{1-\left(\frac{2mMu}{f_y*\emptyset*b*d^2}\right)}\right) \tag{G54}$$

$$0 \geq \frac{0.85*fc'}{f_y}*\left(1-\sqrt{1-\left(\frac{2Mu}{0.85f_c{}'*\emptyset*b*d^2}\right)}\right)-\frac{As}{b*d} \tag{G55}$$

Moreover, section 10.5.4 and section 7.12.2.1 of the ACI require the area of reinforcement not to be less than:

$$As \geq 0.0018*\frac{420}{fy}*b*h \qquad \text{ACI 318M-11}$$

$$0 \geq 0.0018*\frac{420}{fy}*b*h-As \tag{G56}$$

The constraints will be:

$$0 \geq \frac{0.85*fc'}{f_y}*\left(1-\sqrt{1-\left(\frac{2Mu1}{0.85f_c{}'*\emptyset*x(3)*x(4)^2}\right)}\right)-\frac{x(5)}{x(3)*x(4)} \tag{C13}$$

$$0 \geq \left(0.0018*\frac{420}{fy}*x(3)*\left(x(4)+C+\frac{dpl}{2}\right)\right)-x(5) \tag{C14}$$

$$0 \geq \frac{0.85*fc'}{f_y}*\left(1-\sqrt{1-\left(\frac{2Mu2}{0.85f_c{}'*\emptyset*x(3)*x(4)^2}\right)}\right)-\frac{x(6)}{x(3)*x(4)} \tag{C15}$$

$$0 \geq \frac{0.85 * fc'}{f_y} * \left(1 - \sqrt{1 - \left(\frac{2Mu2'}{0.85f_c' \ * \emptyset * x(3) * x(4)^2}\right)}\right) - \frac{x(6)}{x(3) * x(4)} \tag{C16}$$

$$0 \geq \left(0.0018 * \frac{420}{fy} * x(3) * \left(x(4) + C + \frac{dpl}{2}\right)\right) - x(6) \tag{C17}$$

$$0 \geq \frac{0.85 * fc'}{f_y} * \left(1 - \sqrt{1 - \left(\frac{2Mu3}{0.85f_c' \ * \emptyset * x(3) * x(4)^2}\right)}\right) - \frac{x(7)}{x(3) * x(4)} \tag{C18}$$

$$0 \geq \frac{0.85 * fc'}{f_y} * \left(1 - \sqrt{1 - \left(\frac{2Mu3'}{0.85f_c' \ * \emptyset * x(3) * x(4)^2}\right)}\right) - \frac{x(7)}{x(3) * x(4)} \tag{C19}$$

$$0 \geq \left(0.0018 * \frac{420}{fy} * x(3) * \left(x(4) + C + \frac{dpl}{2}\right)\right) - x(7) \tag{C20}$$

$$0 \geq \frac{0.85 * fc'}{f_y} * \left(1 - \sqrt{1 - \left(\frac{2Mu4}{0.85f_c' \ * \emptyset * (x(4) + cb1) * x(4)^2}\right)}\right) - \frac{x(8)}{x(4) * (x(4) + cb1)} \tag{C21}$$

$$0 \geq \left(0.0018 * \frac{420}{fy} * (x(4) + cl1) * \left(x(4) + C + \frac{dpl}{2}\right)\right) - x(8) \tag{C22}$$

$$0 \geq \frac{0.85 * fc'}{f_y} * \left(1 - \sqrt{1 - \left(\frac{2Mu5}{0.85f_c' * \emptyset * (x(4) + cb2) * x(4)^2}\right)}\right) - \frac{x(9)}{x(4) * (x(4) + cb2)} \tag{C23}$$

$$0 \geq \left(0.0018 * \frac{420}{fy} * (x(4) + cl2) * \left(x(4) + C + \frac{dpl}{2}\right)\right) - x(9) \tag{C24}$$

$$0 \geq \left(0.0018 * \frac{420}{fy} * (x(1) + x(2) + l) * \left(x(4) + C + \frac{dpl}{2}\right)\right) - x(10) \tag{C25}$$

f) Another set of constraints are also imposed by the ACI Code, Section 10.3. This defines the concept of tension or compression-controlled sections in terms of net tensile strain in the reinforcement closest to the tension face. Tension-controlled sections are those sections in which the ε^t is equal to or greater than 0.005 just as the concrete in the compression reaches its assumed strain limit of 0.003.

$$\varepsilon^t = 0.003(\frac{d - c}{c}) \geq 0.005 \tag{G57}$$

$$c = a/\beta$$

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$$a = \frac{As * fy}{0.85fc * b} \tag{G58}$$

$$\beta = 0.85 - 0.007(fc' - 28) \tag{G59}$$

$$0.65 \le \beta \le 0.85$$

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$$0.003(\frac{0.85b * d * \beta * fc}{As * fy} - 1) \ge 0.005 \quad \text{(G60)}$$

$$0 \ge 0.005 - 0.003(\frac{0.85b * d * \beta * fc}{As * fy} - 1) \quad \text{(G61)}$$

$$0 \ge 0.005 - 0.003\left(\frac{0.85x(3) * x(4) * \beta * fc}{x(5) * fy} - 1\right) \quad \text{(C26)}$$

$$0 \ge 0.005 - 0.003\left(\frac{0.85x(3) * x(4) * \beta * fc}{x(6) * fy} - 1\right) \quad \text{(C27)}$$

$$0 \ge 0.005 - 0.003\left(\frac{0.85x(3) * x(4) * \beta * fc}{x(7) * fy} - 1\right) \quad \text{(C28)}$$

$$0 \ge 0.005 - 0.003\left(\frac{0.85(x(1) + x(2) + l) * x(4) * \beta * fc}{x(8) * fy} - 1\right) \quad \text{(C29)}$$

$$0 \ge 0.005 - 0.003\left(\frac{0.85(x(1) + x(2) + l) * x(4) * \beta * fc}{x(9) * fy} - 1\right) \quad \text{(C30)}$$

g) The last set of constraints are also imposed by the ACI Code and its concerned about spacing between the reinforcement bars. According to ACI 7.6, the minimum clear spacing between parallel bars in a layer shall be dp, but not less than 25mm. And the maximum spacing should be less than three times the thickness but not more than 450mm.

$$\le 3h \quad \text{(G62)}$$

$$S \leq 0.45 \tag{G63}$$

$$S \geq dp \tag{G64}$$

$$S \geq 0.025 \tag{G65}$$

$$\#\ of\ bars = \frac{As}{Aperbar} \tag{G66}$$

$$S = \frac{b - 2C - \#of\ bars * dp}{\#of\ bars - 1} \tag{G67}$$

$$\left(\frac{x(3) - 2C - \left(\frac{x(5)}{Adpl}\right) * dpl}{\left(\frac{x(5)}{Adpl}\right) - 1} \right) - 3\left(x(3) + C + \frac{dpl}{2}\right) \leq 0 \tag{C31}$$

$$\left(\frac{x(3) - 2C - \left(\frac{x(5)}{Adpl}\right) * dpl}{\left(\frac{x(5)}{Adpl}\right) - 1} \right) - 0.45 \leq 0 \tag{C32}$$

$$dpl - \left(\frac{x(3) - 2C - \left(\frac{x(5)}{Adpl}\right) * dpl}{\left(\frac{x(5)}{Adpl}\right) - 1} \right) \leq 0 \tag{C33}$$

$$0.025 - \left(\frac{x(3) - 2C - \left(\frac{x(5)}{Adpl}\right) * dpl}{\left(\frac{x(5)}{Adpl}\right) - 1} \right) \leq 0 \tag{C34}$$

$$\left(\frac{x(3) - 2C - \left(\frac{x(6)}{Adpl}\right) * dpl}{\left(\frac{x(6)}{Adpl}\right) - 1} \right) - 3\left(x(3) + C + \frac{dpl}{2}\right) \leq 0 \tag{C35}$$

$$\left(\frac{x(3) - 2C - \left(\frac{x(6)}{Adpl}\right) * dpl}{\left(\frac{x(6)}{Adpl}\right) - 1}\right) - 0.45 \leq 0 \tag{C36}$$

$$dpl - \left(\frac{x(3) - 2C - \left(\frac{x(6)}{Adpl}\right) * dpl}{\left(\frac{x(6)}{Adpl}\right) - 1}\right) \leq 0 \tag{C37}$$

$$0.025 - \left(\frac{x(3) - 2C - \left(\frac{x(6)}{Adpl}\right) * dpl}{\left(\frac{x(6)}{Adpl}\right) - 1}\right) \leq 0 \tag{C38}$$

$$\left(\frac{x(3) - 2C - \left(\frac{x(7)}{Adpl}\right) * dpl}{\left(\frac{x(7)}{Adpl}\right) - 1}\right) - 3\left(x(3) + C + \frac{dpl}{2}\right) \leq 0 \tag{C39}$$

$$\left(\frac{x(3) - 2C - \left(\frac{x(7)}{Adpl}\right) * dpl}{\left(\frac{x(7)}{Adpl}\right) - 1}\right) - 0.45 \leq 0 \tag{C40}$$

$$dpl - \left(\frac{x(3) - 2C - \left(\frac{x(7)}{Adpl}\right) * dpl}{\left(\frac{x(7)}{Adpl}\right) - 1}\right) \leq 0 \tag{C41}$$

$$0.025 - \left(\frac{x(3) - 2C - \left(\frac{x(7)}{Adpl}\right) * dpl}{\left(\frac{x(7)}{Adpl}\right) - 1}\right) \leq 0 \tag{C42}$$

$$\left(\frac{x(1) + \frac{cl1}{2} + \frac{x(4)}{2} - C - \left(\frac{x(8)}{Adpt}\right) * dpt}{\left(\frac{x(8)}{Adpt}\right) - 1}\right) - 3\left(x(3) + C + \frac{dpl}{2}\right) \leq 0 \tag{C43}$$

$$\left(\frac{x(1)+\frac{cl1}{2}+\frac{x(4)}{2}-C-\left(\frac{x(8)}{Adpt}\right)*dpt}{\left(\frac{x(8)}{Adpt}\right)-1}\right)-0.45\leq 0 \tag{C44}$$

$$dpt-\left(\frac{x(1)+\frac{c1}{2}+\frac{x(4)}{2}-C-\left(\frac{x(8)}{Adpt}\right)*dpt}{\left(\frac{x(8)}{Adpt}\right)-1}\right)\leq 0 \tag{C45}$$

$$0.025-\left(\frac{x(1)+\frac{cl1}{2}+\frac{x(4)}{2}-C-\left(\frac{x(8)}{Adpt}\right)*dpt}{\left(\frac{x(8)}{Adpt}\right)-1}\right)\leq 0 \tag{C46}$$

$$\left(\frac{x(2)+\frac{cl2}{2}+\frac{x(4)}{2}-C-\left(\frac{x(9)}{Adpt}\right)*dpt}{\left(\frac{x(9)}{Adpt}\right)-1}\right)-3\left(x(3)+C+\frac{dpl}{2}\right)\leq 0 \tag{C47}$$

$$\left(\frac{x(2)+\frac{cl2}{2}+\frac{x(4)}{2}-C-\left(\frac{x(9)}{Adpt}\right)*dpt}{\left(\frac{x(9)}{Adpt}\right)-1}\right)-0.45\leq 0 \tag{C48}$$

$$dpt-\left(\frac{x(2)+\frac{cl2}{2}+\frac{x(4)}{2}-C-\left(\frac{x(9)}{Adpt}\right)*dpt}{\left(\frac{x(9)}{Adpt}\right)-1}\right)\leq 0 \tag{C49}$$

$$0.025-\left(\frac{x(2)+\frac{cl2}{2}+\frac{x(4)}{2}-C-\left(\frac{x(9)}{Adpt}\right)*dpt}{\left(\frac{x(9)}{Adpt}\right)-1}\right)\leq 0 \tag{C50}$$

$$\left(\frac{(1) + x(2) + l - 2C - \left(\frac{x(10)}{Adpt}\right) * dpt}{\left(\frac{x(10)}{Adpt}\right) - 1}\right) - 3\left(x(3) + C + \frac{dpl}{2}\right) \leq 0 \tag{C51}$$

$$\left(\frac{x(1) + x(2) + l - 2C - \left(\frac{x(10)}{Adpt}\right) * dpt}{\left(\frac{x(10)}{Adpt}\right) - 1}\right) - 0.45 \leq 0 \tag{C52}$$

$$dpt - \left(\frac{x(1) + x(2) + l - 2C - \left(\frac{x(10)}{Adpt}\right) * dpt}{\left(\frac{x(10)}{Adpt}\right) - 1}\right) \leq 0 \tag{C53}$$

$$0.025 - \left(\frac{x(1) + x(2) + l - 2C - \left(\frac{x(10)}{Adpt}\right) * dpt}{\left(\frac{x(10)}{Adpt}\right) - 1}\right) \leq 0 \tag{C54}$$

CHAPTER V

NUMERICAL EXPERIMENTATION

The optimization process was formulated in MATLAB and operated using 5 different solvers and algorithms. The MATLAB code is found in the appendices. First, an example case of concentric loaded rectangular combined footing (see appendix G) which involves a restricted side dimension (property line column) with known optimum solution was optimized. Next, the variables which that were considered to affect the cost was modified in various cases and then their respective effect on the objective function was recorded. The design parameters are given in Table 3 below.

The material properties and ranges used in the optimization as upper bounds for some of the variables are shown in Tables 4 and 5, and the cost data based on the US national average for the used materials are shown in Table 6. The conventional design solution is shown in Table 7.

Table III. Input data used in optimization.

		Unit	Symbol
Clear cover	7.50E-02	m	C
Depth of footing	1.52E+00	m	hf
Excavation Margin	5.00E-01	m	E
Soil initial void ratio	7.50E-01		e0
Dead load on column 1	7.56E+02	KN	PD1
LIVE load on column 1	3.34E+02	KN	PL1
Dead load on column 2	8.90E+02	KN	PD2
LIVE load on column 2	5.56E+02	KN	PL2
Column 1 length	4.57E-01	m	cl1
Column 1 width	4.57E-01	m	cb1
Column 2 length	4.57E-01	m	cl2
Column 2 length	4.57E-01	m	cb2
Center to center column spacing	9.14E+00	m	l
Diameter for longitudinal reinforcement	2.50E-02	m	dpl
Diameter for transverse reinforcement	2.50E-02	m	dpt
Diameter for dowels	1.60E-02	m	dpd
Moist nit weight of soil	1.81E+01	KN/m^3	Ws
Unit weight of concrete	2.50E+01	KN/m^3	Wc
Surcharge	0.00E+00	KN/m^2	qanet
Factor of safety for bearing capacity	3.00E+00		FS
Allowable settlement	5.08E-02	m	Sca
Depth of water table from surface	1.83E+00	m	Dw
Depth of soil layer	3.05E+00	m	D
Cohesion of soil	3.35E+01	KN/m^2	C
Soil friction angle	2.00E+01	degree	ø
Compression index	1.50E-01		Cc

Table IV. Material properties.

Concrete compressive strength fc'	27.579 MPa
Steel yield strength fy	413.68 MPa
Specific gravity of steel, γs	7.86 t/m3

Table V. Ranges used in optimization.

		Unit	Symbol
Upper value for Distance between left edge to center column 1, $x(1)$	0.6096	m	r1
Upper value for Distance between right edge to center column 2, $x(2)$	10	m	r2
Upper value for width (B), $x(3)$	10	m	r3

Table VI. Cost data as taken from (Rsmeans 2011) based on national average.

Item	Price	Unit	symbol
Excavation	19.80	$\$/ m^3$	Rexc
Form work	77.18	$\$/ m^2$	Rfw
Concrete(4000psi)	182.56	$\$/ m^3$	Rcon
Insulation	13.35	$\$/ m^2$	Rins
Blinding Concrete (2500 psi)	169.41	$\$/ m^3$	Rb
Reinforcement Steel labor	2524.29	$\$/ ton$	Rst

Table VII. Conventional design.

		Unit	Symbol
Distance between left edge to center column 1	6.10E-01	m	x(1)
Distance between right edge to center column 2	1.89E+00	m	x(2)
Width (B)	3.07E+00	m	x(3)
Effective depth (d)	5.80E-01	m	x(4)
As for -ve moment	1.33E-02	m^2	x(5)
As for +ve moment under column 1	3.75E-03	m^2	x(6)
As for +ve moment under column 2	3.75E-03	m^2	x(7)
As for transverse moment under column 1	1.91E-03	m^2	x(8)
As for transverse moment under column 2	2.88E-03	m^2	x(9)
As for temp and shrinkage	1.42E-02	m^2	x(10)
Objective function value	6.10E-01	$	

CHAPTER VI

RESULTS AND DISCUSSION

Optimum results for the example are given in Tables 8. All optimum dimensions and reinforcements were found using several algorithms. The results from three algorithms (Global search: sqp and interior point, and Multistart: interior point) were identical and the results from the other two solvers (genetic and pattern search) were slightly higher as both solvers got trapped in a local optimum. The solution found by the former three solvers is identical to the conventional design optimum solution.

For instance, in the global search and Multistart optimization approaches, the length from the left edge to the centre of the left column, i.e., *x(1)* was evaluated to be 0.61m (which was equal to the conventional design case). The results from the genetic and pattern search were 0.373 m and 0.390 m, respectively (which corresponded to a difference of almost 37%). Moreover, the calculated values for the effective depth was 0.58 m for all the optimization solvers except the genetic algorithm, which reported a slightly lesser value of 0.572 m.

Table VIII. Optimization results.

Algorithm	Global search: sqp	Global search: interior point	Multistart: interior point	genetic	pattern search
Function value	1.32E+04	1.32E+04	1.32E+04	1.36E+04	1.34E+04
time	1.97E+04	1.97E+04	1.19E+05	1.62E+05	1.86E+01
x(1)	6.10E-01	6.10E-01	6.10E-01	3.73E-01	3.90E-01
x(2)	1.89E+00	1.89E+00	1.89E+00	1.66E+00	1.67E+00
x(3)	3.07E+00	3.07E+00	3.07E+00	3.30E+00	3.21E+00
x(4)	5.80E-01	5.80E-01	5.80E-01	5.72E-01	5.80E-01
x(5)	1.33E-02	1.33E-02	1.33E-02	1.46E-02	1.43E-02
x(6)	3.75E-03	3.75E-03	3.75E-03	3.98E-03	3.94E-03
x(7)	3.75E-03	3.75E-03	3.75E-03	3.98E-03	3.94E-03
x(8)	1.91E-03	1.91E-03	1.91E-03	2.85E-03	2.06E-03
x(9)	2.88E-03	2.88E-03	2.62E-03	3.10E-03	2.93E-03
x(10)	1.42E-02	1.42E-02	1.42E-02	1.35E-02	1.43E-02
Exit Flag	1	1	1	1	1

Table IX. Properties of the optimization process.

Algorithm	GS: sqp	GS: interior point	Multistart: interior point	genetic	pattern search
Num trial points	400000	400000			
Num stage points	80000	80000			
Max Function evaluation			100000		5000000
Max Iteration			10000		100000
Num trial points			1000		
pop size				40000	
Max generation				16000	
Max stall				2000	

Summary of the design variables for the conventional design and the optimization are shown in Table 10. The optimization results from Global Search solver with the interior point algorithm were selected for the rest of the experimentation as the time for the mentioned solver is the minimum in comparison to the other solvers which were able to find the global minimum.

Table X. Conventional design and optimization results.

	Conventional design	optimization
Distance between left edge to center column 1	6.10E-01	6.10E-01
Distance between right edge to center column 2	1.89E+00	1.89E+00
Width (B)	3.07E+00	3.07E+00
Effective depth (d)	5.80E-01	5.80E-01
As for -ve moment	1.33E-02	1.33E-02
As for +ve moment under column 1	3.75E-03	3.75E-03
As for +ve moment under column 2	3.75E-03	3.75E-03
As for transverse moment under column 1	1.91E-03	1.91E-03
As for transverse moment under column 2	2.62E-03	2.62E-03
As for temp and shrinkage	1.42E-02	1.42E-02

Several optimization processes were performed for the same example using multiple steel grades (grade 420, 520, and 550) and for different loads ratios to investigate the overall effect of using higher-grade materials. Here, the research investigated whether the reinforcement steel or concrete grade tremendously affected the total construction cost or not. The load ratios were chosen such that the load varies as a percentage of the original load (100%,150%, and 200%) as shown on table 11.

Table XI. Load ratios.

Load ratio	100%	150%	200%	Unit
Dead load on column 1	7.56E+02	1.13E+03	1.51E+03	KN
LIVE load on column 1	3.34E+02	5.01E+02	6.68E+02	KN
Dead load on column 2	8.90E+02	1.34E+03	1.78E+03	KN
LIVE load on column 2	5.56E+02	8.34E+02	1.11E+03	KN

The influence of using higher steel grade on the objective function can be seen in Figure 11. For the three load percentages (i.e., 100%,150%, and 200%), the objective function value decreased slightly with the increase of steel yield strength. In particular, at the 100% load ratio, the total cost dropped from a value of 13194 to a value of 12287 $. It is worth taking note that this is valid due to the fact that the optimization model allows all the other design variables (which contribute to the cost) to change once the material properties change.

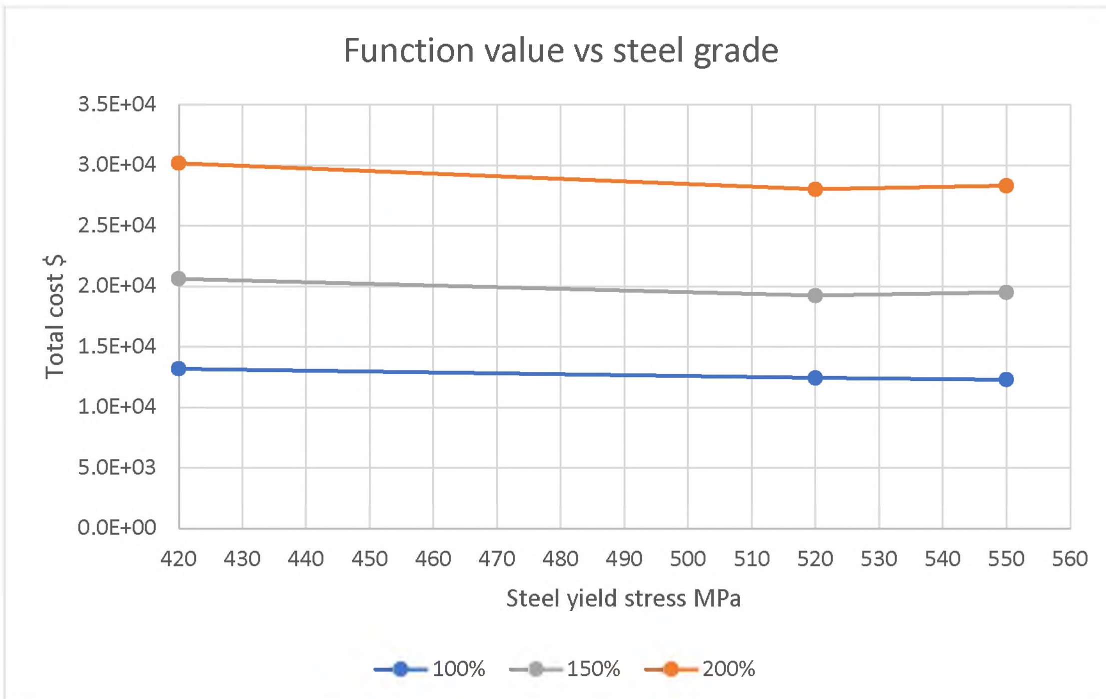

Figure 9. Function value ($) vs Steel grade (MPa).

Several other optimization processes were performed for the same example to investigate the effect of concrete grade on the total cost of the structure. In this case, different concrete grades with compressive strength range from 20 to 82 MPa and for different load ratios were used. The influence of using higher strength concrete on cost can be seen in Figure 12, and for 100%,150%, and 200% load percentages. For compressive strength from 20 to 35 MPa, it does not seem to have a major impact on the total cost of the structure.

For higher-strength concrete, the objective function value tends to increase significantly with the increase of compressive strength. A critical look at all load ratios show that the rise in the objective function was very gentle between 20MPa and 42MPa but rises sharply between 55MPa and 85MPa. This is because concrete with compressive strength above 55MPa is significantly more expensive than concrete with lower strengths.

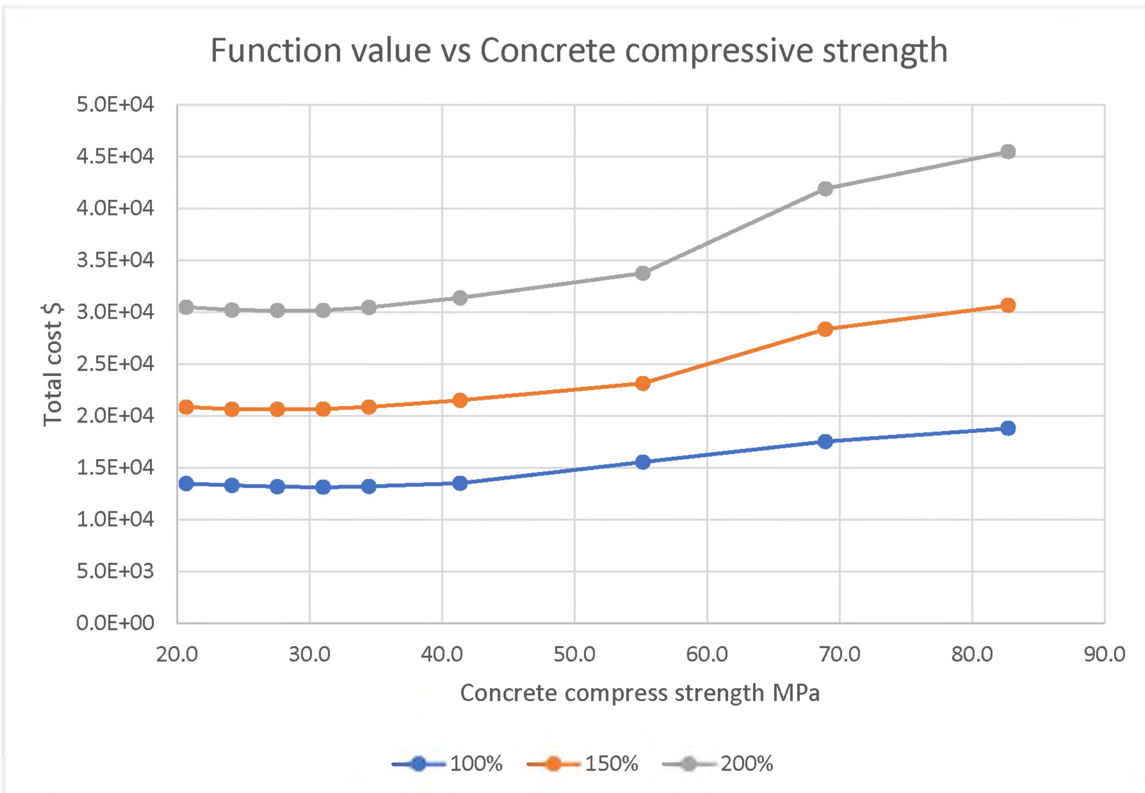

Figure 10. Function value ($) vs Concrete compressive strength (MPa).

A different optimization scenario was performed for the same examples to investigate the effect of foundation depth on the overall cost of the combined footing. In this case, different depths were selected for the combined footing, the depth varied from 0.6096 to 3.048m. The influence of changing the depth on cost can be seen in Figure 12. It seems that the cost decreased significantly in the range between 0.5 to 2.0m, then the cost remained almost constant with the depth increase. The exhibited behavior is due to the fact that the settlement constraint was active until reaching a point where the settlement is no longer controlling, and the constraint is inactive.

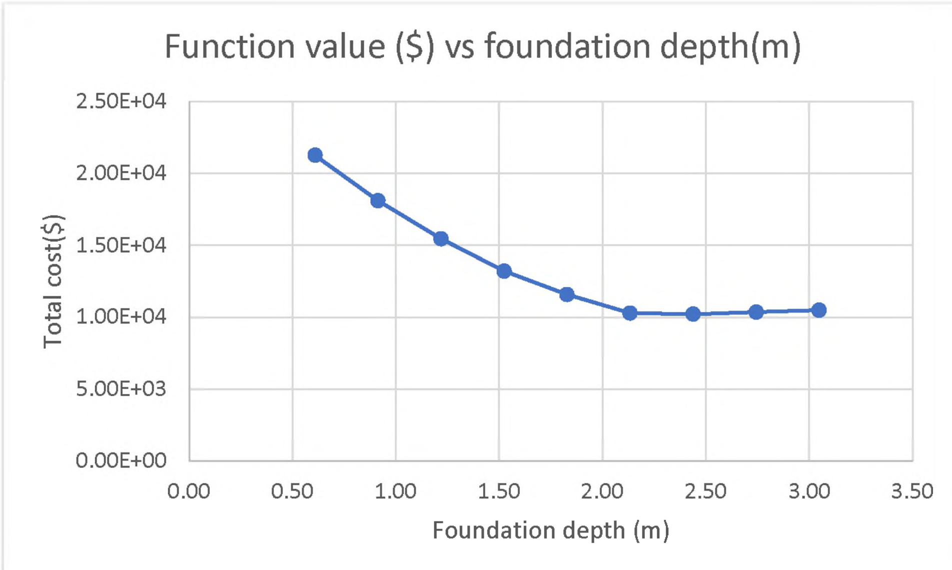

Figure 11. Function value ($) vs Foundation depth (m).

Different optimization design scenario was carried out to investigate the effect of column shape on the total cost. The same parameters in the previous examples were assumed to ensure that circular columns instead of their original square columns were used (but had an equivalent magnitude of the area). The results are illustrated in Table 12 and figure 13. There results showed negligible increase in cost for the combined footing with the circular columns

Table XII. Function value for circular and rectangular columns.

Circular	Rectangular
13192	13178

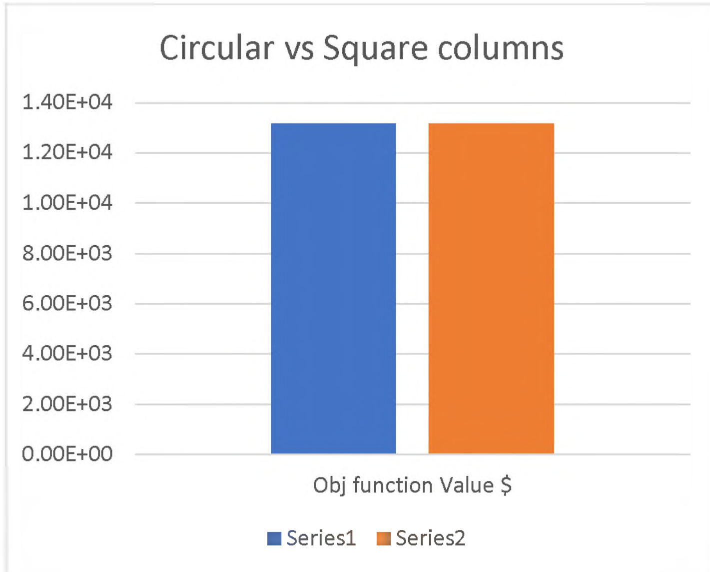

Figure 12. Function value for circular and rectangular columns.

Few assumptions were made in this particular model. For instance, the strain in the extreme steel fibers is assumed to be more than 0.005 (i.e., tension-controlled). Moreover, no shear reinforcement is provided to resist both flexural and punching shear and the shear resistance is solely provided by concrete since it is the most common practice. This model was limited to concentric loaded rectangular footings with uniform soil pressure distribution.

CHAPTER VII

CONCLUSION

The main goal of the research was to formulate an optimization model that can solve concentric loaded rectangular footings in a general manner. This stems from the advantages accrued from using this approach in design jobs instead of the traditional methods. Particularly, it ensures a safe design with the least possible cost in time. The optimization model developed works to find the value of optimization variables, i.e. foundation dimensions and steel reinforcement areas, that gives the minimum value of the objective function (a comprehensive cost function). These values bounded with upper and lower limits and constrained with several sets of inequality constraints that represent the structural, geotechnical, and logical requirements.

Five solvers were applied on one example using MATLAB computer program with the global optimization toolbox. Two solvers did not find the global minima and got trapped in local minima and therefore eliminated. Three solvers showed identical results for the global optimum. However, optimization time varied significantly, and the quickest of the three solvers was used for the rest of the experimentation. The comparison between the conventional design and the optimization results showed identical results. Charts for steel yield strength, concrete compressive strength and foundation depth against the objective function value was built to show the variation of cost for different loads to column spacing ratios. It was found that the cost tended to decrease with the use of higher steel grades for all load cases. There was no major effect for the concrete compressive strength in the range of 20 to 35 MPa on the value of the objective function. However, for higher concrete strengths >35MPa, the objective function value increased significantly for most cases.

The influence of changing the depth on the total cost was significant in terms of cost reduction for the depth ranges between 0.5 to 2.0m, then the cost remained almost constant with the depth increase.

There is no significant impact of the column shape on the cost function value for the combined footings. The optimization model made it easier to investigate the effect of changing material properties and site conditions on the overall cost since you can compare the global minimum for different optimization processes with different material properties. Future studies will be conducted to extend the model capability to cases of optimization in eccentrically loaded foundations with real soil pressure, as well as incorporate material properties as design variables.

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APPENDIX A

OPTIMIZATION MATLAB CODE

```
     clear
clc
tic
%Input variables in the accompaying excel sheet
data = xlsread('opt.xls');

%------------------------------------------------------

%Choose solver
Solver=1;
%1 for GlobalSearch-sqp
%2 for GlobalSearch-Intpoint
%3 for Multistarrt
%4 for GENETIC
%5 for Pattern Search

%------------------------------------------------------

Cshape=data(1,1);% column shape,0 for circular,1 for
rectangular or square
C =data(3,1);%clear cover
hf =data(4,1);%footing depth in (m)
E =data(5,1);%excavation margin in (m)
Rexc=data(6,1);%rate for excavation ($/m3)
Rfw=data(7,1);%rate for form work ($/m2)
Rcon=data(8,1);%rate for concrete ($/m3)
Rins=data(9,1);%rate for insulation ($/m2)
Rb=data(10,1);%rate for blinding concrete ($/m3)
Rst=data(11,1);%rate for reinforcment steel ($/TON)
e0=data(12,1);%Soil initial void ratio
PD1=data(13,1);%Column 1 -Dead load (KN)
PL1=data(14,1);%Column 1 -Live load (KN)
PD2=data(15,1);%Column 2 -Dead load (KN)
PL2=data(16,1);%Column 2 -Live load (KN)
c1=data(17,1);%Column 1 diameter (m)
c2=data(18,1);%Column 2 diameter (m)
cl1=data(19,1);%Column 1 dim along the long axis in (m)
cb1=data(20,1);%Column 1 dim along the short axis in
(m)
cl2=data(21,1);%Column 2 dim along the long axis in (m)
```

```
cb2=data(22,1);%Column 2 dim along the short axis in
(m)
fc=data(23,1);%concrete compressive strength in (MPa)
fy=data(24,1);%steel yield stress in (MPa)
l=data(25,1);%center to center spacing between columns
in(m)
phi1=data(26,1);%one way shear reduction factor
phi2=data(27,1);%two way shear reduction factor
phi3=data(28,1);%flexture reduction factor
r1=data(29,1);%upper bound for x1
r2=data(30,1);%upper bound for x2
r3=data(31,1);%upper bound for B
dpl=data(32,1);%diameter for longitudenal bars in (M)
dpt=data(33,1);%diameter for transverse bars in (M)
dpd=data(34,1)%diameter for the dowels in (M)
FS=data(35,1);%Factor of safety for bearing capacity
Sca=data(36,1);%Allowable settelment
lam=data(37,1);%lightweight-aggregate-concrete factor
surcharge=data(38,1);%surcharge in (kn/m2)
Dw=data(39,1);%Depth of water taple from surface
D=data(40,1);%Depth of soil layer
GAMA=data(41,1);%Moist unit weight of soil
CO=data(42,1);%Cohesion of soil
angle=data(43,1);%Soil friction angle
Cc=data(44,1);%Consolidation coefficant

HC=D-hf;

%-------------------------------------------------------

%starting point
x0(1)=data(45,1);
x0(2)=data(46,1);
x0(3)=data(47,1);
x0(4)=data(48,1);
x0(5)=data(49,1);
x0(6)=data(50,1);
x0(7)=data(51,1);
x0(8)=data(52,1);
x0(9)=data(53,1);
x0(10)=data(54,1);

%-------------------------------------------------------
```

```
%Bars Area import
X= xlsread('ReinforcmentArea.xls');
V= X(X(:,1) == dpl,:);
N= X(X(:,1) == dpt,:);
Adpl= V(2);%area for longitudenal bars in (m2)
Adpt= N(2);%area for transverse bars in (m2)

%-----------------------------------------------------

%Bearing capacity factores import
FNC= xlsread('Nc.xls');
FFNC= FNC(FNC(:,1) == angle,:);
NC= FFNC(2);

FNQ= xlsread('Nq.xls');
FFNQ= FNQ(FNQ(:,1) == angle,:);
NQ= FFNQ(2);

FNG= xlsread('Ng.xls');
FFNG= FNG(FNG(:,1) == angle,:);
NG= FFNG(2);

%-----------------------------------------------------

%Minimum effective depth (d)
dmin=max([0.15,(0.24*fy*dpd/(lam*((fc)^0.5)))+(dpd+dpt+
(dpl/2)),((0.043*fy*dpd)+(dpd+dpt+(dpl/2))),(0.2+(dpd+d
pt+(dpl/2)))]);

%-----------------------------------------------------

%load combination
ff1=[1.4*PD1,((1.2*PD1)+(1.6*PL1))];
ff2=[(1.4*PD2),((1.2*PD2)+(1.6*PL2))];
PU1=max(ff1);%Factored load for column 1
PU2=max(ff2);%Factored load for column 2

%-----------------------------------------------------

%sigma0 calculation
if Dw==0
    sigma0=(GAMA-9.807)*((D+hf)/2)
else
    if 0<Dw<hf
```

```
        sigma0=(Dw*GAMA)+((hf-Dw)*(GAMA-9.807))+((GAMA-
9.807)*((D-hf)/2))
    else
        if hf<Dw<((D+hf)/2)
               sigma0=(Dw*GAMA)+((GAMA-
9.807)*((D/2)+(hf/2)-Dw))
        else sigma0=GAMA*((D/2)+(hf/2))
        end
    end
end

%-------------------------------------------------------

%beta calculation
Beta=0.85-0.007*(fc-28);
if Beta >=0.85
    Beta=0.85;
else
    if Beta<=0.65
        Beta=0.65;
    end
end
%-------------------------------------------------------
if Cshape==1

%two way shear variables
ff3=[cl1/cb1,cb1/cl1];
ff4=[cl2/cb2,cb2/cl2];
B1=max(ff3);
B2=max(ff4);
%Minimum width (B)
Bmin=max([cb1,cb2]);

%-------------------------------------------------------

%objective function
fun = @(x)
objfunr(x,l,E,hf,Rb,Rfw,Rcon,Rins,Rst,Rexc,C,cl1,cl2,cb
1,cb2,dpl);
fun(x0)

%-------------------------------------------------------

%linear constrains
```

```
A=[];
b=[];
Aeq=[];
beq=[];

%----------------------------------------------------------

%variables bounds
lb=[cl1/2,cl2/2,Bmin,dmin,0,0,0,0,0,0];
ub=[r1,r2,r3,hf,inf,inf,inf,inf,inf,inf];

%----------------------------------------------------------

%nonlinear constrains
nonlincon=@(x)
constr(x,l,fc,fy,PU1,PU2,phi1,phi2,phi3,B1,B2,C,cl1,cl2
,cb1,cb2,Cc,HC,e0,sigma0,Sca,FS,Dw,CO,NC,NQ,GAMA,NG,PD1
,PL1,PD2,PL2,dpl,dpt,Adpl,Adpt,hf,Beta,surcharge)

%-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-
if Solver==1
    % fmincon options
options =
optimoptions(@fmincon,'algorithm','sqp','Display','fina
l-detailed','ConstraintTolerance',1e-
8,'MaxFunctionEvaluations',5000,'MaxIterations',2000,'O
ptimalityTolerance',1e-6);
%[c,ceq]=nonlincon(x0);

%problem defenition
problem =
createOptimProblem('fmincon','x0',x0,'objective',fun,'l
b',lb,'ub',ub,'nonlcon',nonlincon,'options',options);

% fmincon SOLVER
%[x,fval,eflag,output] = fmincon(problem);

% Global search solver
gs =
GlobalSearch('Display','final','FunctionTolerance',0,'N
umTrialPoints',400000,'NumStageOnePoints',80000)
rng default % for reproducibility
[xg,fg,exitflag,output,solutions] = run(gs,problem)
```

```
else if Solver==2
        % fmincon options
options = optimoptions(@fmincon,'Display','final-
detailed','ConstraintTolerance',1e-
8,'MaxFunctionEvaluations',5000,'MaxIterations',2000,'O
ptimalityTolerance',1e-6);
%[c,ceq]=nonlincon(x0);

%problem defenition
problem =
createOptimProblem('fmincon','x0',x0,'objective',fun,'l
b',lb,'ub',ub,'nonlcon',nonlincon,'options',options);

% fmincon SOLVER
%[x,fval,eflag,output] = fmincon(problem);

% Global search solver
gs =
GlobalSearch('Display','final','FunctionTolerance',0,'N
umTrialPoints',100000,'NumStageOnePoints',20000)
rng default % for reproducibility
[xg,fg,exitflag,output,solutions] = run(gs,problem)

    else if Solver==3
            % fmincon options
options = optimoptions(@fmincon,'Display','off',
'MaxFunctionEvaluations',10000,'MaxIterations',1000)
%'Algorithm','sqp'
%problem defenition
problem =
createOptimProblem('fmincon','x0',x0,'objective',fun,'l
b',lb,'ub',ub,'nonlcon',nonlincon,'options',options)

%Multistart solver
ms =
MultiStart('PlotFcn',@gsplotbestf,'UseParallel',true)
[xg,fg,exitflag,output,solutions] =
run(ms,problem,100000)

        else if solver==4

options=optimoptions('ga','InitialPopulationMatrix',x0,
'PlotFcn',{@gaplotbestf,@gaplotmaxconstr},'Display','it
er','PopulationSize',40000,'ConstraintTolerance',0.0000
```

```
001,'FunctionTolerance',0.0000001,'MaxGenerations',1600
0,'UseParallel',true,'MaxStallGenerations',2000);
nvars=10;

rng(1,'twister');
[xg,fg,exitflag,output,solutions]=ga(fun,nvars,[],[],[]
,[],lb,ub,nonlincon,options);
            else
                options =
optimoptions('patternsearch','InitialMeshSize',100,'Dis
play','iter','PlotFcn',@psplotbestf,'MaxFunctionEvaluat
ions',5000000,'MaxIterations',100000,'UseParallel',true
,'ConstraintTolerance',1.0000e-
20,'FunctionTolerance',1.0000e-
20,'MeshTolerance',1.0000e-20);
 %x =
patternsearch(fun,x0,[],[],[],[],lb,ub,nonlincon,option
s)
 [x,fval,exitflag,output] =
patternsearch(fun,x0,[],[],[],[],lb,ub,nonlincon,option
s)
            end
        end
    end
end

%++++++++++++++++++++++++++++++++++++++++++++++++++++++
+++++++++++++++++++++++++++++++++++++++++++++++++++++++
++++++++++++++++++++++++++++
else
  %two way shear variables
B1=1;
B2=1;
%Minimum width (B)
Bmin=max([c1,c2]);

%objective function
fun = @(x)
objfunc(x,l,E,hf,Rb,Rfw,Rcon,Rins,Rst,Rexc,C,c1,c2,dpl)
;
fun(x0)
disp(['initial objective;' num2str(fun(x0))])

%linear constrains
```

```
A=[];
b=[];
Aeq=[];
beq=[];

%variables bounds
lb=[c1/2,c2/2,Bmin,dmin,0,0,0,0,0,0];
ub=[r1,r2,r3,hf,inf,inf,inf,inf,inf,inf];

%nonlinear constrains
nonlincon=@(x)
constc(x,l,fc,fy,PU1,PU2,phi1,phi2,phi3,B1,B2,C,c1,c2,C
c,HC,e0,sigma0,Sca,FS,Dw,CO,NC,NQ,GAMA,NG,PD1,PL1,PD2,P
L2,dpl,dpt,Adpl,Adpt,hf,Beta,surcharge)

%-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-=-
=-=-
if Solver==1
    % fmincon options
options =
optimoptions(@fmincon,'algorithm','sqp','Display','fina
l-
detailed','ConstraintTolerance',0.000001,'MaxFunctionEv
aluations',10000,'MaxIterations',4000,'OptimalityTolera
nce',1e-5);
%[c,ceq]=nonlincon(x0);

%problem defenition
problem =
createOptimProblem('fmincon','x0',x0,'objective',fun,'l
b',lb,'ub',ub,'nonlcon',nonlincon,'options',options);

% fmincon SOLVER
%[x,fval,eflag,output] = fmincon(problem);

% Global search solver
gs =
GlobalSearch('Display','final','FunctionTolerance',0,'N
umTrialPoints',1000000,'NumStageOnePoints',200000)
rng default % for reproducibility
[xg,fg,exitflag,output,solutions] = run(gs,problem)

else if Solver==2
        % fmincon options
```

```
options = optimoptions(@fmincon,'Display','final-
detailed','ConstraintTolerance',1e-
14,'MaxFunctionEvaluations',5000,'MaxIterations',2000,'
OptimalityTolerance',1e-8);
%[c,ceq]=nonlincon(x0);

%problem defenition
problem =
createOptimProblem('fmincon','x0',x0,'objective',fun,'l
b',lb,'ub',ub,'nonlcon',nonlincon,'options',options);

% fmincon SOLVER
%[x,fval,eflag,output] = fmincon(problem);

% Global search solver
gs =
GlobalSearch('Display','final','FunctionTolerance',0,'N
umTrialPoints',100000,'NumStageOnePoints',20000)
rng default % for reproducibility
[xg,fg,exitflag,output,solutions] = run(gs,problem)

    else if Solver==3
            % fmincon options
options = optimoptions(@fmincon,'Display','off',
'MaxFunctionEvaluations',10000,'MaxIterations',1000)
%'Algorithm','sqp'
%problem defenition
problem =
createOptimProblem('fmincon','x0',x0,'objective',fun,'l
b',lb,'ub',ub,'nonlcon',nonlincon,'options',options)

%Multistart solver
ms =
MultiStart('PlotFcn',@gsplotbestf,'UseParallel',true)
[xg,fg,exitflag,output,solutions] =
run(ms,problem,100000)

        else if solver==4

options=optimoptions('ga','InitialPopulationMatrix',x0,
'PlotFcn',{@gaplotbestf,@gaplotmaxconstr},'Display','it
er','PopulationSize',40000,'ConstraintTolerance',0.0000
001,'FunctionTolerance',0.0000001,'MaxGenerations',1600
0,'UseParallel',true,'MaxStallGenerations',2000);
```

```
nvars=10;

rng(1,'twister');
[xg,fg,exitflag,output,solutions]=ga(fun,nvars,[],[],[]
,[],lb,ub,nonlincon,options);
            else
                options =
optimoptions('patternsearch','InitialMeshSize',100,'Dis
play','iter','PlotFcn',@psplotbestf,'MaxFunctionEvaluat
ions',5000000,'MaxIterations',100000,'UseParallel',true
,'ConstraintTolerance',1.0000e-
20,'FunctionTolerance',1.0000e-
20,'MeshTolerance',1.0000e-20);
 %x =
patternsearch(fun,x0,[],[],[],[],lb,ub,nonlincon,option
s)
 [x,fval,exitflag,output] =
patternsearch(fun,x0,[],[],[],[],lb,ub,nonlincon,option
s)
            end
        end
    end
end
end

toc
tt=toc
```

APPENDIX B

COMBINED FOOTING WITH CIRCULAR COLUMNS OBJECTIVE FUNCTION MATLAB CODE

```
function
o=objfunc(x,l,E,hf,Rb,Rfw,Rcon,Rins,Rst,Rexc,C,c1,c2,dpl)
o=((((x(1)+x(2)+l+2*E)*(x(3)+2*E)*hf)*Rexc)+(((x(1)+x(2)+l+
0.2)*(x(3)+0.2)*0.1)*Rb)+(((x(1)+x(2)+l+x(3))*(x(4)+C+(dpl/
2))*2)*Rfw)+(((x(1)+x(2)+l)*(x(4)+C+(dpl/2))*x(3))*Rcon)+((
((x(1)+x(2)+l+x(3))*(x(4)+C+(dpl/2))*2)+((x(1)+x(2)+l)*x(3)
)-(c1*0.78571428)-
(c2*0.78571428))*Rins)+(((x(5)*(x(1)+x(2)+l))+(x(6)*(x(4)+c
1))+(x(7)*(x(4)+c2))+(x(8)*x(3))+(x(9)*x(3))+(x(10)*x(3)))*
7.85*Rst))
end
```

APPENDIX C

COMBINED FOOTING WITH RECTANGULAR COLUMNS OBJECTIVE

FUNCTION MATLAB CODE

```
function
o=objfunr(x,l,E,hf,Rb,Rfw,Rcon,Rins,Rst,Rexc,C,cl1,cl2,cb1,
cb2,dpl)
o=((((x(1)+x(2)+l+2*E)*(x(3)+2*E)*hf)*Rexc)+(((x(1)+x(2)+l+
0.2)*(x(3)+0.2)*0.1)*Rb)+((((x(1)+x(2)+l+x(3))*(x(4)+C+(dpl/
2))*2)*Rfw)+((((x(1)+x(2)+l)*(x(4)+C+(dpl/2))*x(3))*Rcon)+((
((x(1)+x(2)+l+x(3))*(x(4)+C+(dpl/2))*2)+((x(1)+x(2)+l)*x(3)
)-(cl1*cb1)-
(cl2*cb2))*Rins)+(((x(5)*(x(1)+x(2)+l))+(x(6)*(x(4)+cl1))+(
x(7)*(x(4)+cl2))+(x(8)*x(3))+(x(9)*x(3))+(x(10)*x(3)))*7.85
*Rst))
end
```

APPENDIX D

COMBINED FOOTING WITH CIRCULAR COLUMNS CONSTRAINTS

FUNCTION

```
function [c,ceq]
=constc(x,l,fc,fy,PU1,PU2,phi1,phi2,phi3,B1,B2,C,c1,c2,
Cc,HC,e0,sigma0,Sca,FS,Dw,CO,NC,NQ,GAMA,NG,PD1,PL1,PD2,
PL2,dpl,dpt,Adpl,Adpt,hf,Beta,surcharge)
%Bearing Capacity
c(1)=((((PL1+PD1+PL2+PD2)*FS)/(((0.5+(0.5*(Dw/(hf+x(3))
)))*((CO*NC)+(hf*NQ*GAMA)+(0.5*GAMA*x(3)*NG))-
(GAMA*hf)-(surcharge)))-((x(1)+x(2)+l)*x(3)));
%settelments
c(2)=(((Cc*HC)/(1+e0))*log10((sigma0+((PD1+PL1+PD2+PL2)
/((x(1)+x(2)+l+(HC/2))*(x(3)+(HC/2)))))/sigma0))-Sca;
%one way shear Vu-?Vc<=0
c(3)=abs(((PU1+PU2)/(x(1)+x(2)+l))*(x(1)-(c1/2)-x(4)))-
(phi1*x(3)*x(4)*1000*(1/6)*((fc)^0.5));
c(4)=abs(PU1-
(((PU1+PU2)/(x(1)+x(2)+l))*(x(1)+(c1/2)+x(4))))-
(phi1*x(3)*x(4)*1000*(1/6)*((fc)^0.5));
c(5)=abs((((PU1+PU2)/(x(1)+x(2)+l))*(x(1)+l-(c1/2)-
x(4)))-PU1)-(phi1*x(3)*x(4)*1000*(1/6)*((fc)^0.5));
c(6)=abs(((PU1+PU2)/(x(1)+x(2)+l))*(x(2)-(c2/2)-x(4)))-
(phi1*x(3)*x(4)*1000*(1/6)*((fc)^0.5));
c(7)=abs(((PU1+PU2)/((x(1)+x(2)+l)*x(3)))*((x(3)/2)-
(c1/2)-x(4))*(x(1)+x(2)+l))-
(phi1*(x(1)+x(2)+l)*x(4)*1000*(1/6)*((fc)^0.5));
%two way shear Vu-?Vc<=0 as interior column
c(8)=PU1-
(((PU1+PU2)/((x(1)+x(2)+l)*x(3)))*(((c1+x(4))^2)*0.7857
142857))-
((1000*phi2/6)*(1+(2/B1))*((fc)^0.5)*x(4)*(((c1+x(4))*3
.14)));
c(9)=PU1-
(((PU1+PU2)/((x(1)+x(2)+l)*x(3)))*(((c1+x(4))^2)*0.7857
142857))-
((1000*phi2/12)*((((40*x(4))/(((c1+x(4))+(c1+x(4)))*2))
)+2)*((fc)^0.5)*x(4)*((c1+x(4))*3.14));
c(10)=PU1-
(((PU1+PU2)/((x(1)+x(2)+l)*x(3)))*(((c1+x(4))^2)*0.7857
```

```
142857))-
((1000*phi2/3)*((fc)^0.5)*x(4)*((c1+x(4))*3.14));
c(11)=PU2-
(((PU1+PU2)/((x(1)+x(2)+l)*x(3)))*(((c2+x(4))^2)*0.7857
142857))-
((1000*phi2/6)*(1+(2/B2))*((fc)^0.5)*x(4)*((c2+x(4))*3.
14));
c(12)=PU2-
(((PU1+PU2)/((x(1)+x(2)+l)*x(3)))*(((c2+x(4))^2)*0.7857
142857))-
((1000*phi2/12)*((((40*x(4))/(((c2+x(4))+(c2+x(4)))*2))
)+2)*((fc)^0.5)*x(4)*((c2+x(4))*3.14));
c(13)=PU2-
(((PU1+PU2)/((x(1)+x(2)+l)*x(3)))*(((c2+x(4))^2)*0.7857
142857))-
((1000*phi2/3)*((fc)^0.5)*x(4)*((c2+x(4))*3.14));
%two way shear Vu-?Vc<=0 as edge column
c(14)=PU1-
(((PU1+PU2)/((x(1)+x(2)+l)*x(3)))*((((c1+x(4))^2)*0.785
7142857)*(1-
((acosd(2*x(1)/(c1+x(4))))/180))+(0.125*((c1+x(4))^2)*s
in(2*(acosd(2*x(1)/(c1+x(4))))))))-
((1000*phi2/6)*(1+(2/B1))*((fc)^0.5)*x(4)*(((c1+x(4))*3
.14)*(1-((acosd(2*x(1)/(c1+x(4))))/180))));
c(15)=PU1-
(((PU1+PU2)/((x(1)+x(2)+l)*x(3)))*((((c1+x(4))^2)*0.785
7142857)*(1-
((acosd(2*x(1)/(c1+x(4))))/180))+(0.125*((c1+x(4))^2)*s
in(2*(acosd(2*x(1)/(c1+x(4))))))))-
((1000*phi2/12)*(((30*x(4))/((2*x(1))+c1+c1+(2*x(4))))+
2)*((fc)^0.5)*x(4)*(((c1+x(4))*3.14)*(1-
((acosd(2*x(1)/(c1+x(4))))/180))));
c(16)=PU1-
(((PU1+PU2)/((x(1)+x(2)+l)*x(3)))*((((c1+x(4))^2)*0.785
7142857)*(1-
((acosd(2*x(1)/(c1+x(4))))/180))+(0.125*((c1+x(4))^2)*s
in(2*(acosd(2*x(1)/(c1+x(4))))))))-
((1000*phi2/3)*((fc)^0.5)*x(4)*(((c1+x(4))*3.14)*(1-
((acosd(2*x(1)/(c1+x(4))))/180))));
c(17)=PU2-
(((PU1+PU2)/((x(1)+x(2)+l)*x(3)))*((((c2+x(4))^2)*0.785
7142857)*(1-
((acosd(2*x(2)/(c2+x(4))))/180))+(0.125*((c2+x(4))^2)*s
in(2*(acosd(2*x(2)/(c2+x(4))))))))-
```

```
((1000*phi2/6)*(1+(2/B2))*((fc)^0.5)*x(4)*(((c2+x(4))*3
.14)*(1-((acosd(2*x(2)/(c2+x(4))))/180))));
c(18)=PU2-
(((PU1+PU2)/((x(1)+x(2)+l)*x(3)))*((((c2+x(4))^2)*0.785
7142857)*(1-
((acosd(2*x(2)/(c2+x(4))))/180))+(0.125*((c2+x(4))^2)*s
in(2*(acosd(2*x(2)/(c2+x(4))))))))-
((1000*phi2/12)*(((30*x(4))/((2*x(2))+c2+c2+(2*x(4))))+
2)*((fc)^0.5)*x(4)*(((c2+x(4))*3.14)*(1-
((acosd(2*x(2)/(c2+x(4))))/180))));
c(19)=PU2-
(((PU1+PU2)/((x(1)+x(2)+l)*x(3)))*((((c2+x(4))^2)*0.785
7142857)*(1-
((acosd(2*x(2)/(c2+x(4))))/180))+(0.125*((c2+x(4))^2)*s
in(2*(acosd(2*x(2)/(c2+x(4))))))))-
((1000*phi2/3)*((fc)^0.5)*x(4)*(((c2+x(4))*3.14)*(1-
((acosd(2*x(2)/(c2+x(4))))/180))));
%-ve Moment Mu1 Mu-?Mn<=0
c(20)=(-100000*(((x(5)/(x(3)*x(4)))-((0.85*fc/fy)*(1-
(1-((0.002*((PU1*(((PU1-
(((PU1+PU2)/(x(1)+x(2)+l))*x(1)))*l)/(((PU1+PU2)/(x(1)+
x(2)+l))*l)))-
(0.5*((PU1+PU2)/(x(1)+x(2)+l))*((x(1)+(((PU1-
(((PU1+PU2)/(x(1)+x(2)+l))*x(1)))*l)/(((PU1+PU2)/(x(1)+
x(2)+l))*l)))^2))))/(0.85*fc*phi3*x(3)*(x(4)^2))))^0.5)
))));
c(21)=(-100000*(((x(5))-
(0.0018*420*x(3)*(x(4)+C+(dpl/2))/fy))));
%+ve Moment Mu2 Mu-?Mn<=0
c(22)=(-100000*(((x(6)/(x(3)*x(4)))-((0.85*fc/fy)*(1-
(1-((0.002*(0.5*((PU1+PU2)/(x(1)+x(2)+l))*((x(1)-
(c1/2))^2)))/(0.85*fc*phi3*x(3)*(x(4)^2))))^0.5)))));
c(23)=(-100000*(((x(6)/(x(3)*x(4)))-((0.85*fc/fy)*(1-
(1-
((0.002*(abs((0.5*((PU1+PU2)/(x(1)+x(2)+l))*((x(1)+(c1/
2))^2))-
(0.5*PU1*c1))))/(0.85*fc*phi3*x(3)*(x(4)^2))))^0.5)))))
;
c(24)=(-100000*(((x(6))-
(0.0018*420*x(3)*(x(4)+C+(dpl/2))/fy))));
%+ve Moment Mu3 Mu-?Mn<=0
c(25)=(-100000*(((x(7)/(x(3)*x(4)))-((0.85*fc/fy)*(1-
(1-((0.002*(-PU1*(l-
(0.5*c2))+(0.5*((PU1+PU2)/(x(1)+x(2)+l))*((x(1)+l-
```

```
(0.5*c2))^2))))/(0.85*fc*phi3*x(3)*(x(4)^2))))^0.5)))))
;
c(26)=(-100000*(((x(7)/(x(3)*x(4)))-((0.85*fc/fy)*(1-
(1-((0.002*(-PU1*(l+(0.5*c2))-
(0.5*PU2*c2)+(0.5*((PU1+PU2)/(x(1)+x(2)+l))*((x(1)+l+(0
.5*c2))^2))))/(0.85*fc*phi3*x(3)*(x(4)^2))))^0.5)))));
c(27)=(-100000*(((x(7))-
(0.0018*420*x(3)*(x(4)+C+(dpl/2))/fy))));
%+ve transverse Moment Mu4 Mu-?Mn<=0
c(28)=(-100000*(((x(8)/(x(4)*(x(4)+c1)))-
((0.85*fc/fy)*(1-(1-((0.002*((PU1/(2*x(3)))*((x(3)/2)-
(c1/2))^2))/(0.85*fc*phi3*(x(4)+c1)*(x(4)^2))))^0.5))))
);
c(29)=(-100000*(((x(8))-
(0.0018*420*(x(4)+C+(dpl/2))*(x(4)+c1)/fy))));
%+ve transverse Moment Mu5 Mu-?Mn<=0
c(30)=(-100000*(((x(9)/(x(4)*(x(4)+c2)))-
((0.85*fc/fy)*(1-(1-((0.002*((PU2/(2*x(3)))*((x(3)/2)-
(c2/2))^2))/(0.85*fc*phi3*(x(4)+c2)*(x(4)^2))))^0.5))))
);
c(31)=(-100000*(((x(9))-
(0.0018*420*(x(4)+C+(dpl/2))*(x(4)+c2)/fy))));
%Temp and shrinkage steel
c(32)=(-100000*(((x(10))-
(0.0018*420*(x(1)+x(2)+l)*(x(4)+C+(dpl/2))/fy))));
%spacing for longitudenal As1
c(33)=(((x(3)-(2*C)-((x(5)/Adpl)*dpl))/((x(5)/Adpl)-
1))-(3*(x(4)+(dpl/2)+C)));
c(34)=(((x(3)-(2*C)-((x(5)/Adpl)*dpl))/((x(5)/Adpl)-
1))-0.45);
c(35)=dpl-((x(3)-(2*C)-((x(5)/Adpl)*dpl))/((x(5)/Adpl)-
1));
c(36)=0.025-((x(3)-(2*C)-
((x(5)/Adpl)*dpl))/((x(5)/Adpl)-1));
%spacing for longitudenal As2
c(37)=(((x(3)-(2*C)-((x(6)/Adpl)*dpl))/((x(6)/Adpl)-
1))-(3*(x(4)-(dpl/2)-C)));
c(38)=(((x(3)-(2*C)-((x(6)/Adpl)*dpl))/((x(6)/Adpl)-
1))-0.45);
c(39)=dpl-((x(3)-(2*C)-((x(6)/Adpl)*dpl))/((x(6)/Adpl)-
1));
c(40)=0.025-((x(3)-(2*C)-
((x(6)/Adpl)*dpl))/((x(6)/Adpl)-1));
%spacing for longitudenal As3
```

```
c(41)=(((x(3)-(2*C)-((x(7)/Adpl)*dpl))/((x(7)/Adpl)-
1))-(3*(x(4)+(dpl/2)+C)));
c(42)=(((x(3)-(2*C)-((x(7)/Adpl)*dpl))/((x(7)/Adpl)-
1))-0.45);
c(43)=dpl-((x(3)-(2*C)-((x(7)/Adpl)*dpl))/((x(7)/Adpl)-
1));
c(44)=0.025-((x(3)-(2*C)-
((x(7)/Adpl)*dpl))/((x(7)/Adpl)-1));
%spacing for longitudenal As3
c(45)=(((x(3)-(2*C)-((x(7)/Adpl)*dpl))/((x(7)/Adpl)-
1))-(3*(x(4)+(dpl/2)+C)));
c(46)=(((x(3)-(2*C)-((x(7)/Adpl)*dpl))/((x(7)/Adpl)-
1))-0.45);
c(47)=dpl-((x(3)-(2*C)-((x(7)/Adpl)*dpl))/((x(7)/Adpl)-
1));
c(48)=0.025-((x(3)-(2*C)-
((x(7)/Adpl)*dpl))/((x(7)/Adpl)-1));
%spacing for longitudenal As4
c(49)=(((x(1)+(c1/2)+(x(4)/2)-C-
((x(8)/Adpt)*dpt))/((x(8)/Adpt)-1))-
(3*(x(4)+(dpl/2)+C)));
c(50)=(((x(1)+(c1/2)+(x(4)/2)-C-
((x(8)/Adpt)*dpt))/((x(8)/Adpt)-1))-0.45);
c(51)=dpl-((x(1)+(c1/2)+(x(4)/2)-C-((x(8)/Adpt)*dpt)));
c(52)=0.025-((x(1)+(c1/2)+(x(4)/2)-C-
((x(8)/Adpt)*dpt)));
%spacing for longitudenal As5
c(53)=(((x(2)+(c2/2)+(x(4)/2)-C-
((x(9)/Adpt)*dpt))/((x(9)/Adpt)-1))-
(3*(x(4)+(dpl/2)+C)));
c(54)=(((x(2)+(c2/2)+(x(4)/2)-C-
((x(9)/Adpt)*dpt))/((x(9)/Adpt)-1))-0.45);
c(55)=dpl-((x(2)+(c2/2)+(x(4)/2)-C-((x(9)/Adpt)*dpt)));
c(56)=0.025-((x(2)+(c2/2)+(x(4)/2)-C-
((x(9)/Adpt)*dpt)));
%spacing for longitudenal As6
c(57)=(((x(1)+x(2)+l-(2*C)-
((x(10)/Adpt)*dpt))/((x(10)/Adpt)-1))-
(3*(x(4)+(dpl/2)+C)));
c(58)=(((x(1)+x(2)+l-(2*C)-
((x(10)/Adpt)*dpt))/((x(10)/Adpt)-1))-0.45);
c(59)=dpl-((x(1)+x(2)+l-(2*C)-
((x(10)/Adpt)*dpt))/((x(10)/Adpt)-1));
```

```
c(60)=0.025-((x(1)+x(2)+l-(2*C)-
((x(10)/Adpt)*dpt))/((x(10)/Adpt)-1));
%Strain >=0.005
c(61)=0.005-
0.003*(((x(4)*x(3)*fc*Beta*0.85)/(x(5)*fy))-1);
c(62)=0.005-
0.003*(((x(4)*x(3)*fc*Beta*0.85)/(x(6)*fy))-1);
c(63)=0.005-
0.003*(((x(4)*x(3)*fc*Beta*0.85)/(x(7)*fy))-1);
c(64)=0.005-
0.003*(((x(4)*(x(1)+x(2)+l)*fc*Beta*0.85)/(x(8)*fy))-
1);
c(65)=0.005-
0.003*(((x(4)*(x(1)+x(2)+l)*fc*Beta*0.85)/(x(9)*fy))-
1);
ceq= ((x(1)+x(2)+l)-
(2*(((PD2+PL2)*l)/(PD1+PL1+PD2+PL2)+x(1))));
end
```

APPENDIX E

COMBINED FOOTING WITH RECTANGULAR COLUMNS CONSTRAINTS FUNCTION

```
function [c,ceq]
=constr(x,l,fc,fy,PU1,PU2,phi1,phi2,phi3,B1,B2,C,cl1,cl
2,cb1,cb2,Cc,HC,e0,sigma0,Sca,FS,Dw,CO,NC,NQ,GAMA,NG,PD
1,PL1,PD2,PL2,dpl,dpt,Adpl,Adpt,hf,Beta,surcharge)
%Bearing Capacity
c(1)=((((PL1+PD1+PL2+PD2)*FS)/(((0.5+(0.5*(Dw/(hf+x(3))
)))*((CO*NC)+(hf*NQ*GAMA)+(0.5*GAMA*x(3)*NG))-
(GAMA*hf)-(surcharge)))-((x(1)+x(2)+l)*x(3)));
%settelments
c(2)=(((Cc*HC)/(1+e0))*log10((sigma0+((PD1+PL1+PD2+PL2)
/((x(1)+x(2)+l+(HC/2))*(x(3)+(HC/2)))))/sigma0))-Sca;
%one way shear Vu-?Vc<=0
c(3)=abs(((PU1+PU2)/(x(1)+x(2)+l))*(x(1)-(cl1/2)-
x(4)))-(phi1*x(3)*x(4)*1000*(1/6)*((fc)^0.5));
c(4)=abs(PU1-
(((PU1+PU2)/(x(1)+x(2)+l))*(x(1)+(cl1/2)+x(4))))-
(phi1*x(3)*x(4)*1000*(1/6)*((fc)^0.5));
c(5)=abs((((PU1+PU2)/(x(1)+x(2)+l))*(x(1)+l-(cl1/2)-
x(4)))-PU1)-(phi1*x(3)*x(4)*1000*(1/6)*((fc)^0.5));
c(6)=abs(((PU1+PU2)/(x(1)+x(2)+l))*(x(2)-(cl2/2)-
x(4)))-(phi1*x(3)*x(4)*1000*(1/6)*((fc)^0.5));
c(7)=abs(((PU1+PU2)/((x(1)+x(2)+l)*x(3)))*((x(3)/2)-
(cb1/2)-x(4))*(x(1)+x(2)+l))-
(phi1*(x(1)+x(2)+l)*x(4)*1000*(1/6)*((fc)^0.5));
%two way shear Vu-?Vc<=0 as interior column
c(8)=PU1-
(((PU1+PU2)/((x(1)+x(2)+l)*x(3)))*(cl1+x(4))*(cb1+x(4))
)-
((1000*phi2/6)*(1+(2/B1))*((fc)^0.5)*x(4)*(((cl1+x(4))+
(cb1+x(4)))*2));
c(9)=PU1-
(((PU1+PU2)/((x(1)+x(2)+l)*x(3)))*(cl1+x(4))*(cb1+x(4))
)-
((1000*phi2/12)*((((40*x(4))/(((cl1+x(4))+(cb1+x(4)))*2
)))+2)*((fc)^0.5)*x(4)*(((cl1+x(4))+(cb1+x(4)))*2));
c(10)=PU1-
(((PU1+PU2)/((x(1)+x(2)+l)*x(3)))*(cl1+x(4))*(cb1+x(4))
```

```
)-
((1000*phi2/3)*((fc)^0.5)*x(4)*(((cl1+x(4))+(cb1+x(4)))
*2));
c(11)=PU2-
(((PU1+PU2)/((x(1)+x(2)+l)*x(3)))*(cl2+x(4))*(cb2+x(4))
)-
((1000*phi2/6)*(1+(2/B2))*((fc)^0.5)*x(4)*(((cl2+x(4))+
(cb2+x(4)))*2));
c(12)=PU2-
(((PU1+PU2)/((x(1)+x(2)+l)*x(3)))*(cl2+x(4))*(cb2+x(4))
)-
((1000*phi2/12)*((((40*x(4))/(((cl2+x(4))+(cb2+x(4)))*2
)))+2)*((fc)^0.5)*x(4)*(((cl2+x(4))+(cb2+x(4)))*2));
c(13)=PU2-
(((PU1+PU2)/((x(1)+x(2)+l)*x(3)))*(cl2+x(4))*(cb2+x(4))
)-
((1000*phi2/3)*((fc)^0.5)*x(4)*(((cl2+x(4))+(cb2+x(4)))
*2));
%two way shear Vu-?Vc<=0 as edge column
c(14)=PU1-
(((PU1+PU2)/((x(1)+x(2)+l)*x(3)))*(x(1)+(cl1/2)+(x(4)/2
))*(cb1+x(4)))-
((1000*phi2/6)*(1+(2/B1))*((fc)^0.5)*x(4)*((2*x(1))+cl1
+cb1+(2*x(4))));
c(15)=PU1-
(((PU1+PU2)/((x(1)+x(2)+l)*x(3)))*(x(1)+(cl1/2)+(x(4)/2
))*(cb1+x(4)))-
((1000*phi2/12)*(((30*x(4))/((2*x(1))+cl1+cb1+(2*x(4)))
)+2)*((fc)^0.5)*x(4)*(((2*x(1))+cl1+cb1+(2*x(4)))));
c(16)=PU1-
(((PU1+PU2)/((x(1)+x(2)+l)*x(3)))*(x(1)+(cl1/2)+(x(4)/2
))*(cb1+x(4)))-
((1000*phi2/3)*((fc)^0.5)*x(4)*((2*x(1))+cl1+cb1+(2*x(4
))));
c(17)=PU2-
(((PU1+PU2)/((x(1)+x(2)+l)*x(3)))*(x(2)+(cl2/2)+(x(4)/2
))*(cb2+x(4)))-
((1000*phi2/6)*(1+(2/B2))*((fc)^0.5)*x(4)*((2*x(2))+cl2
+cb2+(2*x(4))));
c(18)=PU2-
(((PU1+PU2)/((x(1)+x(2)+l)*x(3)))*(x(2)+(cl2/2)+(x(4)/2
))*(cb2+x(4)))-
((1000*phi2/12)*(((30*x(4))/((2*x(2))+cl2+cb2+(2*x(4)))
)+2)*((fc)^0.5)*x(4)*(((2*x(2))+cl2+cb2+(2*x(4)))));
```

```
c(19)=PU2-
(((PU1+PU2)/((x(1)+x(2)+l)*x(3)))*(x(2)+(cl2/2)+(x(4)/2
))*(cb2+x(4)))-
((1000*phi2/3)*((fc)^0.5)*x(4)*((2*x(2))+cl2+cb2+(2*x(4
))));
%-ve Moment Mu1 Mu-?Mn<=0
c(20)=(-100000*(((x(5)/(x(3)*x(4)))-((0.85*fc/fy)*(1-
(1-((0.002*((PU1*(((PU1-
(((PU1+PU2)/(x(1)+x(2)+l))*x(1)))*l)/(((PU1+PU2)/(x(1)+
x(2)+l))*l)))-
(0.5*((PU1+PU2)/(x(1)+x(2)+l))*((x(1)+(((PU1-
(((PU1+PU2)/(x(1)+x(2)+l))*x(1)))*l)/(((PU1+PU2)/(x(1)+
x(2)+l))*l)))^2))))/(0.85*fc*phi3*x(3)*(x(4)^2))))^0.5)
))));
c(21)=(-10000*(((x(5))-
(0.0018*420*x(3)*(x(4)+C+(dpl/2))/fy))));
%+ve Moment Mu2 Mu-?Mn<=0
c(22)=(-100000*(((x(6)/(x(3)*x(4)))-((0.85*fc/fy)*(1-
(1-((0.002*(0.5*((PU1+PU2)/(x(1)+x(2)+l))*((x(1)-
(cl1/2))^2)))/(0.85*fc*phi3*x(3)*(x(4)^2))))^0.5)))));
c(23)=(-100000*(((x(6)/(x(3)*x(4)))-((0.85*fc/fy)*(1-
(1-
((0.002*(abs((0.5*((PU1+PU2)/(x(1)+x(2)+l))*((x(1)+(cl1
/2))^2))-
(0.5*PU1*cl1))))/(0.85*fc*phi3*x(3)*(x(4)^2))))^0.5))))
);
c(24)=(-100000*(((x(6))-
(0.0018*420*x(3)*(x(4)+C+(dpl/2))/fy))));
%+ve Moment Mu3 Mu-?Mn<=0
c(25)=(-100000*(((x(7)/(x(3)*x(4)))-((0.85*fc/fy)*(1-
(1-((0.002*(-PU1*(l-
(0.5*cl2))+(0.5*((PU1+PU2)/(x(1)+x(2)+l))*((x(1)+l-
(0.5*cl2))^2))))/(0.85*fc*phi3*x(3)*(x(4)^2))))^0.5))))
);
c(26)=(-100000*(((x(7)/(x(3)*x(4)))-((0.85*fc/fy)*(1-
(1-((0.002*(-PU1*(l+(0.5*cl2))-
(0.5*PU2*cl2)+(0.5*((PU1+PU2)/(x(1)+x(2)+l))*((x(1)+l+(
0.5*cl2))^2))))/(0.85*fc*phi3*x(3)*(x(4)^2))))^0.5)))))
;
c(27)=(-100000*(((x(7))-
(0.0018*420*x(3)*(x(4)+C+(dpl/2))/fy))));
%+ve transverse Moment Mu4 Mu-?Mn<=0
c(28)=(-100000*(((x(8)/(x(4)*(x(4)+cb1)))-
((0.85*fc/fy)*(1-(1-((0.002*((PU1/(2*x(3)))*((x(3)/2)-
```

```
(cb1/2))^2))/(0.85*fc*phi3*(x(4)+cb1)*(x(4)^2))))^0.5))
)));
c(29)=(-100000*(((x(8))-
(0.0018*420*(x(4)+C+(dpl/2))*(x(4)+cl1)/fy))));
%+ve transverse Moment Mu5 Mu-?Mn<=0
c(30)=(-100000*(((x(9)/(x(4)*(x(4)+cb2)))-
((0.85*fc/fy)*(1-(1-((0.002*((PU2/(2*x(3)))*((x(3)/2)-
(cb2/2))^2))/(0.85*fc*phi3*(x(4)+cb2)*(x(4)^2))))^0.5))
)));
c(31)=(-100000*(((x(9))-
(0.0018*420*(x(4)+C+(dpl/2))*(x(4)+cl2)/fy))));
%Temp and shrinkage steel
c(32)=(-100000*(((x(10))-
(0.0018*420*(x(1)+x(2)+l)*(x(4)+C+(dpl/2))/fy))));
%spacing for longitudenal As1
c(33)=(((x(3)-(2*C)-((x(5)/Adpl)*dpl))/((x(5)/Adpl)-
1))-(3*(x(4)+(dpl/2)+C)));
c(34)=(((x(3)-(2*C)-((x(5)/Adpl)*dpl))/((x(5)/Adpl)-
1))-0.45);
c(35)=dpl-((x(3)-(2*C)-((x(5)/Adpl)*dpl))/((x(5)/Adpl)-
1));
c(36)=0.025-((x(3)-(2*C)-
((x(5)/Adpl)*dpl))/((x(5)/Adpl)-1));
%spacing for longitudenal As2
c(37)=(((x(3)-(2*C)-((x(6)/Adpl)*dpl))/((x(6)/Adpl)-
1))-(3*(x(4)-(dpl/2)-C)));
c(38)=(((x(3)-(2*C)-((x(6)/Adpl)*dpl))/((x(6)/Adpl)-
1))-0.45);
c(39)=dpl-((x(3)-(2*C)-((x(6)/Adpl)*dpl))/((x(6)/Adpl)-
1));
c(40)=0.025-((x(3)-(2*C)-
((x(6)/Adpl)*dpl))/((x(6)/Adpl)-1));
%spacing for longitudenal As3
c(41)=(((x(3)-(2*C)-((x(7)/Adpl)*dpl))/((x(7)/Adpl)-
1))-(3*(x(4)+(dpl/2)+C)));
c(42)=(((x(3)-(2*C)-((x(7)/Adpl)*dpl))/((x(7)/Adpl)-
1))-0.45);
c(43)=dpl-((x(3)-(2*C)-((x(7)/Adpl)*dpl))/((x(7)/Adpl)-
1));
c(44)=0.025-((x(3)-(2*C)-
((x(7)/Adpl)*dpl))/((x(7)/Adpl)-1));
%spacing for longitudenal As3
c(45)=(((x(3)-(2*C)-((x(7)/Adpl)*dpl))/((x(7)/Adpl)-
1))-(3*(x(4)+(dpl/2)+C)));
```

```
c(46)=(((x(3)-(2*C)-((x(7)/Adpl)*dpl))/((x(7)/Adpl)-
1))-0.45);
c(47)=dpl-((x(3)-(2*C)-((x(7)/Adpl)*dpl))/((x(7)/Adpl)-
1));
c(48)=0.025-((x(3)-(2*C)-
((x(7)/Adpl)*dpl))/((x(7)/Adpl)-1));
%spacing for longitudenal As4
c(49)=(((x(1)+(cl1/2)+(x(4)/2)-C-
((x(8)/Adpt)*dpt))/((x(8)/Adpt)-1))-
(3*(x(4)+(dpl/2)+C)));
c(50)=(((x(1)+(cl1/2)+(x(4)/2)-C-
((x(8)/Adpt)*dpt))/((x(8)/Adpt)-1))-0.45);
c(51)=dpl-((x(1)+(cl1/2)+(x(4)/2)-C-
((x(8)/Adpt)*dpt)));
c(52)=0.025-((x(1)+(cl1/2)+(x(4)/2)-C-
((x(8)/Adpt)*dpt)));
%spacing for longitudenal As5
c(53)=(((x(2)+(cl2/2)+(x(4)/2)-C-
((x(9)/Adpt)*dpt))/((x(9)/Adpt)-1))-
(3*(x(4)+(dpl/2)+C)));
c(54)=(((x(2)+(cl2/2)+(x(4)/2)-C-
((x(9)/Adpt)*dpt))/((x(9)/Adpt)-1))-0.45);
c(55)=dpl-((x(2)+(cl2/2)+(x(4)/2)-C-
((x(9)/Adpt)*dpt)));
c(56)=0.025-((x(2)+(cl2/2)+(x(4)/2)-C-
((x(9)/Adpt)*dpt)));
%spacing for longitudenal As6
c(57)=(((x(1)+x(2)+l-(2*C)-
((x(10)/Adpt)*dpt))/((x(10)/Adpt)-1))-
(3*(x(4)+(dpl/2)+C)));
c(58)=(((x(1)+x(2)+l-(2*C)-
((x(10)/Adpt)*dpt))/((x(10)/Adpt)-1))-0.45);
c(59)=dpl-((x(1)+x(2)+l-(2*C)-
((x(10)/Adpt)*dpt))/((x(10)/Adpt)-1));
c(60)=0.025-((x(1)+x(2)+l-(2*C)-
((x(10)/Adpt)*dpt))/((x(10)/Adpt)-1));
%Strain >=0.005
c(61)=0.005-
0.003*(((x(4)*x(3)*fc*Beta*0.85)/(x(5)*fy))-1);
c(62)=0.005-
0.003*(((x(4)*x(3)*fc*Beta*0.85)/(x(6)*fy))-1);
c(63)=0.005-
0.003*(((x(4)*x(3)*fc*Beta*0.85)/(x(7)*fy))-1);
```

```
c(64)=0.005-
0.003*(((x(4)*(x(1)+x(2)+l)*fc*Beta*0.85)/(x(8)*fy))-
1);
c(65)=0.005-
0.003*(((x(4)*(x(1)+x(2)+l)*fc*Beta*0.85)/(x(9)*fy))-
1);
ceq= ((x(1)+x(2)+l)-
(2*(((PD2+PL2)*l)/(PD1+PL1+PD2+PL2)+x(1))));
end
```

APPENDIX F

CODE INPUT EXCEL SHEET FOR WIGHT

choose column shape "0 for circular,1 for rectangular or square"	1		
Clear cover	0.075	M	C
Depth of footing	1.524	(M)	hf
Excavation Margin	0.5	(M)	E
Rate of Excavation	19.8	($/m3)	Rexc
Rate of Form work	77.18	($/m2)	Rfw
Rate of Concrete	182.56	($/m3)	Rcon
Rate of Insulation	13.35	($/m2)	Rins
Rate of Blinding Concrete	169.41	($/m3)	Rb
Rate of Steel	2524.29	($/TON)	Rst
Soil initial void ratio	0.75		e0
Dead load on column 1	756.198	(KN)	PD1
LIVE load on column 1	333.617	(KN)	PL1
Dead load on column 2	889.644	(KN)	PD2
LIVE load on column 2	556.028	(KN)	PL2
"Circular" Column 1 Diameter		(M)	c1
"Circular" Column 2 Diameter		(M)	c2
"Square" Column 1 length	0.4572	(M)	cl1
"Square" Column 1 width	0.4572	(M)	cb1
"Square" Column 2 length	0.4572	(M)	cl2
"Square" Column 2 length	0.4572	(M)	cb2
Concrete compressive strength	27.579	(MPa)	fc
Steel yield strength	413.685	(MPa)	fy
Center to center column spacing	9.144	(M)	l
Reduction factor one way shear	0.75		phi1
Reduction factor twoway shear	0.75		phi2
Reduction factor flexture	0.9		phi3
Upper value for Distance between left edge to center column 1	0.6096	(M)	r1
Upper value for Distance between right edge to center column 2	10	(M)	r2
Upper value for width (B)	10	(M)	r3
Diameter for longitudenal reinforcment	0.025	(M)	dpl
Diameter for transvere reinforcment	0.025	(M)	dpt
Diameter for dowels	0.016	(M)	dpd
Factor of safety for bearing capacity	3		FS
Allowable settelment	0.0508	(M)	Sca
lightweight-aggregate-concrete factor(0.75 fpr light weight)	1		lam
Surcharge	0	(KN/M2)	qanet
Depth of water taple from surface	1.8288	(M)	Dw
Depth of soil layer	3.048	(M)	D
Moist unit weight of soil	18.0651	(KN/M3)	Gama
Cohesion of soil	33.5162	(KN/M2)	C
Soil friction angle	20	degree	ø
Consolidation coefficant	0.15		Cc/Cs
Distance between left edge to center column 1	0.60955	(M)	x(1)
Distance between right edge to center column 2	1.89291	(M)	x(2)
Width (B)	3.07017	(M)	x(3)
Effective depth (d)	0.58	(M)	x(4)
As for -ve moment	0.01329	(M2)	x(5)
As for +ve moment under column 1	0.00542	(M2)	x(6)
As for +ve moment under column 2	0.00375	(M2)	x(7)
As for transverse moment under column 1	0.00191	(M2)	x(8)
As for transverse moment under column 2	0.00288	(M2)	x(9)
As for temp and shrinkage	0.01421	(M2)	x(10)

starting point

APPENDIX G

ADVANCED FOUNDATION CLASS COMBINED FOOTING EXAMPLE

HW : CVE 531 Combined footers

Problem 1: Two column loads are shown at 30 ft apart. Determine

a. The required length of the combined footer to produce uniform soil reaction.

b. The width of the foundation to satisfy bearing FS =3. Assume foundation depth 5′.

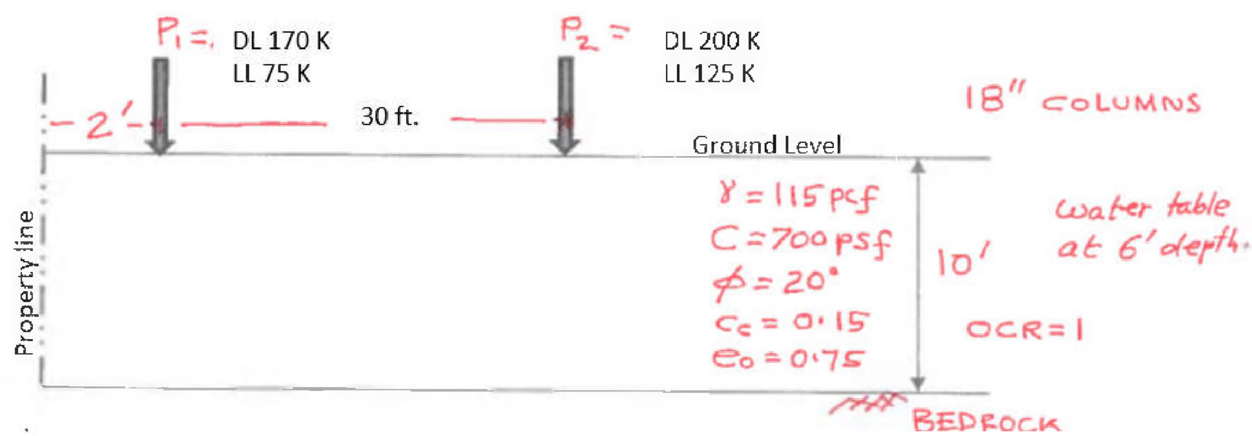
c. The width required to satisfy an allowable settlement of 2 inch.

d. Determine the required thickness of the footer.

e. For the longitudinal direction, draw the shear and bending moment diagrams and determine the desgn moments.

f. Determine the required Steel areas for the longitudial and transverse directions.

g. Show Plan and Elevation stetches of your design.



APPENDIX H

BEARING CAPACITY FACTORS ACCORDING TO TERZAGHI

Friction Angle	Nc	Nq	Nγ
0	5.7	1	
1	6	1.1	0.01
2	6.3	1.22	0.04
3	6.62	1.35	0.06
4	6.97	1.49	0.1
5	7.34	1.64	0.14
6	7.73	1.81	0.2
7	8.15	2	0.27
8	8.6	2.21	0.35
9	9.09	2.44	0.44
10	9.61	2.69	0.56
11	10.16	2.98	0.69
12	10.76	3.29	0.85
13	11.41	3.63	1.04
14	12.11	4.02	1.26
15	12.86	4.45	1.52
16	13.68	4.92	1.82
17	14.6	5.45	2.18
18	15.12	6.04	2.59
19	16.56	6.7	3.07
20	17.69	7.44	3.64
21	18.92	8.26	4.31
22	20.27	9.19	5.09
23	21.75	10.23	6
24	23.36	11.4	7.08
25	25.13	12.72	8.34
26	27.09	14.2	9.84
27	29.24	15.9	11.6
28	31.61	17.81	13.7
29	34.24	19.98	16.18
30	37.16	22.46	19.13
31	40.41	25.28	22.65
32	44.04	28.52	26.87
33	48.09	32.23	31.94
34	52.64	36.5	38.04
35	57.75	41.44	45.41
36	63.53	47.16	54.36
37	70.01	53.8	65.27
38	77.5	61.55	78.61
39	85.97	70.61	95.03
40	95.66	81.27	115.31
41	106.81	93.85	140.51
42	119.67	108.75	171.99
43	134.58	126.5	211.56
44	151.95	147.74	261.6
45	172.28	173.28	325.34
46	196.22	204.19	407.11
47	224.55	241.8	512.84
48	258.28	287.85	650.67
49	298.71	344.63	831.99
50	347.5	415.14	1072.8