2004

Analysis and Implementation of a Synchronous Buck Converter Used as an Intermediate Stage of an Hid Ballast

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ANALYSIS AND IMPLEMENTATION OF A SYNCHRONOUS BUCK CONVERTER USED AS AN INTERMEDIATE STAGE OF AN HID BALLAST

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Bachelor of Electrical Engineering
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May, 2003

Submitted in partial fulfillment of requirement for degree
MASTER OF SCIENCE IN ELECTRICAL ENGINEERING
at the
CLEVELAND STATE UNIVERSITY
December, 2004
ACKNOWLEDGEMENTS

There are many people I would like to thank for helping me achieve this thesis. Dr. Ana Stankovic, as my advisor at CSU, provided many comments, suggestions, and criticisms of this thesis. Dr. Louis Nerone, as my advisor at GE Lighting, suggested the idea for this thesis and constantly helped me along the way. Thank you to Dr. L. Ilyes, who was always willing to answer questions and help me learn something new, as well as the rest of the team at GE Lighting, who made the internship a pleasant experience. Special thanks to Dr. Villaseca and Dr. Simon for their suggestions and comments on the thesis, as well as the rest of the faculty of the EEC department of CSU who got me to this point. Finally, my Lord and Savior, Jesus Christ without Whom, none of this would be possible.
ANALYSIS AND IMPLEMENTATION OF A SYNCHRONOUS BUCK CONVERTER USED AS AN INTERMEDIATE STAGE OF AN HID BALLAST

SERGEY V. VERNYUK

ABSTRACT

The contribution of this thesis is the analysis of a Synchronous Buck converter, used as the second stage (which controls the current through the lamp, and consequently, the lamp power) in a 3-stage High-Intensity Discharge (HID) ballast. Where previously standard Buck converters were used, this new application of the Synchronous Buck converter to a medium-power lighting ballast improves efficiency by operating the converter in a new, modified critical-conduction mode to achieve soft-switching and improve efficiency significantly. The converter and 2-loop controller will be analyzed in the three HID lamp modes of operation: open circuit, short circuit, and power control mode. Simulation and analytical results are compared to the experimental results.
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CHAPTER I

LITERATURE SEARCH

High-Intensity Discharge (HID) lamps are known for their high efficiency, high luminous efficacy, long life-times and good maintenance in a more-or-less compact source. The lamps provide bright, controlled color light in small sizes, and are used in a wide range of applications such as outdoor and street lighting, shop and restaurant illumination, and even automobile headlights [1,3,4].

Literature on the operation of an HID lamp exists and a brief description of the lamp operation is given here. The most common types of lamps are the high pressure mercury lamp, the high pressure sodium lamp, and the metal halide lamp. In a lamp, the electrons undergo different types of collisions with molecules of the filling gas of the lamp when accelerated by an external voltage. One type of collision is elastic, which heats the gas, and another type is inelastic, which ionizes the filling gas. In an inelastic collision, the kinetic energy of the electron is used to excite the filling gas molecules, which may give off their excitation energy as radiation, or light [1,2].

The load characteristic of an HID lamp can be represented as a pure resistor. Since HID lamps have a negative impedance characteristic, ballasts must be used to limit
the current after the lamp starts. Conventionally, line-frequency transformers and inductors, or electromagnetic ballasts have been used. These types of ballasts are inexpensive, simple, and reliable. However, they are also bulky, heavy, and inefficient [3,4,8].

A high frequency electronic ballast is an alternative that can reduce the size and weight of the ballast, improve the system efficacy, prolong the lamp life, eliminate flickering, and introduce other features such as dimming [3,4,5].

1.1 Acoustic Resonance

The problem, however, with operating an HID lamp at high frequency is arc instabilities called acoustic resonance. This instability occurs when the lamp is operated on an AC current with a frequency between a few kHz and a few hundred kHz (Figure 1.3) [3].

The common explanation for acoustic resonance is that the periodic power input (Figure 1.1) from the discharge current causes pressure fluctuations in the gas volume of the lamp. If this power frequency is close to an eigenfrequency of the lamp, traveling pressure waves occur. These waves travel between the discharge tube walls and create large amplitude standing waves. A strong oscillation in the gas can distort the discharge path and cause the arc to vibrate and change shape erratically. This causes the light to move and flicker, the arc may grow to a size that cannot be supported by the ballast and therefore extinguish, or in the worst case, the arc may touch the wall of the tube and extinguish itself or in the worst case, shatter the tube [3-9].
Much research has been done to find a way to deal with acoustic resonance. The most common approaches are described below:

1. **Supply the lamp with a DC current.** Under DC operation, the lamp power is constant and therefore no oscillations or waves occur. However, DC-operated lamps are hampered by the cataphoretic effect, which results in demixing of the gas filling toward the cathode side of the tube and asymmetrically eroding the electrode [2,5].

2. **Supply the lamp with a square-wave current at low frequency.** Since acoustic resonance is believed to happen at high frequencies, the lamp should be stable under low frequency operation. This operation results in a DC power wave for the lamp so no excitation is present and the arc is stable [2,3,4,7,8,9,10].

3. **Supply the lamp with a square-wave current at high frequency.** Again, the assumption is that the DC power wave will result in no excitation for the lamp
(Figure 1.2). However, a perfect square wave is hard to achieve and the efficiency at high frequency is low [4,7].

![Diagram](image)

**Figure 1.2 Constant power input**

4. *Operation in a frequency window free of acoustic resonance.* The eigenfrequencies of the lamp can be determined by solving several nonlinear equations and a window should theoretically exist that is free of acoustic resonance. However, the equations are not trivial and the frequency windows differ for each type of lamp. The lamp is usually operated with a sine wave [2-7].

5. *Using power spectrum spreading control methods.* By modulating the switching frequency or phase angle, the power spectrum of the lamp will expand and lower the energy of a certain frequency that supplies the lamp. These methods, in general, have not had successful results all of the time for all kinds of lamps [4-7].
6. Supplying the lamp with a very-high-frequency current, which is above the acoustic resonance frequency range of the HID lamp. Because of thermal hysteresis, when the time constant of the plasma parameter of the lamp is significantly higher than the time constant of the ballast, the discharge path behaves like a DC-driven arc. The highest frequency at which acoustic resonance exists still must be determined and at this high of a frequency, efficiency is an issue due to the switching losses [2-6].

In practice, the low-frequency square-wave approach is considered the most practical and most effective in eliminating acoustic resonance and is the one used in this paper.

1.2 Lamp Ignition and Starting

The operation of the HID lamp can be described in three distinct phases: starting (or ignition), warm-up, and nominal power operation [8].

Before ignition, the lamp is an open circuit and the electrical resistance between the electrodes is very high. A high ignition voltage must be applied to the lamp in order...
to break down the starting gases within the arc and establish a high-pressure arc. This ionization is achieved by using an igniter (Figure 1.4) [3,8,11].

![Figure 1.4 Block diagram of a system](image)

The ignition voltage may vary from 1 kV to 5 kV for a cold lamp and from 20 kV to 40 kV for a hot lamp typically. Therefore, as the operating frequency of the lamp increases, the re-ignition and extinction peaks disappear and the lamp will not need to be reignited while it is still hot and therefore, lamp life will increase [5,6,7,12].

Ignition techniques for HID lamps are usually either pulse starting or resonant starting. In pulse starting, a certain pulse waveform is applied to the lamp. There are certain pulse width and height requirements that will start a lamp and this method usually is made up of two main blocks: the pulse generator and a high-voltage coil. The pulse generator produces the voltage pulses which are stepped up to the high voltage required for lamp ignition by the high-voltage coil and are applied to the lamp [2,11].

Resonant ignition takes advantage of the fact that the lamp impedance is initially infinite and a resonant voltage wave can be applied to the lamp terminals by carefully designing the ballast. While pulse ignition has been the more popular ignition technique, resonant ignition is coming into its own for HID lamps. One of the reasons is that with resonant ignition, existing ballast components are used to create the ignition voltage rather than the external pulse generator and high-voltage coil, thus reducing component costs [2,3,13].
1.3 Warm-Up

Immediately after ignition, the lamp operates as a rectifier because one electrode warms up faster than the other. After the arc has been formed, the lamp goes into the warm-up phase when the temperature inside the lamp rises as the pressure increases. As the temperature increases, the voltage across the lamp also increases until the lamp reaches its normal operating temperature and the lamp voltage becomes steady (Figure 1.5) [8,11].

During the warm-up process, the resistance of the lamp is very small; it changes from open circuit to almost short circuit until it reaches steady state operation [3,4].

During this phase, the lamp current must be limited to the rated value or else the lamp and possibly the ballast will be destroyed [8,9,11,12].

1.4 Nominal Power Operation

When the lamp reaches its operating temperature and its voltage is steady, the lamp resistance will be a constant value and the current through the lamp will be less than the warm-up current. At this point, the lamp should be operated in constant power control [8-10].

Since the lamp impedance can be represented as a resistance, to keep the lamp operating at a constant power, the lamp current should be controlled to be at a constant value. However, as the lamp ages, the resistance increases and therefore if current control is used, the voltage drop across the lamp will increase as the lamp ages and consequently the lamp power will also increase [4,8].
1.5 Low Frequency Square Wave Electronic Ballasts

For the low frequency square wave electronic ballasts, the traditional design consists of three stages (Figure 1.6). The input of this ballast is connected to an AC voltage supply while the output of this ballast is connected to an HID lamp. The output voltage is a low frequency square wave [8-11].

The first stage consists of rectifying the sinusoidal input and performing power factor correction (PFC). Sometimes, the EMI filter and rectifier are not considered as a stage and in that case, the first stage consists of the PFC. The output of this stage is a constant high voltage [8,9,10,13].

The second stage consists of a DC-DC converter which acts as a controlled current source. Typically, this converter is a Buck converter which steps down the high
DC voltage to a lower DC voltage which, across the lamp that is represented by a resistor, gives the desirable lamp current. This converter operates at a high frequency in order to keep the voltage ripple at a minimum and is usually operated in current-mode control \([8,9,10,12,14]\).

The third stage consists of a DC-AC converter which only performs current steering, or in other words, changes the DC output voltage of the second stage into a bipolar square wave voltage that can be applied to the lamp. Typically, this is a full-bridge inverter \([8,9,10,12]\).

The ignition technique is most often pulse starting and the secondary of the high-voltage transformer is placed in series with the lamp. Usually, Sidacs are used to create ignition pulses. When the lamp is off, a capacitor in the igniter circuit gets charged up until the Sidac breaks, therefore sending a pulse to the high-voltage coil primary, which gets stepped up and applied to the lamp terminals from the secondary. When the lamp is
on, the capacitor in the igniter circuit does not get charged to the point when the Sidac breaks and therefore no ignition pulses occur [8,10,11,12,13].

1.6 One- or Two-Stage Ballasts

Since three power processing stages have many switching components which contribute to losses, much work has been done to combine certain stages. The most common approach is to combine the second and third stage, or the DC-DC and DC-AC converters into one (Figure 1.7). Since the third stage is primarily used for current steering, the two stages can be combined by making the resulting stage a full-bridge step-down converter with the upper switches switching at a high frequency and the lower switches at a lower frequency or vice versa.

![Diagram of Two-stage ballast](image)

Figure 1.7 Two-stage ballast

This is certainly possible and has been used successfully. However, with three separate stages, it is somewhat more straightforward to consider the design in detail and fine-tune the performance without one stage affecting the other directly as is the case when two stages are combined.
In fact, work has been done on combining all three stages into one (Figure 1.8). And while the switching losses do decrease, the design is somewhat more complicated than with three separate stages. Also, if soft-switching is used for the three stage approach, the losses decrease significantly and become comparable to that of the one-stage approach [3,4,8,13].

1.7 Comparisons

As was mentioned previously, the traditional ballast design consisted of three stages and the resulting operation was usually hard-switching. In order to minimize the cost of the ballast and improve the efficiency, two- and one-stage ballasts were designed. These ballasts have less components, resulting in a lower cost, less switching components, resulting in lower switching losses, hence, a higher efficiency, and also a smaller size.

However, two-stage and one-stage ballasts have their own disadvantages. One such disadvantage is that design is more complicated since the stages are all interrelated.
Optimization of stages is impossible since the stages are not separate from each other, like in the three-stage case. Other features such as load universality are simpler to analyze and design when the stages are separate rather than combined.

In a three-stage ballast, there are more components: two extra switches as well as an extra inductor. The secondary of the igniter coil, which is in series with the lamp, is usually a large inductor in the $\text{mH}$ range. However, the large inductor in a two-stage ballast is typically several times larger than the sum of the Buck inductor and the igniter coil in a three-stage ballast, although in a three-stage ballast, two separate cores are needed.

One other reason for choosing a three-stage ballast is that with soft-switching, the efficiency can be increased to a point where it is equal or better than that of the two-stage ballasts. With a two-stage ballast, soft-switching is more difficult to design and implement since the stages are interleaved. With three separate stages, though, soft-switching is more straightforward to design and will increase the efficiency.

### 1.8 Buck Converter Stage

This thesis is the analysis of the second stage of a three-stage ballast, namely the Buck converter stage. The input to this stage is a constant high voltage while the output is also a constant voltage that has been stepped down, as mentioned previously.

Since lamps are rated according to power, the purpose of a ballast is to supply the rated power to the lamp. Since an HID lamp can be represented as a resistor, the ballast must supply a constant current to the lamp in order to deliver rated power. Figure 1.9 illustrates this concept if the input is a DC voltage and output is also a DC voltage. As
mentioned previously, the first stage outputs a DC voltage while the third stage performs current steering and therefore, this is a valid circuit embodiment.

As the input to this stage is a DC voltage, if the input current could be controlled to a DC value, power control would be obtained, assuming a lossless converter. This is the reason that current mode control is used for the Buck converter (Figure 1.10) [9,10,12,14].
1.9 Synchronous Buck Converter

In order to decrease switching losses, soft-switching techniques can be applied to the Buck converter. In order to have zero-current switching, the Buck inductor current must reach zero. This means the Buck converter would be operating in critical conduction mode or discontinuous conduction mode (Figure 1.11) [14,15].

![Figure 1.11 Buck inductor current in discontinuous mode](image)

However, due to the circuit’s parasitic capacitance, the voltage across the rectifier would ring at a high frequency, resulting in EMI. However, if a synchronous Buck converter is used (Figure 1.13), assuming complementary switching, the converter always operates in the continuous conduction mode, therefore eliminating this ringing (Figure 1.12) [15,16,17].

![Figure 1.12 Buck inductor current of a synchronous Buck converter in continuous conduction mode](image)

In a synchronous Buck converter, an active switch, usually a MOSFET or IGBT, replaces the diode. When the primary switch is on, the secondary switch is off and
current flows through the inductor to the output. When the primary switch is turned off, the secondary switch is turned on and current flows through the secondary switch from the source to the drain, which puts the switch in reverse conduction mode. The inductor current decreases and then changes direction, flowing through the secondary switch in its normal way, from drain to source [16,17,20].

Figure 1.13 Synchronous Buck converter

One other advantage of using a synchronous Buck converter is the lower voltage drop across the secondary switch. In the traditional Buck converter, a Schottky diode is used since it has a fast reverse recovery as well as a fairly low forward voltage drop. However, an active switch, namely a MOSFET or IGBT has a voltage drop significantly less compared to the diode [15-19].
1.10 Synchronous Rectification Issues

Using an active switch instead of a regular diode has a few of its own issues however. The first issue is the requirement that both switches do not conduct at the same time. If they do, a short circuit will occur across the input terminals, therefore damaging the ballast. The best way to solve this issue is to carefully design the gate drives of both switches, use complementary switching, and use some dead time between the conduction times of the two switches [15,16,21,22,23].

The second issue, somewhat related to the first, is too long a dead time between the conduction of the two switches. During the dead times, current will flow through the internal diode of the secondary switch. This diode has a slow reverse recovery that adversely affects the converter's efficiency as well as a voltage drop that is higher than that of the active switch.

The solution for this issue is to keep the dead time as short as possible without risking short circuiting the input and also place a Schottky diode in parallel with the secondary switch. This Schottky will have a much better reverse recovery than the internal diode and does not need to be rated for the full voltage and current that the active switch is rated for since the Schottky diode will only conduct for a short time [15,16,18,19,21,22,23].

One other issue with synchronous rectification is the switching frequency. At high frequencies, say over 300 kHz, the switching losses of the active switch make it less efficient than the losses of a Schottky diode. The solution is to keep the switching frequency lower [18,19,22,24].
1.11 Controller

In order for the Buck converter to operate the lamp at the desired power, the controller must be designed and implemented in a way to achieve the desired operation. Typically, simple current-mode PWM control is enough to achieve the desired output current.

Current-mode control has advantages over voltage-mode control such as immediate response to circuit variations, pulse-by-pulse current limiting, and a single pole in the feedback loop versus two for voltage-mode control.

However, there are certain issues of current-mode control as well, such as instability at duty cycles over 50% without slope compensation and noise caused by the leading edge current spike [17,26,29].

In the case of an HID lamp, simple PWM control will not work simply because the lamp has three modes of operation, rather than only two. If there were only two modes of operation such as open circuit mode and power control mode, as in fluorescent lighting, it would be possible to use simple PWM control since in the open circuit mode, the controller duty ratio would be 1 until the lamp starts at which point the controller would keep the power constant.

For HID lamps, the Short Circuit Mode requires a different set-point from Power Control Mode and therefore simple PWM control cannot be used.

Extensive literature on HID ballast controllers does not exist because there seem to be many ways of controlling the ballast. The main characteristics of the controllers seem to be current-mode control and two set-points. It is the way to obtain the two set-points that sets the controllers apart.
One way to have two set-points is to use a Schmitt trigger, as is done in [4]. The lamp current is amplified, filtered and compared with the reference in a PI controller. During ignition and warm-up, the Schmitt trigger sets the reference to the higher limit. As the duty cycle exceeds a preset value, the reference decreases to the power control rating and the lamp enters Power Control Mode. A disadvantage of using a Schmitt trigger is that the two reference levels are discrete and going from one to the other is not continuous.

In [12], a PFC IC is used as the controller turning the switch on when the Buck inductor current is zero, and is set up for power control mode. During Short Circuit Mode, the output voltage breaks down to zero and the zero current detection circuit will not be able to generate restart pulses for the switch. Therefore, a Watch-dog timer is added to keep the converter going. First, according to [12], the PFC IC is not suited well for operating as the controller for this converter. Second, using the Watch-dog timer is not quite as intuitive for setting a correct reference for the Short Circuit Mode as using two loops, reference scheduling, or using a Schmitt trigger with two discrete references.

Digital control has also been implemented, usually using a microcontroller unit [8]. Reference values are calculated and stored in the memory of the microcontroller and are scheduled based on the current lamp operation mode. The lamp voltage and current are compared to the references and the error is sent to PI controllers. The controller outputs are sent to a PWM which controls the switches. Using a microcontroller brings up several issues. First, the A/D converters of the microcontroller must be fast enough to capture the changes of the converter accurately. Second, the code written for the microcontroller must be efficient and must execute quickly. Also, the microcontroller
unit might be more expensive than using discrete devices to perform similar operations. And finally, in order to schedule the reference values correctly, the current lamp operation mode must be determined as well.

It is possible to control the converter without using PWM. Hysteretic control is described somewhat in [13]. Again, two reference values or set-points must be determined and changed based on the current lamp operation mode. The frequency of operation is also changed based on the lamp operation mode. Using hysteretic control ensures that the circuit will be stable since the controller determines the upper and lower values of the outputs. Choosing the correct hysteretic width is important for proper circuit performance and sensing the lamp operation mode is still necessary in order to set the appropriate reference or set-point so the lamp does not burn up by using the Short Circuit Mode reference in Power Control Mode, for example.

The controller analyzed in this paper uses two loops to achieve two set-points for the SCM and PCM. In SCM, the current loop is the dominant control loop and modifies the set-point to the power loop which controls the switches. This ensures a high current through the lamp during warm-up mode. As the lamp warms up and the resistance increases from a small value to its nominal value, the current loop slowly decreases the power loop set-point up to the point where the lamp is in Power Control Mode, at which point the current control loop gets disconnected and the power loop becomes the dominant loop. This controller is stable because the current loop acts as an upper limit controller.

Variable frequency operation is achieved by using the converter operation itself to affect the switching, as will be described in a following chapter. This variable frequency
operation ensures minimal losses during SCM while keeping the ripple low and soft-switching in PCM.

1.12 Organization

Knowing that this thesis will focus on the synchronous Buck converter when used as the second, or intermediate, stage of an HID ballast, the second chapter will focus on the power side of the converter. Further requirements of the converter will be discussed and the analysis of the new mode of operating the Buck converter will be presented.

Chapter 3 will focus on the analysis of the controller for this Buck converter based on the requirements that will be set forth in the second chapter.

Chapter 4 will present the Simulink and PSPICE results, as well as some experimental results from a ballast that was designed with these specifications in mind.

Chapter 5 summarizes the results of the thesis and suggests possibilities for future work.

The contribution of this thesis will be the analysis of the Synchronous Buck converter in the three HID lamp modes of operation using a 2-loop controller. This Buck converter will run in a new mode of operation (a modified critical/continuous conduction mode) in order to achieve soft-switching (zero-voltage switching), and therefore lower the losses significantly.

A Synchronous Buck converter has not been used in this type of medium-power lighting application before because traditionally, a standard Buck converter has been used instead.
CHAPTER II
SYNCHRONOUS BUCK CONVERTER

There are three modes of operation for this Buck converter: Open Circuit Mode (OCM), Short Circuit Mode (SCM), and Power Control Mode (PCM). Even though the input voltage to the Buck converter will remain constant through all of these modes, the operation of the converter will be different for each mode.

The analysis performed in this paper deals with a 400 W HID lamp and the input voltage to the Buck converter is assumed to be a constant 450 V.

2.1 Open Circuit Mode

In this mode, the lamp has not been ionized, the resistance is very high and assumed infinite. The lamp voltage needs to be high for ignition and therefore the Buck converter output voltage should be as high as possible, with an ideal duty ratio of 1.

Since the two switches are driven by a half-bridge driver, the upper or primary switch cannot be kept on all of the time because the half-bridge driver needs to be replenished. Therefore, in this mode, the switching frequency will be low, around 100 Hz. The exact value is not important as long as the driver is replenished and the
switching is low enough to avoid excess losses (since it is hard-switching) and generate less EMI.

2.2 Short Circuit Mode

In this mode, also called the Warm-up Mode, the lamp resistance is very low, approximated by a 5 Ω resistor in this study. A high current is needed to warm up the electrodes quickly and it will be limited to 4.5 A as not to damage the circuit components. When the lamp is in this mode, the lamp voltage goes from 450 V to roughly 30 V based on the lamp resistance and desired current.

Since the soft-switching technique for this converter requires the Buck inductor current to reach zero, the switching frequency in SCM would be very low and result in a high lamp ripple current. Therefore, to avoid acoustic resonance and keep the lamp ripple current low, hard-switching will be used with a switching frequency of 20-25 kHz.

In this mode, the Buck converter must control the lamp current rather than the lamp power that it will need to control in PCM.

2.3 Power Control Mode

In this mode, the lamp resistance is a constant value, approximated by a 50 Ω resistor. The load current needs to have a low ripple content and soft-switching is used in this mode.

A power controller, which will be discussed in the next chapter, is used with a Current-Mode PWM chip to control the switches at a switching frequency of 45-50 kHz. This high frequency will give a small ripple content and decrease the size of the converter
components. The frequency in PCM as well as SCM is variable based on the behavior of the Buck converter and controller but should be approximately as specified.

2.4 Synchronous Buck Components

Figure 2.1 shows a synchronous Buck converter with an inductive load:

![Synchronous Buck Converter Diagram](image)

The input voltage to this stage, as mentioned before, is 450 V DC and the output of the Buck is $V_o$, which is the commutator voltage. The commutator only performs current steering and therefore is ignored mostly in this analysis.

The load is inductive with $L_2$ being the igniter coil and $R_{\text{lamp}}$, also written as $R$, is the HID lamp resistance indicated in the previous sections of this chapter. The igniter was designed by Dr. L. Ilyes from GE Lighting and the igniter coil $L_2$ was set at 1.5 mH.

The diode $D$ is a fast-recovery diode that is anti-parallel with the secondary or lower switch, that allows current to flow during the dead time.

The design objective is to choose the Buck inductor value, $L_1$, and the filtering capacitor value, $C$, based on the desired requirements given earlier in this chapter.
2.5 Modified Critical Conduction Mode and Soft-Switching

As mentioned in Section 1.9, zero-current switching is one soft-switching technique. However, in this application, zero-voltage switching is accomplished by using a Synchronous Buck converter (rather than a standard Buck converter), and running it in a new, modified critical/continuous conduction mode.

This mode greatly reduces the losses so the efficiency increases dramatically. By operating the converter in this mode, the switching frequency can be extended to several hundred kHz, unlike typical Synchronous Buck converters (as mentioned in Section 1.10).

The main part of this mode of operation (to achieve zero-voltage switching) is to have the anti-parallel diodes conduct during the dead time, just before turning on the active switches. Since the diodes are anti-parallel, when they conduct, the voltage across the active switch becomes zero and zero-voltage switching occurs.

The upper switch, S1, is a MOSFET, which has an intrinsic anti-parallel diode. This diode will conduct during the lower peak of the current through the Buck inductor (and hence, the switch).

The lower switch is an IGBT, since it does not have an intrinsic diode. Using a MOSFET, and hence, an intrinsic diode, would not work because the intrinsic diode would not be as fast as the external diode, D, nor have the high current-carrying capability of D.

The diode anti-parallel with S2, D, is a fast-recovery diode. Traditionally, in Synchronous Buck converters, a Schottky diode is used, as was discussed in Section 1.10. However, many of the traditional applications for a Synchronous Buck converter include
low-power applications. For this medium-power application, a Schottky diode is not rated for this much current. Therefore, a fast-recovery diode is used.

The entire concept of this new mode can be best visualized when explained on the Buck inductor current waveform, shown in Figure 2.2:

Initially (at time 0), the primary switch, S1, is turned on, and the current through the Buck inductor (and consequently, through S1), increases.

When it reaches its maximum value at $t_{on}$, the controller will turn off S1 but there will be some dead time before S2 turns on. The current will be flowing through $L_1$ in the positive direction and since both active switches are open, it will flow through diode D. As mentioned previously, this current will be rather high (since it occurs at the peak of the current waveform) and therefore an external, high-current, fast-recovery diode was used.

When this diode conducts, the voltage across S2 will be zero since it is shorted by the diode. So when the controller turns on S2, it will turn on with zero-voltage switching.

With S2 conducting, the current through $L_1$ and S2 will decrease from its peak value to zero. During this interval, the switch is operating in reverse-conduction mode,
as explained in Section 1.9. When the current crosses the zero Amps boundary, it will change directions and flow through S2 in the traditional direction.

At time $T_s$, the controller will turn S2 off and again, dead-time will occur before S1 is turned on. During this interval, the current is flowing through $L_1$ in the opposite direction (towards the source), and since both active switches are open, and D is reverse biased, this current will flow through the intrinsic diode of S1 into the source (or back to the first stage).

With the intrinsic diode forward-biased, the voltage across S1 is zero (since it is shorted) and when the controller turns on S1, it will be operating in zero-voltage switching.

The most important part of this method is to limit the negative excursion of the current. It must go negative but not very high, since the intrinsic diode does not have very good ratings for time or current-handling capability.

Choosing this lower Buck inductor point is accomplished by designing both the Buck converter and the actual controller. This is the method with which soft-switching is accomplished.

2.6 Primary Switch Conducting

During the on-time of the switching period, $S_1$ will be on and the equivalent circuit is shown in Figure 2.3.

For soft-switching, the Buck inductor current should go to zero and slightly negative. For this purpose, the Buck inductor current must be determined.

$$i_L = \frac{1}{L} \int v_L \, dt$$  \hspace{1cm} (2.1)
For the Buck inductor, $v_{L+}$ is the DC voltage $V_d$ and $v_{L-}$ is the output voltage $V_o$. 

$$i_t = \frac{1}{L_1} \int_0^t (V_d - V_o)dt$$  \hspace{1cm} (2.2)

Therefore, as long as the voltage across $L_1$ is positive, the current through the inductor will increase linearly with time.

2.7 Secondary Switch Conducting

During the off-time of the switching period, $S1$ is off and $S2$ is on, resulting in the equivalent circuit shown in Figure 2.4.

![Figure 2.4 Equivalent circuit with secondary switch conducting](image-url)
In this case, the voltage across the Buck inductor is $-V_o$ and the current through the inductor is:

$$i_{L_2} = \frac{1}{L_1} \int_{t_m}^{t} (-V_o) dt$$

(2.3)

Clearly, the inductor current will decrease and, since this is a synchronous converter, the current will change direction after it reaches zero, and become negative, going through the switch $S_2$.

When the Buck inductor current is slightly negative, the secondary switch will turn off and the primary one will turn on (after a small dead-time) and the cycle will repeat.

2.8 Duty Ratios

Since the intermediate stage is a Buck converter, the switching will be controlled by a duty ratio, $D$. This duty ratio will be determined in all three modes of operation.

2.8.1 PCM

As mentioned previously, in this mode, the lamp resistance is approximately 50 $\Omega$ and the lamp power is 400 W. Therefore, the lamp current and voltage can be determined by the following equations:

$$I_R = \sqrt{\frac{P_R}{R}} = \sqrt{\frac{400}{50}} = 2.83 A$$

(2.4)

$$V_R = I_R R = 2.83(50) = 141.5 V$$

(2.5)
For a Buck converter, the duty ratio $D$ is the ratio between the output voltage of the Buck converter and its input voltage. As is known, in steady state, the voltage across an inductor is zero. Therefore, in steady state, $V_R$ is equal to $V_o$.

\[ D = \frac{V_o}{V_d} = \frac{V_R}{V_d} \]  

\[ I_R = \frac{V_R}{R} = \frac{D \cdot V_d}{R} = \frac{450 \cdot D}{50} = 9D \]  

\[ D = \frac{I_d}{I_o} \]  

Since the load is in series, the output Buck current is also the load current.

\[ I_d = D \cdot I_R = 9 \cdot D^2 \]  

Since the lamp is a 400 W lamp, assuming a lossless converter, the input power should also be 400 W. Since the input voltage is a constant value, by controlling the input current, the input power will be controlled.

\[ P_i = I_d \cdot V_d = 450 \cdot I_d \]  

\[ I_d = \frac{P_i}{V_d} = \frac{400}{450} = 0.889 A \approx 0.9 A \]  

At this point, knowing $I_d$, $I_R$, $V_d$, and $V_R$, $D$ can be determined. When solving Equation 2.7:

\[ D = \frac{I_R \cdot R}{V_d} = \frac{I_R}{9} = \frac{2.83}{9} = 0.314 \]  

Also, solving Equation 2.9:

\[ D = \sqrt{\frac{I_d \cdot R}{V_d}} = \sqrt{\frac{I_d}{9}} = \sqrt{\frac{0.9}{9}} = 0.316 \]
As there will inherently be power losses in the Buck converter, the higher D will be taken, 0.316.

### 2.8.2 SCM

In this mode of operation, the lamp resistance will be approximated by 5 Ω. By Equation 2.4, the lamp current should be 8.9 A. However, a current this high will damage the circuit and the lamp, and therefore will be limited to 4.5 A. At this current value, the lamp voltage will be 22.5 V.

The duty ratio in this mode can be derived from Equation 2.12 and is equal to 0.05. From the input current side, the duty ratio can be determined from Equation 2.13 and is equal to 0.1. In this mode, if the input power was allowed to produce 400 W, the lamp current would be the above-mentioned 8.9 A. Therefore, in this mode, the controller must limit the current to 4.5 A.

As will be further discussed in the next chapter, the controller will have two loops: a power loop and a current loop. In this mode, the current loop will be the controlling one.

### 2.8.3 OCM

In this mode of operation, the resistance is assumed to be infinite. From Equation 2.4, the lamp current will be zero (or very close to zero in actuality) and the voltage across the load will need to be very large, hence, the large ignition voltage pulses.

The commutating voltage, in this mode, will be the full input voltage of 450 V. From Equation 2.13, with the desired input current at 0.9 A, the duty ratio will be infinite.
From Equation 2.12, the duty ratio is also infinite theoretically. However, the duty ratio can only be between 0 and 1 and therefore, in this mode it will be limited at the full value of 1. This means that S1 should be on and S2 off during this mode.

2.9 Buck Inductor Current

Looking at Equations 2.2 and 2.3, the Buck inductor current depends on the input and output voltages (which are now established), the on- and off-times (which are derived from the switching frequencies and duty ratios at the appropriate modes, which are also established), and the Buck inductor value.

Therefore, the Buck inductor should be selected so that the current through it goes slightly negative, as specified previously, in order to have soft-switching. Since soft-switching will occur in PCM only, this mode will be analyzed first.

2.9.1 PCM Buck Inductor Current

In this mode, as stated previously, the input voltage is 450 V, the output voltage is 141.5 V, the duty ratio is 0.316, and the switching frequency is assumed to be 50 kHz, which results in a $T_s$ of 20 $\mu$s.

Even though $V_o$ will have some ripple, it is assumed constant in Equations 2.2 and 2.3 to result in a triangular Buck inductor current (shown in Figure 2.2) because the $V_o$ ripple is so small that it is negligible compared with the average value of $V_o$.

For PCM, the ripple needs to be small in order to avoid acoustic resonance and therefore the Buck converter will be designed to result in a small $V_o$, and therefore $I_R$, ripple.
Looking at the decreasing half of the Buck inductor waveform and integrating, the inductor current ripple can be obtained:

\[ \Delta i_{L_i} = \frac{V_o}{L_i} \cdot (T_s - t_{on}) = \frac{V_o}{L_i} \cdot (1 - D) \cdot T_s \]  

(2.14)

since the duty ratio \( D \) is the ratio between the on time \( t_{on} \), and the switching period \( T_s \).

The integral of the increasing half of the Buck inductor waveform will result in the same inductor current ripple.

Knowing the Buck inductor current ripple is not enough to find the lower Buck inductor current point, where it goes negative: the average of this current must also be determined.

Since a capacitor does not pass DC current but rather filters out all of the harmonics, it can be assumed that the average Buck inductor current is the same as the average load, or lamp current, which was determined to be 2.83 A in PCM.

All of the information is now available to find the lower point of the Buck inductor current:

\[ i_{L_i} = I_R - \frac{\Delta i_{L_i}}{2} \]  

(2.15)

\[ = \frac{V_o}{R} \cdot \frac{V_o (T_s - t_{on})}{2L_i} \]

\[ = \frac{V_o t_{on}}{RT_s} \cdot \frac{V_o t_{on} (T_s - t_{on})}{2L_i T_s} \]

\[ = \frac{V_o t_{on} (2L_i - RT_s + R t_{on})}{2T_s} = \frac{V_o D (2L_i + R (D - 1) T_s)}{2L_i T_s R} \]

From Equation 2.15, since all of the variables except for \( L_1 \) are known, the Buck inductor current lower point can be selected simply by choosing an \( L_1 \) value. Table I
gives the theoretical lower Buck inductor current point in PCM at various switching frequencies and Buck inductor values assuming a 400 W lamp and $I_R$ is 2.83 A.

### Table I Lower Buck inductor current point in PCM

<table>
<thead>
<tr>
<th>$L_1$ (μH)</th>
<th>15 kHz</th>
<th>20 kHz</th>
<th>25 kHz</th>
<th>30 kHz</th>
<th>35 kHz</th>
<th>40 kHz</th>
<th>45 kHz</th>
<th>50 kHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 μH</td>
<td>-29.6</td>
<td>-21.5</td>
<td>-16.6</td>
<td>-13.4</td>
<td>-11.1</td>
<td>-9.31</td>
<td>-7.96</td>
<td>-6.88</td>
</tr>
<tr>
<td>200 μH</td>
<td>-13.4</td>
<td>-9.31</td>
<td>-6.88</td>
<td>-5.26</td>
<td>-4.10</td>
<td>-3.24</td>
<td>-2.56</td>
<td>-2.02</td>
</tr>
<tr>
<td>250 μH</td>
<td>-10.1</td>
<td>-6.88</td>
<td>-4.94</td>
<td>-3.64</td>
<td>-2.71</td>
<td>-2.02</td>
<td>-1.48</td>
<td>-1.05</td>
</tr>
<tr>
<td>300 μH</td>
<td>-7.96</td>
<td>-5.26</td>
<td>-3.64</td>
<td>-2.56</td>
<td>-1.79</td>
<td>-1.21</td>
<td>-0.758</td>
<td>-0.398</td>
</tr>
<tr>
<td>350 μH</td>
<td>-6.42</td>
<td>-4.10</td>
<td>-2.71</td>
<td>-1.79</td>
<td>-1.13</td>
<td>-0.630</td>
<td>-0.244</td>
<td>0.065</td>
</tr>
<tr>
<td>400 μH</td>
<td>-5.26</td>
<td>-3.24</td>
<td>-2.02</td>
<td>-1.21</td>
<td>-0.630</td>
<td>-0.196</td>
<td>0.142</td>
<td>0.412</td>
</tr>
<tr>
<td>450 μH</td>
<td>-4.36</td>
<td>-2.56</td>
<td>-1.48</td>
<td>-0.758</td>
<td>-0.244</td>
<td>0.142</td>
<td>0.442</td>
<td>0.683</td>
</tr>
<tr>
<td>500 μH</td>
<td>-3.64</td>
<td>-2.02</td>
<td>-1.05</td>
<td>-0.398</td>
<td>0.065</td>
<td>0.412</td>
<td>0.683</td>
<td>0.899</td>
</tr>
</tbody>
</table>

In PCM, the frequency, as mentioned previously is 45-50 kHz so the right side of the table is more relevant.

Since the desired lower current point at 50 kHz is slightly negative but close to zero, from Table I, a good Buck inductor value would be 300 μH.

From Equation 2.14, the Buck inductor current peak-to-peak ripple, or $|\Delta i_L|$ can be calculated. The results in PCM are shown in Table II in a style similar to Table I:

### Table II Buck inductor current peak-to-peak ripple in PCM

<table>
<thead>
<tr>
<th>$L_1$ (μH)</th>
<th>15 kHz</th>
<th>20 kHz</th>
<th>25 kHz</th>
<th>30 kHz</th>
<th>35 kHz</th>
<th>40 kHz</th>
<th>45 kHz</th>
<th>50 kHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 μH</td>
<td>64.8</td>
<td>48.6</td>
<td>38.9</td>
<td>32.4</td>
<td>27.8</td>
<td>24.3</td>
<td>21.6</td>
<td>19.5</td>
</tr>
<tr>
<td>200 μH</td>
<td>32.4</td>
<td>24.3</td>
<td>19.5</td>
<td>16.2</td>
<td>13.9</td>
<td>12.2</td>
<td>10.8</td>
<td>9.73</td>
</tr>
<tr>
<td>250 μH</td>
<td>25.9</td>
<td>19.5</td>
<td>15.6</td>
<td>13.0</td>
<td>11.1</td>
<td>9.73</td>
<td>8.65</td>
<td>7.78</td>
</tr>
<tr>
<td>300 μH</td>
<td>21.6</td>
<td>16.2</td>
<td>13.0</td>
<td>10.8</td>
<td>9.26</td>
<td>8.11</td>
<td>7.20</td>
<td>6.48</td>
</tr>
<tr>
<td>350 μH</td>
<td>18.5</td>
<td>13.9</td>
<td>11.1</td>
<td>9.26</td>
<td>7.94</td>
<td>6.95</td>
<td>6.18</td>
<td>5.56</td>
</tr>
<tr>
<td>400 μH</td>
<td>16.2</td>
<td>12.2</td>
<td>9.73</td>
<td>8.11</td>
<td>6.95</td>
<td>6.08</td>
<td>5.40</td>
<td>4.86</td>
</tr>
<tr>
<td>450 μH</td>
<td>14.4</td>
<td>10.8</td>
<td>8.65</td>
<td>7.20</td>
<td>6.18</td>
<td>5.40</td>
<td>4.80</td>
<td>4.32</td>
</tr>
<tr>
<td>500 μH</td>
<td>13.0</td>
<td>9.73</td>
<td>7.78</td>
<td>6.48</td>
<td>5.56</td>
<td>4.86</td>
<td>4.32</td>
<td>3.89</td>
</tr>
</tbody>
</table>
2.9.2 SCM Buck Inductor Current

In this mode, as stated previously, the input voltage is 450 V, the output voltage is 22.5 V, the duty ratio is 0.05, and the switching frequency is assumed to be 20 kHz, which results in a $T_s$ of 50 μs.

Using Equation 2.15, the lower Buck inductor current point can also be determined for SCM. Table III shows the results of this equation for a 400 W lamp and a lamp current of 4.5 A. Since the lamp is in SCM for only a brief time, the hard-switching that will occur in this mode is tolerable.

As mentioned previously, the frequency could have been decreased even further in order to allow soft-switching but this would have created a large lamp current ripple which is unacceptable due to acoustic resonance issues.

<table>
<thead>
<tr>
<th>Table III Lower Buck inductor current point in SCM (in Amps)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lower Buck inductor current point in SCM (in Amps)</strong></td>
</tr>
<tr>
<td><strong>Switching frequency, $f_s$</strong></td>
</tr>
<tr>
<td>$L_1$</td>
</tr>
<tr>
<td>100 μH</td>
</tr>
<tr>
<td>200 μH</td>
</tr>
<tr>
<td>250 μH</td>
</tr>
<tr>
<td>300 μH</td>
</tr>
<tr>
<td>350 μH</td>
</tr>
<tr>
<td>400 μH</td>
</tr>
<tr>
<td>450 μH</td>
</tr>
<tr>
<td>500 μH</td>
</tr>
</tbody>
</table>

In this mode, the left side of the table is more relevant. As seen from the table, a lower Buck inductor value would also allow soft-switching in SCM but in PCM, the ripple would be too great and the PCM switching frequency would need to be increased dramatically to counter this.
Table IV shows the Buck inductor current peak-to-peak ripple in SCM based on Equation 2.14:

<table>
<thead>
<tr>
<th>Table IV Buck inductor current peak-to-peak ripple in SCM (in Amps)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Buck inductor current peak-to-peak ripple in SCM (in Amps)</strong></td>
</tr>
<tr>
<td><strong>Switching frequency, ( f_s )</strong></td>
</tr>
<tr>
<td><strong>L_1</strong> (( \mu )H)</td>
</tr>
<tr>
<td></td>
</tr>
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<td></td>
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<tr>
<td></td>
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<tr>
<td></td>
</tr>
</tbody>
</table>

As can be seen from the table, the ripple decreases as the switching frequency increases, as is expected from theory. The reason for the low switching frequency in SCM is that with a higher frequency, the current through the switches will not decrease as low at the switching times as with a lower frequency: it will not have enough time to decrease that low. Therefore, the losses at a higher frequency will be higher.

In this mode, as long as the ripple does not trigger acoustic resonance, it is tolerated if the switching losses can be decreased.

2.9.3 OCM Buck Inductor Current

In this mode, as stated previously, the input and output voltage of the Buck converter is 450 V and the duty ratio is 1. Since the impedance is so high, ideally no current flows and therefore issues such as Buck inductor current ripple and soft-switching are irrelevant.
2.10 Current Division

With $L_1$ chosen, the Buck inductor current is known for all modes and the lamp current is needed. An equivalent circuit can be derived:

![Figure 2.5 Equivalent current divider circuit](image)

All of the variables in the circuit are known except for $C$, the capacitor value. If $i_{L1}$ is the input to the circuit, then $i_R$ can be determined by current division since $C$ and $L_2$ form a second-order low-pass filter.

\[
\frac{1}{sC + R + sL_2} \cdot i_{L1} = \frac{1}{s^2CL_2 + sCR + 1} \cdot i_{L1} = \frac{1}{1 + j2\pi fRC - (2\pi f)^2CL_2} \cdot i_{L1} \quad (2.16)
\]

As is shown in Equation 2.16, passing the Buck inductor current through the filter will result in the lamp current determined by the parameters in the equation, the only one of which must be selected is $C$.

However, Equation 2.16 deals with sinusoidal inputs and therefore the Buck inductor current must be broken down from a triangular waveform to a sum of sinusoidal waveforms in a Fourier Series Expansion.

2.11 Fourier Series Expansion of the Buck Inductor Current

The Buck inductor current is derived from Equations 2.2 and 2.3 and can be expressed in the following forms:
\begin{align}
\left. i_{L_1} \right|_{t_0} &= \frac{V_d - V_o}{L_1} t + i_{L_1^-} = \frac{V_d (1 - D)}{L_1} t + i_{L_1^-} \\
\left. i_{L_1} \right|_{t_{on}} &= \frac{-V_o}{L_1} t + \frac{V_o}{L_1} T_s + i_{L_1^-} = \frac{-DV_d}{L_1} t + \frac{DV_d T_s}{L_1} + i_{L_1^-} \\

2.11.1 \quad a_n \text{ Between } 0 \text{ and } t_{on} \\
&= 2 \sum_{T_2} \left[ \frac{V_d - V_o}{L_1} t + i_{L_1^-} \right] \cos(n \omega x) dt \\
&= 2 \sum_{T_2} \left[ \frac{V_d - V_o}{L_1} \int_0^{t_{on}} t \cos(n \omega x) dt + i_{L_1^-} \int_0^{t_{on}} \cos(n \omega x) dt \right] \\
&= 2 \sum_{T_2} \left[ \frac{V_d - V_o}{L_1} \left( \cos(n \omega x_{on}) - \frac{1}{n \omega} \right) \right] + i_{L_1^-} \left( \frac{\sin(n \omega x_{on})}{n \omega} \right) \\
&= \frac{2}{n \omega T_2} \left[ \frac{V_d - V_o}{L_1} \left( \frac{\sin(n \omega x_{on})}{n \omega} - t_{on} \cos(n \omega x_{on}) \right) + i_{L_1^-} (1 - \cos(n \omega x_{on})) \right]

2.11.2 \quad b_n \text{ Between } 0 \text{ and } t_{on} \\
&= 2 \sum_{T_2} \left[ \frac{V_d - V_o}{L_1} t + i_{L_1^-} \right] \sin(n \omega x) dt \\
&= 2 \sum_{T_2} \left[ \frac{V_d - V_o}{L_1} \int_0^{t_{on}} t \sin(n \omega x) dt + i_{L_1^-} \int_0^{t_{on}} \sin(n \omega x) dt \right] \\
&= 2 \sum_{T_2} \left[ \frac{V_d - V_o}{L_1} \left( \sin(n \omega x_{on}) - \frac{1}{n \omega} \right) \right] + i_{L_1^-} \left( -\cos(n \omega x_{on}) \right) \\
&= \frac{2}{n \omega T_2} \left[ \frac{V_d - V_o}{L_1} \left( \frac{\sin(n \omega x_{on})}{n \omega} - t_{on} \cos(n \omega x_{on}) \right) + i_{L_1^-} (1 - \cos(n \omega x_{on})) \right]
2.11.3 \( a_n \) Between \( t_n \) and \( T_s \)

\[
a_n = \frac{2}{T_s} \int_t^T \left[ \frac{-V_n}{L_1} t + \frac{V_n T_s}{L_1} + i_{L_n} \right] \cos(n\omega) dt \tag{2.21}
\]

\[
= \frac{2}{T_s} \left[ -\frac{V_n}{L_1} \int_t^T t \cos(n\omega) dt + \left( \frac{V_n T_s}{L_1} + i_{L_n} \right) \int_t^T \cos(n\omega) dt \right]
\]

\[
= \frac{2}{T_s} \left[ -\frac{V_n}{L_1} \left( \frac{\cos(n\omega)}{n^2 \omega^2} + t \sin(n\omega) \right) \bigg|_t^T + \left( \frac{V_n T_s}{L_1} + i_{L_n} \right) \left( \frac{\sin(n\omega)}{n \omega} \right) \bigg|_t^T \right]
\]

\[
= \frac{2}{T_s} \left[ -\frac{V_n}{L_1} \left( \frac{\cos(n\omega T_s) - \cos(n\omega t)}{n^2 \omega^2} + T_s \sin(n\omega T_s) - t \sin(n\omega t) \right) \right] +
\]

\[
\left( \frac{V_n T_s}{L_1} + i_{L_n} \right) \left( \frac{\sin(n\omega T_s) - \sin(n\omega t)}{n \omega} \right)
\]

At this point, a simplification can be made. Since,

\[
\omega = 2\pi f = \frac{2\pi}{T_s}
\tag{2.22}
\]

then

\[
\cos(n\omega T_s) = \cos \left( \frac{2\pi n T_s}{T_s} \right) = \cos(2\pi n) = 1 \tag{2.23}
\]

\[
\sin(n\omega T_s) = \sin \left( \frac{2\pi n T_s}{T_s} \right) = \sin(2\pi n) = 0 \tag{2.24}
\]

for all integers \( n \).
Therefore, Equation 2.21 can be simplified further:

\[
a_n = \frac{2}{n \omega T_s} \left[ \frac{V_n}{L_1} \left( \frac{\cos(n \omega x_m) - 1}{n \omega} + t_{on} \sin(n \omega x_m) \right) - \left( \frac{V_{n, T_s}}{L_1} + i_{L_{on}} \right) \sin(n \omega x_m) \right] \tag{2.25}
\]

2.11.4 \( b_n \) Between \( t_{on} \) and \( T_s \\

\[
b_n = \frac{2}{T_s} \int_{t_{on}}^{T_s} \left[ \frac{-V_n}{L_1} t + \frac{V_{n, T_s}}{L_1} + i_{L_{on}} \right] \sin(n \omega x) \, dt
\]

\[
= \frac{2}{T_s} \left[ \frac{-V_n}{L_1} \int_{t_{on}}^{T_s} \sin(n \omega x) \, dt + \left( \frac{V_{n, T_s}}{L_1} + i_{L_{on}} \right) \int_{t_{on}}^{T_s} \sin(n \omega x) \, dt \right]
\]

\[
= \frac{2}{T_s} \left[ \frac{-V_n}{L_1} \left( \frac{\sin(n \omega T_s) - \sin(n \omega x_m)}{n \omega} - T_s \cos(n \omega T_s) - t_{on} \cos(n \omega x_m) \right) + \right.
\]

\[
\left. \left( \frac{V_{n, T_s}}{L_1} + i_{L_{on}} \right) \left( -\cos(n \omega T_s) + \cos(n \omega x_m) \right) \right]
\]

\[
= \frac{2}{n \omega T_s} \left[ \frac{-V_n}{L_1} \left( \frac{\sin(n \omega T_s) - \sin(n \omega x_m)}{n \omega} - T_s \cos(n \omega T_s) + i_{L_{on}} \cos(n \omega x_m) \right) + \right.
\]

\[
\left. \left( \frac{V_{n, T_s}}{L_1} + i_{L_{on}} \right) \left( -\cos(n \omega T_s) + \cos(n \omega x_m) \right) \right]
\]

By simplification from Equations 2.22 through 2.24,

\[
b_n = \frac{2}{n \omega T_s} \left[ \frac{V_n}{L_1} \left( \frac{\sin(n \omega x_m)}{n \omega} + T_s - t_{on} \cos(n \omega x_m) \right) + \left( \frac{V_{n, T_s}}{L_1} + i_{L_{on}} \right) \left( \cos(n \omega x_m) - 1 \right) \right] \tag{2.27}
\]
2.11.5 Combining the $a_n$ Terms

Combining Equation 2.19 and 2.25, the complete $a_n$ coefficient equation can be determined:

\[
\begin{align*}
    a_n &= \frac{2}{n\omega T_s L_1} \left[ V_d - V_o \left( \frac{\cos(n\omega x_{om}) - 1}{n\omega} + i_{om} \sin(n\omega x_{on}) \right) + i_{m-} \sin(n\omega x_{on}) + \
    & \quad \frac{V_o}{L_1} \left( \frac{\cos(n\omega x_{om}) - 1}{n\omega} + i_{om} \sin(n\omega x_{on}) \right) - \frac{V_o T_s}{L_1} \sin(n\omega x_{on}) - i_{m+} \sin(n\omega x_{on}) \right] \quad \text{(2.28)} \\
    &= \frac{2}{n\omega T_s L_1} \left[ V_d \left( \frac{\cos(n\omega x_{om}) - 1}{n\omega} + i_{om} \sin(n\omega x_{on}) \right) - V_o T_s \sin(n\omega x_{on}) \right] \\
    &= \frac{2}{n\omega T_s L_1} \left[ V_d \cos(n\omega x_{om}) - V_o \right] + (V_d t_{om} - V_o T_s) \sin(n\omega x_{om}) \right] \quad \text{(2.31)}
\end{align*}
\]

Another way to write Equation 2.6 is:

\[
D = \frac{t_{om}}{T_s} = \frac{V_o}{V_d} \quad \text{(2.29)}
\]

and therefore,

\[
V_d t_{om} = V_o T_s \quad \text{(2.30)}
\]

Equation 2.28 can be simplified further by using Equation 2.30:

\[
a_n = \frac{2V_d}{n^2 \omega^3 T_s L_1} \left[ \cos(n\omega x_{om}) - 1 \right] \quad \text{(2.31)}
\]

and when Equation 2.22 is also used,

\[
a_n = \frac{2V_d T_s}{4\pi^2 n^2 L_1} \left[ \cos \left( \frac{2\pi n t_{om}}{T_s} \right) - 1 \right] = \frac{V_d T_s}{2\pi^2 n^2 L_1} \left[ \cos(2\pi D n) - 1 \right]
\]

for $n = 1, 2, 3, \ldots$
2.11.6 Combining the $b_n$ Terms

Likewise, Equations 2.20 and 2.27 can be combined to give the resulting $b_n$ coefficient equation:

$$b_n = \frac{2}{n \omega T_s} \left[ \frac{V_d - V_o}{L_1} \left( \frac{\sin(n \alpha_{on})}{n \omega} - t_{on} \cos(n \alpha_{on}) \right) + i_{L_{on}} \left( 1 - \cos(n \alpha_{on}) \right) \right] +$$

$$\frac{V_o}{L_1} \left( \frac{\sin(n \alpha_{on})}{n \omega} - t_{on} \cos(n \alpha_{on}) + T_s \right) - \frac{V_o T_s}{L_1} \left( 1 - \cos(n \alpha_{on}) \right) - i_{L_{on}} \left( 1 - \cos(n \alpha_{on}) \right) \right] \quad (2.33)$$

$$= \frac{2}{n \omega T_s L_1} \left[ \frac{V_d}{n \omega} \left( \frac{\sin(n \alpha_{on})}{n \omega} - t_{on} \cos(n \alpha_{on}) \right) + V_o T_s - V_o T_s \left( 1 - \cos(n \alpha_{on}) \right) \right]$$

$$= \frac{2}{n \omega T_s L_1} \left[ \frac{V_d}{n \omega} \left( \frac{\sin(n \alpha_{on})}{n \omega} - (V_o T_s - V_o T_s) \cos(n \alpha_{on}) \right) \right]$$

Again, using Equation 2.30, Equation 2.33 can be simplified further:

$$b_n = \frac{2V_d}{n^2 \omega^2 T_s L_1} \left[ \sin(n \alpha_{on}) \right] \quad (2.34)$$

and when Equation 2.22 is also used,

$$b_n = \frac{2V_d T_s}{4 \pi^2 n^2 L_1} \left[ \sin \left( \frac{2 \pi t_{on}}{T_s} \right) \right] = \frac{V_d T_s}{2 \pi^2 n^2 L_1} \left[ \sin(2 \pi d T) \right]$$

for $n = 1, 2, 3, \ldots \quad (2.35)$

2.11.7 Average

The average value of the waveform analyzed, which is the Buck inductor current, is $a_0$ and is calculated as follows:

$$a_0 = \frac{1}{T_s} \left[ \int_0^{T_s} \left( \frac{V_d - V_o}{L_1} + i_{L_{on}} \right) dt + \int_0^{T_s} \left( \frac{-V_o}{L_1} + \frac{V_o T_s}{L_1} + i_{L_{on}} \right) dt \right] \quad (2.36)$$
Using Equation 2.29,

\[ a_0 = \frac{V_{aT_s}}{2L_1} - \frac{V_{aT_m}}{2L_1} + \frac{V_{aT_s}}{2L_1} + i_{t_n} \]

\[ = \frac{V_{aT_s}}{2L_1} - \frac{V_{aT_m}}{2L_1} + i_{t_n} = V_{aT_s} (1 - T_m) + i_{t_n} \]

Combining Equation 2.37 with Equation 2.15, the following is obtained for \( a_0 \):

\[ a_0 = \frac{V_d T_s D (1 - D)}{2L_1} + \frac{V_d D}{2L_1 R} (2L_1 + RT_s (D - 1)) \]

\[ = \frac{V_d D}{2L_1} \left[ \frac{(1 - D)T_s R + 2L_1 + (D - 1)T_s R}{R} \right] = \frac{V_d D}{2L_1 R} [T_s R (1 - D + D - 1) + 2L_1] \]

\[ = \frac{V_d D}{R} = \frac{V_a}{R} = I_k \]
Equation 2.38 matches with Equation 2.7 confirming that the average of the Buck inductor current is also the average load or lamp current.

**2.11.8 Combining the Fourier Coefficients**

Now having all of the coefficient formulas, the complete formula for the Buck inductor current can be written.

\[
i_{t_1} = a_0 + \sum_{n=1}^{\infty} \left[ a_n \cos(n\omega t) + b_n \sin(n\omega t) \right]
\]

\[
= a_0 + \sum_{n=1}^{\infty} \left[ a_n \cos(2\pi f t) + b_n \sin(2\pi f t) \right]
\]

The Fourier Expansion can also be performed with complex exponential coefficients. Conversion from the trigonometric form is given by the following formulas:

\[
F_n = \sqrt{\frac{a_n^2 + b_n^2}{2}}
\]

\[
\phi_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right)
\]

\[
\vec{f}_n = F_n e^{j\phi_n}
\]

\[
F_0 = a_0 = I_R
\]

\[
i_{t_1} = F_0 + \sum_{n=1}^{\infty} \vec{f}_n
\]

\[
= F_0 + \sum_{n=1}^{\infty} \text{Re}\left[\sqrt{2}F_n e^{j(n\omega t - \phi_n)}\right]
\]

\[
= F_0 + \sum_{n=1}^{\infty} \text{Re}\left[\sqrt{2}F_n e^{j(2\pi f t - \phi_n)}\right]
\]
2.12 Buck Inductor Current Harmonics

The results of Section 2.11 will be used to obtain the fundamental and higher-order harmonics of the Buck inductor current in PCM and SCM. Since no current flows in OCM (theoretically), this mode will not be considered.

Equations 2.32, 2.35, 2.38, and 2.40 are used and their results tabulated.

<table>
<thead>
<tr>
<th>n</th>
<th>( a_n )</th>
<th>( b_n )</th>
<th>( F_n ) (A)</th>
<th>( \varphi_n ) (rad/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.8300</td>
<td></td>
<td>2.8300</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-2.1322</td>
<td>1.3910</td>
<td>1.8002</td>
<td>0.5780</td>
</tr>
<tr>
<td>2</td>
<td>-0.6366</td>
<td>-0.2812</td>
<td>0.4921</td>
<td>-0.4159</td>
</tr>
<tr>
<td>3</td>
<td>-0.0089</td>
<td>-0.0542</td>
<td>0.0388</td>
<td>-1.4080</td>
</tr>
<tr>
<td>4</td>
<td>-0.1033</td>
<td>0.0946</td>
<td>0.0990</td>
<td>0.7415</td>
</tr>
<tr>
<td>5</td>
<td>-0.1141</td>
<td>-0.0293</td>
<td>0.0833</td>
<td>-0.2514</td>
</tr>
<tr>
<td>6</td>
<td>-0.0087</td>
<td>-0.0257</td>
<td>0.0192</td>
<td>-1.2444</td>
</tr>
<tr>
<td>7</td>
<td>-0.0237</td>
<td>0.0301</td>
<td>0.0271</td>
<td>0.9038</td>
</tr>
<tr>
<td>8</td>
<td>-0.0471</td>
<td>-0.0042</td>
<td>0.0334</td>
<td>-0.0889</td>
</tr>
<tr>
<td>9</td>
<td>-0.0083</td>
<td>-0.0156</td>
<td>0.0125</td>
<td>-1.0818</td>
</tr>
<tr>
<td>10</td>
<td>-0.0071</td>
<td>0.0128</td>
<td>0.0104</td>
<td>1.0644</td>
</tr>
</tbody>
</table>

In PCM, the duty ratio is again 0.316 at a switching frequency of 50 kHz with \( L_1 \) being the previously chosen 300 \( \mu \)H inductor.

Looking at the magnitudes of the harmonics in the \( F_n \) column, there is a clear fundamental at 50 kHz as well as a smaller harmonic at 100 kHz. The harmonics higher than this frequency become significantly smaller.

In SCM, the duty ratio is 0.05 at a switching frequency of 20 kHz with \( L_1 \) being the previously chosen 300 \( \mu \)H inductor.

As shown in Table VI, in this mode, the harmonics seem to decay in significance somewhat slower than in PCM. However, the average current is also significantly higher.
2.13 Lamp Current Ripple

Since the purpose of the Buck converter is to deliver a constant current to the lamp with low ripple, there needs to be a criterion of measure to compare the currents with different component values.

The measurement of interest is the ripple of the lamp current or $R_{ir}$.

$$R_{ir} = \sqrt{i_{\text{RMS}}^2 - I_R^2}$$ \hspace{1cm} (2.45)

where $i_R$ is the RMS value of the lamp current and $I_R$ is the average or DC value of the lamp current.

However, the RMS value of an entire signal can also be written as:

$$F_{\text{RMS}} = \sqrt{F_0^2 + \sum_{n=1}^{\infty} F_n^2}$$ \hspace{1cm} (2.46)

where $F_0$ is the DC value of the signal and $F_n$'s are the RMS values of the higher order harmonics.

<table>
<thead>
<tr>
<th>n</th>
<th>$a_n$</th>
<th>$b_n$</th>
<th>$F_n$ (A)</th>
<th>$\phi_n$ (rad/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4.5000</td>
<td>4.5000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-0.1860</td>
<td>1.1741</td>
<td>0.8406</td>
<td>1.4137</td>
</tr>
<tr>
<td>2</td>
<td>-0.1814</td>
<td>0.5583</td>
<td>0.4151</td>
<td>1.2566</td>
</tr>
<tr>
<td>3</td>
<td>-0.1740</td>
<td>0.3415</td>
<td>0.2710</td>
<td>1.0996</td>
</tr>
<tr>
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<td>0.2258</td>
<td>0.1974</td>
<td>0.9423</td>
</tr>
<tr>
<td>5</td>
<td>-0.1520</td>
<td>0.1520</td>
<td>0.1520</td>
<td>0.7854</td>
</tr>
<tr>
<td>6</td>
<td>-0.1382</td>
<td>0.1004</td>
<td>0.1208</td>
<td>0.6283</td>
</tr>
<tr>
<td>7</td>
<td>-0.1231</td>
<td>0.0627</td>
<td>0.0977</td>
<td>0.4711</td>
</tr>
<tr>
<td>8</td>
<td>-0.1074</td>
<td>0.0349</td>
<td>0.0799</td>
<td>0.3142</td>
</tr>
<tr>
<td>9</td>
<td>-0.0915</td>
<td>0.0145</td>
<td>0.0655</td>
<td>0.1572</td>
</tr>
<tr>
<td>10</td>
<td>-0.0760</td>
<td>0.0000</td>
<td>0.0537</td>
<td>0.0000</td>
</tr>
</tbody>
</table>
Therefore, simplifying Equation 2.45, the ripple of the lamp current becomes:

\[ R_{rx} = \sqrt{\sum_{n=1}^{\infty} i_{Rn}^2} \]  

(2.47)

where \( i_{Rn} \) are the RMS magnitudes of the harmonics of the lamp current.

A more useful measurement of the ripple would be the ripple factor which is:

\[ k_{ix} = \frac{\sqrt{i_{Rms}^2 - I_R^2}}{\sqrt{I_R^2}} \]  

(2.48)

Using the simplification of Equation 2.46, the ripple factor can be written as:

\[ k_{ix} = \frac{\sqrt{\sum_{n=1}^{\infty} i_{Rn}^2}}{I_R} \]  

(2.49)

which is just the ripple of the lamp current over the average value of that current.

**2.14 Filter Design**

Figure 2.5, shown earlier, gives the schematic of the filter. Since the lamp resistance changes, the filter design will need to be considered in both modes. The input to the filter will be the fundamental component of the Buck inductor current, \( i_{L1} \).

The output will be the fundamental component of the lamp current, \( i_{R1} \) which will be the dominant contributor to the lamp current ripple. This will simplify Equation 2.49 to:

\[ k_{ix} = \frac{i_{R1}}{I_R} \]  

(2.50)

which is just the fundamental RMS component divided by the DC value of the current.
The criterion for this filter is that the ripple factor be less than 0.5% for both modes. Therefore, rearranging Equation 2.50 for the unknown:

\[ i_{R1} = I_{Rk} k_i \]  
(2.51)

2.14.1 SCM

The SCM will be considered first because it will likely be the dominant mode in filter design, seeing as how the switching frequency in SCM is significantly lower than in PCM, and therefore the ripple content will be higher in SCM.

With a lamp current of 4.5 A in SCM and a desired 0.005 ripple factor, using Equation 2.51, the RMS magnitude of the fundamental component of the lamp current can be no more than 0.0225 A.

From Table VI, the fundamental input to the filter is 0.8406 A. With this information, the desired gain of the filter at the switching frequency can be obtained:

\[ A_v = \frac{i_{R1}}{i_{L1}} = 0.0268 \]  
(2.52)

\[ A_{vdb} = 20 \cdot \log A_v = -31.45 \text{ dB} \]  
(2.53)

Since the two-pole filter will have a 40 dB/dec attenuation, the corner frequency should be set around 3 kHz in order to ensure that at 20 kHz (the switching frequency for SCM) the gain is -31.45 dB.

Since for this application, linear phase is not important, but rather a flat DC response, a 2nd-order Butterworth filter will be used [32]. For a Butterworth filter, knowing the order and break frequency of the filter, the attenuation at any frequency can be determined by:
\[ A_{dB} = 10 \log \left[ 1 + \left( \frac{f_x}{f_c} \right)^{2n} \right] \]  

(2.54)

where \( n \) is the order of the filter, \( f_c \) is the break frequency, and \( f_x \) is the frequency at which the attenuation is desired to be calculated. Since the break frequency is the desired variable and the other variables are known in this case, Equation 2.54 can be rearranged as:

\[ f_c = \frac{f_x}{\frac{A_{dB}}{2}\sqrt{10^{10} - 1}} \]  

(2.55)

Using Equation 2.55, for a 2\(^{nd}\)-order filter, to achieve a desired attenuation of 31.45 dB at 20 kHz, the break frequency should be 3.3 kHz.

The component values for Butterworth filters of various orders are widely known and tabulated in literature and are used to assist and simplify the design process. They will also be used here. However, to simplify the tabulations, the values are normalized and therefore the scaling factors will need to be determined and applied.

The frequency scaling factor, \( FSF \), will need to be determined first. It is the desired break frequency divided by the normalized, or standard reference break frequency. In most tables, this standard break frequency is 1 rad/sec. Therefore, for SCM, the FSF will be the break frequency converted to radians per second, or 20737 rad/sec.

This scaling factor will be used to convert the normalized \( L \) and \( C \) values to actual component values by the following formulas:

\[ R = R_c Z \]  

(2.56)
$$L = \frac{L_n Z}{FSF} \quad (2.57)$$

$$C = \frac{C_n}{FSF \cdot Z} \quad (2.58)$$

where $R_n$, $L_n$, and $C_n$ are the normalized values from the tables and $Z$ is the load impedance scaling factor, or the lamp resistance in this case.

Using the table for a Butterworth filter in [32], for a 2nd-order filter with $R=5$, the normalized capacitor value is 0.1557 and the normalized inductor value is 7.7067. Using Equations 2.57 and 2.58, the resulting $L_2$ is 1.85 mH and $C$ is 1.5 μF.

Since $L_2$ is the secondary of the igniter coil, this value was already chosen as 1.5 mH and is close to the desired 1.85 mH value. The capacitor is taken to the next standard value of 2 μF, which helps compensate for the slightly smaller inductor value. The resonant frequency of this circuit, which is also the break frequency, is:

$$f_r = \frac{1}{2\pi \sqrt{L_2 C}} = 2.9 \text{ kHz} \quad (2.59)$$

which is close to the desired break frequency of 3.3 kHz.

2.14.2 PCM

The design process is repeated in PCM to ensure that the filter designed for SCM will also be sufficient in PCM. From Table V, the fundamental input to the filter will be 1.8 A while the maximum allowed fundamental lamp current can be 0.0142 A, according to Equation 2.51. Using Equations 2.52 and 2.53, the gain of the filter at the switching frequency should be 0.00789 or -42.06 dB. This can be met with a two-pole filter with a break frequency less than 5 kHz.
The exact corner frequency, using Equation 2.55, should be 4.4 kHz, which is higher than the necessary break frequency for SCM. Therefore, SCM is the dominant mode for this filter design and will meet the PCM criteria.

The FSF in PCM is 27646 rad/sec and will be used to calculate the actual components from the normalized values. The Butterworth table does not have values for R=50: the highest R value on there is 10. Therefore, the reading will be made for R=∞. For this case, \( C_n = 1.4142 \) and \( L_n = 0.7071 \).

Using Equations 2.57 and 2.58 with \( Z=50 \), the resulting filter component values are 1.3 mH for \( L_2 \) and 1 \( \mu \)F for \( C \). Using a slightly bigger inductor (1.5 mH) and capacitor (2 \( \mu \)F) will ensure that the ripple requirements will be met in this mode and in SCM.

A higher \( C \) would lower the ripple even more but the advantage is offset by several disadvantages. First, a higher \( C \) means a more expensive capacitor and a bigger size.

Also, time becomes an issue. The larger the capacitor, the longer it takes for the lamp current to reach steady state and overshoot becomes a consideration. With the third stage of the ballast being a commutator which changes direction of the lamp current every certain time period, balancing \( C \) becomes an important issue.
CHAPTER III

POWER CONTROLLER FOR SYNCHRONOUS BUCK

Sections 2.1 through 2.3 and 2.7 describe the desired operation of the synchronous Buck converter in the three modes of operation of the HID lamp. The Buck converter has been designed to perform under these conditions as long as it is controlled properly. The purpose of this chapter is to discuss the controller for the Buck converter.

The controller was proposed and designed by Dr. Louis R. Nerone from GE Lighting and therefore, this paper will focus mostly on analyzing the implementation of the controller.

As mentioned previously, the HID lamp is specified by rated power and therefore, a power controller is the ideal way to ensure the lamp performs under rated specifications. Since the first stage of the ballast rectifies the AC line voltage and outputs a DC voltage to the second stage, and since the third stage only performs current steering, it is the function of the second stage to take the DC voltage input and convert it in such a way that the lamp receives constant power.

A simple way to do this is by controlling the input current to the second stage. Since the input voltage to the second stage is set at a constant value and is not dependent
on the second and third stages, if the input current to the second stage was controlled then 
the product of the two would result in power control.

Since the lamp power should remain constant and the input voltage to the Buck 
converter $V_d$, is also constant, the input current $i_d$ should also ideally be constant. 
However, this will not be the case since the input current is also the current through the 
primary switch, which obviously is discontinuous and certainly not a DC value.

Therefore, the input current to the Buck converter needs to be low-pass filtered so 
that what is measured is the average input current, or the DC component of the input 
current. Then, the product of the average input current and input voltage results in a 
constant power.

As discussed in Section 2.8.2, in SCM the lamp impedance is significantly 
smaller than in PCM and, if power control is used, the currents through the ballast will be 
high enough to damage components. Therefore, there must also be an upper limit for the 
current, especially the current through the lamp.

The basic block diagram of the system is shown in Figure 3.1.

![Figure 3.1 Control loops of the Buck converter](image)

As is shown in the figure, there are two loops: the outer current loop and the inner 
power loop. The set-point of the current loop is simply the maximum current that should 
go through the lamp. If the lamp current is less than this set-point, the error is positive
and after being integrated, the output of the current controller increases theoretically to infinity but practically to the upper limit of the device used.

This output of the current controller also happens to be the input to the power loop. However, a problem is posed here: when in PCM, the current should be 2.83 A. And therefore the output of the current controller will increase until the lamp current reaches 4.5 A. If this was the case, the lamp power would be roughly 1 kW. And besides, the current loop is there purely to limit the lamp current in SCM.

This is the reason for the saturation block in Figure 3.1. This saturation point is set at the power set-point, or 400 W. If the output of the current controller exceeds this value, the saturation block ensures that the control loop is disconnected from the power loop and the power loop set-point is kept at 400 W. Actually, this power loop set point is really the average input current set point of 0.9 A as is mentioned in Section 2.8.1.

The power controller is also an integral controller, the output of which goes to the Current-Mode PWM chip, as mentioned in Section 2.3. The chip used for this ballast is the Unitrode UCC 3813. This chip converts the control signal into a PWM signal which is used by the half-bridge driver to turn the switches on and off.

3.1 Current Sensors

In order to compare the actual currents with the set-points, current sensing needs to occur. Since discrete devices will be used for the controller, this means the current measurements need to be converted into voltage values that may be used by the op amps.

For this purpose, current sense resistors will be used to convert the currents into voltages. Since the average Buck input current should be at 0.9 A in PCM and even less
in SCM, a 1 Ω sense resistor for this current will suffice. At 0.9 A sensed, the power
dissipated through this resistor will be 0.81 W:

\[ P_R = I_d^2 R_{s1} = 0.9^2 \cdot 1 \]  

(3.1)

Therefore, a 1 W sense resistor should suffice for this purpose. The maximum current it
could measure would be 1 A.

However, as mentioned previously, the Buck input current is not a DC value.
Therefore a 10 µF capacitor is placed in parallel with the sense resistor in order to filter
out the harmonics and allow only the DC current to be measured.

Measuring the lamp current is somewhat more difficult because of the location of
the lamp with respect to the second stage: the third stage is between the two. However,
for a Buck converter, the average output current is equal to the average of the Buck
inductor current.

The highest lamp current will occur in SCM with the current reaching 4.5 A.
From Equation 3.1, it is clear that either a high power sense resistor must be used or the
resistor must have a low resistance. Even if a 0.1 Ω resistor was used, it would need to
have a power rating greater than 2 W, which means it would be expensive.

Therefore, the solution is to use several resistors in parallel. Their power ratings
add and the equivalent resistance decreases. In this ballast, three 0.39 Ω resistors rated at
1 W are used. This results in an equivalent 3 W resistor of 0.13 Ω. By Equation 3.1, the
power dissipated in this resistor in SCM would be 2.63 W, which is less than the rated
equivalent 3 W.
The highest current this combination of resistors can measure is 4.8 A. Therefore, for both the lamp current and the input current, a safety margin is included for the sense resistors.

Figure 2.1 showed the circuit of the synchronous Buck converter. With the sense resistors and filtering capacitor included, the resulting circuit would be:

![Synchronous Buck converter circuit diagram]

Figure 3.2 Synchronous Buck converter with sense resistors and controller block diagram

In Figure 3.2, \( f_i \) is the measured lamp current while \( f_p \) is the negative of the measured input current. The reason for measuring the input current negatively will be explained in a later section.

### 3.2 Controller Implementation

Figure 3.3 shows the component implementation of the controller. The inputs to the controller are \( f_i \) and \( f_p \) as discussed above and the output of the controller is \( v_{con} \) which goes into the UCC3813 to be converted to a PWM signal.
As can be seen from Figure 3.3, the controller can be divided into 5 smaller parts: the power set-point, current set-point, current controller, buffer, and power controller. The functions of these parts will be discussed.

3.2.1 Power Set-Point

The power set-point circuit is shown in Figure 3.4:

![Figure 3.4 Power set-point](image)

The purpose of this circuit is to generate a constant power set-point, which is the saturation limit shown in Figure 3.1. This set-point was set at 4 V for a 400 W lamp. The input to the op amp is a reference voltage of 2.5 V, which was derived by a voltage divider from the 5 V supply voltage.

For the op amp with an input at the non-inverting terminal, the equation is:

\[
  v_p = \left( \frac{Z_f + R_f}{R_f} \right) v_{ref}
\]  

(3.2)

where \( Z_f \) is the feedback impedance of the feedback resistor in parallel with the feedback capacitor:

\[
  Z_f = \frac{R_f}{sC} = \frac{R_f}{1 + sCR_f}
\]  

(3.3)
Combining Equation 3.2 with 3.3, the resulting equation is:

$$v_p = \left[ \frac{\left( \frac{R_f}{R_i} + 1 \right) \left( 1 + s \frac{CR_f R_i}{R_f + R_i} \right)}{1 + sCR_f} \right] V_{ref}$$ \hspace{1cm} (3.4)

As is shown in Equation 2.4, the op amp gives a DC gain of \((R_f/R_i + 1)\) and two break frequencies, one a zero and one a pole, with the pole occurring at a lower frequency.

If \(v_p\) is desired to be 4 V and \(V_{ref}\) is 2.5 V, the DC gain, by Equation 3.4, should be 1.6. Therefore, if \(R_f\) is 100 kΩ, \(R_i\) must be 167 kΩ.

The pole break frequency is:

$$\frac{1}{2\pi CR_f}$$ \hspace{1cm} (3.5)

while the zero break frequency is:

$$\frac{R_f + R_i}{2\pi CR_f R_i}$$ \hspace{1cm} (3.6)

With C being a smoothing or filtering capacitor of 100 nF, the pole break frequency is 15.9 Hz while the zero break frequency is 25.4 Hz.

### 3.2.2 Current Set-Point

The current set-point is simply a voltage divider from the power set-point. Section 3.1 discussed the selection of the sense resistors and the equivalent sense resistor for the lamp current was 0.13 Ω. Since the current loop is supposed to only limit the lamp current at 4.5 A, the sensed voltage would be 0.585 V.
The input to this current set-point voltage divider is the power set-point, which is 4 V and the output should be 0.585 V. A simple voltage division equation can be used to calculate the voltage divider resistor values.

A filtering capacitor is placed in parallel with the resistor across which the current set-point is taken. The voltage division equation can be written as:

\[
\begin{align*}
    \frac{R}{sC} + R & \quad \frac{1}{sC} + R \\
    \frac{1}{sC} + R \quad + R & \quad \frac{1}{sC} + R \quad + R \\
    \frac{1}{sC} + R & \quad \frac{1}{sC} + R \\
\end{align*}
\]

where R is the resistor between \( v_i \) and ground, and in parallel with C, and \( R_2 \) is the resistor between \( v_p \) and \( v_i \).

As is seen from Equation 3.7, the DC gain is \( \frac{R}{R + R_2} \). Based on the input and desired output, this gain should be 0.146. If \( R_2 = 110 \text{ k} \Omega \), R would need to be 18.7 kΩ. C is also set at 100 nF to give a pole corner frequency of 100 Hz after which the filter attenuates at 20 dB/dec.

3.2.3 Current Controller

Figure 3.5 shows the current controller with the set-point and sensed lamp current as the inputs and the current control signal \( v_c \) as the output.

The transfer function of this op amp is:

\[
v_c = \left( -\frac{Z_f}{R_i} \right) f_i + \left( \frac{Z_f + R_i}{R_i} \right) v_i
\]
Since \( Z_f \) is only the impedance of the capacitor in the feedback, Equation 3.8 can be written as:

\[
V_c = \left( \frac{-1}{sC} \right) f_i + \left( \frac{1}{sC} + \frac{R_i}{R_f} \right) V_i = \left( \frac{1 + sCR_i}{sCR_i} \right) V_i - \left( \frac{1}{sCR_i} \right) f_i \quad (3.9)
\]

\[
= (V_i - f_i) \left( \frac{1}{sCR_i} \right) + V_i
\]

As is shown in Equation 3.9, the current error is passed through an integral controller with a gain of \( 1/CR_i \) and an offset of \( V_i \) is added to result in the current control signal \( v_c \).

The capacitor in the feedback is small to ensure a fast response to the lamp current and therefore prevent damage to the components when the current gets too high. The input resistor is high to limit the current going into the op amp and therefore limit the power losses of the controller.

Taking the 10 nF capacitor value and 100 k\( \Omega \) resistor value, the controller gain is 1000, which will ensure a fast response. The \( V_i \) offset signal is not that much of a problem because typically this controller will be saturated at the positive supply voltage of this op amp, 5 V since in PCM, the lamp current will be significantly less than the
current limit of 4.5 A. Therefore, the error of 1.67 A (or 0.368 V), after being passed through the integrator with such high gain, will be much greater than the 0.581 V of $v_i$.

Therefore, if the offset of $v_i$ can be ignored, the transfer function of the current controller output to the current error input can be written as:

$$\frac{v_c}{e_i} = \frac{1}{sCR_i}$$

(3.10)

where $e_i$ is the difference between the upper lamp current limit and the actual lamp current.

A Bode plot of the current controller is shown in Figure 3.6.

![Bode Diagram](image)

**Figure 3.6 Bode plot of current controller**

As can be seen from the Bode plot, there is a high DC gain which ensures that the average of the Buck inductor current will be compared against the upper lamp current
limit. And, as mentioned previously, the average of the Buck inductor current is also the average of the lamp current.

A brief description of the current controller is as follows. As long as the average inductor current, hence the lamp current, is less than the upper limit, the error will be positive and the current controller output will increase. In OCM and PCM, this will be the case and therefore, $v_c$ will saturate at the 5 V supply voltage.

When, in SCM, the lamp current reaches the limit and exceeds it, the error will turn negative and $v_c$ will therefore start decreasing from its present value. Naturally, as the control signal decreases, the lamp current should also decrease until a stable point is reached when the error is zero. When that is the case, the control signal will stop and remain there until the error changes.

3.2.4 Buffer

Figure 3.7 shows the buffer stage of the controller.

![Figure 3.7 Buffer](image)

The purpose of this buffer stage is to perform the saturation function that is shown on Figure 3.1 and isolate the current loop from the power loop when necessary. This is done using a diode and a voltage follower. The two 100 kΩ resistors are there purely to limit the current and help minimize the power losses of the controller.
The input to the voltage follower op amp will be called $v_+$ since it is applied at the non-inverting terminal and the output is called $p_r$ since that will be the reference point for the power controller.

The operation of a voltage follower is simply that the output voltage is the same as the input voltage. The reason for using a voltage follower rather than just a wire is the impedance properties of the op amp.

The inputs see an impedance of infinity looking into the op amp and therefore the part of the circuit at the output does not affect the input part. Likewise, the output sees an impedance of close to zero looking into the op amp, also isolating or buffering the output part from the input part of this voltage follower.

The saturation operation is performed primarily by the diode. As was discussed previously, $v_p$ is the power set-point of 4 V. This is the voltage at the anode of the diode.

If the lamp current is less than its upper limit, the current loop should be disconnected and only the power loop should be active. As discussed in the previous section, when the lamp current is less than its limit, the output of the current controller saturates at its positive supply terminal, or 5 V. This is the voltage at the cathode of the diode.

Therefore, if the output of the current controller is greater than the power set-point, the diode will be reverse biased and therefore an open circuit. In that case, the current loop will be disconnected from the power loop and the input to the buffer will be only the power set-point of 4 V. As mentioned above, the output of a voltage follower is equal to the input and therefore the power set-point $p_r$ will be equal to 4 V.
In the case that the lamp current is greater than its upper limit, the current controller output will be significantly less than the 5 V supply voltage and also less than the 4 V of \( v_p \). In that instance, the diode becomes forward biased and shorted.

Since the diode is now shorted, this means that \( v_- \) is equal to \( v_c \) and the input to the voltage follower is the output of the current controller. Again, the output of the voltage follower is equal to the input and so the power set-point will be set at the control signal of the current loop. This new set-point will obviously be less than the PCM set-point of 4 V and consequently the duty ratio should decrease and insure that the lamp current is within bounds.

In this case, there will be a voltage drop across the resistor between \( v_p \) and \( v_- \) and since this resistor is high, a small current will flow through the diode into the current controller op amp.

### 3.2.5 Power Controller

The power controller is shown on Figure 3.8.

![Figure 3.8 Power controller](image-url)
The inputs to the power controller are the power set-point and the average input current, which indirectly controls power. So it may be assumed that the power set-point is really the average input current set-point.

The output of the power controller $v_{con}$ is the control voltage of the entire controller and is the input to the PWM chip.

As was mentioned in Section 3.1, the input current is sensed negatively. The reason behind this is because this signal can be summed directly with the set-point by connecting both inputs to the inverting terminal of the power controller op amp.

$$v_{con} = -Z_f \left( \frac{p_f}{R_1} - \frac{f_p}{R_2} \right)$$

(3.11)

The feedback impedance is just the capacitor making this also an integral controller. Therefore Equation 3.11 can be written as:

$$v_{con} = -\frac{1}{sC} \left( \frac{p_f}{R_1} - \frac{f_p}{R_2} \right)$$

(3.12)

The sense resistor measuring the input current is 1 $\Omega$ and with the desired input current at 0.9 A, this would give a sensed voltage of 0.9 V. However, the power set-point is at 4 V and therefore needs to be scaled down appropriately.

In order to get a controller with a high gain and also to limit the currents flowing through the controller, $R_2$ was chosen as 100 k$\Omega$. Therefore, $R_1$ was calculated to be 430 k$\Omega$ in order to match the two signals. Therefore, Equation 3.12 can be rewritten as:

$$v_{con} = \left[ f_p - p_f \left( \frac{R_2}{R_1} \right) \right] \left( \frac{1}{sCR_2} \right)$$

(3.13)

The ratio of $R_2/R_1$ is purely to scale $p_f$ down to $f_p$. The controller gain is $1/CR_2$. The response of this control loop does not need to be as fast as of the current loop and
therefore C was chosen as 100 nF. This resulted in an integral gain of 100. The Bode plot of this controller is shown in Figure 3.9.

![Bode Diagram](image)

*Figure 3.9 Bode plot of power controller*

Again, like in the current controller, the DC gain of the power controller is high in order to deal with the average of the input current rather than the harmonics which will be present.

The operation of this controller is also quite simple. When the average input current is less than the set-point (scaled), the input to the op amp will be positive. Since the input is applied at the inverting terminal, the output of the op amp will go towards the negative rail.

When the input current is greater than the set-point, the error will be negative and the output of the controller will increase towards the positive supply voltage. When the
currents match, the error will become zero and the control signal will stop and stay at its current value for as long as the error remains at zero.

At this point, it is obvious that the control signal should be inverted so that when the input current is too high, the control signal should decrease and therefore lower the duty ratio. The control signal will indeed be inverted when it goes into the UCC3813 chip and that will be discussed in a later section.

### 3.3 Simulink Model

At this point, the block diagram shown in Figure 3.1 can be modified as shown in Figure 3.10:

![Figure 3.10 Block diagram of control loops](image)

Since transfer functions were derived for all of the op amps and sub-circuits, it is fairly straight-forward to develop a Simulink model of the controller. This model is shown on Figure 3.11.
Figure 3.11 Simulink controller model
3.4 PWM Generation

A simplified block diagram of the UCC3813 Current Mode PWM chip is given in Figure 3.12 [27-29].

![Figure 3.12 UCC3813 block diagram](image)

The important inputs to this chip are RC, FB, COMP, and the output OUT. $V_{ref/2}$ is internally derived from the internal UCC3813 reference voltage of 5 V so the input to the non-inverting terminal of the error amplifier (U1 in Figure 3.12) is 2.5 V.

### 3.4.1 Error Op Amp

FB is the inverting terminal of the error amplifier while COMP is the output of the error amplifier. Having a constant voltage applied to the non-inverting terminal and a measured signal applied to the inverting terminal is the same design idea as in the current controller in Section 3.2.3.

The idea is that this error op amp can be used as a controller op amp. However, this controller would have been inadequate for the operation of this Buck converter and
since a controller has already been developed, this error op amp is used for a different purpose.

As was stated at the end of Section 3.2.5, it seems that the control signal needs to be inverted. This is the function of the error op amp in this case.

Let there be a resistor connected from COMP to FB named $R_f$ and another resistor connected between FB and $v_{\text{con}}$, which is the controller output, called $R_i$. An equation can be written for this op amp:

\[
COMP = -\frac{R_f}{R_i}FB + \frac{R_f + R_i}{R_i}V_{\text{ref}/2}
\]  

(3.14)

If the desired COMP output is the inverse of $v_{\text{con}}$, and both signals range from 0 to 5 V, then it is apparent that the desired equation for COMP is $(5 - v_{\text{con}})$. Since $V_{\text{ref}/2}$ is set at 2.5 V, the non-inverting gain must equal 2. From Equation 3.14, the way to obtain this gain is to set $R_f$ equal to $R_i$. When this is done, the gain for FB becomes -1 and since FB is connected to $v_{\text{con}}$ through a resistor, this is exactly what is needed.

Therefore, if $R_f$ and $R_i$ are set at 20 kΩ, Equation 3.14 can be rewritten as:

\[
COMP = 5 - FB
\]  

(3.15)

3.4.2 PWM Comparator

At this point, there is a controller with an output that is proportional to the desired duty ratio that will keep the Buck converter operating at the set-point. However, the signals going to the Buck converter must be digital square waves and not an analog continuous signal. The purpose of a Pulse Width Modulator is to convert this analog control signal into a digital square wave signal that can drive the switches in the Buck converter.
The Buck inductor current is exactly the signal that is used as a triangle wave in this controller. Since it is already measured to limit the lamp current, the same signal can be sent into the CS pin to provide a triangular waveform.

There seems to be a common practice to keep the amplitude range of the controller to 1 V in Current-Mode PWM chips. That is the purpose of the zener in Figure 3.12.

Also, the output of the error op amp is passed through a voltage divider with a gain of 1/3 to reduce this control signal before it gets limited by the zener. Therefore, working backwards, the working range of the error op amp is 0 to 3 V. If this was taken one step further, the control signal $v_{\text{con}}$ would be limited from 2 to 5 V by Equation 3.15.

From Chapter 2, looking at Tables III and IV, in SCM with the highest inductor peak current, at 20 kHz, this peak will be at 6.28 A. With a sense resistor of 0.13 ohm, this results in a maximum voltage of 0.816 V for CS. Therefore, the 1 V zener limit should not be an issue if the controller designed limits the current at the right value.

### 3.4.3 PWM Latch

A PWM comparator is not enough to convert the analog control signal into a digital control pulse sequence. The reason is when the triangular waveform increases and becomes greater than the control voltage, the output of the comparator will go low turning off the primary switch.

When the primary switch is turned off, the current through the Buck inductor will decrease and therefore CS will drop and become less than COMP, which is the COMP signal after the voltage divider and zener. At this moment, the output of the comparator
will go high again turning on the primary switch. This process will repeat itself at a very high frequency.

Typically, a simple comparator can be used as described above without this problem. However, those instances are when a stable waveform generator is used to create the triangular waveform which will not decrease after a switch has been turned off, as is the case when the Buck inductor current is used. Therefore, an RS Latch is used.

When the S input of the latch is high, the output Q is also high. When S goes low, Q does not change. In fact, the only way Q will go low is when the R input is low. At that point Q goes low and stays low until S goes high again.

Using a latch, the inputs to the comparator must be switched so the triangular waveform is applied at the non-inverting terminal while the control signal is applied at the inverting terminal. Also, the comparator output must be connected to the R input of the latch as is shown on Figure 3.12.

Now, when CS becomes greater than COMPs, the comparator output becomes high, the latch is reset, and Q goes low and stays low. The only way it will go high is if S is applied.

S is the output of the oscillator, the frequency of which is controlled by connecting a resistor and capacitor to the RC pin.

At the beginning of each switching period, the oscillator pulses high and therefore sets the latch so OUT becomes high. As the Buck inductor current increases, so does CS until it becomes greater than the control signal, COMPs, at which time the PWM comparator output goes high, therefore resetting the latch and causing OUT to go low,
turning off the primary switch. At this point, the Buck inductor current decreases until the next oscillator pulse when the primary switch turns on again.

### 3.5 Variable Frequency Operation

Sections 2.1 through 2.3 describe the desired operating frequency of the Buck converter. The UCC3813 PWM has an internal oscillator, the frequency of which can be set by connecting a resistor and capacitor to the RC pin. However, this would make the operation frequency constant for all modes.

The UCC3813 datasheet [27] shows some application information, some of which is used for this Buck converter in order to create variable frequency operation. The block diagram of the oscillator is shown in Figure 3.13.

![Figure 3.13 UCC3813 oscillator block diagram](image)

When the signal at the RC pin is at ground, the oscillator latch is low and it does not oscillate. When the RC signal is at 5 V, the oscillator latch is set, which causes Q to go high and also turn on the internal transistor Q1. At that moment, the signal at the drain of the transistor is 5 V (from the RC pin), the signal at the source is at ground, and the gate pulse turns on the transistor. A high current will flow through the transistor and
possibly damage it. Therefore, a constant high signal should not be applied to the RC pin. However, by applying pulses to the RC pin at a certain frequency, the oscillator can be forced to operate at that given frequency.

According to the UCC3813 datasheet, the oscillator rise time is set by the time constant of $R_t$ and $C_t$ while the fall time is set by the time constant of $C_t$ and the internal transistor on-resistance, which is approximately 125\,Ω.

The datasheet specifies a timing capacitor value between 1 and 0.1 nF and a timing resistor between 10 and 200 kΩ. It is obvious that the rise time is at least 80 times as long as the fall time and therefore the operating frequency depends primarily on choosing the timing resistors and capacitors to achieve the correct rise time.

Since the frequency in PCM will be higher than the frequency of SCM, the switching period in PCM will be significantly shorter than in SCM. Therefore, by choosing the timing resistor and capacitor for the lower frequency for SCM, one mode will be taken care of. Then for PCM, the switching period just needs to be shortened and the timing resistor and capacitor bypassed.

From Section 3.4.2, it was explained that the PWM output will be low when the triangular waveform becomes greater than the control signal and the active switch will be turned off. Another way to look at this is limiting the positive peak of the Buck inductor current.

Since the Buck inductor current rises and falls based on Equations 2.1 through 2.3, the approximate rise and fall time of the Buck inductor current can be calculated. Therefore, if the negative peak of the Buck inductor current could also be controlled (just
like the positive peak is), enough information would be available to determine the switching frequency in PCM.

Controlling this negative Buck inductor current peak means setting the PWM latch at the appropriate time, therefore turning on the active switch. And the best way to set the PWM latch is to apply a pulse of 5 V at the RC pin when the Buck inductor current reaches the desired lower level.

From Table I in Chapter 2, the lower Buck inductor points were tabulated for different frequencies. In the case of PCM at 50 kHz, this lower point would be -0.4 A. Using a 0.13 Ω sense resistor, this results in a voltage of -0.05 V. Therefore the PWM latch must be set somewhere around this level.

Figure 3.14 shows the resulting design.

The parts of primary interest in Figure 3.14 are the components on the left side. They are the means with which variable frequency operation is achieved.

The comparator is used to detect the lower Buck inductor current point. Since the input is applied at the inverting terminal, a positive input will result in the output at the
negative rail and vice versa. The output of the comparator should go high when the lower Buck inductor current point is reached.

The way to achieve this is by having is a constant positive input being added to the sensed Buck inductor current voltage. Since the Buck inductor current becomes negative, and at that point the primary switch should be turned off, the two voltage inputs to the comparator should be scaled such that the input to the comparator becomes negative when the lower Buck inductor current point is reached.

This will ensure that the comparator output will be low until the Buck inductor current reaches the desired negative value.

The most common available DC voltage is the 5 V supply voltage. This voltage is scaled through a 100 kΩ resistor to equal the desired Buck inductor current negative limit which is therefore scaled through a 200 Ω resistor.

Since for soft-switching, the primary switch should turn on when the current through it is as close to zero as possible, the resistor scaling $f_1$ must be chosen so that the Buck inductor current is negative, yet close to zero. This is the reason for 200 Ω, which results in the primary switch turning on at -0.08 A Buck inductor current.

Having considered all of the individual parts, the basic operation is as follows. In PCM, at $t_{on}$, the primary switch is turned on. The voltage across the RC pin is at ground and the PWM latch is high until the Buck inductor current reaches the control signal, at which point the latch output becomes low.

When the latch output becomes low, the primary switch turns off and the voltage across the RC pin starts building up through $R_t$. Since the rise time constant is roughly 50 µs, the Buck inductor current will reach its lower peak much quicker. The off time at
50 kHz in PCM is 13.7 μs. When the Buck inductor current reaches the desired lower point, the comparator output in Figure 3.14 becomes high, causing the voltage on pin RC to be high.

At this point, the primary switch turns on again and also turns on S3 in Figure 3.14. Since the primary switch is on, the Buck inductor current starts increasing and therefore the comparator output goes low again and the voltage on the RC pin is reset to ground. This continues as long as the lamp is in PCM and the resulting operation is roughly at 50 kHz.

In SCM, when the primary switch is turned on, the Buck inductor current quickly reaches the control signal due to the low lamp resistance at which point the PWM latch will go low turning off the primary switch.

At this point, the Buck inductor current will decrease slowly and the voltage on pin RC will increase slowly through R_c. In SCM, the voltage across C_i will build up much faster than the time it takes for the Buck inductor current to drop to the negative level at which the comparator can go high. The comparator will always be set low in SCM and can therefore be replaced with an equivalent ground connection at the C_i terminal, resulting in fixed frequency operation. With a rise time constant of 50 μs, this results in operation slightly above 20 kHz.

3.5.1 OCM

Open Circuit Mode is a special instance where the variable frequency design for the other two modes does not apply. The reason being is that in OCM, theoretically no current will flow and therefore never reach the control signal. This results in the PWM
latch never being reset and therefore having OUT set high constantly, resulting in a duty ratio of 1.

A duty ratio of 1 is desired, naturally. However, according to Section 2.1, the switches need to be operated at a non-DC frequency in order to replenish the half-bridge driver.

The way to deal with this problem is shown in Figure 3.15, using a 555 timer.

The formula for the frequency of the 555 timer is given:

\[ f_{355} = \frac{1.44}{C(R_a + 2R_b)} \]  

(3.16)

As mentioned in Section 2.1, the desired frequency in OCM should be fairly low, around 100 Hz. By letting \( C \) equal 10 nF, \( R_a \) equal 1 M\( \Omega \), and \( R_b \) equal 330 \( \Omega \), the 555 timer frequency will equal 144 Hz.

Figure 3.15 OCM variable frequency implementation
The operation is as follows: when OUT (from UCC3813) is low, it is clear that one of the inputs to the NOR gate becomes high, which results in the output of the NOR gate being low. In this case, the 555 timer output O is also low.

When OUT is high, CON will be the output of the 555 timer. When OUT is held high, the timer oscillates with a 50% duty cycle at roughly 140 Hz.

In OCM, OUT always will be high, as discussed previously and therefore the switching frequency in this mode will be around 140 Hz.

In any of the other two modes, the 555 timer behaves in the following way: when OUT is high, signaling the primary switch should turn on, the 555 Reset’ terminal is also high and therefore the output of the timer goes high until the voltage at pin TR exceeds 2/3 of VCC, or 3.334 V.

Since the 555 timer is set to oscillate at 140 Hz, the rise time of the capacitor across TR is very slow. Therefore, the voltage on pin TR would have barely increased when the UCC3813 OUT signal goes low, therefore resetting the timer and sending its output to ground.

When OUT goes high again, the capacitor across TR will need to start charging again from ground voltage and therefore the 555 output will go high again.

Therefore, when the frequency of the UCC3813 is significantly higher than the 555 timer frequency, the output of the timer is identical to the output of the PWM chip, effectively removing the 555 timer stage shown in Figure 3.15. This is how variable frequency control is attained in all three modes of lamp operation.
CHAPTER IV

MATHEMATICAL, SIMULATION AND EXPERIMENTAL RESULTS

In the previous chapters, the analysis and implementation of the synchronous Buck converter and its controller were discussed. In this chapter, the results of the analysis are given.

First, the mathematical results are given based on the equations in the previous two chapters. Then, the circuit was simulated using OrCAD’s Capture CIS and PSPICE as well as MATLAB’s Simulink. Finally, lab results are given using a preliminary low-frequency three-stage HID ballast developed at GE Lighting.

The lamp, as discussed in the previous two chapters, is a 400 W lamp that operates in the three modes: OCM, SCM, and PCM.

4.1 Mathematical Results

In Chapter 2, mathematical equations were derived to obtain the Buck inductor current. Fourier Series Expansion was performed to decompose the triangular waveform into sinusoidal components. The tables in Chapter 2 show the results of the calculations.
Based on the equations of Chapter 2, a MATLAB program was written to determine the Buck inductor current as well as the lamp current based on the mode of operation and component values. The code is attached in the Appendix.

For PCM, at 50 kHz, using L₁ at 300 μH, and C at 2 μF, Figure 4.1 shows the Buck inductor current as well as the output current. The Fourier trigonometric and exponential coefficients were calculated to the 100th harmonic.

The Buck inductor current clearly reaches a negative value for a short amount of time during which the Buck switches reverse conduction.

For SCM at 20 kHz with the same component values, Figure 4.2 shows the Buck inductor current and the output current.

In both cases, the triangular Buck inductor current is clearly visible as well as the much smoother output current, after it has been passed through the low-pass filter.

In OCM, no current flows and therefore no waveform plot is shown.
Figure 4.1 Buck inductor and output current in PCM (MATLAB result)

X Axis = time in sec x 10^-5
Y Axis = current in Amps
Figure 4.2 Buck inductor and output current in SCM (MATLAB result)

X Axis = time in sec x 10^-4
Y Axis = current in Amps
4.2 Simulink Results

MATLAB's Simulink is an ideal environment for working on control problems. Chapter 3 gives several Simulink models for the controller, the results of which will be shown in this section.

In order to close the loop, a model of a Buck converter was needed. Since a model for the controller was developed up to the PWM chip, the other parts would also need a model in order to control a real Buck converter model. For this reason, a mathematical model of a Buck converter was developed using the equations in section 2.8.1 (Figure 4.3).

Since the output of the controller is a signal between 0 and 5 V and is inverted (as explained in the previous chapter), this control signal needs to be scaled and inverted in order to be used as the duty ratio input in the mathematical Buck converter. Figure 4.4 shows the conversion blocks.
Finally, the controller model of Figure 3.11 is connected to this mathematical Buck converter in order to test how the controller operates. The resulting circuit is shown in Figure 4.5.

Figure 4.6 shows the magnitude of the lamp voltage in PCM and Figure 4.7 shows the duty ratio in PCM. It should be remembered that the actual lamp voltage will be a square wave oscillating at a slow 75 Hz between a positive and negative value. In the simulations, the commutator/inverter was ignored since it only changes the polarity of the lamp voltage and direction of the lamp current. When looking at the lamp voltage at the switching period level, it will seem like a DC value rather than the square wave because the Buck converter switching frequency is much higher than the commutator frequency.

Figure 4.8 shows the Buck output current in PCM while Figure 4.9 shows the average input current in PCM. The result in Figure 4.8, which shows the Buck output current, can be compared to Figure 4.1 and both results are around 2.8 A.

As is seen from the graphs, in PCM, the average input current is roughly 0.9 A while the output current is around 2.85 A with a duty cycle of 0.32.

In short-circuit mode, the commutator voltage is shown in Figure 4.10, the duty ratio in Figure 4.11, the Buck output current in Figure 4.12, and the average input current in Figure 4.13.
In SCM, the duty ratio, as desired, is roughly 0.5 while the lamp current gets limited at 4.5 A. Figure 4.12 and 4.2 can be compared again.

In open-circuit mode, the commutator voltage is shown in Figure 4.14, the duty ratio in Figure 4.15, the Buck output current in Figure 4.16, and average input current in Figure 4.17.

As shown on the graphs, no current flows, the duty ratio is 1 and the commutator voltage is the DC input bus voltage of 450 V.
Figure 4.5 Simulink Controller and mathematical Buck converter
Figure 4.6 Magnitude of lamp voltage in PCM (Simulink result)

X Axis = time in sec

Y Axis = voltage in Volts
Figure 4.7 Duty ratio in PCM (Simulink result)

X Axis = time in sec
Y Axis = duty ratio
Figure 4.8 Buck output current in PCM (Simulink result)

X Axis = time in sec
Y Axis = current in Amps
Figure 4.9 Average input current in PCM (Simulink result)

X Axis = time in sec
Y Axis = current in Amps
Figure 4.10 Magnitude of lamp voltage in SCM (Simulink result)

X Axis = time in sec
Y Axis = voltage in Volts
Figure 4.11 Duty ratio in SCM (Simulink result)

X Axis = time in sec
Y Axis = duty ratio
Figure 4.12 Buck output current in SCM (Simulink result)

X Axis = time in sec
Y Axis = current in Amps
Figure 4.13 Average input current in SCM (Simulink result)

X Axis = time in sec
Y Axis = current in Amps
Figure 4.14 Magnitude of lamp voltage in OCM (Simulink result)

X Axis = time in sec
Y Axis = voltage in Volts
Figure 4.15 Duty ratio in OCM (Simulink result)

X Axis = time in sec
Y Axis = duty ratio
Figure 4.16 Buck output current in OCM (Simulink result)

X Axis = time in sec
Y Axis = current in Amps
Figure 4.17 Average input current in OCM (Simulink result)

X Axis = time in sec
Y Axis = current in Amps
4.3 PSPICE Results

The synchronous Buck converter was built in Capture CIS and simulated in PSPICE. The synchronous PCM Buck converter is shown in Figure 4.18.

![Figure 4.18 Buck converter in PCM](image)

Figure 4.18 shows the open-loop Buck converter in SCM. The difference is the lamp resistance value and switching period and duty ratio.

![Figure 4.19 Buck converter in SCM](image)

Finally, Figure 4.20 shows the Buck converter in OCM. The lamp resistor is removed and therefore no current flows through the igniter coil, L2. In actuality, however, the resistance will not be infinite and a small current will flow until the lamp enters warm-up mode.
When the circuit in Figure 4.18 (PCM) is simulated, Figures 4.21 through 4.24 show the results. Figure 4.21 shows the Buck inductor current which can be compared to Figure 4.1. Figure 4.22 shows the Buck output current which can be compared to Figure 4.1 and 4.8. Figure 4.23 shows the average input current which can be compared to Figure 4.9 and Figure 4.24 shows the magnitude of the lamp voltage, which can be compared to Figure 4.6. Again, the commutator was not simulated and therefore the value is a DC value rather than a square wave.

In SCM, the simulated figure is 4.19 and the results are shown in Figures 4.25 through 4.28. The Buck inductor current is shown in Figure 4.25 and is comparable to Figure 4.2 while the output current is shown in Figure 4.26 and can be compared to Figures 4.2 and 4.12. The average input current is shown in Figure 4.27 and is comparable to Figure 4.13 while the magnitude of the lamp voltage is shown in Figure 4.28 and can be compared to Figure 4.10.

The output current has a significantly higher ripple in this mode than in PCM. It is also much higher in amplitude than in PCM, while the average input current is significantly lower in amplitude than in PCM.
In OCM, for the simulation to work, the igniter coil also had to be removed as not to cause a floating node. With the switching frequency at 140 Hz, the currents were practically at zero and are not shown. Figure 4.29 shows the commutator voltage which is at 450 V, as is required for ignition purposes. This figure is comparable to Figure 4.14 which is also at 450 V.

Closed-loop simulation was attempted in PSPICE but proved impossible to accomplish due to various convergence errors and other issues in PSPICE, even after a significant time investment.
Figure 4.21 Buck inductor current in PCM (PSPICE result)

X Axis = time in msec
Y Axis = current in Amps
Figure 4.22 Buck output current in PCM (PSPICE result)

X Axis = time in msec
Y Axis = current in Amps
Figure 4.23 Average Buck input current in PCM (PSPICE result)

X Axis = time in msec
Y Axis = current in Amps
Figure 4.24 Magnitude of lamp voltage in PCM (PSPICE result)

X Axis = time in msec
Y Axis = voltage in Volts
**Figure 4.25** Buck inductor current in SCM (PSPICE result)

X Axis = time in msec

Y Axis = current in Amps
Figure 4.26 Buck output current in SCM (PSPICE result)

X Axis = time in msec
Y Axis = current in Amps
Figure 4.27 Average Buck input current in SCM (PSPICE result)

X Axis = time in msec
Y Axis = current in Amps
Figure 4.28 Magnitude of lamp voltage in SCM (PSPICE result)

X Axis = time in msec  
Y Axis = voltage in Volts
Figure 4.29 Magnitude of lamp voltage in OCM (PSPICE result)

X Axis = time in msec
Y Axis = voltage in Volts
4.4 Experimental Results

A low-frequency, three-stage HID ballast was designed at GE Lighting by Dr. L. Nerone and Dr. L. Ilyes using components with values similar to the ones obtained in this analysis.

The entire ballast was connected to an AC line voltage of 277 V and the output was connected to a 50 Ω resistive load instead of an HID lamp for PCM. For OCM, the output was left unconnected and for SCM, a 2 Ω resistor was used.

The input voltage was taken from a Pacific Smart Source 108-AMX AC Power Source. The ballast (not Buck converter) input and output measurements were taken by a Xitron Technologies 2503AH Power Analyzer System with current and voltage probes. Certain signal waveforms were captured by a Tektronix TDS 544A 4-channel Digitizing Oscilloscope with current and voltage probes.

Table X shows the resulting measurements in all three modes as taken by the Power Analyzer. The voltage and current measurements are RMS values.

<table>
<thead>
<tr>
<th>Table X Experimental measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Power Analyzer Measurements</strong></td>
</tr>
<tr>
<td>-----------------------------------</td>
</tr>
<tr>
<td><strong>PCM</strong></td>
</tr>
<tr>
<td>$V_{in}$ 275.4 V</td>
</tr>
<tr>
<td>$I_{in}$ 1.505 A</td>
</tr>
<tr>
<td>$P_{in}$ 408 W</td>
</tr>
<tr>
<td>$V_R$ 139.2 V</td>
</tr>
<tr>
<td>$I_R$ 2.759 A</td>
</tr>
<tr>
<td>$P_{out}$ 384 W</td>
</tr>
<tr>
<td><strong>SCM</strong></td>
</tr>
<tr>
<td>$V_{in}$ 276.5 V</td>
</tr>
<tr>
<td>$I_{in}$ 0.313 A</td>
</tr>
<tr>
<td>$P_{in}$ 67 W</td>
</tr>
<tr>
<td>$V_R$ 10.1 V</td>
</tr>
<tr>
<td>$I_R$ 4.108 A</td>
</tr>
<tr>
<td>$P_{out}$ 35 W</td>
</tr>
<tr>
<td><strong>OCM</strong></td>
</tr>
<tr>
<td>$V_{in}$ 276.5 V</td>
</tr>
<tr>
<td>$I_{in}$ 0.194 A</td>
</tr>
<tr>
<td>$P_{in}$ 10 W</td>
</tr>
<tr>
<td>$V_R$ N/A</td>
</tr>
<tr>
<td>$I_R$ N/A</td>
</tr>
<tr>
<td>$P_{out}$ N/A</td>
</tr>
</tbody>
</table>
As can be seen from the output section in Table X, the values are very close to the desired requirements. In PCM, the output current of 2.76 A and output power of 384 W are close to the desired 2.83 A and 400 W. In SCM, the lamp current is limited to 4.1 A while in OCM, there are no output measurements since the load is disconnected. However, some power losses do occur in the ballast, as seen by the 10 W input power drawn by the ballast.

As mentioned previously, voltage and current waveforms were captured. In PCM, Figure 4.30 shows the lamp current. As mentioned previously, the captured lamp current is a square wave at 75 Hz while simulation results ignored the commutator. Therefore, in order to compare the results more accurately, the magnitude of the experimental results should be used to compare these results to the simulation results. If the scope was to zoom in on the lamp current so as not to show the commutation, the results would be the same as in the simulation. Figure 4.30 should be compared to Figures 4.1, 4.8, and 4.22 with the average equal to 2.8 A.

Figure 4.31 shows the lamp voltage. The commutator switching is seen clearly at roughly 77 Hz. This figure should be compared to Figure 4.6 and Figure 4.24. Again, if the magnitude of the lamp voltage was taken and compared with the other results, the lamp voltage is in all three cases roughly 140 V.

Figure 4.32 shows the Buck converter input bus voltage, which should be at 450 V. The measured current is at 460 V, which is 2.22% greater than the desired value. This input voltage was measured for all three modes and remains constant.

Figure 4.33 shows the Buck converter input current at a frequency of 43.48 kHz, which is slightly less than the desired 45-50 kHz operating frequency range.
In SCM, Figure 4.34 shows the lamp current, which is limited to roughly 4.1 A. This is slightly less than the controller set-point of 4.5 A from the comparable Figures 4.2, 4.12, and 4.26. Figure 4.35 shows the lamp voltage, also with a commutation frequency of 75 Hz. This figure can be compared to Figures 4.10 and 4.28. However, the comparison is not exact since for experimentation the resistance was 2 Ω while for simulation, the resistance was 5 Ω.

The input current of the Buck converter is shown in Figure 4.36. From these plots of the input current, the duty ratio can be approximated visually based on the time during which the input current is not zero and the switching frequency is seen as 24 kHz, which is slightly higher than the desired 20 kHz.

In OCM, the lamp terminals are unconnected and therefore no current flows. However, this is the mode during which ignition pulses are applied to the terminals. Figure 4.37 shows the lamp or commutator voltage. As can be seen, the controller keeps this voltage at 450 V in OCM and this figure can be compared to Figures 4.14 and 4.29. Figure 4.38 shows the same commutator voltage only zoomed in on an ignition spike with an amplitude of 3.8 kV.

Finally, Figure 4.39 shows the Buck converter input current in OCM. When the experiment was performed, zooming in on the waveform showed the Buck input current to be very low in magnitude except for the time when commutation occurred, during which a current spike is observed in the figure.
Figure 4.30 Lamp current in PCM (Experimental result)

X Axis = time in msec
Y Axis = current in Amps
Figure 4.31 Lamp voltage in PCM (Experimental result)

X Axis = time in msec
Y Axis = voltage in Volts
Figure 4.32 Buck input voltage (Experimental result)

X Axis = time in μsec
Y Axis = voltage in Volts
Figure 4.33 Buck input current in PCM (Experimental result)

X Axis = time in μsec
Y Axis = current in Amps
Figure 4.34 Lamp current in SCM (Experimental result)

X Axis = time in msec
Y Axis = current in Amps
Figure 4.35 Lamp voltage in SCM (Experimental result)

X Axis = time in msec
Y Axis = voltage in Volts
Figure 4.36 Buck input current in SCM (Experimental result)

X Axis = time in μsec

Y Axis = current in Amps
Figure 4.37 Lamp/Commutator voltage in OCM (Experimental result)

X Axis = time in msec
Y Axis = voltage in Volts
Figure 4.38 Ignition voltage spike (Experimental result)

X Axis = time in μsec
Y Axis = voltage in Volts
Figure 4.39 Buck input current in OCM (Experimental results)

X Axis = time in msec
Y Axis = current in Amps
CHAPTER V
CONCLUSION

This thesis presents the analysis and implementation of a synchronous Buck converter which is used as an intermediate stage in a 3-stage HID ballast. Based on the three modes of HID lamp operation, the Buck converter components were selected to give the desired performance.

A detailed analysis of the synchronous Buck converter in the new modified critical/continuous conduction mode was the primary contribution. Based on the Buck inductor and output currents, components were chosen to give soft-switching during regular operation and low ripple during warm-up.

The analysis and implementation of the controller for this Buck converter was also given with an inner power loop and an outer current loop. All three modes of operation were accounted for and explained.

A controller model was developed in Simulink and tested with a “mathematical” Buck converter in the three modes. The synchronous Buck converter was also simulated in all three modes in PSPICE and the results were in agreement with the theoretical results that were obtained mathematically.
A preliminary three-stage ballast was tested in the laboratory at GE Lighting with results that are very close to the theoretical and simulation results. These results reaffirm the analysis and design of this synchronous Buck converter were accurate.

5.1 Future Work

Closed-loop simulation was attempted in PSPICE but ultimately failed even after many hours of tuning the schematic and trying various approaches. Many parts were tried from various libraries but perhaps even more libraries should be tried to see if the convergence errors can be overcome.

Finally, the intermediate stage could be built from beginning rather than using an existing ballast for experimental results. The component values could be fine-tuned even more in the Buck converter to have the experimental and simulation results match closer. The controller might also be improved with some tuning of gains or scaling values.


[27] Unitrode, Low Power Economy BiCMOS Current Mode PWM UCC3813 technical data.

[28] Fairchild Semiconductor, SMPS Controller UC3842 technical data.


[31] International Rectifier, Half-Bridge Driver IR21834 technical data.

APPENDIX

The MATLAB code written to obtain the mathematical results and plots in Chapter 4 is included here.

% This .m file calculates the trigonometric Fourier coefficients
% and uses them to calculate and plot the Buck inductor current.
% It also converts the trigonometric Fourier coefficients into
% exponential
% Fourier coefficients and again plots the Buck inductor current.
% Then, it takes the Buck inductor current and passes it through the
% L2C low-pass filter to result in the lamp current harmonic phasors.
% Finally, the harmonics are summed and lamp current is plotted.
% The inputs are the switching frequency, duty ratio, average current,
% number of harmonics to consider, input voltage, Buck inductor value,
% igniter value, filtering capacitor value, and lamp resistance.
% Outputs can be an and bn coefficients, and harmonic phasors for
% Buck inductor current and lamp current, ripple content, ripple factor
% as well as the plots of both currents.

f=140;     % switching frequency
d=1.0;     % duty ratio
aO=0.0;    % average output current
r=500e6;   % lamp resistance

n=100;     % how many harmonics
vd=450;    % bus voltage 450V
l1=300e-6; % buck inductor value
l2=1.5e-3; % igniter coil value
c=2e-6;    % filtering capacitor value
t=0:1e-7:(2/f);  % plot two cycles of switching period

% initialization
i=0;
nsquared=0;
an=0;
bn=0;
fn=0;
theta=0;
il=0;
ili2=0;
ilrec=0;
x=0;
y=0;
har=0;
har2=0;
har3=0;
ir=0;
ir2=0;
ir3=0;
theta2=0;
rir=0;
kir=0;

% Buck inductor current
% establish n^2
i=1;
while i<(1+n)
    nsquared(i)=i^2;
    i=i+1;
end

% calculating an
i=1;
while i<(1+n)
    an(i)=vd*(1/f)*(cos(2*pi*d*i)-1)/pi^2/ll/nsquared(i);
    i=i+1;
end

% calculating bn
i=1;
while i<(1+n)
    bn(i)=vd*(1/f)*sin(2*pi*d*i)/pi^2/ll/nsquared(i);
    i=i+1;
end

% harmonics
i=1;
har=zeros(n,(2/(f*1e-7)+1));
while i<(1+n)
    har(i,:)=an(i)*cos(2*pi*f*i*t)+bn(i)*sin(2*pi*f*i*t);
    i=i+1;
end

% add all of the harmonics to the dc value
i=1;
il=a0;
while i<(1+n)
    il=il+har(i,:);
    i=i+1;
end

% plot the buck inductor current
plot(t, il)

% find the rms magnitudes of the exponential coefficient
% and angle
i=1;
while i<(1+n)
    fn(i)=sqrt((an(i)^2+bn(i)^2)/2);
    theta(i)=atan(-bn(i)/an(i));
    i=i+1;
end
% exponential harmonics
i=1;
har2=zeros(n,(2/(f*1e-7)+1));
while i<(1+n)
    har2(i,:)=real(sqrt(2)*fn(i)*exp(j*(2*pi*i*t*f-theta(i))));
    i=i+1;
end

% add all of the harmonics to the dc value
i=1;
il2=a0;
while i<(1+n)
il2=il2+har2(i,:);
i=i+1;
end

% graph the buck inductor current using exponential Fourier
% coefficients
hold on
%plot(t, il2)

% Lamp Current

% change the inductor current harmonics from polar to rectangular
% phasors
i=1;
while i<(n+l)
x(i)=fn(i)*cos(theta(i));
y(i)=fn(i)*sin(theta(i));
ilrec(i)=x(i)+j*y(i);
i=i+1;
end

% low-pass L2C filter
i=1;
while i<(n+l)
    ir(i)=ilrec(i)/(1-(2*pi*f)^2*c*12+j*2*pi*f*r*c);
i=i+1;
end

% converting back to polar mode
% rms magnitude and angle of lamp current harmonics
i=1;
while i<(l+n)
    ir2(i)=sqrt((real(ir(i))^2+(imag(ir(i)))^2));
    theta2(i)=atan((imag(ir(i)))/(real(ir(i))));
i=i+1;
end
% ripple content and ripple factor
i=1;
while i<(n+1)
    rir=sqrt((ir2(i))^2+(rir)^2);
    i=i+1;
end
kir=rir/a0*100;

% lamp current harmonics
i=1;
har3=zeros(n,(2/(f*1e-7)+1));
while i<(1+n)
    har3(i,:)=real(sqrt(2)*ir2(i)*exp(j*(2*pi*i*t*f-theta2(i))));
    i=i+1;
end

% add all of the harmonics to the dc value
i=1;
ir3=a0;
while i<(1+n)
    ir3=ir3+har3(i,:);
    i=i+1;
end

% graph the lamp current using exponential Fourier coefficients
plot(t, ir3)
hold off

% display the ripple content and ripple factor
rir
kir