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Frequency-Hopped Multiple-Access Communications With Noncoherent $M$-ary OFDM-ASK

A. Al-Dweik, Member, IEEE, and F. Xiong, Senior Member, IEEE

Abstract—A noncoherent, bandwidth-efficient modulation scheme is proposed for frequency-hopping multiple-access (FH-MA) networks. The proposed scheme is a combination of noncoherent $M$-ary amplitude-shift keying (NMASK) and orthogonal frequency-division multiplexing (OFDM). Using this scheme will minimize the required data bandwidth. The number of frequency slots available to the users will increase significantly for a fixed spread-spectrum bandwidth ($\text{BW}_{\text{SS}}$). The effect of the multiple-access interference will be reduced. Simple and accurate bit error rate expressions have been derived for FH-OFDM-MASK in additive white Gaussian noise channels and for FH-OFDM-ASK in Rayleigh fading channels.

Index Terms—Amplitude-shift keying (ASK), frequency hopping (FH), $M$-ary amplitude-shift keying (MASK), noncoherent, orthogonal frequency-division multiplexing (OFDM), Rayleigh channels.

I. INTRODUCTION

In this letter, a slow frequency-hopping multiple-access (FH-MA) system will be considered. The $\text{BW}_{\text{SS}}$ is divided into $q$ subbands called frequency slots, with one carrier frequency available in each slot. The bandwidth of each subband ($\text{BW}_{\text{skt}}$) is equal to the bandwidth of the modulated signal, as shown in Fig. 1. Noncoherent $M$-ary frequency-shift keying (NMFSK) is the most common modulation scheme used in FH systems [1]–[3]. The main disadvantage of MFSK is its poor bandwidth efficiency that leads to a wide and small . For MFSK, $\text{BW}_{\text{skt}} \approx (2^k + 1)R_s$, where $R_s$ is the symbol rate, $R_s = R_b/k$, and $R_b$ is the bit rate. The large bandwidth expansion decreases $q$, hence, increases the chance of collision between different users. In the literature, little work has been done to solve the MFSK bandwidth problem. Lately, a bandwidth-efficient modulation scheme has been proposed in [4], which is a combination of multicarrier and noncoherent on–off keying. In this letter, we will extend the work done in [4] to include the $M$-ary case in additive white Gaussian noise (AWGN) channels. The binary case ($M = 2$) will also be considered in Rayleigh fading channels, the results from this letter and the results obtained in [4] will be compared.

II. CHANNEL MODEL AND SYSTEM DESCRIPTION

The channel is assumed to be AWGN, and the noise $n(t)$ has a two-sided power spectral density $N_0/2$. At any given time, we will assume that there are $K$ active users transmitting through the channel. Let us call one of the active users as the reference user. The reference user signal will be jammed (hit) when any of the $K - 1$ active users transmits on the frequency that the reference user is using during any symbol period; we call that user, in this case, an interferer. The probability of having $n$ interferers out of $K - 1$ users ($P(n)$) can be represented by a binomial distribution [5]. In this model, the FH system is assumed to be synchronous, i.e., only full hits will be considered. The proposed system is shown in Fig. 2. We can see that each group of $k_2$ data bits is assigned a unique amplitude $A_i$, $i \in \{1, 2, \ldots, M\}$, using the level generator (LG) block, where $M = 2^{k_2}$. The $k_1$ parallel data streams will modulate $k_1$ subcarriers: $\omega_1, \omega_2, \ldots, \omega_{k_1}$ with a frequency separation of $1/T_s$, which is the minimum tone spacing for noncoherently detected orthogonal signals [6]. The transmitted signal during any symbol period can be expressed as

$$s(t) = \sum_{i=1}^{k_1} A_i \cos(\omega_i t + \theta_i), \quad 0 \leq t \leq T_s \tag{1}$$

where $\theta_i$ is an arbitrary initial phase. The null-to-null bandwidth of the signal $s(t)$, i.e., $\text{BW}_{\text{skt}}$, can be expressed as $\text{BW}_{\text{skt}} = R_b \cdot (1 + k_3)/(k_1 \cdot k_2)$. After frequency dehopping, the signal is applied to $k_1$ sections of a noncoherent quadrature receiver with a structure as shown in Fig. 2. The last stage consists of a decision circuit, analog-to-digital (A/D) converter, and parallel-to-serial (P/S) converter. The output of the decision circuit will be one of $M$ levels, each level is converted to $k_2$ bits using the A/D converter.

III. PERFORMANCE OF OFDM-$M$-ARY AMPLITUDE-SHIFT KEYING (MASK) IN AWGN CHANNELS

Due to the subcarriers orthogonality, each subchannel at the receiver can be considered as an independent channel. Hence, it is sufficient to consider only one of the $k_1$ channels. Assuming that the amplitude spacing is uniform and is equal to $d$, and all the $M$ levels are equiprobable, the total energy transmitted can be expressed as

$$E_{\text{total}} = \sum_{j=0}^{M-1} \frac{T_s A_j^2}{2} \tag{2}$$

where each amplitude $A_j$ is equal to $j \times d$, $j = 0, 1, \ldots, M-1$. The average symbol energy ($E_s$) can be calculated by substi-
Substituting for $A_j$ in (2) and dividing the result over the number of all possible amplitudes $M$. The result is

$$E_s = \frac{d^2 T_s}{4} \left[ \frac{2}{3} M^2 - M + \frac{1}{2} \right] = \frac{E_{\min}}{2 B_M} \quad (3)$$

where $E_{\min} = d^2 T_s/2$ is the energy of the symbol with minimum (nonzero) amplitude, and

$$B_M = \frac{3}{2M^2 - 3M + 1}. \quad (4)$$

The system probability of error ($P_e$) can be calculated by averaging the probability of error over all possible symbols. For equally likely symbols, $P_e$ can be expressed as

$$P_e = \sum_{i=0}^{M-1} P(e | A_i) P(A_i). \quad (5)$$

To evaluate (5), we should notice that when $A_0$ is transmitted, the probability density function (pdf) of the sufficient statistics ($\eta$) is Rayleigh, and it is Rician for all other amplitudes. In addition, each of the Rician pdfs is centered at $j \times \sqrt{E_{\min}}$. 

Fig. 1. Spectrum of the proposed system.

Fig. 2. Proposed system.
0 < j ≤ M − 1. Therefore, it is straightforward to find all the thresholds in the system; they are calculated in terms of $E_{\text{min}}$. Since we know all the pdfs and all the thresholds in the system, it can be shown that the probability of error ($P_e$) can be expressed as [7]

\[
P_e = P_{e|0} = \frac{1}{M} \left[ \exp \left( -\frac{k_2 E_b}{2N_0} B_M \right) \right. \\
+ \left. (2M - 3)Q \left( \sqrt{\frac{k_2 E_b}{N_0}} B_M \right) \right] \tag{6}
\]

where $Q(\cdot)$ is the well-known complementary error function.

**IV. EFFECT OF MULTIPLE-ACCESS INTERFERENCE (MAI)**

At the receiver, the dehopped signal can be expressed as

\[
r(t) = \sum_{i=1}^{k_1} A_i \cos(\omega_it + \theta_i) + I(t) + n(t)
\]

\[
I(t) = \sum_{j=1}^{n} s_j(t) \tag{7}
\]

where $I(t)$ is the MAI and $s_j(t)$ is the $j$th interfering signal, and it has the form of (1), and $n$ is the number of interferers. The value of $P_e$ can be calculated as the mean of several situations corresponding to the possible hit pattern produced by the MA process. The value of $P_e$ without MAI was given in (6), where $P_{e|0}$ is the probability of error, given that $n = 0$.

The value of $P_e$ for FH-MASK depends on the symbol of the reference user ($S_{r,i}$, $i = 1, 2, \cdots, M$), the hit pattern for a given number of interferers ($H_{j,n}$), and the number of interferers ($n$). Thus, $P_e$ can be expressed as

\[
P_e = \sum_{n=0}^{K-1} \sum_{j=1}^{M} P(e|n, S_{r,i}, H_{j,n})
\]

\[
\cdot P(H_{j,n}) \cdot T(S_{r,i}) \cdot P(n) \tag{8}
\]

where $H_{(n)}^{(total)}$ is the number of all possible hit patterns for a given $n$. $H_{(n)}^{(total)} = M^n$. To simplify the solution of (8), we will consider five different hit situations. The first represents the case where no hit occurs, $P(e|n = 0) = P_{e|0}$. The second represents the case where all the interferers are transmitting the zero-amplitude symbol, therefore, $P(e|n, H_{1,n}) = P_{e|0}$, where $H_{1,n}$ is the all-zeros hit pattern, given that we have $n$ interferers. The third represents the case where the reference user transmits the zero-amplitude symbol ($S_{r,1}$) and any hit pattern excluding $H_{1,n}$ occurs. It is clear that the probability of error in this case can be closely approximated by one. The fourth represents the case where the reference user transmits the symbol with maximum amplitude ($S_{r,M}$) and any hit pattern excluding $H_{1,n}$ occurs. In this case, the interfering signals will have a limited effect, since the decision circuit has only one threshold to compare $\eta$ with [8]. Therefore, the probability of error for this case is very small compared to other terms in (8) and can be closely approximated by zero. The fifth case represents the case where at least one of the interfering signals has nonzero amplitude and the symbol of the reference user is not $S_{r,M}$; the probability of error in this case is very high and can be closely approximated by one. Therefore, $P_e$ can be expressed as

\[
P_e \approx \sum_{n=0}^{K-1} \left( P_{e|0} M^n + \left( 1 - \frac{1}{M^n} \right) \left( 1 - \frac{1}{M} \right) \right) \cdot P(n). \tag{9}
\]

The bit error rate (BER) can be calculated by noticing that $P_{e|0} = P_{e|0}/k_2$ (gray coding is still valid). For the third and fifth cases, we can see that all symbol errors are equiprobable, since all transmitted symbols are equiprobable, then $P_b = P_e \cdot M/(2(M - 1))$ [9]. Hence, the BER can be approximated by

\[
P_b \approx \sum_{n=0}^{K-1} \left( \frac{P_{e|0}}{M^n} + \left( \frac{M}{2(M - 1)} \right) \cdot \left( 1 - \frac{1}{M^n} \right) \cdot \left( 1 - \frac{1}{M} \right) \right) \cdot P(n). \tag{10}
\]

**V. FH-OFDM WITH BINARY ASK IN RAYLEIGH FADING CHANNELS**

The BER for the binary case is given by $M = 2$ in (10). Hence

\[
P_b \approx \sum_{n=0}^{K-1} \left( \frac{P_{e|0}}{2^n} + \frac{M - 1}{2^{n+1}} \right) \cdot P(n) \tag{11}
\]

where $P_{e|0}$ is the BER for ASK signals in Rayleigh fading channels, which will be determined next. The channel is assumed to be a slow frequency-nonselective Rayleigh fading channel [4]. Thus, the received signal can be expressed as

\[
r(t) = \sqrt{2P} \sum_{i=1}^{k_1} \alpha_i A_i \cos(\omega_it + \varphi_i) + n(t), \quad 0 \leq t \leq T_s \tag{12}
\]

where $\alpha_i$ and $\varphi_i$ are the attenuation and the phase shift of the received $i$th subcarrier. The attenuation has a Rayleigh distribution, and the phase is uniformly distributed between $0, 2\pi$.

The optimum receiver of ASK signals in Rayleigh fading channels can be implemented as a quadrature receiver, similar to the receiver used for ASK signals in AWGN channels, as shown in Fig. 2. The main difference is that the threshold is significantly dependent on $E_b/N_0$. The optimum threshold can be expressed as [5], [8]

\[
\eta^2 > \frac{\ln \left( 1 + \frac{E_b}{N_0} \right) \cdot \frac{E_b}{E_b/N_0}}{\frac{S_1}{S_0}} = \gamma^2 \tag{13}
\]

where $\gamma = 2 \cdot E \{ \alpha^2 \} \cdot E_b$. Using (13), it can be expressed as [8]

\[
P_{e|0} = \frac{1}{2} \left( 1 + \frac{E_b}{N_0} \right) - \frac{1}{2} \frac{1}{2} \left( 1 + \frac{E_b}{N_0} \right)^{-1} \cdot \frac{1}{E_b/N_0}. \tag{14}
\]
The performance in FH channels can be obtained by substituting (14) in (11). Fig. 3 shows the system performance using (11), simulations, and the exact results derived by [4], for $q = 100$, $K = 30$, and 15.

VI. NUMERICAL RESULTS AND CONCLUSION

The bandwidth required for each subchannel ($B_{ch}$) will be considered as the null-to-null bandwidth, $B_{ss}$ will be considered as $1024 R_b$, $K = 31$. Since $B_{ss} = q \cdot B_{skt}$, the value of $q$ can be obtained by substituting for $B_{skt}$ and $B_{skt}$. Thus, $q = 1024(k_1 \cdot k_2)/(k_3 + 1)$. The BER at very high $E_b/N_0$ ($\infty$), i.e. the error floor, will be used to compare the performance of OFDM-MASK and MFSK in MA environment with background AWGN. The two systems were compared for different values of $k_2$ (or $M$) and $K$. Since the MASK is a function of $k_1$ and $k_2$, we used $k_1 = 32$. Fig. 4 shows the BER of both systems, it also shows the simulations results for the OFDM-MASK system.

It is clear that the proposed system can be considered as an efficient alternative for MFSK in FH systems. Analytical and simulation results have shown that the MAI can be reduced significantly using the proposed system. Reducing the MAI can be achieved by increasing the number of parallel branches transmitted ($k_3$), or by increasing the modulation order ($M$), or by increasing both.

The derived approximation for FH-OFDM with ASK in Rayleigh fading channels is much simpler than the expression derived by [4] without any loss in accuracy.

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