Road Surface Condition Detection and Identification and Vehicle Anti-Skid Control

Maosheng Ye
Cleveland State University

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ROAD SURFACE CONDITION DETECTION AND IDENTIFICATION AND VEHICLE ANTI-SKID CONTROL

MAOSHENG YE

BACHELOR OF SCIENCE INSTRUMENTATION AND CONTROL ENGINEERING
AT THE
BEIJING JIAOTONG UNIVERSITY, BEIJING, CHINA
JULY, 2004

Submitted in partial fulfillment of the requirements for the degree

MASTER OF SCIENCE IN MECHANICAL ENGINEERING
AT THE
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Department of Mechanical Engineering and the
College of Graduate Studies by

Thesis/Dissertation Committee Chairperson, Dr. Paul P. Lin

Department/Date

Dr. Zhiqiang Gao

Department/Date

Dr. Majid Rashidi

Department/Date
I would like to thank my advisor Dr. Paul P. Lin, who provided essential support and assistance throughout my graduate study, and also for his guidance which immensely contributed towards the completion of this thesis. I would also like to thank Dr. Zhiqiang Gao and Dr. Majid Rashidi for being in my thesis committee.

Thanks are also due to my family and friends who have encouraged, supported and inspired me.
ROAD SURFACE CONDITION DETECTION AND IDENTIFICATION AND VEHICLE ANTI-SKID CONTROL
MAOSHENG YE

ABSTRACT

Road surface condition is greatly dependent on the surface’s friction coefficient. The abrupt change of the coefficient results in variation of wheel slip which likely leads to vehicle instability. Vehicle steering model and the dynamic equations for four-wheel drive vehicle is developed. A new observer, called Extended State Observer (ESO) is used to estimate the longitudinal velocity, lateral velocity and yaw rate, and more importantly an additional quantity known as system dynamics. A trained neural network was employed to help determine the friction coefficient. Fuzzy logic was employed to quickly detect the change of road surface condition and further classify the surface condition. The presented methods were simulated with a vehicle encountering a significant change from a uniform-μ (i.e. uniform friction coefficient) surface to a split-μ surface (i.e. different friction coefficient on each side of the wheels) during cornering. The results obtained show that the developed techniques could effectively detect and identify the road surface condition. Furthermore, a new anti-skid controller by means of Active Disturbance Rejection Controller (ADRC) and the ESO is proposed. The simulation results show that the controller can effectively control the vehicle’s yaw rate while cornering.
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CHAPTER I

INTRODUCTION

The tire-road friction coefficient, $\mu$ plays a significant role in vehicle stability control. With estimated friction coefficients, vehicle motion can be estimated more accurately. Dieckman [1] developed a method to determine the road surface variation based on the measurements of the wheel slip ratio. Gustafsson [2] designed an algorithm to estimate the tire-road friction during normal driving using the measured wheel slip ratio along with Kalman filter. Eichhorn and Roth [3] introduced the estimation of road friction using optical and noise sensors near the front-end of the tire, and stress-strain sensors inside the tire tread. In terms of control, Ray [4] proposed a nonlinear estimator and controller to estimate the vehicle state, and friction coefficient, and implement the stability controller by distributing driving torques on the wheels.

Road surface can be generally classified into four possible conditions: dry, wet, snowy and icy. Each of these conditions has distinct characteristics in friction coefficient. There
is a relationship between the surface friction coefficient and the vehicle’s wheel slip ratio. The relationship has been experimentally determined by researchers, such as [2].

An observer can be designed to estimate the vehicle velocities and yaw rate when direct measurements are not available, or inaccurate due to signal noise and drift. However, the existing observers are either for linear systems or requiring exact knowledge of dynamic system model. The Extended State Observer (ESO) introduced in this paper is different from conventional observers. The ESO can augment both unknown system dynamics and disturbances as extended state and estimate them in real time by using given input and output data. The extended state in ESO is found to be capable of providing the insight to the system dynamics which can be used to detect the change of road surface condition.

Active Disturbance Rejection Control (ADRC) was first proposed by Han [5-8] and later simplified by Gao [9]. The controller is designed to be inherently robust against plant variations. It treats the unknown system dynamics and external disturbances as a total disturbance, which can be estimated by ESO. With the estimated total disturbance, the controller can be designed to reject this disturbance.

This thesis first presents the dynamic model of a vehicle, followed by detection and identification of the road surface condition, and vehicle anti-skid control. In the anti-skid control, the ADRC is described and implemented. More specifically, Chapter I gives general introduction. Chapter II shows how the vehicle dynamic model is built using the Newton’s Law. Chapter III gives an introduction to ESO. Chapter IV presents an intelligent method of road surface condition detection and identification by means of the
ESO, artificial neural network and fuzzy logic techniques. Chapter V introduces the ADRC and the ADRC-based anti-skip controller.
A vehicle dynamics model is built and introduced in this chapter. In this model, it is assumed that only the front wheels are used for the purpose of steering.

The vehicle dynamic model was built by applying Newton’s second law to the lumped vehicle mass longitudinally and laterally through the center of the mass. The vehicle model takes the steering angle and applied forces on the wheels as inputs, and generates the vehicle longitudinal velocity, lateral velocity and yaw rate as outputs.

In this chapter, the vehicle steering model is first introduced, followed by the vehicle dynamic equations, and the procedure for the vehicle dynamic modeling.

2.1 Vehicle Steering Model

The vehicle steering model during a counterclockwise turning maneuver is developed as shown in Figure 1.
where

$V_x$: the longitudinal velocity at the vehicle’s center of mass

$V_y$: the lateral velocity at the vehicle’s center of mass

$\gamma$: yaw rate at the vehicle’s center of mass

$F_{Lij}$: longitudinal force of axle $i$ and side wheel $j$

$F_{Sij}$: lateral forces of axle $i$ and side wheel $j$

$\delta$: steering angle

$\beta$: slip angle

$\alpha_{ij}$: the side slip angle of axle $i$ and side wheel $j$

Toyota Camry Sedan [10] was used. The vehicle’s parameters are listed in Table I.
Table I. Vehicle’s Key Parameters

<table>
<thead>
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<th>Parameter</th>
<th>Value</th>
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<tr>
<td>Vehicle mass, m</td>
<td>1500.45 kg</td>
</tr>
<tr>
<td>Wheel radius, R</td>
<td>0.334 m</td>
</tr>
<tr>
<td>Distance between the vehicle center of mass and the</td>
<td>2.71 m</td>
</tr>
<tr>
<td>front axle of the wheels, $l_f$</td>
<td></td>
</tr>
<tr>
<td>Distance between the vehicle center of mass and the</td>
<td>2.06 m</td>
</tr>
<tr>
<td>back axle of the wheels, $l_r$</td>
<td></td>
</tr>
<tr>
<td>Half of the distance between the left wheels and the</td>
<td>0.91 m</td>
</tr>
<tr>
<td>right wheels, $l_t$</td>
<td></td>
</tr>
<tr>
<td>Moment of inertia at the vehicle center of mass, J</td>
<td>3294 kg-m²</td>
</tr>
<tr>
<td>Moment of inertia at each wheel, $J_w$</td>
<td>1 kg-m²</td>
</tr>
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2.2 Vehicle Dynamic Equations

Without considering air resistance, load transfer between the axles, and some other disturbances, the dynamic equation for a vehicle during cornering can be expressed as follows:

$$m\ddot{V}_x = m\gamma V_y + F_{LLF}\cos\delta + F_{LRF}\cos\delta - F_{SLF}\sin\delta - F_{SRF}\sin\delta_{ir} + F_{LLR} + F_{LRR}$$  \hspace{1cm} (2.1)

$$m\ddot{V}_y = -m\gamma V_x + F_{LLF}\sin\delta + F_{LRF}\sin\delta + F_{SLF}\cos\delta + F_{SRF}\cos\delta + F_{SLR} + F_{SRR}$$  \hspace{1cm} (2.2)

$$J\ddot{\gamma} = (F_{LLR}\cos\delta - F_{LRF}\cos\delta + F_{LRR} - F_{SLF}\sin\delta - F_{SRF}\sin\delta)l_f$$

$$+ (F_{LLF}\sin\delta + F_{LRF}\sin\delta + F_{SLF}\cos\delta + F_{SRF}\cos\delta)l_r - (F_{SLR} + F_{SRR})l_f$$ \hspace{1cm} (2.3)
To solve the equations, the longitudinal and lateral forces applied to each wheel need to be determined. Dugoff’s tire model developed in [11] is used to determine the values of these forces. Due to the requirement of relatively small parameters and the capability of simulating pure cornering maneuver, in this model, the effects of wheel camber and tire relaxation length are neglected in this model. The longitudinal and lateral forces for axle \(i\) and side tire \(j\) can be expressed by [11]

\[
F_{Lij} = \frac{C_L \sigma_{ij}}{(1 + \sigma_{ij})} f(\lambda_{ij})
\]

\[
F_{Sij} = \frac{C_S \tan \alpha_{ij}}{(1 + \sigma_{ij})} f(\lambda_{ij})
\]

where the \(C_L\) and \(C_S\) represent the longitudinal and lateral cornering stiffness, \(\sigma_{ij}\) is the slip ratio of axle \(i\) and side wheel \(j\), and \(\alpha_{ij}\) is the side slip angle of axle \(i\) and side wheel \(j\). For each wheel, \(\lambda_{ij}\) and \(f(\lambda_{ij})\) are given by

\[
\lambda_{ij} = \frac{\mu F_n (1 + \sigma_{ij})}{2 \{ (C_L \sigma_{ij})^2 + (C_S \tan \alpha_{ij})^2 \}^{1/2}},
\]

\[
f(\lambda_{ij}) = \begin{cases} 
2 - \lambda_{ij} & (\lambda_{ij} < 1) \\
1 & (\lambda_{ij} \geq 1)
\end{cases}
\]

where \(F_n\) is the normal force on each tire. In this study, the value of normal force is assumed the same for each tire, which in reality could vary with weight of the driven and that of each passenger, along with other factors.

Cornering stiffnesses \(C_L\) and \(C_S\) play an important role during vehicle cornering, which could directly affect the vehicle path. These stiffnesses could be estimated by a
GPS device [12]. In this study, these cornering stiffness values are assumed 10,000 for \( C_L \) and 80,000 for \( C_S \) during the cornering.

For each driving wheel, the relationship between the longitudinal force \( (F_{Lij}) \), and the applied torque \( (\tau_{ij}) \), is governed by

\[
\tau_{ij} - RF_{Lij} = J_w \omega
\]

(2.8)

where \( R \) is the wheel radius, \( J_w \) is the mass moment of inertia of the wheel and \( \omega \) is the angular velocity of the wheel.

When the vehicle is cornering, the longitudinal velocity of each wheel is different from others. The longitudinal velocity of the right and left front wheels \( (V_{x,WRF} \) and \( V_{x,WLF} \) respectively) and that of the right and left rear wheel \( (V_{x,WRR} \) and \( V_{x,WLR} \) respectively) can be calculated by

\[
V_{x,WRF} = (V_x + \gamma l_f) \cos \delta + (V_y + \gamma l_f) \sin \delta
\]

(2.9)

\[
V_{x,WLF} = (V_x - \gamma l_f) \cos \delta + (V_y + \gamma l_f) \sin \delta
\]

(2.10)

\[
V_{x,WRR} = V_x + \gamma l_f
\]

(2.11)

\[
V_{x,WLR} = V_x - \gamma l_f
\]

(2.12)

Slip ratio is another important variable during the vehicle cornering. The Dugoff tire model requires the use of the slip ratio value to calculate the wheel forces, as described by Equations (2.4), (2.5), and (2.6). The slip ratio can also be used to estimate the road-tire friction coefficient which will be described in the Chapter IV. The slip ratio of each wheel can be determined by
\[ \sigma_{ij} = \frac{\omega_{ij} R - V_{x,Wij}}{\omega_{ij} R} \]  \hspace{1cm} (2.13)

where \( \omega_{ij} \) is the wheel speed at axle \( i \) and side wheel \( j \), and \( V_{x,Wij} \) is the longitudinal velocity at axle \( i \) and side wheel \( j \).

The side slip angle of each wheel can be calculated by

\[ \alpha_{RF} = \delta - \tan^{-1} \left( \frac{V_x + \gamma l_f}{V_x + \gamma l_t} \right) \]  \hspace{1cm} (2.14)

\[ \alpha_{RR} = -\tan^{-1} \left( \frac{V_y - \gamma l_f}{V_x + \gamma l_t} \right) \]  \hspace{1cm} (2.15)

\[ \alpha_{LF} = \delta - \tan^{-1} \left( \frac{V_x + \gamma l_f}{V_y - \gamma l_t} \right) \]  \hspace{1cm} (2.16)

\[ \alpha_{LR} = -\tan^{-1} \left( \frac{V_y - \gamma l_f}{V_y - \gamma l_t} \right) \]  \hspace{1cm} (2.17)

All these equations introduced in this section are combined in the Matlab/Simulink environment as illustrated in Figure 2. Input variables for this model are steering angle, wheels torques, and coefficient of friction at each wheel.
Figure 2 shows the procedure for modeling the vehicle. The modeling requires two system inputs: the steering angle, \( \delta \), and wheel torques, \( \tau_{ij} \). With system outputs \( V_x \), \( V_y \) and \( \gamma \), and system input \( \delta \), Equations 2.9-2.12 are used to calculate the longitudinal velocities of the wheels \( V_{x,WRF} \), \( V_{x,WLF} \), \( V_{x,WRR} \) and \( V_{x,WLR} \). The other system inputs, the applied torques, are used to calculate the corresponding wheels’ angular velocities by using Equation 2.8. In this procedure, the applied forces on the wheels can be calculated.
by Dugoff’s tire model. With the calculated wheels’ longitudinal velocities and angular
delocities by, the slip ratios of the wheels can be estimated using Equation 2.13. The
friction coefficient can be estimated using the relationship between slip ratios and the
friction coefficient. The detailed estimation will be explained in Chapter III. The slip
ratio values affect the wheels’ applied forces. Equations 2.4-2.7 are the Dugoff’s tire
model equations for determining the longitudinal and lateral forces on wheels. The last
step of this model is to calculate the vehicle’s longitudinal velocity, lateral velocity and
yaw rate using Equations 2.1-2.3.

The model introduced in this Chapter is to be used later for road surface detection and
identification, and anti-skid control. To detect and identify the road surface condition, the
applied forces on the driving wheels need to be calculated, and the longitudinal, lateral
velocities and yaw rates at the center of mass are to be estimated using the ESO which
will be introduced in the next chapter.
CHAPTER III
INTRODUCTION TO EXTENDED STATE OBSERVER

In control theory, a state observer is used to estimate the internal states of the system by using the given measurements of the inputs and outputs. To solve the control problems, it is necessary to know the system state. The approach of using state feedback to stabilize a system is a commonly used control technique. In the history of observers, two classes of observer design have emerged. One relies on mathematical plant models to produce state estimates, whereas the other uses available plant knowledge to estimate the part of the physical process that is not described in the plant model, such as unknown disturbances.

For the first type of the observers, the more accurate the plant information is incorporated into the observer, the better state estimate could be obtained. The information includes knowledge of noise and disturbances characterized by deterministic, differential, polynomial, bounded and stochastic descriptions. Consequently, many of
these enhancements were proposed at the cost of detailed model information. However, in practice, an accurate mathematical model of the plant is often unavailable, and the physical systems are usually nonlinear and time varying. Under these circumstances, the second type of observers is designed to estimate the disturbance. Extended State Observer (ESO) introduced in this chapter belongs to this class. The major advantage of ESO is that it can estimate both system unknown disturbance and states.

3.1 Extended State Observer (ESO) Formulation

The concept of Extended State Observer (ESO) was originally proposed by Han in [5]. At the beginning, ESO design was rather complicated due to the need of tuning the variety of parameters. Later, Gao [9] made it more practical by using a single bandwidth parameterization method that reduces the number of tuning parameters to one.

Consider a general second-order plant

\[ \dot{y} + a_1 \dot{y} + a_2 y = F_d + bu \]  

(3.1)

where \( y, F_d, u \) represent the system output, the external disturbance and the system input, respectively, \( a_1 \) and \( a_2 \) are the parameters that can be unknown, and \( F_d \) is also usually unknown. ESO requires the some knowledge of parameter \( b \), so that the plant can be rewritten as

\[ \ddot{z} = f + b_0u \]  

(3.2)

where \( f = a_1 \dot{y} + a_2 y + F_d + (b - b_0)u \), which is called generalized disturbance, or disturbance. Unknown internal states and external disturbance represented by \( F_d \) are parts of the disturbance \( f \).
The ESO can be used to find gives an idea of obtaining $\hat{f}$ (estimate of $f$), and $\hat{y}$ (estimate of $y$). First of all, the plant can be expressed by the following equations:

$$\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 + b_0 u \\
\dot{x}_3 &= h = \dot{f} \\
y &= x_1
\end{align*}$$

(3.3)

where $x_3 = f$, and both $f$ and $h$ are unknown. For a single input system, equation (3.3) can be written in matrix form as

$$\begin{align*}
\dot{x} &= Ax + Bu + Eh \\
y &= Cx
\end{align*}$$

(3.4)

where

$$\begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
0 & 0 & 0
\end{bmatrix},
\begin{bmatrix}
b_0 \\
0 \\
0
\end{bmatrix},
\begin{bmatrix}
1 & 0 & 0
\end{bmatrix},
\begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix},
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}$$

Thus, the ESO can be expressed as

$$\begin{align*}
\dot{z} &= Az + Bu + L(y - \hat{y}) \\
\hat{y} &= Cz
\end{align*}$$

(3.5)

where $\hat{y}$ is the estimation of the system output $y$, and $L$ is the observer gain vector. For the third-order observer, $L$ can be denoted as

$$L = \begin{bmatrix}
\beta_1 \\
\beta_2 \\
\beta_3
\end{bmatrix}$$

(3.6)

Therefore, three parameters $\beta_1$, $\beta_2$ and $\beta_3$ need to be tuned in the observer. As the order of the plant on the observer increases, the number of parameters that need to be
tuned also increases. To simplify the observer tuning, a parameterization method developed by Gao[9] is used.

For a third-order observer described above, the parameterization method places all three of the observer poles at $-\omega_0$, which can be written as

$$\lambda(s) = s^3 + \beta_1 s^2 + \beta_2 s + \beta_3 = (s + \omega_0)^3 \quad (3.7)$$

From where the observer parameters can be replaced by $\omega_o$ by $\beta_1 = 3\omega_o$, $\beta_2 = 3\omega_o^2$, and $\beta_3 = \omega_o^3$. The $\omega_o$ is known as the observer’s bandwidth. The parameterization method can be extended to an ESO of nth order, in which tuning the observer gain L is essentially the same as tuning the bandwidth $\omega_o$.

### 3.2 Extended State Observer (ESO) Used in This Thesis

The ESO has ability to estimate the external disturbance of the system with limited knowledge of the plant model. The application of the ESO in this thesis is two-fold. First of all, the ESO is used to estimate the vehicle’s longitudinal velocities, lateral velocities and yaw rate, and more importantly, the system dynamics. These estimations are essential to road surface detection and identification, which will be described in the next chapter. Second of all, the estimated system dynamics plays a critical role in the vehicle’s anti-skid control by means of ADRC.
CHAPTER IV

ROAD SURFACE CONDITION DETECTION AND IDENTIFICATION

Road surface condition, represented by friction coefficient, plays a significant role in vehicle stability control. The stability control is focused mainly on controlling the vehicle’s yaw rate during vehicle cornering, where the yaw rate is affected by the surfaces’ friction coefficient.

Since the surface’s coefficient of friction is difficult to measure in real-time, it is necessary to relate the surface friction coefficient to certain variables that can be estimated. The relationship between the coefficient of friction and slip ratio has been experimentally determined [13]. However, this relationship is difficult to be represented by a polynomial function. To overcome this drawback, it is proposed to train the relationship by an artificial neural network.

The wheel longitudinal velocities can be used to calculate the slip ratio of each wheel if the rotating speed of each wheel is known. This speed can be estimated or measured from the wheel speed sensors equipped in anti-brake systems (ABS). The longitudinal
velocities of a vehicle may be measured by a GPS device even though it often contains signal noise and drift. The yaw rate can be measured by a gyro sensor. Alternatively, the velocities and yaw rate can be estimated by means of an observer. However, the existing observers are either designed for a linear system or requiring substantial knowledge of the dynamic system model. It is proposed that Extended State Observer (ESO) be used in this study. The ESO can augment both unknown dynamics and disturbances as an extended state and estimate it in real time by using input-output data. Such extended state provides insight to the system dynamics that is crucial for detecting the change of road surface condition.

This chapter presents a method that can quickly detect the change of the road surface condition by means of the ESO, fuzzy logic and neural networks, followed by a method to identify the road surface. The presented methods are demonstrated with a simulation.

4.1 ANN-Based Friction Coefficient Estimation

4.1.1 Introduction to Artificial Neural Network

Artificial Neural Network (ANN) is the information processing paradigm that is originally inspired by biological nerves. As in nature, the network function is determined largely by the connections between elements, as [14] introduced. The elements are called neurons, which work in unison to solve any given specific problem. The ANN learned by examples and can be trained to perform a particular function by making adjustments to the synaptic connections between the neurons. The ANN possesses the remarkable ability to derive meaning from complex or imprecise data and can be used to extract pattern or
detect trend that are too complex to be noticed by either humans or other computer techniques.

ANN has been trained to perform complex function in various field of application including pattern recognition, identification, classification, speech, vision, and control system.

An artificial neuron has many inputs and one output. The neuron has two modes of operation; training mode and the using mode. In the training mode, the neuron is trained to fire or not fire. For particular input patterns, in the using mode, when an input pattern is detected at the input the output associated with it is produced.

As shown in Figure 3, a neuron with a single scalar input and bias has scalar input p, which is transmitted through a connection that multiplies its strength by the scalar weight w, to form the product wp (a scalar). The weighted input wp is argument of transfer function f, which produces the scalar output a. A neuron with R-element vector is shown in Figure 4.

Figure 3 A neuron with a single scalar input
There are many types of transfer functions. Two functions used for training in this chapter are tangent sigmoid transfer function and linear transfer function, as shown in Figure 5.

Most ANN designs use multi-layer network with several neurons on each layer. Figure 6 shows the structure of multi-layer ANN with a weight matrix $W$, a bias vector $b$ and output vector $a$ on each layer.
After establishing an ANN, the network needs to be trained to perform some task. During the training procedure, the weights and bias are adjusted so that the error between the output and the actual output is reduced. The backpropagation algorithm is the most widely used method for determining the derivative of the weights, which can be used to see how each weight is increased or decreased slightly during the training. Standard backpropagation is a gradient descent algorithm, as is the Widrow-Hoff learning rule, in which the network weights are moved along the negative of the gradient of the performance function. The term backpropagation refers to the manner in which the gradient is computed for nonlinear multilayer networks.

The network is trained by the comparison of the output and the target, until the network output matches the target. In Batch training of a network weights and bias is changed based on the entire batch of input vector whereas in incremental or adaptive training the weights and biases of a network are changed as needed after the presentation of each individual input vector. Many training algorithms were developed, such as...
Resilient Backpropagation, Fletcher-Reeves Update, Scaled Conjugate Gradient, Levenberg-Marquardt, etc, [14]. For a network that contains up to a few hundred weights, the Levenberg-Marquardt algorithm will have the fastest convergence. This advantage is especially noticeable if very accurate training is required. In many cases, Levenberg-Marquardt algorithm is able to obtain lower mean square errors than any of the other algorithms tested. In this paper, Levenberg-Marquardt algorithm is chosen as training algorithm.

4.1.2 ANN for Friction Coefficient Estimation

The road surface condition affecting the vehicle dynamics very much is always difficult to determine in real time. To address this problem, an ANN-based road friction coefficient estimation method is proposed.

First of all, the relationship between coefficient of friction and slip ratio depends on the road surface condition. The four surface conditions under study are dry, wet, snowy and icy. An experimental data published in [13] is shown in Figure 7, where the highest values of road surface friction coefficients are 1.09, 0.90, 0.44, and 0.38 under dry, wet, snowy and icy surface conditions, respectively.

A backpropagation neural network was built to train simulate the relationship between the slip ratio and the friction coefficient for each of the four road surface conditions. More specifically, the input of the neural networks is the slip ratio, and the output of the networks is from the friction coefficients. It is worthwhile to note that for a given slip ratio, there are four corresponding friction coefficients. Thus, the relationship between
the single input and the four outputs is very nonlinear which will require multiple hidden layers.

![Graph of friction coefficients](image)

**Figure 7** Slip ratio to surface friction coefficient

Four layers including input and output layer are designed for the network. The numbers of neurons on the input layer, hidden layer 1, hidden layer 2, and output layer are 4, 10, 3, and 4, respectively. The “tangent sigmoid (tansig)” transfer function is used for the input layer, hidden layer 1 and hidden layer 2, while the “linear (purelin)” transfer function is used for the output layer. To train the ANN, 100 pairs of slip ratio and friction coefficient values are collected from the experimental data (see Appendix A). The training goals were set to be 10e-5, with learning rate 0.05 and the use of Levenberg-Marquardt backpropagation training algorithm. The goals were successfully achieved during the training. Figure 8 shows the training process. The ANN met the training goal after the 104 epochs (i.e. iterations).
To verify the trained neural network’s generation, 30 pairs of unseen data were tested (see Appendix B). The average error for the test data was 0.0506, which signified that the network had been successfully trained.

4.2 ESO Formulation for Vehicle Dynamics System

Since the vehicle dynamics is highly nonlinear and time varying, estimated state observer, the ESO was used to actively observe the system output and the internal states. As mentioned earlier, the ESO can augment both unknown dynamics and disturbances as an extended state and estimate it in real time by using input-output data. Such extended state provides insight to the system dynamics that is crucial for detecting the change of road surface condition. ESO also functions as to a filter to reduce the measurement noise.

For the first-order system, the state space system can be represented by
\[
\begin{align*}
\dot{x}_1 &= x_2 + b_0 u \\
\dot{x}_2 &= h = \dot{f} \\
y &= x_1
\end{align*}
\] (4.1)

where \( x_2 = f \), and both \( f \) and \( h \) are unknown. Alternatively, for a single input system,

Equation (4.1) can be written in matrix form as

\[
\begin{align*}
\dot{x} &= Ax + Bu + Eh \\
y &= Cx
\end{align*}
\] (4.2)

where

\[
x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}; \quad A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}; \quad B = \begin{bmatrix} b_0 \\ 0 \end{bmatrix}; \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}; \quad E = \begin{bmatrix} 0 \\ 1 \end{bmatrix}
\]

Thus, the extended state observer (ESO) can be expressed as

\[
\begin{align*}
\dot{\hat{z}} &= Az + Bu + L(y - \hat{y}) \\
\hat{y} &= Cz
\end{align*}
\]

where \( L = [\beta_1, \beta_2]^T \), \( \beta_1 \) and \( \beta_2 \) are the observer gains; \( \hat{y} \) is the estimation of the system output \( y \). Through the parameterization of ESO [9], for the second-order observer, the observer gains are \( \beta_1 = 2 \omega_0 \), and \( \beta_2 = \omega_0^2 \), where \( \omega_0 \) is the observer’s bandwidth.

The ESO formulation for the vehicle dynamic model can be rewritten as

\[
\begin{align*}
\dot{V}_x &= f_1(\delta, V_x, V_y, \gamma) + \frac{F_{LLR}}{m} + \frac{F_{LRR}}{m} \\
\dot{V}_y &= f_2(\delta, V_x, V_y, \gamma) \\
\dot{y} &= f_3(\delta, V_x, V_y, \gamma) + \frac{F_{LLR} J_1}{J} - \frac{F_{LRR} J_1}{J}
\end{align*}
\] (4.3)

where
\[
f_1(\delta, V_x, V_y, \gamma) = \gamma V_y + \frac{1}{m} (F_{LLF} \cos \delta + F_{LRF} \cos \delta - F_{SLF} \sin \delta - F_{SRF} \sin \delta)
\]
\[
f_2(\delta, V_x, V_y, \gamma) = -\gamma V_x + \frac{1}{m} (F_{LLF} \sin \delta + F_{LRF} \sin \delta + F_{SLF} \cos \delta
\]
\[
+ F_{SRF} \cos \delta + F_{SLR} + F_{SRR})
\]
\[
f_3(\delta, V_x, V_y, \gamma) = \frac{1}{J} ((F_{LLF} \cos \delta - F_{LRF} \cos \delta + F_{SLF} \sin \delta - F_{SRF} \sin \delta)l_t
\]
\[
+ (F_{LLF} \sin \delta + F_{LRF} \sin \delta + F_{SLF} \cos \delta + F_{SRF} \cos \delta)l_f - (F_{SLR} + F_{SRR})l_r)
\]

and the system output \( y(t) \) and system input \( u(t) \) are:
\[
y(t) = \begin{bmatrix} V_x \\ V_y \\ \gamma \end{bmatrix}, \quad u(t) = \begin{bmatrix} F_{LLR} \\ F_{LLR} \\ 0 \end{bmatrix}
\]

The \( V_x, V_y \) and \( \gamma \) are the vehicle’s longitudinal velocity, the lateral velocity and the yaw rate, respectively. The system’s inputs are the force applied on each driving wheel.

Rear drive is assumed in this study.

Referring to ESO formulation, the \( f \) and \( b_0 \) can be expressed as
\[
f = \begin{bmatrix} f_1(\delta, V_x, V_y, \gamma) \\ f_2(\delta, V_x, V_y, \gamma) \\ f_3(\delta, V_x, V_y, \gamma) \end{bmatrix}, \quad b_0 = \begin{bmatrix} \frac{1}{m} & \frac{1}{m} & 0 \\ 0 & 0 & 0 \\ \frac{l_t}{J} & -\frac{l_f}{J} & 0 \end{bmatrix}
\]

The applied forces on the driving wheels are the input \( u \).

Thus, the observer for the presented vehicle dynamic system can be designed as
\[
\begin{align*}
\dot{x} &= Ax + Bu + L(y - \hat{y}) \\
\hat{y} &= Cx
\end{align*}
\]

where
\[
x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{6 \times 1}, \quad A = \begin{bmatrix} 0 & I_{3 \times 3} \\ 0 & 0 \end{bmatrix}_{6 \times 6}, \quad B = \begin{bmatrix} b_0 \\ 0 \end{bmatrix}_{6 \times 3}, \quad C = [I_{3 \times 3} \ 0]_{3 \times 6}, \quad L = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}_{6 \times 1},
\]

\[
\beta_1 = \begin{bmatrix} 2\omega_0 & 0 & 0 \\ 0 & 2\omega_0 & 0 \\ 0 & 0 & 2\omega_0 \end{bmatrix}, \quad \beta_2 = \begin{bmatrix} \omega_0^2 & 0 & 0 \\ 0 & \omega_0^2 & 0 \\ 0 & 0 & \omega_0^2 \end{bmatrix}.
\]

Using the values of \( l_t, m \) and \( R \) from Table I, the elements of \( b_0 \) can be determined.

\[
b_0 = \begin{bmatrix} 6.66 \times 10^{-4} & 6.66 \times 10^{-3} & 0 \\ 0 & 0 & 0 \\ 2.76 \times 10^{-4} & -2.76 \times 10^{-4} & 0 \end{bmatrix}.
\]

The only parameter needs to be tuned in the observer is \( \omega_o \), denoted as the observer bandwidth. The performance of the observer can be adjusted by tuning only this observer bandwidth.

4.3 Intelligent Road Surface Change Detection

4.3.1 Simulation Results on Dry Surface Condition

With properly designed ESO, the system outputs \((V_x, V_y, \text{and } \gamma)\), and system dynamics \((f_1, f_2, \text{and } f_3)\) can be estimated. The system dynamics, which are also called generalized disturbance, will be used to monitor road surface condition.

It has been found in this study through a computer simulation that abrupt change of system dynamics is a reflection of the change of road surface condition. The simulation assumes that the vehicle first drives on a dry road for two seconds, then turns counter clockwise for one radian (57.3°) during the next three second. It assumes that the vehicle first drives on a dry road for six seconds, then the right-side wheels are encountered with
ice road while the left-side wheels remain on the same dry road surface. This is referred
to as split-$\mu$ road. The observer bandwidth, $\omega_o$ is chosen as 5.

The assumed driving conditions are listed below and also shown in Figure 9.

**Initial surface condition**: Dry ($\mu=1$) on both sides of wheels, $t = 0 \rightarrow 6$ sec.

**Final surface condition**: dry on left-side and icy on right-side of wheels, $t = 6 \rightarrow 10$ sec.

**Initial slip ratio**: 0.15

**Initial vehicle speed**: 0.1 (m/s)

**Initial wheel speed**: 0.12 (m/s)

**Steering angle** $\delta$: $0 \rightarrow 57.3^\circ$ (1 radian) CCW.

![Figure 9 Assumed input steering angle](image)

It is further assumed that the torques are applied only on the rear wheels, and the maximum torque on each wheel is 247 Nm. Since the input steering angle is given counterclockwise, it means vehicle is turning to the left so that the right wheel torque
should be a little larger than the left torque during the vehicle turning. (See Figure 10 for details)

![Figure 10 Assumed input applied torques](image)

The ESO is also capable of filtering the measurement noise. The effects of noise filtering are shown in Figures 11-13. It should be noted that these figures were generated assuming that the road surface condition remained dry all the way.

![Figure 11 Measured and filtered longitudinal velocity](image)

(a) Measured longitudinal velocity                  (b) Filtered longitudinal velocity
Figure 12 Measured and filtered lateral velocity

(a) Measured lateral velocity  
(b) Filtered lateral velocity

Figure 13 Measured and filtered yaw rate

(a) Measured yaw rate  
(b) Filtered yaw rate

The system dynamics has been observed by the ESO. Figures 14-16 show their respective values assuming that the road surface condition has remained the same as dry surface all the way. The next section will investigate how the system dynamics will change if the vehicle encounters with an icy road surface.
Figure 14 System dynamics $f_1$

Figure 15 System dynamics $f_2$
The observer also adequately observes the system dynamics, as shown in Figures 14-16.

4.3.2 Detection of Road Surface Condition

The detection and classification are simulated using Matlab/Simulink. The following shows one of our vehicle dynamic simulations assuming that the vehicle first drove straight for two seconds and made a counterclockwise turn for a radian in the next second, and continued to drive on a dry road for three more seconds, then the vehicle’s right side wheels encountered icy road surface while the left-side wheels remain on the same dry road surface. This is known as split-μ road.

As mentioned earlier, the generated disturbance $f$ in ESO contains the system internal states and the unknown disturbances. The change of $f$ reflects the change of system
dynamics. In other words, the road surface condition change can be detected by observing the change of $f$.

In the simulation for road surface condition, all the initial conditions were the same as those mentioned in the previous section, except that the surface condition on the right side of the vehicle encountered icy surface in six seconds, as shown in Figure 17.

![Figure 17 Surface’s friction coefficient on the right-side wheels](image)

Figures 18-20 give the comparisons of system dynamics between with and without the road surface change.
Figure 18 Comparison on the System Dynamics Regarding $V_x$

Figure 19 Comparison on the System Dynamics Regarding $V_y$
At $t = 6$ second, when the vehicle encountered a surface condition change from dry to icy, the system dynamics for $V_x$, $V_y$ and $\gamma$ exhibit abrupt changes. The magnitude of change is greatly dependent on the change of the road surfaces friction coefficient. For example, if the icy surface was replaced by a wet surface, the abrupt change of the system dynamics would not be large. Figures 21-24 show the results.
Figure 21 Surface’s friction coefficient on the right side of the vehicle

Figure 22 Comparison on the System Dynamics of $V_x$
The internal dynamics is closely related to the driving conditions such as acceleration and cornering. The external dynamics is heavily dependent on the road surface condition. Thus, when an abrupt change of the system dynamics, $\Delta f$ occurs and exceeds the preset
threshold value, it can be reasoned that the road surface condition likely has changed. The \( \Delta f_1, \Delta f_2 \) and \( \Delta f_3 \) in Table II indicate the changes of the system dynamics of \( V_x, V_y \) and \( \gamma \), respectively. This table contains the differences between the new surface (icy at 6 seconds) and the referenced surface condition. (i.e. Any or both sides of wheels)

**Table II Changes of System Dynamics**

<table>
<thead>
<tr>
<th>Surface Condition</th>
<th>Max. Change (Time period 6→6.25 sec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \Delta f_1/</td>
</tr>
<tr>
<td>Left Dry</td>
<td>ref</td>
</tr>
<tr>
<td>Left Wet</td>
<td>0.0420</td>
</tr>
<tr>
<td>Left Snow</td>
<td>0.1074</td>
</tr>
<tr>
<td>Left Ice</td>
<td>0.1671</td>
</tr>
<tr>
<td>Wet Dry</td>
<td>0.0009</td>
</tr>
<tr>
<td>Wet Wet</td>
<td>0.0521</td>
</tr>
<tr>
<td>Wet Snow</td>
<td>0.1166</td>
</tr>
<tr>
<td>Wet Ice</td>
<td>0.1863</td>
</tr>
<tr>
<td>Snow Dry</td>
<td>0.0088</td>
</tr>
<tr>
<td>Snow Wet</td>
<td>0.0623</td>
</tr>
<tr>
<td>Snow Snow</td>
<td>0.1260</td>
</tr>
<tr>
<td>Snow Ice</td>
<td>0.1960</td>
</tr>
<tr>
<td>Ice Dry</td>
<td>0.1224</td>
</tr>
<tr>
<td>Ice Wet</td>
<td>0.0751</td>
</tr>
<tr>
<td>Ice Snow</td>
<td>0.1508</td>
</tr>
<tr>
<td>Ice Ice</td>
<td>0.2107</td>
</tr>
</tbody>
</table>

When considering the possibility of encountering a split-\( \mu \) road surface, there are a total of 16 scenarios or combinations. Each has distinct characteristics, which can be distinguished by fuzzy logic. The next section will describe how a fuzzy logic method can be used to help identify the road surface condition.
4.4 Introduction to Fuzzy Logic

Fuzzy logic was developed by Zadeh in mid 1960’s to represent approximate reasoning, and is introduced to processing industry, traffic control and household applications in the 1970’s. It uses fuzzy set theory, in which a fuzzy set is represented by a membership function.

4.4.1 Fuzzy Sets

A fuzzy set is a set without a crisp, clearly defined boundary. It can contain elements with only a partial degree of membership. Unlike an ordinary set where each object or element either belong or does not belong to the set, a partial membership in a fuzzy set is possible. Thus, in fuzzy logic, the truth of any statement becomes a matter of degree. For example, if \( X \) is a collection of objects denoted generically by \( x \), then a fuzzy set \( A \) in \( X \) is defined as a set of ordered pairs:

\[
A = \{(x, \mu_A(x)) \mid x \in X\}
\]

(4.8)

Where \( \mu_A(x) \) is called the membership function (MF) for the fuzzy set \( A \).

4.4.2 Membership functions

A membership function is a curve that defines how each point in the input space is mapped to a membership value (or degree of membership) between 0 and 1. It associates a weighting with each of the inputs that are processed, define functional overlap between inputs, and ultimately determines an output response. The fuzzy rules use the input membership values as weighting factors to determine their influence on the fuzzy output sets of the final output conclusion. Once the functions are inferred, scaled, and combined, they are defuzzified into a crisp output which drives the system. There are many types of...
membership functions. Figure 25 shows eleven types of membership functions that are most commonly used in the fuzzy logic system design. In this thesis, the triangular (trimf) and trapezoidal (trapmf) membership functions are used in the fuzzy logic system design for road surface detection.

![Commonly used membership functions](image)

When triangular membership function is defined, three parameters need to be determined. The mathematical expression for triangular membership function specified parameters $a$, $b$, and $c$ can be expressed as Equation (39).

$$trimf(a, b, c) = \begin{cases} 0, & x \leq a \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ \frac{c-x}{c-b}, & b \leq x \leq c \\ 0, & c \leq x \end{cases}$$

(4.9)
Similar to triangular membership function, the trapezoidal membership function is specified by four parameters. Equation (40) gives a mathematical expression for a trapezoidal membership function specified by number a, b, c, and d.

\[
\text{trapmf}(a,b,c,d) = \begin{cases} 
0, & x \leq a \\
\frac{x-a}{b-a}, & a \leq x \leq b \\
1, & b \leq x \leq c \\
\frac{d-x}{d-c}, & c \leq x \leq d \\
0, & d \leq x 
\end{cases} 
\]  
(4.10)

4.4.3 Fuzzy Operations

Fuzzy operations are operations on fuzzy sets. These operations are generalization of crisp set operations. The basic operations are fuzzy complements, fuzzy intersections, and fuzzy unions, as Equation (41), (42), and (43) showed, respectively.

\[
\mu_{\overline{A}}(x) = 1 - \mu_A(x) 
\]  
(4.11)

\[
\mu_{A \cup B}(x) = \max[\mu_A(x), \mu_B(x)] 
\]  
(4.12)

\[
\mu_{A \cap B}(x) = \min[\mu_A(x), \mu_B(x)] 
\]  
(4.13)

4.4.4 Fuzzy If-Then Rules

Fuzzy sets and fuzzy operators are the subjects and verbs of fuzzy logic, and the if-then rules are used to formulate the conditional statements that comprise fuzzy logic. A fuzzy if-then rule is of the form

\[
\text{If } X=A \text{ and } Y=B, \text{ Then } Z=C
\]

Where X, Y, and Z are linguistic variables and A, B, and C are linguistic terms. The ‘If’ part is called the antecedent or premise, while the ‘Then’ part is called the consequence or conclusion.
4.5 Fuzzy Inference for Fast Road Surface Classification

A fuzzy logic system can be designed based on the different values of the system dynamics changes as shown in Table II. For each $\Delta f_1$, $\Delta f_2$ and $\Delta f_3$, five membership functions are designed.

The ranges of the membership functions for $\Delta f_1$ are:

- PS (Positive Small): $\leq 0.06$;
- PMS (Positive Medium Small): $0.05 \rightarrow 0.1$;
- PM (Positive Medium): $0.09 \rightarrow 0.14$;
- PML (Positive Medium Large): $0.13 \rightarrow 0.18$;
- PL (Positive Large): $\geq 0.17$.

The ranges of the membership functions for $\Delta f_2$ are:

- PS (Positive Small): $\leq 0.03$;
- PMS (Positive Medium Small): $0.02 \rightarrow 0.05$;
- PM (Positive Medium): $0.04 \rightarrow 0.07$;
- PML (Positive Medium Large): $0.06 \rightarrow 0.09$;
- PL (Positive Large): $\geq 0.08$.

The ranges of the membership functions for $\Delta f_3$ are:

- PS (Positive Small): $\leq 0.015$;
- PMS (Positive Medium Small): $0.01 \rightarrow 0.025$;
- PM (Positive Medium): $0.02 \rightarrow 0.035$;
- PML (Positive Medium Large): $0.03 \rightarrow 0.045$;
- PL (Positive Large): $\geq 0.04$.

Figure 26-28 show the designed input membership functions of $\Delta f_1$, $\Delta f_2$, and $\Delta f_3$. 
Figure 26 Membership function for $\Delta f_1/|f_1|$

Figure 27 Membership function for $\Delta f_2/|f_2|$
Table III Fuzzy rule table for all cases of surface change

<table>
<thead>
<tr>
<th>Surface Change</th>
<th>Max. Change (Time period 6 → 6.25 sec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left Dry</td>
<td>PS</td>
</tr>
<tr>
<td>Dry Snowy</td>
<td>PM</td>
</tr>
<tr>
<td>Dry Icy</td>
<td>PML</td>
</tr>
<tr>
<td>Wet Dry</td>
<td>PS</td>
</tr>
<tr>
<td>Wet Snowy</td>
<td>PM</td>
</tr>
<tr>
<td>Wet Icy</td>
<td>PL</td>
</tr>
<tr>
<td>Snow Dry</td>
<td>PS</td>
</tr>
<tr>
<td>Snow Wet</td>
<td>PMS</td>
</tr>
<tr>
<td>Snow Snowy</td>
<td>PM</td>
</tr>
<tr>
<td>Snow Icy</td>
<td>PL</td>
</tr>
<tr>
<td>Ice Dry</td>
<td>PL</td>
</tr>
<tr>
<td>Ice Snowy</td>
<td>PML</td>
</tr>
<tr>
<td>Ice Icy</td>
<td>PML</td>
</tr>
</tbody>
</table>

Figure 28 Membership function for $\Delta f_3/|f_3|$
Table III essentially lists 15 fuzzy rules. The purpose of designing a fuzzy inference system was to determine the surface condition on the left and right wheels.

The output membership function was designed and shown in Figure 29.

![Figure 29 Output membership function](image)

The output variables are named based on surface conditions on left and right sides of the vehicle. For instance, Dry-Wet indicates the surface condition with dry on the left side of the vehicle and wet on the right side of the vehicle. The fifteen fuzzy logic rules are listed below:

Rule 1: If \( \Delta f_1/|f_1| \) is PS and \( \Delta f_2/|f_2| \) is PS and \( \Delta f_3/|f_3| \) is PMS then (Dry-Wet is true) (others are untrue).

Rule 2: If \( \Delta f_1/|f_1| \) is PM and \( \Delta f_2/|f_2| \) is PMS and \( \Delta f_3/|f_3| \) is PM then (Dry-Snowy is true) (others are untrue).

Rule 3: If \( \Delta f_1/|f_1| \) is PML and \( \Delta f_2/|f_2| \) is PML and \( \Delta f_3/|f_3| \) is PL then (Dry-Icy is true) (others are untrue).
Rule 4: If \(\Delta f_1/|f_1|\) is PS and \(\Delta f_2/|f_2|\) is PS and \(\Delta f_3/|f_3|\) is PS then (Wet-Dry is true) (others are untrue).

Rule 5: If \(\Delta f_1/|f_1|\) is PMS and \(\Delta f_2/|f_2|\) is PS and \(\Delta f_3/|f_3|\) is PMS then (Wet-Wet is true) (others are untrue).

Rule 6: If \(\Delta f_1/|f_1|\) is PM and \(\Delta f_2/|f_2|\) is PMS and \(\Delta f_3/|f_3|\) is PML then (Wet-Snowy is true) (others are untrue).

Rule 7: If \(\Delta f_1/|f_1|\) is PL and \(\Delta f_2/|f_2|\) is PMS and \(\Delta f_3/|f_3|\) is PM then (Wet-Icy is true) (others are untrue).

Rule 8: If \(\Delta f_1/|f_1|\) is PS and \(\Delta f_2/|f_2|\) is PL and \(\Delta f_3/|f_3|\) is PS then (Snowy-Dry is true) (others are untrue).

Rule 9: If \(\Delta f_1/|f_1|\) is PMS and \(\Delta f_2/|f_2|\) is PL and \(\Delta f_3/|f_3|\) is PM then (Snowy-Snowy is true) (others are untrue).

Rule 10: If \(\Delta f_1/|f_1|\) is PL and \(\Delta f_2/|f_2|\) is PL and \(\Delta f_3/|f_3|\) is PML then (Snowy-Icy is true) (others are untrue).

Rule 11: If \(\Delta f_1/|f_1|\) is PS and \(\Delta f_2/|f_2|\) is PL and \(\Delta f_3/|f_3|\) is PL then (Icy-Dry is true) (others are untrue).

Rule 12: If \(\Delta f_1/|f_1|\) is PMS and \(\Delta f_2/|f_2|\) is PMS and \(\Delta f_3/|f_3|\) is PMS then (Icy-Wet is true) (others are untrue).

Rule 13: If \(\Delta f_1/|f_1|\) is PML and \(\Delta f_2/|f_2|\) is PML and \(\Delta f_3/|f_3|\) is PM then (Icy-Snow is true) (others are untrue).
Rule 15: If ($\Delta f_1/|f_1|$ is PL) and ($\Delta f_2/|f_2|$ is PML) and ($\Delta f_3/|f_3|$ is PL) then (Icy-Icy is true) (others are untrue).

The Fuzzy Logic Toolbox in Matlab was used to develop the fuzzy inference system. Figure 30 shows the rule editor in the interface of the toolbox.

To verify that the fuzzy logic system can indeed be used to identify the road surface condition, some arbitrary numbers were used within in each range of the input membership function. The fuzzy inference logic system will automatically determine which fuzzy logic rule to be fired, and generate the output value for each case. The output value ranges from 0 to 1, which essentially indicate the degree of confidence.
shows several examples of this verification. The four verified surface changes are Dry-Wet, Dry-Snowy, Dry-Icy, and Wet-Dry.

Table IV Inputs to the fuzzy logic system

<table>
<thead>
<tr>
<th></th>
<th>Case 1</th>
<th></th>
<th>Case 2</th>
<th></th>
<th>Case 3</th>
<th></th>
<th>Case 4</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Value</td>
<td>Range</td>
<td>Value</td>
<td>Range</td>
<td>Value</td>
<td>Range</td>
<td>Value</td>
<td>Range</td>
</tr>
<tr>
<td>$\Delta f_1/f_1$</td>
<td>0.04</td>
<td>PS</td>
<td>0.11</td>
<td>PM</td>
<td>0.16</td>
<td>PML</td>
<td>0.035</td>
<td>PS</td>
</tr>
<tr>
<td>$\Delta f_2/f_2$</td>
<td>0.02</td>
<td>PS</td>
<td>0.03</td>
<td>PMS</td>
<td>0.07</td>
<td>PML</td>
<td>0.025</td>
<td>PS</td>
</tr>
<tr>
<td>$\Delta f_3/f_3$</td>
<td>0.017</td>
<td>PMS</td>
<td>0.03</td>
<td>PM</td>
<td>0.042</td>
<td>PL</td>
<td>0.012</td>
<td>PS</td>
</tr>
</tbody>
</table>

Table V Corresponding outputs for each case

<table>
<thead>
<tr>
<th>Condition</th>
<th>Fuzzy System Estimation Output</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Case 1</td>
</tr>
<tr>
<td>Dry-Dry</td>
<td>0.0605</td>
</tr>
<tr>
<td>Dry-Wet</td>
<td>0.9395</td>
</tr>
<tr>
<td>Dry-Snowy</td>
<td>0.0605</td>
</tr>
<tr>
<td>Dry-Icy</td>
<td>0.0605</td>
</tr>
<tr>
<td>Wet-Dry</td>
<td>0.0605</td>
</tr>
<tr>
<td>Wet-Wet</td>
<td>0.0605</td>
</tr>
<tr>
<td>Wet-Snowy</td>
<td>0.0605</td>
</tr>
<tr>
<td>Wet-Icy</td>
<td>0.0605</td>
</tr>
<tr>
<td>Snowy-Dry</td>
<td>0.0605</td>
</tr>
<tr>
<td>Snowy-Wet</td>
<td>0.0605</td>
</tr>
<tr>
<td>Snowy-Snowy</td>
<td>0.0605</td>
</tr>
<tr>
<td>Snow-Icy</td>
<td>0.0605</td>
</tr>
<tr>
<td>Icy-Dry</td>
<td>0.0605</td>
</tr>
<tr>
<td>Icy-Wet</td>
<td>0.0605</td>
</tr>
<tr>
<td>Icy-Snowy</td>
<td>0.0605</td>
</tr>
<tr>
<td>Icy-Icy</td>
<td>0.0605</td>
</tr>
<tr>
<td><strong>Accuracy</strong></td>
<td>100%</td>
</tr>
</tbody>
</table>

Table V shows that the fuzzy inference system accurately identified the road surface condition.
4.6 Road Surface Identification

As mentioned in the previous section, a fuzzy inference system can be used to quickly classify the road surface. The system could work well if and only if the reference system is dry on both sides of wheels. In other words, the fuzzy inference system is based on the relative difference of system dynamics. It is, however, more desired to be able to identify the road surface regardless of the reference surface condition. To do so, a road surface identification scheme is presented in this section.

The ESO plays an important role in estimating the vehicle’s velocities and yaw rate from one surface to another. The proposed identification scheme requires measurements of the longitudinal velocity ($V_x$), the lateral velocity ($V_y$), the yaw rate ($\gamma$) at the center of mass, and speed of each wheel. The later is usually provided by the popular ABS (anti-brake system). The yaw rate can be measured by a gyro yaw rate sensor, but it needs to compensate for the difference due to the road surface’s bank angle.

For each side of the wheels, The ANN generates four friction coefficients corresponding to dry, wet, snowy and icy surface conditions. These four possible friction coefficient values were used as inputs to the vehicle model, generating four sets of velocities and yaw rates values for each possible weather condition. Then, these values were compared with the observed values.

The flow chart shown in Figure 31 outlines the road surface identification procedure.
Figure 31 Road Surface Identification Scheme

The last step of this identification is to compare the errors between the observed and the expected output values. Mean Square Error (MSE) was used to quantify the discrepancy. Equation (4.14) shows the formula for calculating the error on $V_x$.

$$
MSE(V_x) = \frac{1}{n} \sum_{i=1}^{n} (\hat{V}_x - \tilde{V}_x)^2
$$

Where $t$ is the duration time period of simulation, $n$ is the number of sampled data taken, $\hat{V}_x$ is the ESO observed longitudinal velocity, $\tilde{V}_x$ is the system identified longitudinal velocity. This MSE calculation method is also used for lateral velocity and yaw rate error calculation. The SSE (Sum of Square Errors) for $\Delta V_x$, $\Delta V_y$ and $\Delta \gamma$ are:
\[ SSE = \Delta V_x^2 + \Delta V_y^2 + \Delta \gamma^2 \]  

(4.15)

Table V shows the simulation result that correctly identifies the road surface, which is dry on front-left wheel and icy on front-right wheel. This is a road surface identification scheme that begins right after the change of road surface condition is detected and classified. Not only can it be thought of a stand-alone identification method, but also can be used to verify the result of the intelligent surface condition detection and classification. As shown in Table V, the smallest SSE corresponds to the combination of dry on the left and icy on the right, which matches the assumed road surface condition.

Table VI Identification of road surface condition

<table>
<thead>
<tr>
<th>Possibilities</th>
<th>Error Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ΔVx</td>
</tr>
<tr>
<td>Left</td>
<td>Right</td>
</tr>
<tr>
<td>Dry</td>
<td>Dry</td>
</tr>
<tr>
<td>Wet</td>
<td>Wet</td>
</tr>
<tr>
<td>Dry</td>
<td>Snow</td>
</tr>
<tr>
<td>Wet</td>
<td>Ice</td>
</tr>
<tr>
<td>Wet</td>
<td>Dry</td>
</tr>
<tr>
<td>Wet</td>
<td>Wet</td>
</tr>
<tr>
<td>Wet</td>
<td>Snow</td>
</tr>
<tr>
<td>Wet</td>
<td>Ice</td>
</tr>
<tr>
<td>Snow</td>
<td>Dry</td>
</tr>
<tr>
<td>Snow</td>
<td>Wet</td>
</tr>
<tr>
<td>Snow</td>
<td>Snow</td>
</tr>
<tr>
<td>Snow</td>
<td>Ice</td>
</tr>
<tr>
<td>Ice</td>
<td>Dry</td>
</tr>
<tr>
<td>Ice</td>
<td>Wet</td>
</tr>
<tr>
<td>Ice</td>
<td>Snow</td>
</tr>
<tr>
<td>Ice</td>
<td>Ice</td>
</tr>
</tbody>
</table>

Once the road surface condition has been detected, an anti-skid controller is needed to prevent the vehicle from skidding and spinning on a low friction coefficient surface, such as snow or ice.
Passive systems for vehicle safety control has been investigated by many researchers and developed by the automobile manufacturers. In the passive vehicle control family, several technologies have found their way into production commercial vehicles, such as Anti-Brake Systems (ABS), Traction Control (TC), and Vehicle Stability Control (VSC). The ABS system is designed to prevent vehicle wheel skidding during braking, whereas the TC system is to prevent vehicle wheel skidding during acceleration. VSC is a technology of applying electronic control to vehicles. It was developed to improve the vehicle safety by preventing vehicles from spinning and drifting out with proper control system design. It is also referred to as yaw stability control system or electronic stability control systems. Anti-skid control (ASC) proposed in this thesis can be classified under the category of VSC technology. In other words, the ASC is about skid preventing due to sudden road surface change, while the VSC is about maintaining the vehicle stability during cornering.
When a vehicle is cornering without proper control, the road surface condition may affect the vehicle trajectory. If the road surface’s tire friction coefficient suddenly becomes very small, the driving wheels will slip and the vehicle will likely skid. Under this circumstance, there are several ways to control the vehicle’s yaw motion. They are such as differential braking systems, and steer-by-wire systems, etc. The differential braking systems utilize the ABS brake system on the vehicle to apply differential braking between the left and right wheels to control yaw rate, while the steer-by-wire systems track the driver’s steering angle input by adding an assistant steering angle to the wheels.

Some vehicle stability control methods based on the slip ratio estimation have been developed. Fujimoto, Fuji, and Takahashi in [16] proposed a method for estimating the slip ratio so as to control the vehicle by properly distributing the torque based on wheel’s slip ratio. The drawback of this method is the necessity of knowing the vehicle model in order to properly control the vehicle stability. Hallowell and Ray [4] developed a traction control algorithm by using independent torque control on each wheel.

The proposed anti-skid control is ADRC-based, which does not require much knowledge of the vehicle dynamics. The controller uses the desired yaw rate as reference, and rejects all system dynamics and external disturbances other than the vehicle yaw rate. The controller minimizes the difference between the actual and the desired yaw rate by applying torques to each wheel. A torque distribution algorithm is designed to optimize the control performance.
5.1 Active Disturbance Rejection Controller (ADRC)

Active Disturbance Rejection Controller, known as ADRC, was originally proposed by Han in [7] for nonlinear control. Later, Gao [9] simplified control law and tuning method. ADRC is a new design methodology that uses a very simple model, typically an integrator or a double integrator for a first-order or second-order system, for the controller design and treat any discrepancy between this model and the unknown, nonlinear or time-varying plant as disturbance to be estimated and rejected. The result is a high performance control system that is tuned only with one parameter: the bandwidth. ADRC is built by using the feedback states which can be observed by the ESO described earlier.

For a general second-order plant, the dynamics equation can be written as

$$\dot{y} = f + b_0 u$$  \hspace{1cm} (5.1)

The basic idea is to find an estimate of $f$, called $\hat{f}$, and use it in the control law as

$$u = (u_0 - \hat{f})/b_0$$ \hspace{1cm} (5.2)

By doing so, the control law reduces the original plant to an integral plant

$$\dot{y} = (f - \hat{f}) + u_0 \approx u_0$$ \hspace{1cm} (5.3)

which can be easily controlled by a Proportional-Derivative (PD) controller as

$$u_0 = k_p (r - \hat{y}) - k_d \dot{\hat{y}}$$ \hspace{1cm} (5.4)
The ESO is used to observe the disturbance and system output. The states $z_1$, $z_2$, and $z_3$ represent the estimated system output $\hat{y}$, its derivative $\dot{\hat{y}}$, and system dynamics $\hat{f}$. With the observer being properly designed, the control law can be expressed as

$$u = \frac{k_p (r - \hat{y}) - k_d \dot{\hat{y}} - \hat{f}}{b_0} = \frac{k_p (r - z_1) - k_d z_2 - z_3}{b_0}$$  \hspace{1cm} (5.5)$$

Where $r$ is the reference input, $k_p$ and $k_d$ are proportional and derivative the controller gains that can be selected as $k_p = \omega_c^2$, and $k_d = 2\xi \omega_c$, where $\omega_c$ and $\xi$ are the desired closed-loop natural frequency and damping ratio. Critical damping (i.e. $\xi = 1$) is chosen to avoid system oscillations. The $\omega_c$ is usually chosen as $\frac{1}{5}$ to $\frac{1}{3}$ of $\omega_o$.

By relating $\omega_c$ to $\omega_o$, the controller can be easily designed when the observer bandwidth $\omega_o$ is properly tuned.

An nth-order plant with unknown dynamics and external disturbance can be written as

$$y^{(n)} = f(t, y, \dot{y}, \cdots, y^{(n-1)}, u, \dot{u}, \cdots, u^{(n-1)}, w) + bu$$  \hspace{1cm} (5.6)$$

Rewrite the plant to the state space model form

$$\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
\vdots
\dot{x}_n &= x_{n+1} + b_0 u \\
\dot{x}_{n+1} &= f \\
y &= x_1
\end{align*}$$  \hspace{1cm} (5.7)$$
\[
\begin{align*}
\dot{x} &= Ax + Bu + Ef \\
y &= Cx
\end{align*}
\]

where

\[
A = \begin{bmatrix}
0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 1 \\
0 & 0 & 0 & \cdots & 0
\end{bmatrix}_{(n+1) \times (n+1)}, \quad
B = \begin{bmatrix}
\vdots \\
b_0 \\
\vdots \\
0 \\
1
\end{bmatrix}_{(n+1) \times 1}, \quad
C = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}_{1 \times (n+1)}, \quad
E = \begin{bmatrix} 0 \\
\vdots \\
0 \\
1 \end{bmatrix}_{(n+1) \times 1}
\]

The ESO can be constructed as

\[
\begin{align*}
\dot{z} &= Az + Bu + L(y - \hat{y}) \\
\hat{y} &= Cz
\end{align*}
\]  

(5.9)

where

\[
z = \begin{bmatrix}
z_1 \\
z_2 \\
\vdots \\
z_n \\
z_{n+1}
\end{bmatrix}, \quad
L = \begin{bmatrix}
\beta_1 \\
\beta_2 \\
\vdots \\
\beta_n \\
\beta_{n+1}
\end{bmatrix}
\]

The observer bandwidth, \(\omega_o\) can be designed by using a parameterization method by placing all the observer poles at \(-\omega_0\), which can be written as

\[
s^n + \beta_1 s^{n-1} + \cdots + \beta_{n-1}s + \beta_n = (s + \omega_0)^n
\]

(5.10)

The parameters in \(L\) can then be determined from Equation (5.10). In the case of \(n = 3\),

\[
\beta_1 = 3\omega_0, \quad \beta_2 = 3\omega_0^2, \quad \beta_3 = \omega_0^3.
\]

With the observer proper designed, the ADRC control law for a \(n\)th-order plant can be designed as

\[
u = \frac{k_p(r - \hat{y}) - k_{d_1}\hat{y} - \cdots - k_{d_{n+1}}\hat{y}^{(n-1)} - \hat{f}}{b_0} = \frac{k_p(r - z_1) - k_{d_1}z_2 - \cdots - k_{d_{n+1}}z_n - z_{n+1}}{b_0}
\]

55
where the controller gains are determined by setting the poles at \(-\omega_c\),

\[ s^n + k_{d,n}s^{n-1} + \cdots + k_{d,1}s + k_p = (s + \omega_c)^n \]  

(5.11)

where \(\omega_c\) is chosen as \(\frac{1}{5}\) to \(\frac{1}{3}\) of \(\omega_o\).

### 5.2 ADRC-Based Anti-Skid Controller Design

#### 5.2.1 Control Objective

As described earlier, the anti-skid controller uses the desired yaw rate value as reference. Thus, the control objective is to track the actual yaw rate and minimize its difference from the desired one.

Note that the bank angles on a slant road and vehicle yaw resonance are neglected. The desired yaw rate for steady state can be calculated [11] by the following formula.

\[
\gamma_{\text{desired}} = \frac{V_x \delta}{(l_f + l_r)(1 + \frac{V_x^2}{V^2_x + V^2_y})}
\]  

(5.12)

where \(\delta\) is the steering angle (the driver’s input), \(l_f\) and \(l_r\) are distances between the vehicle center of mass and the front axle of the wheels, and distance between the vehicle center of mass and the back axle of the wheels, respectively. \(V_x\) and \(V_y\) are the longitudinal and lateral velocities, respectively.
5.2.2 Controller Design

In this section, an ADRC-based anti-skid controller is proposed. The vehicle dynamic equations calculate the slip ratio of each wheel, the actual steering angle $\delta$ and $V_x$, $V_y$, and $\gamma$ which are fed into the ESO formulation to estimate the next state of $V_x$, $V_y$, and $\gamma$, called $\hat{V}_x$, $\hat{V}_y$, and $\hat{\gamma}$. The calculated desired yaw rate and the actual yaw rate are fed back to the ADRC.

In Chapter IV, the applied longitudinal forces were used to observe the surface condition change. However, in using the ADRC control, the applied forces on wheels cannot be directly used as control inputs. The direct control inputs need to be the torques that are directly applied to each wheel. The required total torque for a vehicle has been defined by Osborn and Shim [15] as

$$T_{total} = mR\sqrt{(\mu g)^2 - a_y^2}$$  \hspace{1cm} (5.13)

The calculated total torque is constantly checked to make sure that it does not exceed the maximum torque available to a vehicle. Since yaw rate is the only control objective, the controller works only when the vehicle is turning where the yaw rate is non-zero. It does not apply to a vehicle driving straight with a constant speed, acceleration or deceleration.

According to ESO formulation:

$$\dot{y}(t) = f + b_0u(t)$$
Based on the vehicle model proposed in Chapter II and the control signal selection introduced in the last paragraph, the system outputs and the control inputs respectively are:

\[
y(t) = \begin{bmatrix} V_x \\ V_y \\ \gamma \end{bmatrix}, \quad u(t) = \begin{bmatrix} \tau_{LRR} \\ \tau_{LLR} \\ \tau_{LRF} \\ \tau_{LLF} \end{bmatrix}.
\]

The vehicle dynamic model equations can then be written as an ESO formulation with the torque applied to each wheel becomes part of the control inputs.

\[
\begin{align*}
\dot{V}_x &= f_4(\delta, V_x, V_y, \gamma) + \frac{\tau_{LRR}}{mR} + \frac{\tau_{LLR}}{mR} + \frac{\tau_{LRF}}{mR} + \frac{\tau_{LLF}}{mR} \\
\dot{V}_y &= f_5(\delta, V_x, V_y, \gamma) + \frac{\tau_{LRF}}{mR} + \frac{\tau_{LBF}}{mR} \\
\dot{\gamma} &= f_6(\delta, V_x, V_y, \gamma) + \frac{\tau_{LRF}}{JR} + \frac{\tau_{LRF}}{JR} + \frac{\tau_{LRF}}{JR} + \frac{\tau_{LRF}}{JR}
\end{align*}
\]

(5.14)

where

\[
\begin{align*}
f_4(\delta, V_x, V_y, \gamma) &= f_1(\delta, V_x, V_y, \gamma) + \frac{F_{LRR}}{m} + \frac{F_{LRF}}{m} - \frac{\tau_{LRF}}{mR} - \frac{\tau_{LRF}}{mR} - \frac{\tau_{LRF}}{mR} - \frac{\tau_{LRF}}{mR} \\
f_5(\delta, V_x, V_y, \gamma) &= f_2(\delta, V_x, V_y, \gamma) - \frac{\tau_{LRF}}{mR} - \frac{\tau_{LRF}}{mR} \\
f_6(\delta, V_x, V_y, \gamma) &= f_3(\delta, V_x, V_y, \gamma) + \frac{F_{LRF}}{J} - \frac{F_{LRF}}{J} - \frac{\tau_{LRF}}{JR} - \frac{\tau_{LRF}}{JR} - \frac{\tau_{LRF}}{JR} - \frac{\tau_{LRF}}{JR}
\end{align*}
\]

(5.15) (5.16) (5.17)

where \(f_1(\delta, V_x, V_y, \gamma)\), \(f_2(\delta, V_x, V_y, \gamma)\), and \(f_3(\delta, V_x, V_y, \gamma)\) were defined by Equations 4.4, 4.5, and 4.6 in Chapter IV. Since the control signals, which are the applied torques on the wheels does not directly affect the system outputs (\(V_x, V_y, \) and \(\gamma\)). Thus, the torque
related terms that were added in Equation 5.14 will need to be subtracted in the modified
system dynamics Equations 5.15-5.17.

The system dynamics and the control signal gain are

\[
\begin{align*}
   f &= \begin{bmatrix} 
   f_4(\delta, V_x, V_y, \gamma) \\
   f_5(\delta, V_x, V_y, \gamma) \\
   f_6(\delta, V_x, V_y, \gamma)
   \end{bmatrix},
   b_0 = \begin{bmatrix}
   \frac{1}{mR} & \frac{1}{mR} & \frac{1}{mR} & \frac{1}{mR} \\
   0 & 0 & \frac{1}{mR} & \frac{1}{mR} \\
   \frac{l_1}{JR} & \frac{l_1}{JR} & \frac{l_2}{JR} & \frac{l_2}{JR}
   \end{bmatrix},
\end{align*}
\]

Thus, the observer can be designed as

\[
\begin{align*}
   \dot{x} &= Ax + Bu + L(y - \hat{y}) \\
   \hat{y} &= Cx
\end{align*}
\]  

(5.18)

where

\[
   x = \begin{bmatrix} x_1 \\
   x_2 \end{bmatrix}_6, 
   A = \begin{bmatrix} 0 & I_{3 \times 3} \\
   0 & 0 \end{bmatrix}_6, 
   B = \begin{bmatrix} b_0 \\
   0 \end{bmatrix}_6, 
   C = [I_{3 \times 3} \ 0]_{3 \times 6}, 
   L = \begin{bmatrix} \beta_1 \\
   \beta_2 \end{bmatrix}_{2 \times 3}
\]

\[
\begin{align*}
   \beta_1 &= \begin{bmatrix} 2\omega_0 & 0 & 0 \\
   0 & 2\omega_0 & 0 \\
   0 & 0 & 2\omega_0
   \end{bmatrix}, 
   \beta_2 &= \begin{bmatrix} \omega_0^2 & 0 & 0 \\
   0 & \omega_0^2 & 0 \\
   0 & 0 & \omega_0^2
   \end{bmatrix}
\end{align*}
\]

Substituting the values for m and R, gives the \(b_0\) value as

\[
   b_0 = \begin{bmatrix} 0.002 & 0.002 & 0.002 & 0.002 \\
   8.28 \times 10^{-4} & 8.28 \times 10^{-4} & 8.28 \times 10^{-4} & 8.28 \times 10^{-4}
   \end{bmatrix}
\]

As mentioned earlier, the control input of ADRC design can be represented by

\[
   u = \frac{k_p (y_d - \hat{y}) - f_6}{b_0(3,:)}
\]

(5.19)
where $b_0(3,:) = [8.28 \times 10^{-4} \quad 8.28 \times 10^{-4} \quad 8.28 \times 10^{-4} \quad 8.28 \times 10^{-4}]$, the $k_p$ is the controller gain for a proportional controller. It can be described in terms of natural frequency $\omega_c$.

$$k_p = \omega_c$$  \hspace{1cm} (5.20)

The $\omega_c$ is closely related to the bandwidth $\omega_o$.

The slip ratio for each tire is independent of each other, and can be all different. It can be reasoned that a smaller torque should be applied to the wheel with a larger slip ratio. In other words, no torque should be applied to any wheel which completely slips (i.e. with slip ratio of 1). It is proposed that extent of torque distribution should be inversely proportional to the wheels slip ratio. Thus, four individual gains, $K_{RF}$, $K_{RR}$, $K_{LF}$, and $K_{LR}$, were added in the controller. These gains are calculated by

$$K_{ij} = K \frac{1 - \sigma_{ij}}{\sum \sigma}$$  \hspace{1cm} (5.21)

where $K_{ij}$ is the torque distribution gain for the axle $i$ and side $j$ wheel; $K$ is the proportional gain; $\sigma_{ij}$ is the slip ratio of axle $i$ side $j$ wheel; $\sum \sigma$ is the sum of four slip ratios of the wheels.

The schematic diagram of the proposed controller is shown in Figure 32 and described below.
The torques applied to the wheels are the control inputs of the system. Based on Equation 2.8 in Chapter II, the applied torque on each wheel indirectly affects the corresponding wheel’s angular acceleration. The changed angular velocity will then change the vehicle speed and consequently the slip ratio, which will eventually change the system outputs, $V_x$, $V_y$ and $\gamma$.

5.3 Simulation Results

The proposed anti-skid control was simulated using Matlab/Simulink. To better demonstrate the control performance, three situations were simulated, system without the controller, ADRC with equal-torque distribution, and ADRC with unequal-torque distribution. The driving conditions are assumed the same as described in Chapter IV. The vehicle was driven on a dry road for the first two seconds, then turning...
counterclockwise for 1 radian in one second. At t = 6 second, the vehicle entered in a split-μ road (i.e. the left side on dry surface while the right side on icy surface).

First of all, the desired and actual yaw rate with and without control are compared. (See Figures 33-36)

![Figure 33 Desired and actual yaw rate without control](image)

PID (Proportional-Integral-Derivative) control is seldom used in vehicle stability control due to its poor robustness. The controller is hard to stabilize the system when the external condition changes. As shown in Figure 34, with a PID controller, with $K_p = 4.5$, $K_i = 0$, and $K_d = 0$, the vehicle system remains stable prior to the cornering maneuver. But, when the vehicle encounters a low-friction surface, the controller does not perform well. In addition, the poor robustness also causes the increase of the desired yaw rate. When the road surface condition changes while the vehicle is turning, the magnitude of desired yaw rate with PID control is about 20 degrees per second larger than that without control. Due to the unsatisfactory performance of the PID controller, this chapter only
compares the ADRC-based anti-skid controller with the vehicle system without the controller.

Figure 34 Desired and actual yaw rate with PID control

Figure 35 Desired and actual yaw rate on ADRC with equal-torque distribution
Figure 36 Desired and actual yaw rate on ADRC with unequal-torque distributed

\[ K_p = 4.5 \]

Since the desired yaw rate value is a function of longitudinal and lateral velocity, the desired value could vary with how the torques are distributed to wheels.

Figure 37 Desired and actual yaw rate on ADRC with unequal-torque distributed
With a different selection of $K_p$, the system response varies. In Figure 35, the control gain $K_p$ is

$$K_p = \omega_c = 0.3 \cdot \omega_o = 4.5 \quad (5.22)$$

Figure 36 gives the system response by choosing

$$K_p = 0.1 \cdot \omega_o = 1.5 \quad (5.23)$$

In comparison, the first controller gives faster response, but the desired yaw rate and actual yaw rate are a little larger. Since a fast response is more desired in anti-skid control, the first controller is chosen.

Figure 38 Actual yaw rate comparison
Figure 38 and 39 give the comparisons between the desired and actual yaw rates under three different conditions. As it can be seen, the yaw rate without control quickly diverged soon after the vehicle encountered a road surface change at $t = 6$ seconds. The yaw rate with the ADRC controller can reject the disturbance caused by the surface change. When comparing the two ADRC designs, of the same bandwidth, the one with unequal-torque distribution converges faster.

The slip ratio of each wheel plays an important role in the anti-skid control. Figures 39-41 show the slip ratio of each wheel. In the simulation, the vehicle began to turn counterclockwise at $t = 2$ seconds, and encountered an icy surface at $t = 6$ seconds. One can easily see that without control, the slip ratio of right front wheel quickly increases and diverges. With the ADRC equal-torque distribution control, the slip ratio of the right-front wheel slightly increases to a little over 0.4 and comes down to below 0.2, but does not stabilize it at $t = 10$ seconds. With the ADRC unequal-torque distribution control, the
slip ratio of the right-front wheel increases to a little over 0.3 and quickly stabilizes to about 0.14 in one second.

Figure 40 Slip ratio of each wheel without control

Figure 41 Slip ratios on ADRC with equal-torque distribution
The slip ratio comparisons among the three scenarios for each wheel are given in Figures 42-45. Once again, with the control, the slip ratio of any wheel quickly diverges.
Figure 44 Slip ratio comparisons on the right front wheel

Figure 45 Slip ratio comparisons on the right rear wheel
Figure 46 Slip ratio comparisons on the left rear wheel

Applied torque to each wheel is the control input to the system without control. The assumed torque is applied to all four wheels as shown in Figure 46.

Figure 47 Applied torques to all wheels without control
Figures 47, 48 and 49 show how the controller generated torque to each wheel.

To better compare the control performance between equal-torque distribution and unequal-torque distribution, the vehicle course is investigated.
The actual vehicle course is defined as the sum of the vehicle’s slip angle and the vehicle’s yaw angle.

\[ \psi = \beta + \varphi \]  

(5.24)

Where slip angle \( \beta \) is represented by

\[ \beta = \arctan \left( \frac{V_y}{V_x} \right) \]  

(5.25)

And \( \varphi \) is the vehicle’s yaw angle which is the time integral of yaw rate \( \gamma \). The vehicle course estimated by the ESO can be expressed as

\[ \hat{\psi} = \hat{\beta} + \hat{\varphi} \]  

(5.26)

Where

\[ \hat{\beta} = \arctan \left( \frac{\dot{V}_y}{\dot{V}_x} \right) \]  

(5.27)

The actual vehicle course for each scenario is shown in Figures 50-52. One will notice a sudden change soon after \( t = 6 \) seconds when the vehicle first encountered an icy surface. Without control the vehicle continues to spin to \( 90^\circ \) at \( t = 10 \) seconds. With control, the vehicle’s course is confined. Note that in this simulation, the reference or desired steering angle is \( 57.3^\circ \). Thus, a good controller should quickly approach and stabilize around that angle. Figure 53 shows the comparison of vehicle course among the three scenarios. The ADRC via unequal-torque distribution is better than via equal-torque distribution in terms of tracking errors.
Figure 50 Actual vehicle course without control

Figure 51 Actual vehicle course with ADRC via equal-torque distribution
Figure 52 Actual vehicle course with ADRC via unequal-torque distribution

Figure 53 Actual vehicle course comparisons of different controllers
Figure 54 Vehicle course comparison of using ADRC and PID controller

Figure 54 shows the vehicle course comparison between using ADRC with unequal-torque distribution and PID control. Because of the poor robustness, the PID controller does not respond as quickly as ADRC does when road surface suddenly changes at $t = 6$ seconds. The PID controller also did not respond to the new steering angle at $t = 2$ seconds. At $t = 3$ second, the PID controller responds slowly while the vehicle remains turning, and after the sudden change of surface at $t = 6$ second, the PID controller essentially overshoots the reference vehicle course. In contrast, the ADRC with unequal-torque distribution keeps the vehicle under control all the time by rejecting external disturbances.

The controller using different control gain $K_p$ responds differently. Figure 55 shows the comparison between using two values of controller gains.
Both of the controllers are ADRC via unequal-torque distribution controller. One of controllers is with gain $K_p = 4.5$, while the other is with $K_p = 1.5$. The control responses were compared earlier in Figures 35 and 36. The vehicle with the first controller ($K_p = 4.5$) has a shorter response time to the set point (i.e. reference steering angle) than the second controller. Both of the controllers can reject the system disturbance, in a very short period of time and track the input steering angle. For the purpose of real-time anti-skid control, a faster response is more desired. Therefore, the first controller with $K_p = 4.5$ is chosen.

Sideslip angle on each wheel is another variable that can be used to evaluate the control performance. Figures 56-58 show the comparisons with control, the sideslip angle of the right-front wheel quickly settles around $13.5^\circ$. The sideslip angle is important because it affects the actual steering angle $\delta$, which ultimately affects the desired yaw rate of the vehicle.
Figure 56 Sideslip angles without control

Figure 57 Sideslip angles on ADRC with equal-torque distribution
Figures 59-62 show the sideslip angle comparison for the scenarios on left front wheel, right front wheel, right rear wheel, and left rear wheel, respectively.

Figure 58 Sideslip angles on ADRC with unequal-torque distribution

Figure 59 Sideslip angle comparisons on left front wheel
Figure 60 Sideslip angle comparisons on right front wheel

Figure 61 Sideslip angle comparisons on right rear wheel
The unstable yaw rate and sideslip angles will cause unwanted vehicle spinning. The longitudinal and lateral accelerations are also important variables because they may cause the vehicle drifting inward or outward during the cornering maneuver. The ADRC controller with or without equal-torque distribution has the ability to quickly bring the lateral acceleration to zero, and stabilizes at the zero value. Figures 62-64 show the longitudinal and lateral accelerations among the three scenarios. There is no significant difference between equal-torque distribution and unequal-torque distribution. Nevertheless, the latter exhibits a faster response.
(a) Longitudinal acceleration   (b) Lateral acceleration

Figure 63 Accelerations on the system without control

(a) Longitudinal acceleration   (b) Lateral acceleration

Figure 64 Acceleration on ADRC with equal-torque distribution
(a) Longitudinal acceleration  
(b) Lateral acceleration  

Figure 65 Acceleration on ADRC with unequal-torque distribution

It is found that the anti-skid control can be accomplished by the presented ADRC-based controller. It is also found that the ADRC with unequal-torque distribution better improves the control performance as compared to the ADRC with equal-torque distribution.
The dynamic modeling of an all-wheel drive vehicle has been presented. The Extended State Observer (ESO) has been successfully used to observe or estimate the vehicle’s longitudinal velocities, lateral velocities and yaw rate. The major advantage of using the ESO is its ability to augment both unknown dynamics and disturbances as an extended state, and estimate it in real time by using input-output data. The augmented state provides a physical insight to the system dynamics, which was used to monitor the change of road surface condition. Unlike many other observers, the presented ESO requires tuning only one parameter, the observer’s bandwidth. With the ESO, the abrupt change of system dynamics leads to the detection of road surface condition.

The employment of fuzzy logic and neural networks makes the detection and classification of road surface condition more quickly and intelligently. In addition to the detection and classification of road surface condition, and identification technique has also been successfully developed.
An ADRC-based anti-skid control via equal-torque or unequal-torque distribution has demonstrated its effectiveness. The ADRC via unequal-torque exhibits shorter response time in tracking the system and reaching the set point. The simulation results have shown that the controller with or without equal-torque can actively control the vehicle yaw rate while cornering with or without encountering low friction coefficient surface. The controller can also minimize the slip ratio and side slip angle. The ADRC-based control via equal torque distribution has also worked reasonably well, but not as good as that with unequal-torque distribution.

The future work will include the effect of the road surface’s bank angle on the desired yaw rate which is used as the controller’s set point. It is also desired to develop an algorithm to optimally distribute the torques to each wheel. The algorithm might be different from what is presented in this thesis.


## ANN Training Data

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# Appendix B

## Testing data for the Trained ANN

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